Linear isentropic Equation of State in formation of Black hole and Naked singularity

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We analyze the physical process of gravitational collapse of a spherically symmetric perfect fluid space-time with a linear isentropic equation of state $p = k\rho$. We propose two models, with ansatzes (i) $v'(t, r)/v(t, r) = \xi'(r)$ and (ii) $rv'(t, r)/v(t, r) = g(v)$ that give rise to a family of solutions to Einstein equations with equation of state that evolves from a regular initial data satisfying weak energy conditions. The model with first ansatz leads to homogeneous collapse that terminates into the formation of black hole. We establish that as the parameter $k \to 1$ in the range $-1/3 < k \leq 1$, the formation of black hole gets accelerated in time, revealing the significance of equation of state in black hole formation.

In the second model, the end state of collapse in marginally bound spacetime is investigated in the range $0 < k \leq 1$. It is shown that end state of the inhomogeneous collapse culminates into formation of black hole and naked singularity, and that solely depends on the generic regular initial data and the role played by the pressure through parameter k. These studies give us deeper insights into the final states of collapse with a physically relevant equation of state in the light of cosmic censorship conjecture.

Keywords: Gravitational Collapse, Equation of state, Black holes and Naked singularities.

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1. Introduction

The cosmic censorship conjecture (CCC) formulated by Penrose [1], is stated as '*a singularity of gravitational collapse of a massive star developed from a regular initial surface must always be hidden behind the event horizon of the gravity*' [2]. This conjecture advocates the formation of black hole (BH) only as against the formation of naked singularity (NS) (wherein the time of formation of singularity precedes the epoch of formation of trapped surfaces). The CCC is fundamental to the well developed theory and astrophysical applications of black hole physics today.

The physical attributes of the matter field constituting a star are described by an equation of state, but an equation of state describing super dense states of matter close to the end stages of the collapse where the physical region has ultra-high densities, energies and pressures is not precisely known. To describe the collapse of a massive star, we can choose the equation of state to be linear isentropic or polytropic after it loses its equilibrium configuration. The gravitational collapse of a perfect fluid with a linear equation of state is of interest from both theoretical as well as numerical relativity perspectives. Over the decade many authors have shown existence of counter examples to CCC [3-9]. Recently studies regarding gravitational lensing in the strong field limit from the perspective of cosmic censorship have been investigated [10-11]. Further attempts are made to know whether or not naked singularities, if at all they exist in nature, can be distinguished from black holes [12,13].

Author has studied the model I with ansatz $\nu'(t, r)/\nu(t, r) = \xi'(r)$ [14] and model II with Saraykar and Joshi, with the choice $r\nu'(t, r)/\nu(t, r) = g(\nu)$ [15] to analyze the effect of pressure through parameter k. And further to know what if the value of k increases in its range when a BH/NS appears as collapse final state for the underlying spacetime, will the BH/NS formed sustain its nature? If, it is so, will the formation of BH/NS precede in time as $k \rightarrow 1$? We believe answers to these and similar issues are important in the understanding of the physical aspects and the role of an equation of state in gravitational collapse of a star.

The paper is organized as follows: In section 2, the dynamical equations of collapse with linear equation of state are presented. These equations are further used in understanding the collapse of homogeneous matter field in section 3 and the corresponding apparent horizon is discussed in subsection 3.1. The special solution obtained due to ansatz (ii) is used in analyzing the inhomogeneous collapse of the cloud in section 4. The conclusions and remarks are specified in section 5.

2. Collapse dynamics with linear equation of state

The general spherically symmetric metric

$$ds^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t,r)d\Omega^2 \qquad (1)$$

in comoving coordinates (t, r, θ, φ), describes space-time geometry of a collapsing cloud where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ is the metric on a two-sphere. The stress energy-momentum tensor for the type I matter fields for perfect fluids in diagonal form is expressed by, $T^i{}_j = \mathrm{diag}[-\rho, p, p, p]$ where the physical entities ρ and p represent energy density and pressure respectively. The weak energy condition gives rise to requirements $\rho \geq 0$; $\rho + p \geq 0$. Cloud having perfect fluid relation is described through the linear equation of state $p(t, r)=k\,\rho(t, r)$.

The Einstein field equations for the metric (1) are written as $(8\pi G = c = 1)$ [16]

$$\rho = \frac{F'}{R'R^2} = \frac{-\dot{F}}{k\,R^2\dot{R}} \tag{2}$$

$$\nu = -\frac{k}{(k+1)}\left[\ln(\rho)\right]' \tag{3}$$

$$R'\dot{G} - 2\dot{R}G\nu' = 0 \tag{4}$$

$$1 - G + H = \frac{F}{R} \tag{5}$$

where the functions G and H are defined as $G(t, r) = e^{-2\psi} R'^2$ and $H(t, r) = e^{-2\nu} \dot{R}^2$ and the arbitrary function $F(t, r)$ has an interpretation of the mass function for the star. On any spacelike hypersurface $t = const.$, $F(t, r)$ determines the total mass of the star in a shell of comoving radius r. The weak energy conditions restricts F, namely by $F(t, r) \geq 0$, and we have $F(t, 0) = 0$ to preserve the regularity of the model at all the epochs [7].

We introduce a new function $v(t, r)$ by $v(t, r) = R(t, r)/r$, and using the scaling independence of the comoving coordinate r, we write $R(t, r) = r\, v(t, r)$ [5]. In the continual collapse of the star, we have $\dot{R} < 0$, it specifies that the physical radius R of the collapsing cloud keeps decreasing in time and ultimately, it reaches $R = 0$, and it denotes spacetime singularity, namely the shell-focusing singularity at $R = 0$, where all the matter shells collapse to a vanishing physical radius at the epoch $t = t_s$. The mass function $F(t, r)$ acts suitably at the regular center so that the density remains finite and regular there at all times till the occurrence of singular epoch [7]. The Misner-Sharp mass function for the cloud can be written in general as, $F(t, r) = r^3 M(r, v)$ where the function $M(r, v)$ is regular and continuously twice differentiable. On using Misner-Sharp mass in equation (2), we have

$$\rho = \frac{3M + r\left[M_{,r} + M_{,v}\, v'\right]}{v^2\left(v + r\, v'\right)} = -\frac{M_{,v}}{k\, v^2} \tag{6}$$

We rearrange the terms in equation (6) and express it as

$$k\, r\, M_{,\, r} + \left[\left(k+1\right) r v^{'} + v\right] M_{,\, v} = -3kM \,. \tag{7}$$

Let $A(r, v)$ be a suitably differentiable function defined by $A(r, v)_{,\, v} = v' / R'$.

Now solving field equations (4), we obtain $G(t, r) = d(r)\, e^{2rA}$ where $d(r) = 1 + r^2 b(r)$ and $b(r)$ is at least twice continuously differentiable. Now, in order to know the nature of $R(t, r)$, the field equation (5) can be expressed in the form

$$\dot{R} = -e^{\nu} \sqrt{\frac{F}{R} + G - 1} \tag{8}$$

where $G - 1 = r^2 b(r)$, and so $b(r)$ basically characterizes the energy distribution for the collapsing shells.

3. Model I: Collapse of Homogeneous matter field

In this model, we use the function $v(t, r)$ as a catalyst to find solution of field equation (2) and thereafter, (t, r) coordinates are being used in understanding the collapse of the dense star with equation of state $p = k\rho$ where $k \in (-1/3, 1]$. Now, to obtain the general solution of equation (7), we consider here the ansatz [14],

$$\frac{v'}{v} = \xi'\left(r\right) \tag{9}$$

due to which equation (7) has a general solution of the form,

$$M(r, v) = m_o \frac{e^{\left[3\left(k+1\right)\xi\left(r\right)\right]}}{v^{3k}} \tag{10}$$

where m_o is an arbitrary positive constant, and $\xi(r)$ is a continuously differentiable function restrained by compatibility condition and no-trapped surface condition $F(t_i, r)/R(t_i, r) < 1$ at the beginning of the collapse, this allows for the formation of

trapped surfaces during the collapse. $M(r, v)$ expressed in equation (10) represents many classes of solutions of equation (7) but only those classes are physically realistic which satisfy the energy conditions, which are regular and which give $\rho \to \infty$ as $v \to 0$.
We have $M_{,r}(0,v) = 0$ under the conditions $\xi(0) = \text{const.}$, $\xi'(0) = 0$. Here $M_{,r}(0,v) = 0$ is in accordance with the requirement that the energy density has no cusps at the center.

Integrating equation (9), we obtain $v(t, r) = e^{\xi(r)} S(t)$ where $S(t)$ is an arbitrary function due to integration. Hence, the physical quantities in (t, r) take the form

$$F(t,r) = m_o r^3 e^{3\xi}(r) S(t)^{-3k}\ , \rho(t) = 3\ m_o S(t)^{-3(k+1)} \tag{11}$$

and it is easy to verify that these equations together satisfy field equation (2). At the dynamical equilibrium event $t = t_i$, $R(t_i, r) = re^{\xi(r)} S(t_i) = r_1$, and $0 < r_1 < r_b$ where r_b is the radius of the collapsing cloud. Since $\rho_0 \geq 0$, we have $m_o \geq 0$ with $S(t_i) = \text{const.}$

Since the density profile is homogeneous, on integrating equation (3), and solving equation (8), the metric takes the form,

$$ds^2 = -dt^2 + \frac{R'^2}{1 + r^2 b(r)} dr^2 + R(t,r)^2 d\Omega^2 \tag{12}$$

Further, at some $t = t_b$, $r = r_b$ which is the boundary of the cloud where pressure is zero, and where the interior is matched initially with Vaidya radiating metric by exhibiting that pressure vanishes at the boundary [14].

In the study of Einsteins field equations with equation of state, the system of equations gets closed but still, we have introduced equation (9) so it needs its compatibility with the field equations. It is found that for the case $b(r) = 0$, the function $\xi(r)$ remains arbitrary in satisfying the compatibility condition. While for the case $b(r) = 0$, the choice of function $\xi(r)$ is restricted by the condition $b(r) = \pm b_o e^{2\xi(r)}$ where b_o is a positive constant, thus shrinking the domain of the solution set. The collapse condition $\dot{R} < 0$ becomes $\dot{S}(t) < 0$. At the singular epoch $t = t_S$, $S(t)$ should converge to zero and so that density would diverge as $t \to t_S$. So, next we aim to find such $S(t)$ satisfying all the above conditions.

We integrate equation (8) using physical quantities in equation (11), and obtain

$$t(r,S) = t_i + \int_{S}^{S(t_i)} \frac{e^{\zeta(r)}\, dS}{\sqrt{\dfrac{m_o e^{2\zeta(r)}}{S(t)^{(1+3k)}} + b(r)}} \tag{13}$$

where the variable **r** is treated as a constant. Let $t_0 = t(0,0)$ be the time at which the central shell becomes singular. The time taken by the central shell to reach the singularity should be positive and finite, and hence we have the model realistic condition (MRC) for any $k \in (-1/3, 1]$ which compels us to take $b(0) = -b_o e^{2\xi(0)}$, giving rise to the range of $S(t)$ as $0 \le S(t) < [m_o/b_o]^{1/(1+3k)}$ [14]. Thus the initial data of mass and density profiles is restricted by the introduction of the equation (9) through the condition $b(r) = -b_o e^{2\xi(r)}$. For $b(0) = 0$, the MRC takes the form $S(t) \ge 0$.

Now using $b(r) = -b_o e^{2\xi(r)}$ and integrating equation (13), we have

$$t(S) = t_i + \frac{2\left[S(t_i)^{\frac{3(k+1)}{2}} H_1 - S(t)^{\frac{3(k+1)}{2}} H_2 \right]}{3\sqrt{m_o}\,(k+1)} \tag{14}$$

Where $H_1 = hypergeom([1/2, K_1],[K_2], b_o S(t_i)^{(K_3)}/m_o)$,

$H_2 = hypergeom([1/2, K_1],[K_2], b_o S(t_i)^{(K_3)}/m_o)$,

$$K_1 = \frac{3(k+1)}{2(3k+1)}, K_2 = \frac{(9k+5)}{2(3k+1)}, K_3 = 3k+1.$$

The hypergeometric series mentioned above is convergent for $|b_o S(t)^{K_3}/m_o| < 1$ and $1/3 < k \le 1$. The convergence condition on $S(t)$ augurs well with the MRC restriction $0 \le S(t) < [m_o/b_o]^{1/(1+3k)}$. From above equation, we find $\dot{S}(t) < 0$, and thus the desired collapse condition is satisfied for the dense cloud as $t \to t_S$ and this indicates perpetual gravitational collapse of the star.

The time taken by the central shell to reach the singularity is given by

$$t_0 = t_i + \frac{2H_1 s(t_i)^{\frac{a(k+1)}{2}}}{3\sqrt{m_0}(k+1)} \tag{15}$$

The time for other collapsing shells to arrive at the singularity can be expressed by $t_S(r) = t(r, 0)$ and since the energy density has no cusps at the singularity curve then this gives us $t_s(r) = t_0$ [14]. This indicates that time of formation of central singularity $(t = t_S, r = 0)$ and the non-central singularity $(t = t_S, r = r_C > 0)$ in the neighbourhood of the center $r = 0$ is same. Clearly these events are simultaneous and it is understood that in such scenario the singularity be remain covered behind the event horizon, thus confirming the formation of BH as the end state of collapse.

3.1 Apparent horizon

For a naked singularity to come into existence the trapped surfaces should form later, especially after the formation of singularity. Thus for a naked singularity to form we need, $t_{ah}(r) > t_0$ for $r > 0$, near $r = 0$ where t_0 is the epoch at which the central shell hits the singularity [14]. When the singularity curve is constant (χ_1 and other higher order terms are all vanishing), or would be decreasing, then a black hole will necessarily form as the collapse final state.

We know since the collapsing shells are simultaneous in model I, the end state of collapse is bound to be a black hole but the intriguing question is that, what is the role of parameter k of equation of state in the formation of the black hole. Is their a certain range of k in which the formation of black hole will be accelerated in time?

Since we have $b(r) = -b_o e^{\xi(r)}$ with positive b_o, therefore, we have only two cases to study namely that spacetime is bound or marginally bound. The time of occurrence of apparent horizon in a bounded spacetime is written as

$$t_{bah} = t_{bs} - \int_{0}^{S_{ah}} \frac{dS}{\sqrt{\frac{m_0}{S(t)^{(1+3k)}} - b(r)}} \tag{16}$$

where $t_{bs} = t_0$ is given by equation (15) and $S(t_{ah}) \equiv S_{ah}$ is determined from $F/R = 1$. On solving equation (16), we have

$$t_{bah} = t_{bs} - t_{bk} \quad where \quad t_{bk} = \frac{2R_{ah}H_{2rah}}{3(1+k)} \tag{17}$$

and $H_{2rah} = hypergeom\left(\left[\frac{1}{2}, K_1\right], [K_2], z\right)$, $z = b_o r_{ah}^2 \; e^{2\zeta(r_{ah})}$.

It is clear that t_{bk} is a positive quantity for all $k \in (-1/3, 1]$, and therefore $t_{bah} < t_{bs}$ for any $r > 0$, near the center $r = 0$. The collapse progresses to culminate into the formation of trapped surfaces first and eventually the singularity forms later, leading to formation of BH as a final state of collapse for all $k \in (-1/3, 1]$. Now, we study the characteristics of the parameter k in the formation of the BH. It is indeed possible to testify whether formation of trapped surfaces of such a star would accelerate or decelerate in time relative to change in parameter of equation of state. In view of these aspects the theorem follows [14]:

Theorem 1*: Consider $t_{bk} = t_{bk}(k, r_{ah})$, r_{ah} depends on k and $0 < S_{ah} < S(t_i) < 1$. We prove that both t_{bk} and t_{bs} are positive decreasing time functions as $k \to 1$ and that $t_{bs} > t_{bk}$ for all $k \in (-1/3, 1]$. Further $t_{bah} < t_{bs}$ for any $r > 0$, near the center $r = 0$ and t_{bh} is a positive decreasing time function as $k \to 1$.*

Theorem 1 holds under the conditions that $0 < S_{ah} < S(t_i) < 1$ and $|z| < 1$. These physically realistic conditions are possible with the appropriate choice of the function $\xi(r)$ such as $[1 + r_{ah}\xi'(r_{ah})] > 0$. Clearly indicating that trapped surfaces are being formed first, and the event of the formation of singularity is taking place at the later time. Thus black hole forms for all k. Further since t_{bah} is a positive decreasing function as $k \to 1$.

Therefore, the equation of state is stimulating the formation of apparent horizon of gravity to take place at the earlier epoch and further strengthening this characteristic as k increases as compared to the usual process of formation of trapped surfaces in the final stages of collapse of the sufficiently large star, culminating it into the black hole at the earlier time. This process is accelerated in time as $k \to 1$ with the physically plausible choice of the function $\xi(r)$ [14].

In the marginally bound case that is when $b(r) = 0$, on integrating equation (13), we have

$$S(t) = \left[\frac{3}{2}\sqrt{m_o}\,(1+k)(t_s - t)\right]^{2/[3(1+k)]} \tag{18}$$

then $\dot{S}(t) < 0$ and as $t \to t_S$, $\dot{S}(t) \to -\infty$. The Theorem 1 and results thereof follows for marginally bound space-time, for details refer [14].

4. Model II: Inhomogeneous collapse

To study the spherical gravitational collapse of a perfect fluid, we consider here a linear isentropic equation of state, $p = k\rho$ with $0 \le k \le 1$. We have $R(t, r) = rv(t, r)$, at the initial surface $v(t_i, r) = 1$, and the singular surface $t = t_S$, $v(t_S(r), r) = 0$. The collapse condition is now written as $\dot{v} < 0$. The time $t = t_S(r)$ corresponds to the shell-focusing singularity at $R = 0$, where all the matter shells collapse to a vanishing physical radius. We set r and $v \in [0, 1]$ as independent coordinates by performing a transformation from (r, t) to (r, v), thus considering $t = t(r, v)$. We consider here the ansatz [15]

$$\frac{rv'}{v} = g(v) \tag{19}$$

due to which the equation (7) has a general solution of the form,

$$M(r, v) = mo\ f(x)e^{-3Z(v)} \quad \text{where} \quad x = re^{-Z(v)} \tag{20}$$

and $$Z(v) = \int_1^v \frac{k}{v[(k+1)g(v)+1]}dv \tag{21}$$

where m_o is a positive constant. The density profile for this class of models then takes the form,

$$\rho(r,v) = \frac{m_o\, e^{-3Z(v)}\left[3f(x) + x\, f'(x)\right]}{v^3\left[(k+1)g(v)+1\right]}. \tag{22}$$

Such a density profile diverges at $t = t_s$, and decreases away from the center $r = 0$, which is a physically reasonable feature for the collapsing matter cloud and this is possible through the appropriate choice of function $f(x)$ and obtaining $g(v)$ through the compatibility condition, for details refer [15]. The requirement of energy condition on the surface $v = 1$ is fulfilled through $\rho_o(r) > 0$. Such a decreasing density of the cloud finally becomes zero at some $r = r_b$, and at the epoch $t = t_b$. Hence we would take such a value of $r = r_b$ as the boundary of the cloud where the energy density is zero, and where the interior is matched to a suitable exterior metric.

Existence of a solution of equation (7) is an important question which is answered through the Theorem 2. Also it is very crucial in the study of gravitational collapse that collapse commences from the non-singular initial data, and this issue is addressed through the Theorem 3 [15].

Theorem 2: *The general solution of equation (7) exists in domain of $v \in (0, 1]$ and $0 \le r \le r_b$ if $f(x)$ and $g(v)$ are continuously differentiable functions in the domain such that the $\lim_{v\to 0} g(v)$ exists.*

Theorem 3: *Consider the equation of state $p = k\rho$, the mass profile $F(t, r) = r^3 M(r, v)$ and the density profile as in equation (22). Now, if $rv'(t, r) = v(t, r)g[v(t, r)]$ is introduced as an additional equation in the set of field equations then the initial data of mass function, and thereof density is non-singular at the initial epoch $t = t_i$*

We note, from equation (19) that $g(1) = 0$ and assume that $g(0)=\lim_{v\to 0} r v'/v = \alpha_o$ exists and note that at the initial epoch, $g(1) = 0$, $Z(1) = 0$. Solving field equations, we can write the metric in the neighborhood of the center $r = 0$ of the cloud as,

$$ds^2 = -\rho^{-2l}\, dt^2 + \frac{R'^2}{e^{2rA}}\, dr^2 + R(t,r)^2\, d\Omega^2 \tag{23}$$

As per the ansatz (19) g(v) need not be zero in general, for $r = 0$. If we assume g(v) = 0 for $r = 0$, then we obtain $M(0, v) = m_o / v^{3k}$. This can be obtained directly also from equation (7) by putting $r = 0$. We now analyze the outcome of end state of collapse in this particular case where g(v) = 0 for $r = 0$ i.e. g(v) vanishes at the central shell at all regular points of the spacetime.

Integrating equation (8), we have

$$t\left(r,v\right)=t_i+\int_v^1 \frac{e^{-\nu}\, dv}{\sqrt{\frac{M\left(r,v\right)}{v}+\frac{e^{2rA}-1}{r^2}}} \tag{24}$$

Regularity ensures that, in general, $t(r, v)$ is at least C^2 near the singularity and therefore can be expanded around the center as,

$$t\left(r,v\right)=t\left(0,v\right)+r\,\chi_1\left(v\right)+\frac{r^2}{2!}\chi_2\left(v\right)+O\left(r^3\right)$$

where
$$\chi_1\left(v\right)=\frac{dt}{dr}|_{r=0,}\quad \chi_2\left(v\right)=\frac{d^2t}{dr^2}|_{r=0.} \tag{25}$$

Now, for examination of the nature of central singularity at $R = 0, r = 0$, we consider the equation of outgoing radial null geodesics, given by, $dt/dr=e^{\Psi-\nu}$

Further, we write the null geodesic equation in terms of the variables $(u = r^{\beta}, R)$, choosing $\beta = 1/(1-k)\,[5/3 - k]$ for $k \in (0, 1)$, and using equation (5), we obtain

$$x_o^{\,3(1-k)/2}=\frac{3(1-k)}{2}\sqrt{m_o}\left(\frac{1}{3m_o}\right)^{\frac{k}{k+1}}\chi_1(0) \tag{26}$$

for $k \in (0, 1)$. The radial null geodesic emanating from the singularity in (R, u) co-ordinates is $R = x_o u$. Therefore, $x_o > 0$ iff $\chi_1(0) > 0$, and hence $\chi_1(0) > 0$ is a sufficient condition for the occurrence of the NS at the center of the cloud as the end state of gravitational collapse of a sufficiently dense star. Further using equation (24), we can obtain

$$\chi_1(0) = 4l3^l\, m_o^{(l-3/2)} f'''(0)\left[\frac{1}{(5+3k)(5-3k)} - \frac{20l(7+4k)}{m_o(7+9k)(7+5k)(7+3k)(7-k)}\right] \tag{27}$$

where $l = k/(k+1)$. Now from above equation, we clearly observe dependency of $\chi_1(0)$ on the initial data of mass function through the function f, m_o and the parameter k of equation of state. Thus the sign of $\chi_1(0)$ will be decided by the initial mass profile of the collapsing star and the pressure profiles expressed through $p = k\rho$.

Now for the choice say $m_o = 1$, if $f'''(0) = 1$ then we have formation of NS whereas $f'''(0) = -1$ propels formation of BH as the end state of collapse. Next, in case the collapse begins with higher initial central density through m_o together with $f'''(0) = 1$ then formation of NS takes over for higher values of m_o. This is illustrated through Fig. 1. When $\chi_1(0) = 0$ then analysis is carried out through $\chi_2(0)$. Also these results are authenticated through the study of apparent horizon [15]. When we consider $k = 0$, the model here reduces to the dust collapse model. In the special class of dust collapse obtained here, the final state is a black hole, because with a vanishing k, we obtain a homogeneous dust collapse model. The detail analysis and results thereof pertaining to this aspect are presented in [15].

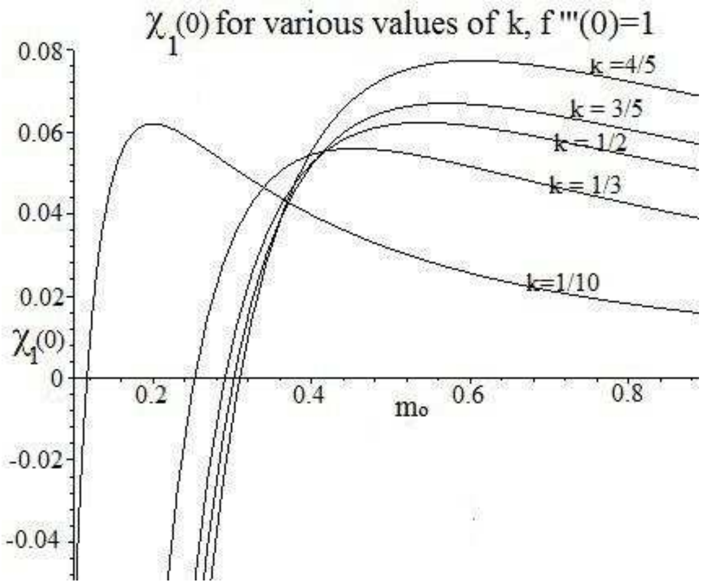

Fig.1 The above illustration of $\chi_1(0)$ indicates the role played by initial central density through the mass the function. $f'''(0) = 1$ together with higher initial central density through m_o propels formation of NS for all k.

5. Conclusions and Remarks

Let us summarize the results, firstly we have obtained the solution of Type I matter field equations through the ansatzs introduced in equations (9) and (19). Certainly, this has led to a special class of solutions with an isentropic equation of state $p = k\rho$ that satisfy weak energy conditions and evolve as the collapse begins according to the homogeneous/inhomogeneous distribution of matter.

With the varied choices of function $\xi(r)$ in model I satisfying physically realistic conditions, we can study different models, for eg. in the case of $b(r) = 0$ if we choose $\xi(r) = 0$, metric (12) gives us Einstein-deSitter model with equation of state while for $b_0 = 1$, $\xi(r) = 0$, we have a closed Friedman model, and so on. So, we have a class of bound and marginally bound space-times which can be explored further. It is shown that how the choice of initial data of mass function and the physical radius through the function $\xi(r)$ lead to the formation of BH.

The study of gravitational collapse with a linear equation of state has revealed the role of the parameter k in terms of formation of BH in homogeneous collapse in the range of $-1/3 < k < 1$ and further strengthening it by accelerating the formation of trapped surfaces in time, in both the bound and marginally bound space-times as $k \to 1$.

In model II, in marginally bound inhomogeneous collapse with $b(r) = 0$, end state of collapse leads to BH/NS subject to the choice of regular initial data of mass function and pressure profiles $k \in [0, 1)$.

The parameter value $k = 1$ depicts the case of stiff fluid (that the equation of state becomes rigid enough) which itself may halt the progress of the collapse at some stage [17]. Therefore our results are more significant in the range of $-1/3 < k < 1$.

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