Fundamental constants, quantum metrology and electrodynamics

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The paper analyzes the relationship between the fundamental constants and systems of units. It is shown that the fundamental dimensional constants as natural units of physical quantities predetermine our choice of the system based on five fundamental units. The paper also reviews the history of systems of natural units based on fundamental constants, proposed by J.C.Maxwell, G.Stoney, M.Planck, D.Hartree, U.Stille et al. The evolution of metrology is directed to the transition from artificial measures to quantum metrology and leads to unification of systems of units in electrodynamics and physics in whole. The modern reform of metrology requires of modification of SI and CGS system by the way of allocation of fine-structure constant in the general laws of electrodynamics in explicit form. It is shown the necessity for the introduction of such physical quantity as "concentration of the potential", introduced earlier by Maxwell in electrostatics.

Keywords: fundamental physical constants, natural units, electrodynamics, fine-structure constant, speed of light, Planck constant, elementary charge.

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1. Classification of the physical constants

All physical constants can be divided by their dimensionality into the two main classes: *dimensional* and *dimensionless* constants. Dimensionless constants such as the fine-structure constant 1/137 and the mass ratios of the particles are given by the laws of Nature and do not depend on the choice of the units. All of them should be purely mathematically justified in the "Theory of Everything." The numerical values of the dimensional constants such as the speed of light c, Planck constant h and others, on the contrary, are arbitrary and depend on the choice of units. Russian physicist M.P. Bronstein in 1935 rightly noted that the problem of the numerical values of the *dimensional constants* does not exist [1], they are due to selected units of measurement. However, this "axiom of Bronstein" needs to be clarified. The fact is that the dimensionless constants form some closed class of the physical constants (the class is denoted by symbol A) since any combination of them is a dimensionless constant too. For the dimensional constants it is not the case: some combinations of dimensional constants are dimensionless ones (for example, the mass ratio of the particles). Therefore, for a proper classification the dimensional constants should be also divided into the two classes: constants, none of the combinations of which forms a dimensionless number (class C), and all remaining dimensional constants (class B). Thus, the whole set of fundamental physical constants = $A \vee B \vee C$. Obviously, the constants of class C

are metrologically independent and they actually should be called *the fundamental constants*. First of all, they include such constants as speed of light c and the Planck constant h that play a fundamental role in the theory of relativity and quantum mechanics.

Each constant of class B can be shown to form *one and only one* combination with constants of class C which is a dimensionless number (if combinations > 1 then constants of class C are not metrologically independent). Therefore, this combination due to its uniqueness should be considered as the definition of class B constants, i.e. all constants of class B are secondary and any of them can be expressed as: $b_i \equiv a_i \times \prod_j c_j^n$, where a_i is a dimensionless constant, and c_i is fundamental dimensional constants in some n degrees (in fact, a_i is numerical value and combination $\prod_{\mathrm{j}} c_{\mathrm{j}}^{\mathrm{n}}$ is a fundamental dimension of b_{i}). For example, the Stefan-Boltzmann constant $\sigma \equiv \dfrac{\pi^2}{60} \cdot \dfrac{k^4}{\hbar^3 c^2} = \dfrac{\pi^2}{60}\left(\dfrac{k^4}{\hbar^3 c^2}\right)$, constant in Coulomb's law $k_{\mathrm{e}} \equiv \alpha \dfrac{\hbar c}{e^2} = \alpha \left(\hbar c/e^2\right)$, where α^{-1}=137,035999139(31), etc. In natural system c=1, $\hbar$=1, e=1 constants $\sigma = \dfrac{\pi^2}{60}$ and $k_{\mathrm{e}} = \alpha$. It should be noted that P.W. Bridgman and some physicists believed that such natural system is impossible in principle [2]. However, with the philosophical point of view it is in essentially denial of the unity of Nature, because in Nature all these constants are natural units simultaneously. From a physical point of view there are no problems in the selection of such system of units (see below).

2. The number of fundamental units is 5.

What is the number of the most fundamental constants (constants of class C), and what kind of constants should be related to this class? It is generally accepted that the number of fundamental constants is equal to the number of basic (or fundamental, as are called by Sommerfeld) units of measurement (and that is inherent in the definition of class C constants). It is believed that the number of basic units is arbitrary. In fact, in different problems we successfully use different systems such as kinematic system of units (LT), e.g., in celestial mechanics, the mechanical system (LTM) (e.g., Gaussian system in electromagnetism) and systems based on a larger number of basic units (e.g., SI).

The equality of the number of fundamental physical constants to the number of main units should be interpreted as the fact that in nature there are a certain number of fundamental physical constants *as fundamental units of the appropriate physical quantities* (e.g., speed of light is natural limit of velocity of interactions, Planck constant $\hbar$ and elementary charge e are natural units respectively for angular momentum and electric charge) and this determines the preferable system of units. The fundamental physical constants as some natural amounts of corresponding physical quantities help us to choose the number of basic units. In fact, if we use the kinematic system of units LT (e.g., centimeters and seconds) the Planck constant h in such a system has dimension of L^5/T^3 but arbitrary numerical value because mass unit can be ambiguously reduced to the units of length and time: h = any number × cm^5/s^3. But when using the system with the three basic LTM units the Planck constant will have certain numerical value. Similar arguments (the certainty of the numerical values of the elementary charge e and the Boltzmann constant k) lead to the need to introduce two more basic units – for electromagnetism and thermodynamics.

Note that the dimension of the elementary charge has never been written in mechanical units due to the ambiguity of the numerical value of the elementary charge e in mechanical units: e^2 = any number × gram·cm^3/s^2 (for example, as in the Gaussian system, and the Lorentz-Heaviside the main units are centimeter, gram and second but the numerical values of the elementary charge are different). Thus, the very existence of such natural constants as c, h, e and k requires five basic units of measurement (and these four constants are not enough for a complete set). It has actually determined the transition to modern metrology based on the choice of these constants as units (and, additionally, a certain frequency).

3. Development of metrology as a transition from arbitrary measures to absolute natural standards

Humanity has used originally random, arbitrary measures, anthropomorphic as a rule, convenient for practice but caused large errors in the standards themselves. However, there has always been an idea of the need to find and use some more fundamental natural standards. Such an opportunity was offered with the discovery of the fundamental constants – the speed of light, Planck's constant, the elementary charge, the Boltzmann constant and others as absolute natural quantities.

Consequently, physicists have begun offering different natural systems of units based on these constants. In 1832 C.F. Gauss proposed the idea of a mechanical system of units, which later, after its modernization by W. Weber, became widespread. From the point of view of the constants, the meaning of this system is the reduction of units of nonmechanical quantities to the three mechanical units by the choice of coefficients in laws, in which mechanical action is manifested, equal to 1 (for example, the choice of the coefficient $k_e = 1$ in Coulomb's law $F = k_e \cdot \frac{q_1 q_2}{r^2}$). In 1870 and 1873 J.C. Maxwell proposed two systems of units, from which the two classes of modern natural systems of units – atomic and gravitational [3, 4] – were originated. Systems proposed by G.J. Stoney (c, G, e; k_e=1) in 1874/1881 and M. Planck (c, G, h, k) in 1899/1906 should be attributed to the gravitational system (G=1), and systems of D. Hartree ($\hbar$, e, m_e; k_e=1), A. Ruark (c, $\hbar$, m_e; k_e=1), U. Stille (c, h, e, m_p, k) and electronic system ($\hbar$, e, m_e; k_e=1) to the atomic ones (mass of some elementary particle as unit of mass) [5-12]. Planck values were forgotten and rediscovered again in 1950s as the limits of applicability of modern physical theories. Hartree system is widely used in atomic physics and the system (c, $\hbar$, eV; k_e=1/4π) arisen from the system of Ruark is widely used in modern quantum electrodynamics. It should be noted that J.C. Maxwell besides the velocity of light also discussed two constants: the elementary charge e as the most natural unit of electricity and the Boltzmann constant k as universal constant for different substances.

Stille's system had not received the recognition at that time and had been forgotten but it underlies in the modern QSI (quantum SI) which is implemented nowadays (at least, he was the first one who suggested the system of units in which all four constants c, h, e, and k were chosen as units).

4. On the problem of simultaneous fundamentality of c, $\hbar$ and e

During the XX century a few ideas on the development of physics based on reduction of some physical constants to the others have been proposed by some theoretical physicists: 1) reduction of the Planck constant h to the constants e and c (A. Einstein, J. Jeans, H. Lorentz, P.A.M. Dirac et al. [13-15]), 2) reduction of the elementary charge e and c to the constant h (M. Planck, A. Sommerfeld, M. Born, M.P. Bronstein, W. Pauli et al. [1, 16-19]). Such a debate took place, in particular, on the 1st Solvay Congress (1911). Other physicists as H. Weyl, W. Heisenberg contrarily considered these three constants as fundamental due to their fundamental role in particle physics [20]. Indeed, all of the secondary constants must be reduced to a combination of the most fundamental constants (e.g., Rydberg constant discovered independently was reduced to the combination of other constants) but there is no reason to regard the elementary charge e as a secondary constant because of its independence from mechanical units of measurement. The fallacy was caused by use of the Gaussian system of units. Thus, all of these ideas on the reduction of constants were unproductive.

Furthermore, some prominent physicists such as P.W. Bridgman, D. Hartree, F. Wilczek and others, argued that the system of units c=1, $\hbar$=1, e=1 is impossible in principle since the combination $\frac{e^2}{\hbar c}$ is a dimensionless constant equal to 1/137 [2, 21, 22]. However, in fact, $\frac{e^2}{\hbar c} = \alpha$ is not a law of Nature, this is a *conventional* relation and true only for Gaussian system; for example, $\frac{e^2}{\hbar c} = 4\pi\alpha$ in the Heaviside-Lorentz system, $\frac{e^2}{\hbar c} = 4\pi\varepsilon_0\alpha$ in SI, and in general $\frac{k_e e^2}{\hbar c} = \alpha$, where k_e is coefficient in Coulomb's law. The physical meaning of these equations is obvious. It is nothing as the *definition* of the elementary charge in the mechanical units: $e^2 \equiv \alpha\hbar c$ in the Gaussian system, $e^2 \equiv 4\pi\alpha\hbar c$ in the Heaviside-Lorentz system, and $e^2 = 4\pi\varepsilon_0\alpha\hbar c$ in the modern SI system. Therefore, there is no problem to choose the units so that c=1, $\hbar$=1 and e=1 simultaneously [23], and, moreover, such a system of units has already been proposed by U. Stille in 1949 [10]. As well a number of erroneous statements were also caused by use of the Gaussian system of units (such as Dirac's approval of non-fundamental status of Planck constant and uncertainty relations [15]).

5. Modernization of the SI and Gaussian systems of units due to the modern reform of metrology

A number of well-known theoretical physicists at different times argued for the Gaussian system and against the use of the SI in theoretical physics. However, their arguments were, in substance, completely incorrect or applied to the version of the SI system legally adopted in 1960. In fact, from the metrological point of view, the Gaussian system of units, being quite correct in mechanics, violates almost all metrological principles in the theory of electromagnetism. For example, in mechanical systems it is impossible to express the units of electromagnetic values uniquely as some combination of mechanical units; *different* physical quantities are related to the *same* dimension (e.g., charge and magnetic flux) whereas *different* units are introduced for the quantities of the *same* dimension (Franklin as electrostatic unit of charge and Maxwell as a unit of magnetic flux). On the contrary, the SI met the requirements of metrology in the electromagnetism and the principal possibility of its modernization. The transition to the modern quantum metrology occurs only on basis of the SI that leads to some modernization of the SI [24-30]. Firstly, the metrological reform finally overcomes one of the drawbacks of the SI subjected to fair criticism in the 1960s that the fourth main unit – ampere – is in fact determined by the mechanical units because such a natural quantity as the elementary charge e irreducible to mechanical units is selected as unit of measurement. Secondly, we must abandon the two electromagnetic constants ε_0 and μ_0 in favor of one of them since they are connected by the well-known relationship $\varepsilon_0\mu_0 = \frac{1}{c^2}$ (as it has already been done by R. Feynman in his lectures). Besides, it should be noted that the inverse of dimensional constant ε_0^{-1} has the clear physical meaning as *dimensional constant characterizing the strength of the electromagnetic interaction* and the SI should be added with this constant defined as $\varepsilon_0^{-1} \equiv 4\pi\alpha \cdot \frac{\hbar c}{e^2}$ (the formula $\alpha = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{\hbar c}$ was discovered by A. Sommerfeld in 1935 [31]). Also it is reasonable to reject symbol ε_0^{-1} and to use another constant instead of it such as $k_e \equiv \varepsilon_0^{-1}$ or $k_e \equiv (4\pi\varepsilon_0)^{-1}$.

As can be seen, the modernization of the SI induced by the requirements of quantum metrology goes in the direction to improve it and to identify the deep physical meaning in the equations, quantities and constants of electromagnetism. At the same time, this metrological reform leads to a principal contradiction with the Gaussian system of units because simultaneous choice of the constants c, $\hbar$ and e as the units of measurement is impossible in this system. This presents the need for modernization of Gaussian system: namely, explicit allocation of fine-structure constant α in the equations of electromagnetism, for example, in the Coulomb law: $F = \alpha \cdot \frac{q_1 q_2}{r^2}$, etc. But it is just the form that represents the physical meaning of the fine-structure constant α as the strength of electromagnetic interaction.

Thus, such a Gaussian system upgraded with the appearance of α in the laws of electromagnetism leads to the fact that the Gaussian system becomes more physically justified in the electromagnetism and allows to transit easily to the natural system of units c=1, $\hbar$=1, e=1. It eliminates the principal contradictions between the SI and CGS system. In this case the relation $e^2 \equiv \hbar c$ (i.e. essentially the formula $h = e^2 / c$ written by A. Einstein [13]) is fulfilled in such modernized CGS-system.

Thus, the requirements of modern quantum metrology based on the existence in Nature of fundamental physical constants as some natural absolute standards, completely determine the path to modernization and convergence different systems of units used nowadays to the unified system of units based on fundamental constants.

6. The physical quantities and laws of electrodynamics in the natural system of units

The modern representation of classical electrodynamics is inadmissible because in the textbooks such fundamentally different objects as definitions of physical quantities, mathematical identities, physical laws, space-time metric, hypothesis, conventional agreements and empirical elements are not differ and mixed. All of these should be clearly and uniquely separated. It should be noted the important works on axiomatics of classical electrodynamics [32-36].

Definitions of physical quantities. The natural classification of physical quantities should be based on the consistent insertion of physical quantities using differential operators (with taking in account of the space-time metric) and the principle of Mie-Sommerfeld – separation of "*intensive*" (Intensitätsgrößen) physical quantities such as 4-potential **A**, 4-tensor of electromagnetic field **F** and "*extensive*" physical quantities (Quantitätsgrößen) such as 4-vector of current density **J**, 4-tensor of excitation **G** (e.g. definitions of electromagnetic tensor $\mathbf{F}_{\mu\nu} = \dfrac{\partial \mathrm{A}_\nu}{\partial x^\mu} - \dfrac{\partial \mathrm{A}_\mu}{\partial x^\nu}$ and dual tensor of excitation $\dfrac{\partial \tilde{\mathbf{G}}^{\mu\nu}}{\partial x^\mu} = \partial_\mu \tilde{\mathbf{G}}^{\mu\nu} = \mathbf{J}^\nu / c$).

Dimensions of *extensive quantities* are directly proportional of dimension of charge Q and does not include of dimension of mass M. On the contrary dimensions of *intensive quantities* are inversely proportional of dimension of charge Q and directly proportional of dimension of mass M.

Also it is necessary to restore in electrodynamics the Maxwell's physical quantity "*concentration of the potential*" as one of the fundamental quantities. The concept of the *concentration* $\nabla^2 q = -\Delta q$ was introduced by J.C. Maxwell as mathematical quantity which "indicates the excess of the value of *q* at that point over its mean value in the neighbourhood of the point" [4, p.29]. Then J.C. Maxwell introduced in electrostatics the concept *concentration of the potential* [4, p.80]. A special symbol for the concentration of the potential Maxwell did not used, and the symbol **V** he used to denote the scalar potential. Since the symbol **A** is usually used for designation of the 4-potential, so the symbol **V** will use for designation of concentration of the potential. Nowadays it should be obviously generalized as a 4-vector **V** (or as a 3-form, see below) in taking into account of the 4-dimensional space-time and defined mathematically as $\mathbf{V}^\nu = \dfrac{\partial \mathbf{F}^{\mu\nu}}{\partial x^\mu} = \partial_\mu \mathbf{F}^{\mu\nu}$ or as the d'Alembertian of electromagnetic four-potential **V** = □**A** (for the Lorenz gauge), where operator $\Box = \dfrac{1}{c^2}\dfrac{\partial^2}{\partial \mathrm{t}^2} - \Delta$.

Laws. The general laws of classical electrodynamics are *linear* relationships between intensive and extensive physical quantities, in which the coefficient of proportionality is $4\pi\alpha$ (in natural units), where α – fine-structure constant characterizing the strength of the electromagnetic interaction. The general law of electrodynamics for tensors:

$$\mathrm{F}^{\mu\nu} = Z_0 \cdot \tilde{\mathrm{G}}^{\mu\nu} = 4\pi\alpha \cdot \tilde{\mathrm{G}}^{\mu\nu}.$$

Maxwell law of proportionality between concentration of the potential and charge density introduced by him for electrostatics [4, p.80] is generalized as the physical law of linear proportionality between four-concentration of the potential **V=**(V_0/c, **v**) and four-current density **J=** ($c\rho$, **j**):

$$\mathrm{V}^{\nu} = 4\pi\alpha\, \mathrm{J}^{\nu}$$

(in natural units c=1, $\hbar$=1, e=1). In general, the coefficient is dimensional constant, in SI:

$$\mathrm{V} = \mu_0\, \mathrm{J}$$

where the magnetic constant $\mu_0 = 4\pi\alpha\left(\hbar/(e^2c)\right) = 0{,}091701236853\left(\hbar/(e^2c)\right)$.

Also vacuum impedance $\mathrm{Z}_0 = 4\pi\alpha\left(\hbar/e^2\right) = 0{,}091701236853\left(\hbar/e^2\right)$

and the electric constant $\varepsilon_0^{-1} = 4\pi\alpha\left(\hbar c/e^2\right) = 0{,}091701236853\left(\hbar c/e^2\right)$.

These formulae are universal and true in any systems of units (SI, CGS etc.). The empirical constant $4\pi\alpha = 0{,}091701236853(21)$.

There are also *continuity equations* for concentration of the potential and current density: $\frac{\partial \mathrm{V}^{\nu}}{\partial x^{\nu}} = \partial_{\nu}\mathrm{V}^{\nu} = 0$ and $\frac{\partial \mathrm{J}^{\nu}}{\partial x^{\nu}} = \partial_{\nu}\mathrm{J}^{\nu} = 0$, which express conservations laws of magnetic flux and electric charge.

The definitions of physical quantities and laws of electromagnetism with Hodge star operator are as follows. 2-form of the electromagnetic field **F** is determined by 1-form potential A using of the operation of exterior differentiation **d**:

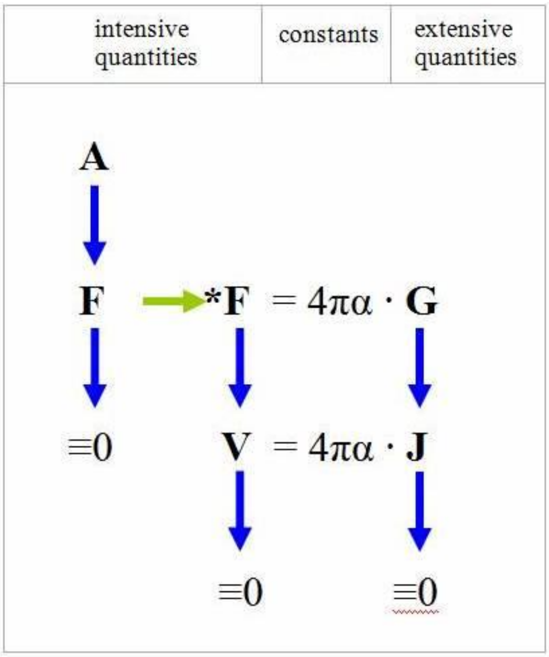
$$\mathrm{F} = d\,\mathrm{A}$$

Note that due to the fact that $\mathrm{d}^2 = 0$, the mathematical identity $\mathrm{d}\,\mathbf{F} = 0$ is valid (in vector form: $\mathrm{rot}\mathbf{E} + \dfrac{\partial \mathbf{B}}{\partial t} = 0$).

3-form of the concentration of the potential **V** is determined by the operation of external differentiation of the dual 2-form ***F**:

$$V = d * F$$

The excitation tensor **G** should be determined as $\mathrm{d}\,\mathbf{G} = \mathbf{J}$. Since $\mathrm{d}^2 = 0$ the continuity equations (the laws of conservation of magnetic flux and electric charge) $\mathrm{d}\,\mathbf{V} = 0$ and $\mathrm{d}\,\mathbf{J} = 0$ are valid.



Thus the system of physical quantities and laws of electromagnetism can be expressed in the following short form (in natural units), where the vertical arrows show the definitions of physical quantities with using of differential operators, and the horizontal – the conversion to dual tensor using the Hodge star. **A** – the potential, **F** – the electromagnetic field tensor, **V** – the concentration of the potential, **J** – the current density, **G** – the tensor of excitation (the denotation belongs to A. Sommerfeld).

For these purely mathematical definitions of physical quantities (taking into account space-time metric), the basic laws of electromagnetism can be expressed in the form of linear equations (in natural units):

$$*F = 4\pi\alpha \cdot G$$
$$V = 4\pi\alpha \cdot J$$

where α – fine-structure constant, the *empirical* constant, which characterizes the strength of the electromagnetic interaction. In SI: $\mathbf{V} = \mu_0\,\mathbf{J}$. This form of laws clearly demonstrates the fact that the laws of electromagnetism are linear, and the fine-structure constant multiplied by 4π acts as a coefficient of proportionality in electromagnetic laws. Later, in the justification (for example, geometric) fine-structure constant, it can lead to a change in the definitions of physical quantities.

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