Movement of a relativistic particle in a spherical potential box

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The problem of a relativistic particle in a spherical potential box with impenetrable walls has investigated. The power spectrum has found. Comparison with the specified characteristics for a similar not relativistic problem has executed. Essential distinctions have revealed. Possibilities of application of the solved problem in astrophysics are considered.

Keywords: potential box, relativistic particle, a power spectrum, black hole, maximon, temperature of maximons.

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Introduction

Potential boxes make a basis of many microscopic models of physical objects. As many physical objects have three spatial measurements, but special interest represent 3-D potential boxes. Among such models the greatest symmetry the box spherical forms possesses. Such box is the natural candidate for modelling of a black hole. With various potentials and construction and the analysis of various models of black holes many works [1-11] are devoted questions of the analysis of this model and this direction intensively develops. The purpose of our work is the analysis of a spectrum of microparticles when their movement is relativistic and application of the received results to model of a black hole.

The Klein-Gordon equation and power spectrum

We use the Klein-Gordon equation [1]. As in [2] we investigate the symmetric decision. We enter dimensionless time and dimensionless radial coordinate by means of following formulas

$$\tau = \frac{E_R t}{\hbar}, \quad \xi = \frac{r}{R}, \qquad (1)$$

where $E_R = \frac{\hbar c}{R}$, $E_R -$ characteristic energy, $2\pi\hbar = h -$constant Planck, $t -$ time, $r -$ radial coordinate, $R -$radius of a spherical potential box. Then the Klein-Gordon equation accepts the following initial form

$$\left(\frac{\partial^2}{\partial\tau^2}-\frac{\partial^2}{\partial r^2}-\frac{2}{\xi}\frac{\partial}{\partial\xi}+\varepsilon^2\right)\Psi=0, \qquad (2)$$

where $\varepsilon=\dfrac{mc^2}{E_R}$, m − mass of a particle in own system of readout.

The decision of the equation (2) on the Furie's method looks like

$$\Psi=\mathrm{X}(\xi)\mathrm{T}(t) \qquad (3)$$

Function $\mathrm{X}(\xi)$ has been set in the form corresponding to not relativistic variant of a similar problem, namely

$$\mathrm{X}_n(\xi)=A\frac{\sin(\pi n\xi)}{\xi}, n=1,2,3,... \qquad (4)$$

where A is constant and n is number of a quantum condition.

This function automatically satisfies to a boundary condition for potential box

$$\mathrm{X}_n(\xi=1)=0, n=1,2,3,... \qquad (5)$$

Substituting (4) in (3) and (3) in (2) we receive the equation for function $\mathrm{T}_n, n=1,2,3,...$

$$\left(\frac{\partial^2}{\partial\tau^2}+\varepsilon^2+(\pi n)^2\right)\mathrm{T}_n=0, \qquad (6)$$

The decision of the equation (6) looks like

$$\mathrm{T}_n(\tau)=C_{1n}e^{\lambda_{1n}\tau}+C_{2n}e^{\lambda_{2n}\tau}, n=1,2,3,... \qquad (7)$$

where C_{jn} are constants, $\lambda_{jn}=(-1)^j i(\varepsilon^2+(\pi n)^2)^{1/2}; j=1,2; n=1,2,3,....$ Therefore the power spectrum of a particle in spherical potential box is defined by the formula

$$E_n = \pm E_R(\varepsilon^2 + (\pi n)^2)^{1/2}, n = 1,2,3,... \qquad (8)$$

In a relativistic spectrum there are two branches corresponding to particles (+) and antiparticles (-).

Not relativistic case

Consider a symmetrical potential box of spherical form. Then the Shredinger's equation [1] looks like

$$\left(i\frac{\partial}{\partial \tau} + \varepsilon_R \left(\frac{\partial^2}{\partial \xi^2} + \frac{2}{\xi}\frac{\partial}{\partial \xi} \right) \right) \Psi = 0, \qquad (9)$$

where $\varepsilon_R = \frac{\hbar}{2cmR}$.

Under the same boundary condition the equation (9) has the decision

$$\Psi_n(\xi) = A\frac{\sin(\pi n \xi)}{\xi} \mathrm{T}_{nNR}(\tau), n = 1,2,3,... \qquad (10)$$

where there is a decision of a following equation

$$\left(i\frac{\partial}{\partial \tau} - (\pi n)^2 \varepsilon_R \right) \mathrm{T}_{nNR} = 0, \qquad (11)$$

and the power spectrum is defined by the formula

$$\mathrm{E}_n = \mathrm{E}_R \varepsilon_R n^2 = \frac{(\pi \hbar n)^2}{2mR^2}, n = 1,2,3,... \qquad (12)$$

Compare power spectra (8) and (12). At big numbers the relativistic power spectrum aspires to linear dependence on level number, and not relativistic power spectrum remains square-law. .

Power spectrum of a particle in a box and substance of a black hole

Consider the appendix of the executed calculations to the analysis of a condition of substance in a black hole. As properties of area in a black hole are known hypothetically and possible the various points of view, suppose, that black hole there is a potential box with impenetrable walls and it has the sphere form.

For radius of a black hole we use formula Laplas-Shvartzshilda [3]

$$R = \frac{2Gm_{\otimes}}{c^2}, \qquad (13)$$

where $m_{\otimes}$ is a mass of a black hole, G is a gravitational constant.

Entering radius of a black hole (13) in formulas for power spectra (8), (12), we receive

$$E_n = \pm((mc^2)^2 + \left(\frac{\pi\hbar c^3 n}{2Gm_{\otimes}}\right)^2)^{1/2}, n = 1,2,3,\ldots \ , \qquad (14)$$

$$E_n = E_R \varepsilon_R n^2 = \left(\frac{\pi^2}{8m}\right)\left(\frac{\hbar c^2 n}{m_{\otimes}}\right)^2, n = 1,2,3,\ldots, \qquad (15)$$

In the resulted calculations it has supposed, that in a potential box force fields were absent. If in a black hole there are force fields (not including of surface) the problem can be considered a method of the theory of indignations.

We yet did not mention a question on mass of particles moving in potential box. For a case of a black hole it is possible to assume equality of mass of such particle to mass of maximon on Markov or the Plank's mass m_{Pl}. Then own energy of this particle is equal

$$E_0 = m_{Pl} c^2 = \left(\frac{\hbar c^5}{G}\right)^{1/2}, \qquad (16)$$

and the formula for a power spectrum accepts the following form:

$$\mathrm{E}_n = \left(\frac{\hbar c^5}{G} \right)^{1/2} \left(1 + \frac{\pi^2}{4} \left(\frac{m_{Pl}}{m_\otimes} \right)^2 n^2 \right)^{1/2}, \qquad (17)$$

Let's estimate the second composed in brackets. As the relation of mass of a black hole to mass of the sun more than four [3,4], and mass of the sun about $\mathrm{m} = 2*10^{30}\,\mathrm{kg}$ [4] the mass of a black hole satisfies an inequality $\mathrm{m}_\otimes > 8*10^{30}\,\mathrm{kg}$. Considering mass maximon $(\mathrm{m}_{\mathrm{Pl}} = 2*10^{-8}\,\mathrm{kg})$ we receive, that the estimated composed has an order of a square of the relation of mass of maximon to mass of a black hole. As a result both composed become sizes of one order under a condition

$$n = \frac{2}{\pi} \frac{m_\otimes}{m_{Pl}} \rightarrow n \geq 3*10^{38}, \qquad (18)$$

On the other hand it is possible to estimate number of maximons, containing in a black hole. As an estimation it is used the simple formula

$$N_{\max} = \frac{m_\otimes}{m_{Pl}}, \qquad (19)$$

Applying the formula (19), we receive

$$N_{\max} \geq 4*10^{38} \qquad (20)$$

In a black hole it is a lot of maximons. Therefore it is possible to enter temperature of maximons.

Energy and temperature of particles in a black hole

We will propose that the maximons temperature is great enough and it is possible to use the Boltzman's distribution. Therefore for energy of system of maximons we have the formula

$$\mathrm{E} = N_{\max} \sum P_n \mathrm{E}_n , \tag{21}$$

where $\mathrm{P_n} = \dfrac{\exp(-\mathrm{E_n}/\mathrm{KT})}{\sum \exp(-\mathrm{E_n}/\mathrm{KT})}$ is weight of condition, K is the Boltzman's constant, T is an absolute temperature. The formula (21) allows to receive the equation for temperature of maximons and, hence, the formula for temperature in a black hole. We will pass to reception of this formula. Equating average energy of thermal movement of gas and average energy from distribution Больцмана, we receive the required formula

$$\mathrm{T} - \frac{2}{3}\frac{\mathrm{E}_{Pl}}{\mathrm{K}} \frac{\sum f(n)\exp(-\beta f(n))}{\sum \exp(-\beta f(n))} = 0 , \tag{22}$$

$$\text{where } f(n) = \left(1 + \frac{\pi^2}{4}\left(\frac{m_{Pl}}{m_{\otimes}}\right)^2 n^2\right)^{1/2}, \beta = \frac{\mathrm{E}_{Pl}}{\mathrm{KT}}, n = 1,2,3,\ldots\infty.$$

From the formula (22) follows that the temperature of maximon gas depends on mass of a black hole. The temperature is essential parametre for research of thermal processes in a black hole, including possibility of evaporation of the black holes, for the first time considered by Howking and gravitation waves [8-9], and also a problem of stability of black holes [5-7,10-11].

Conclusion

The spectrum of maximonsin the spherical potential box, having radius of a black hole is received. The model of the black hole filled maximons is developed. With use of discrete distribution of Boltzman the equation defining temperature of maximons in a black hole is deduced. It is shown, that this temperature depends on weight of a black hole.

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