Phenomenological description of dark energy and dark matter via vector fields

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A simple Lagrangian (with squared covariant divergence of a vector field as a kinetic term) turned out an adequate tool for macroscopic description of dark sector. The zero-mass field acts as the dark energy. Its energy-momentum tensor is a simple additive to the cosmological constant. Space-like and time-like massive vector fields describe two different forms of dark matter. The space-like field is attractive. It is responsible for the observed plateau in galaxy rotation curves. The time-like massive field displays repulsive elasticity. In balance with dark energy and ordinary matter it provides a four parametric diversity of regular solutions of the Einstein equations describing different possible cosmological and oscillating non-singular scenarios of evolution of the Universe. In particular, the singular "big bang" turns into a regular inflation-like transition from contraction to expansion with accelerated expansion at late times. The fine-tuned Friedman-Robertson-Walker singular solution is a particular limiting case at the boundary of existence of regular oscillating solutions (in the absence of vector fields). The simplicity of the general covariant expression for the energy-momentum tensor allows analyzing the main properties of the dark sector analytically, avoiding unnecessary model assumptions. It opens a possibility to trace how the additional attraction of the space-like dark matter, dominating in the galaxy scale, transforms into the elastic repulsion of the time-like dark matter, dominating in the scale of the Universe..

Keywords: Dark sector, vector fields, regular cosmology, galaxy rotation curves.

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Introduction

The two most intriguing long standing problems in astrophysics (plateau in galaxy rotation curves [1,2] and accelerated expansion of the Universe [3,4]) strictly pointed to the existence of "hidden sector", containing "dark energy" and "dark matter", whose interaction with the ordinary matter (baryons and leptons) is observed only via gravitation.

At first glance, these two problems had nothing to do with one another. The accelerated expansion of the Universe indicated the existence of a hidden mechanism of repulsion, while a plateau of galaxy rotation curves was the result of some additional attraction. Nevertheless, the macroscopic approach to the dark sector problems [5], based on the analysis of vector fields in general relativity, provided an appropriate universal tool for theoretical description of both these phenomena. The space-like massive vector field is attractive. It is responsible for the observed plateau in galaxy rotation curves. The time-like massive vector field displays repulsive elasticity. In the scale of the whole Universe it is the source of accelerated expansion. Naturally, the previous solutions of the Einstein equations, describing the expansion of the Universe filled with the mutually attracting matter only, inevitably contained a singularity. Inclusion of the repulsive dark matter into consideration allows the existence of nonsingular solutions describing various possible regular scenarios of evolution of the Universe.

My review article [5] contains the macroscopic theory of dark sector, based on the analysis of vector fields in general relativity. The step by step derivations are accompanied by the references to the benchmark achievements of the predecessors. The main attention was paid to clarify the validity of basing assumptions. This text of my talk contains a discussion of physical nature of manifestations of dark sector. Analytical derivations are presented briefly only by final results.

Regularity in General Relativity

In regular solutions of the Einstein equations all invariants of the Riemann curvature tensor are finite. Hence, the invariants of the Ricci tensor $\boldsymbol{R_{IK}}$ are finite too. By virtue of Einstein equations the requirement of regularity automatically excludes a possibility to achieve an infinite value for all the invariants of the energy-momentum tensor $\boldsymbol{T_{IK}}$. In General Relativity, the distribution/motion of matter and the curvature of space-time are mutually balanced. Necessary restrictions, if any, on the signs of existing parameters arise as a consequence of the condition of regularity.

The requirement, that all the invariants of the Riemann curvature tensor are finite, is a necessary condition of regularity in General Relativity.

Vector fields describing dark sector

Vector fields are widely used to describe quantum particles of the ordinary matter. Equations for ordinary particles are easily established in accordance with the properties of their free motion in plane geometry. This approach is convenient for description of already known particles. However, it does not help to describe the unknown substance of dark sector.

In general relativity, the standard approach, starting from a general form of the Lagrangian of a vector field, is capable to describe not only the already known particles. Starting from a general form of the Lagrangian of a vector field in general relativity, one should derive vector field equations and energy-momentum tensor. Then, excluding the terms associated with the ordinary matter, one gets a chance to separate a Lagrangian describing the dark sector. The separation of the Lagrangian of dark sector is necessary, especially if the ordinary matter is considered as a continuous medium with the macroscopic energy-momentum tensor

$$T_{\text{om}\,IK} = \left(\varepsilon + p\right) u_I u_K - p g_{IK}\,.$$

Otherwise, the ordinary matter would be taken into account twice: as a medium with the energy-momentum tensor $T_{\text{om}\,IK}$, and as quantum particles described by a vector field.
It turns out that the simplest Lagrangian of a vector field ϕ_L,

$$L_{\text{dark}} = a\left(\phi^M_{;M}\right)^2 - V\left(\phi^L \phi_L\right) \tag{1}$$

allows describing the main observed manifestations of dark sector completely within the frames of minimal general relativity. In this case, the massless field corresponds to the dark energy, the massive space-like field ($\phi^L \phi_L$<0) is responsible for a plateau in galaxy rotation curves, and the massive time-like vector field ($\phi^L \phi_L$>0) displays a repulsive elasticity. The competition of repulsive dark matter and attractive ordinary matter leads to a variety of possible regular scenarios of evolution of the Universe. In case of Proca equations, describing ordinary particles, the term with covariant divergence is set to zero (a = 0). For this reason L_{dark} gets separated from a Lagrangian of ordinary matter.

In accordance with (1) the field equations and the energy-momentum tensor are

$$a\frac{\partial \phi^M_{;M}}{\partial x^I} = -V'\phi_1\,, \tag{2}$$

$$T_{\text{dark}\,IK} = g_{IK}\left[a\left(\phi^M_{;M}\right)^2 + V\right] + 2V'\left(\phi_I\phi_K - g_{IK}\phi^L\phi_L\right). \tag{3}$$

Here $V' \equiv dV/d\left(\phi^L \phi_L\right)$. The energy-momentum tensor $T_{\text{dark}\,IK}$ of a zero-mass (V' = 0) vector field reduces to

$$T_{(0)IK} = g_{IK}(a(\phi_0')^2 + V(0)), \tag{4}$$

where $\phi_0{}' \equiv \phi^M_{;M}(0)$ is the constant divergence of a zero-mass vector field. $T_{(0)\,IK}$ acts in the Einstein equations as a simple addition to the cosmological constant, changing Λ to

$$\tilde{\Lambda} = \Lambda - \varkappa(a(\phi_0{}')^2 + V(0)).$$

$\varkappa$ is the gravitational constant.

In the case of weak vector fields the second and higher derivatives of the potential $V(\phi^L\phi_L)$ can be neglected, and the energy-momentum tensor of a massive field is

$$T_{\text{dark}\,IK} = a\left(\phi^M_{;M}\right)^2 g_{IK} + V_0^{'}\left(2\phi_I\phi_K - g_{IK}\phi^L\phi_L\right).$$

In general, it is necessary to consider two independent vectors: $\phi^K_{(s)}$ and $\phi^K_{(t)}$ for a space-like and a time-like massive fields with different potentials $V_{(s)}\left(\phi^K_{(s)}\phi_{(s)K}\right)$ and $V_{(t)}\left(\phi^K_{(t)}\phi_{(t)K}\right)$.

As far as the dark energy is taken into account by $\tilde{\Lambda}$, the energy-momentum tensor of the dark sector is the sum

$$T_{darkIK} = T_{(s)\text{IK}} + T_{(t)\text{IK}}.$$

In the scale of a galaxy (~ 10 kpc) the space-like vector field ($\phi^L\phi_L$<0) dominates. It is responsible for the plateau in galaxy rotation curves. The time-like field ($\phi^L\phi_L$>0) dominates at the scales much larger than the distance between the galaxies, where the Universe can be considered uniform and isotropic. The time-like field displays repulsive elasticity. Together with the dark energy and the ordinary matter it gives rise to a variety of possible regular scenarios of evolution of the Universe, and rules out the problem of fine tuning. In particular, the singular “big bang” turns into a regular inflation-like bounce with accelerated expansion at late times.

It would be interesting to trace how the additional attraction of the space-like dark matter, dominating in the galaxy scale, transforms into the elastic repulsion of the time-like dark matter, dominating in the scale of the whole Universe. Both types of massive fields $\phi^K_{(s)}$ and $\phi^K_{(t)}$ are active in the intermediate region.

The study of the structure of the Universe in the intermediate range (Mpc to hundred Mpcs) had been initiated in the pioneering papers by Zel'dovich [6]. Continuous research by his followers [7] shows that dark energy and dark matter significantly affect the structural dynamics of galaxies and clusters in this range. Utilizing the energy-momentum tensor $T_{\mathrm{dark}\,IK}$ in the analysis of the large scale structure of the Universe would allow avoiding unnecessary model assumptions.

Galaxy rotation curves

The velocity V of a star, orbiting around the center of a galaxy and satisfying the balance between the centrifugal V^2/r and centripetal $GM(r)/r^2$ accelerations, should decrease with radius r of its orbit as $\boldsymbol{V(r)\sim 1/\sqrt{r}}$ at $r \to \infty$. However, numerous observed dependences $V(r)$, named galaxy rotation curves, practically remain constant at far periphery of a galaxy. It had been a fundamental problem for a long time, because General Relativity reduces to Newton's theory in the limit of nonrelativistic velocities and weak gravitation.

Applying general relativity to the galaxy rotation problem it is reasonable to consider a static centrally symmetric metric

$$ds^2 = g_{IK} dx^I dx^K = e^{\nu(r)}(dx^0)^2 - e^{\lambda(r)} dr^2 - r^2 d\Omega^2$$

It contains two metric functions $\nu(r)$ and $\lambda(r)$ depending on only one coordinate - circular radius r. It is the same metric as for a Schwarzschild solution.

Real distribution of stars and planets in a galaxy is neither static, nor centrally symmetric. However, most galaxies are concentrated around super heavy objects, be it a black hole, or a neutron star. The deviation from central symmetry, caused by peripheral stars, is small. In the background of centrally symmetric metric the vector $\phi^{\boldsymbol{I}}$ is longitudinal. Its only non-zero component $\phi^{\boldsymbol{r}}$ depends on r.

Omitting details (one can see a complete derivation in my review article [5]), I present here the following analytical formula for the velocity $\boldsymbol{V(r)}$ of a star, rotating around a black hole far outside the Schwarzschild radius $\boldsymbol{r_{\mathrm{Sch}}}$:

$$\mathrm{V}(\mathrm{r}) = \sqrt{\mathrm{V}_{\mathrm{pl}}^2\left(1 - \frac{\sin 2\mathrm{mr}}{2\mathrm{mr}}\right) + \frac{\mathrm{c}^2}{2}\frac{\mathrm{r}_{\mathrm{Sch}}}{\mathrm{r}}}, \qquad \mathrm{r} \gg \mathrm{r}_{\mathrm{Sch}}. \tag{5}$$

Here $V_{\mathrm{pl}} = c\sqrt{\frac{\varkappa|a|}{2}\frac{\phi_0'}{m}}$ is the plateau velocity at $r \to \infty$, and $m = \sqrt{|V_0'/a|}$ is the mass of a space-like vector field. Recall that $V_0' = dV/d\left(\phi^L\phi_L\right)$ at $\phi^L\phi_L = 0$. Without dark matter ($\phi_0' = 0$) (5) would give the Newton's $V(r) \sim r^{-1/2}$ at $r \to \infty$. In the presence of dark matter ($\phi_0' \neq 0$) the velocity of rotation $V(r)$ tends to V_{pl} at $r \to \infty$ with damping oscillations.

The deviation from the Newton's law due to dark matter takes place at $r \gtrsim r_{\mathrm{Sch}}\left(c/V_{\mathrm{pl}}\right)^2$. At $r \gg r_{\mathrm{Sch}}\left(c/V_{\mathrm{pl}}\right)^2$ the curve of rotation around a black hole is a universal function, see Figure 1. In dimensionless units V/V_{pl} and $x = mr$ there are no parameters.

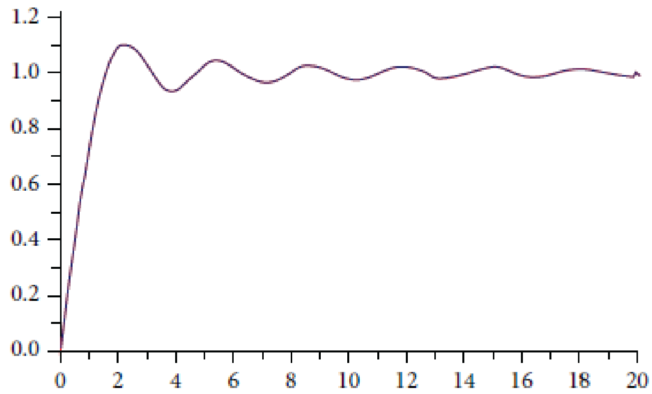

Fig. 1. Universal function $\left(1-\sin 2x/2x\right)^{1/2}$.

Though the rotation curves of galaxies differ from one another, the deviation from the Newton's $\boldsymbol{r^{-1/2}}$ **on the periphery of a galaxy** is their common feature. In order to compare with observations, it looks natural to choose the galaxies having stars outside the main disc. Fitting the rotation curves of two such galaxies by the universal function $(\mathbf{1 - \sin 2}\boldsymbol{x}/\mathbf{2}\boldsymbol{x})^{-\mathbf{1/2}}$ is shown in Figure 2.

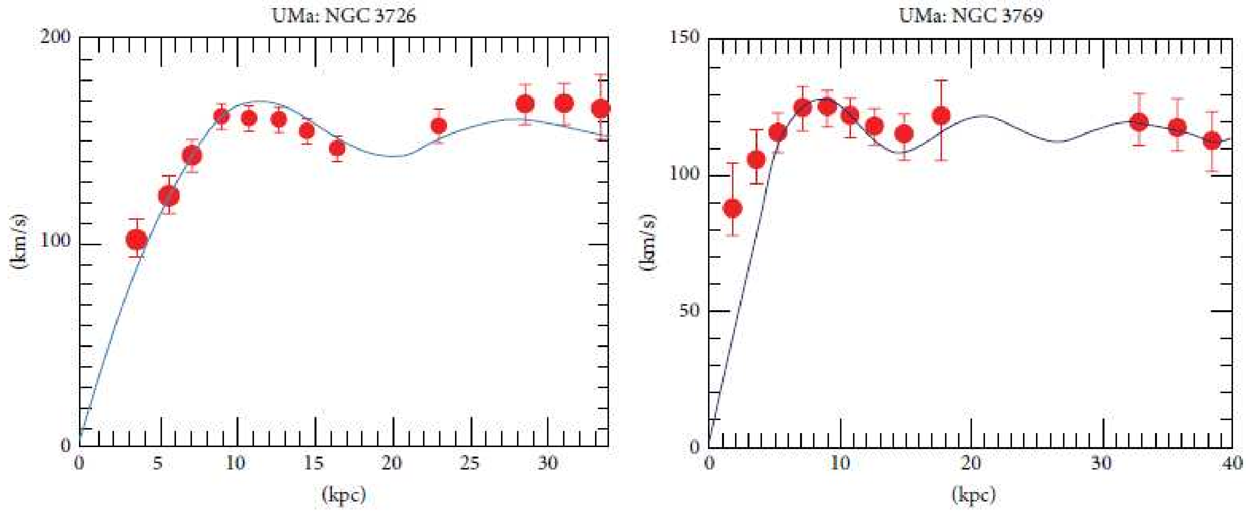

Fig. 2. Fitting the rotation curves of two galaxies in the Ursa Major cluster by the universal curve $(\mathbf{1} - \mathbf{sin2}x/\mathbf{2}x)^{-\mathbf{1/2}}$

These spiral galaxies are located in the Ursa Major cluster (UMa). Their numbers are taken from "The New General Catalogue of Nebulae and Clusters of Stars" (abbreviated as NGC). It is a catalogue of deep-sky objects in astronomy compiled by John Louis Emil Dreyer in 1888 [8], as a new version of John Herschel's Catalogue of Nebulae and Clusters of Stars.

Damping oscillations of a rotation curve at far periphery of a galaxy I consider as a "signature of dark matter", and I strongly recommend this observational test. It confirms the existence of dark matter, along with its adequate description by a longitudinal non-gauge vector field.

From the physical point of view, a more strong attraction to the center at the periphery of a galaxy (than predicted by the Newton's theory) is a consequence of a finite velocity of propagation of interactions. In the Newton's theory any variation of a gravitating object immediately changes the gravitational field everywhere in the whole space. In General Relativity retardation is taken into account, and propagation of interactions has a wave-like character. Roughly speaking, the static Newton's potential $\boldsymbol{\varphi}(\boldsymbol{r}) \sim 1/\boldsymbol{r}$ takes place in the near zone. In the wave zone $\boldsymbol{mr} \gtrsim \mathbf{1}$ it gets proportional to $(\mathbf{cos}\,\boldsymbol{mr})/\boldsymbol{r}$. Accordingly, the force of attraction in the wave zone $\sim\boldsymbol{m}(\mathbf{sin}\,\boldsymbol{mr})/\boldsymbol{r}$ at $\boldsymbol{mr} \gg \mathbf{1}$ decreases more slowly than the Newton's $\sim 1/\boldsymbol{r}^{\mathbf{2}}$.

As a matter of fact, appearance of a plateau in a rotation curve can be interpreted as a manifestation of gravitational waves in the galactic scale. Meanwhile, huge efforts and funds are being spent in vain to detect gravitational waves on the Earth, just to prove their existence.

Regular evolution of the Universe

Discovery of the accelerated expansion of the Universe [3],[4] shows that the source of acceleration continues to exist for a long time after the “big bang”. Naturally, the fact of accelerated expansion gave rise to the assumption that the physical vacuum is not just the absence of the ordinary matter. The existence of dark energy and dark matter, as the unknown source of the Universe's expansion, is widely discussed in modern literature.

If we include into consideration a dark sector providing a mechanism of repulsion, then a singularity ceases to be an inevitable property of evolution of the Universe. It is reasonable to analyze possible scenarios of the Universe evolution in frames of regular solutions of the Einstein equations. The approach to the theory of regular evolution of the Universe driven by vector fields looks most successful among numerous attempts to guess the riddle of accelerated expansion. It allows avoiding unnecessary model assumptions like “$f(R)$”, quintessence, phantom-like cosmologies, …. It allows remaining in the classical frames of the Einstein's general relativity. The solutions have additional parametric freedom, allowing forgetting the fine-tuning problem.

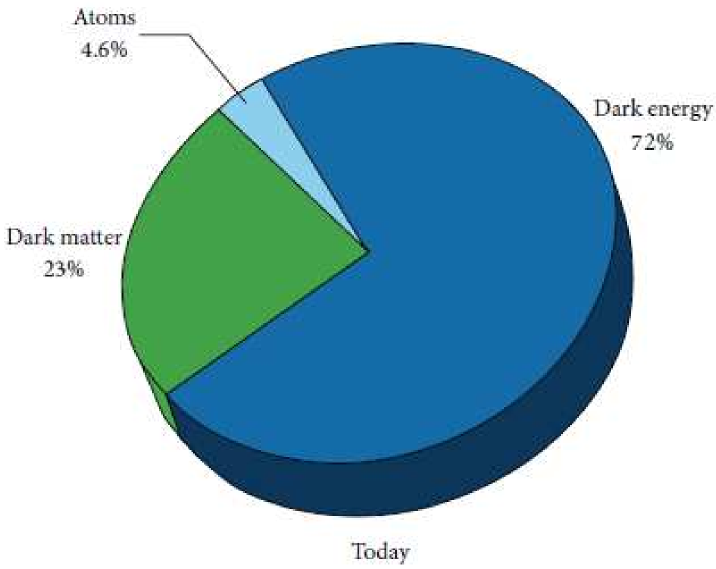

Fig. 3. Stuff of the Universe

Today it is generally accepted that among the staff of the Universe only 4.5% is the ordinary matter, see Figure 3. Remaining 95.5% is dark sector, consisting of dark energy (zero-mass field, 72%) and dark matter (massive fields, 23%).

According to observations, the Universe expands, and its large scale structure remains homogeneous and isotropic. Consider the space-time with metric

$$\boldsymbol{ds}^2 = \boldsymbol{g}_{IK}\boldsymbol{dx}^I\boldsymbol{dx}^K = \left(\boldsymbol{dx}^0\right)^2 - \boldsymbol{e}^{2\boldsymbol{F}\left(\boldsymbol{x}^0\right)}\sum_{\boldsymbol{I}=1}^{3}\left(\boldsymbol{dx}^I\right)^2$$

depending on only one time coordinate $\boldsymbol{x^0 = ct}$. The uniform and isotropic expansion is characterized by a single metric function $\boldsymbol{F}(\boldsymbol{x^0})$, and $\frac{\boldsymbol{dF}}{\boldsymbol{dx^0}} \equiv \boldsymbol{F}'(\boldsymbol{x^0})$ **is** the rate of expansion. Longitudinal massive vector field $\boldsymbol{\phi_I}$ in this case is time-like: as it follows from (2), the only nonzero component is $\boldsymbol{\phi_0}$. In contrast to a space-like field, a massive time-like field demonstrates elastic repulsion.

Role of dark energy

The energy-momentum tensor (4) of a massless field acts in the Einstein equations as a part of the cosmological constant $\boldsymbol{\tilde{\Lambda}}$:

$$R_{IK} - \frac{1}{2}g_{IK}R + \tilde{\Lambda}g_{IK} = 0.$$

The contribution of the zero-mass field to the curvature of space-time remains constant in the process of the Universe evolution. The metric function

$$F\left(x^0\right) = \pm H\left(x^0 - x_0^0\right), \qquad H = \sqrt{-\tilde{\Lambda}/3}$$

is a regular solution of the Einstein equations, provided that $\boldsymbol{\tilde{\Lambda} < 0}$. This solution belongs to de Sitter (1917). It describes either expansion of the Universe at a constant rate $\boldsymbol{F' = H}$ (green horizontal line $\boldsymbol{F'/H = +1}$ in Figure 4), or contraction (blue horizontal line $\boldsymbol{F'/H = -1}$ in Figure 4) **.** *H* is the Hubble constant.

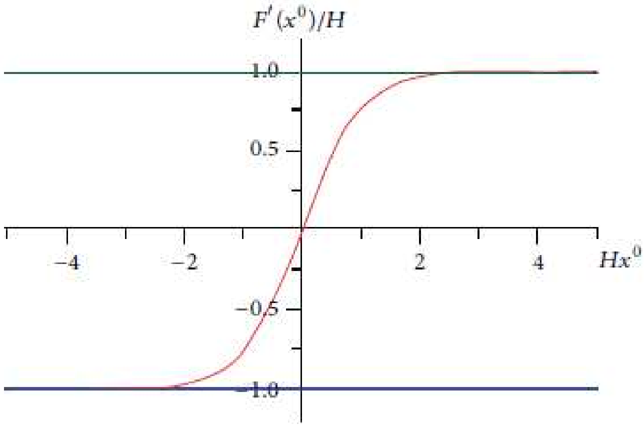

Fig. 4. Rate of evolution of the Universe. Upper green horisontal line is expansion. Lower blue horisontal line is compression. Red curve is a transition from compression to expansion.

As long as the physical nature of vacuum is not known, the "geometrical" origin of the cosmological constant Λ and the "material" contribution to $\tilde{\boldsymbol{\Lambda}}$ by a zero-mass vector field can not be separated from each other. The combined action of the massless field and/or the cosmological constant is described by the single parameter – Hubble constant *H*.

Without a massive field $\boldsymbol{F''=0}$. The dark energy by itself can be responsible only for either contraction, or expansion at a constant rate. In particular, a zero-mass longitudinal vector field alone can not explain the observed switch from deceleration to acceleration at about a half of the age of the Universe [9].

Role of dark matter

With account of a massive time-like field $\boldsymbol{\phi_0}$

$$\boldsymbol{F''=|a|\varkappa m^2{\phi_0}^2}. \tag{6}$$

$\boldsymbol{F''}$ is positive, it is repultion. We conclude, that the massive time-like vector field makes the rate of evolution $\boldsymbol{F'(x^0)}$ a monotonically growing function from $\boldsymbol{-H}$ in the past to $\boldsymbol{+H}$ in future (red line in Figure 4). If we set the origin $\boldsymbol{x^0=0}$ at the moment when $\boldsymbol{F'=0}$, then the Universe contracts at $x^0 < 0$, and expands at $x^0 > 0$. $x^0 = 0$ is the moment of maximum compression. The field equations (2) for a longitudinal time-like field reduce to the only one equation

$$\left(\phi_0' + 3F'\phi_0\right)' + m^2\phi_0 = 0. \tag{7}$$

In the case of a small mass, $m \ll H$ (in dimensional units $mc^2 \ll \hbar H$) a symmetric compression-to-expansion transition is described by the analytical solution

$$F'\left(x^0\right) = H\tanh\left(3Hx^0\right),\ \ \phi_0\left(x^0\right) = \sqrt{\varkappa\left|\tilde{\Lambda}/a\right|}\left[m\cos\left(3Hx^0\right)\right]^{-1},\ \ m \ll H.$$

One can find complete derivations, including the analysis of other cases, in my review article [5].

Dark energy, dark matter, and ordinary matter acting together

Equation (6) takes into account only elastic repultion of a time-like longitudinal vector field (dark matter). With account of attraction of the ordinary matter equation (6) is replaced by

$$F'' = |a|\varkappa m^2\phi_0^{\ 2} - \frac{1}{2}\varkappa\varepsilon_0 e^{(-3F)}. \tag{8}$$

Here $\boldsymbol{\varepsilon_0} = \boldsymbol{\varepsilon}(\boldsymbol{x}^{\mathbf{0}*})$ is the energy density of ordinary matter now. The present moment $\boldsymbol{x}^{\mathbf{0}*}$ is determined by $\boldsymbol{F}(\boldsymbol{x}^{\mathbf{0}*}) = \mathbf{0}$. Remind, that $\boldsymbol{F}' = \mathbf{0}$ at the moment $\boldsymbol{x}^{\mathbf{0}} = \mathbf{0}$ of maximum compression. In the process of expansion the metric function is negative in the past: $\boldsymbol{F}(\boldsymbol{x}^{\mathbf{0}*}) < \mathbf{0}$ at $\boldsymbol{x} < \boldsymbol{x}^{\mathbf{0}*}$.

Equations (7),(8) with initial conditions

$$\frac{\varkappa a}{\tilde{\Lambda}}\left[\phi'^2_0\left(0\right) + m^2\phi_0^2\left(0\right)\right] = 1 + \Omega e^{-3F_0}, F'(0) = 0, F(0) = 0, \tilde{\Lambda} < 0, \mathrm{a} < 0 \tag{9}$$

are easily integrated numerically. As usual, parameter Ω,

$$\Omega = -\frac{\varkappa\varepsilon_0}{\tilde{\Lambda}} = \frac{\varkappa\varepsilon_0}{3H^2},$$

denotes the ratio of today's energy density of the ordinary matter to the density of kinetic energy of expansion at a constant rate H. Regular solutions are free from any fine tuning. Moreover, the existing parametric freedom leads to a great variety of possible regular scenarios of evolution. See details of numerical and analytical analysis in [5] and [10].

With ordinary matter taken into account, there are two kindes of regular solutions: cosmological, and oscillating.

Regular cosmological solutions ($\widetilde{\boldsymbol{\Lambda}} < \mathbf{0}$)

Cosmological solutions describe a transition from contraction to expansion. The parameter $\boldsymbol{F_0} = \boldsymbol{F}(\mathbf{0}) < \mathbf{0}$ determines the degree of maximum compression at the turning point $\boldsymbol{F}'(\mathbf{0}) = \mathbf{0}$. The peak value of the rate of expansion grows exponentially with increasing negative value of $\boldsymbol{F_0}$, while the width of the transition decreases exponentially. It resembles inflation, except that there is no singularity. The regular contraction-to-expansion transition is often referred to as "nonsingular bounce".

In the most interesting case of small $\boldsymbol{m} \ll \boldsymbol{H}$ the transition from contraction to expansion, resembling inflation, can be described analytically. For the rate of evolution $\boldsymbol{F}'(\boldsymbol{x^0})$, and for the scale factor $\boldsymbol{R}(\boldsymbol{x^0}) = \boldsymbol{e^{F(x^0)}}$ we have

$$F'(x^0) = H\frac{\sinh(3Hx^0)}{\cosh(3Hx^0) - (1 + (2/\Omega)e^{3F_0})^{-1}}, \tag{10}$$

$$R(x^0) = \left[(e^{3F_0} + \frac{1}{2}\Omega)\cosh(3\mathrm{H}\mathrm{x}^0) - \frac{1}{2}\Omega\right]^{1/3}, m << H.$$

According to the "sliced cake" diagram (Figure 3) $\Omega \sim 0.06$. A transition, resembling inflation, is shown in Figure 5. For the parameters $F_0 = -10$, $m/H = 10$, $\Omega = 0.06$ the peak is very sharp, there is 10 order difference in horizontal and vertical scales. The numerical result (blue dashed curve) coincides with the analytical solution (10) (red solid curve). It is because the analytical solution, derived for $\boldsymbol{m} \ll \boldsymbol{H}$, is applicable as well for $\boldsymbol{m} \sim \boldsymbol{H}$ in the vicinity of the turning point, provided that $|F_0| \gg 1$.

In the process of compression the repulsing term $\sim\boldsymbol{e^{-6F}}$ increases faster than the compressing term $\sim\boldsymbol{e^{-3F}}$. It is the reason why a regular bounce replaces the singularity independently of how big the negative F_0 is. After the bounce the repulsing term decreases faster than the compressing one, leading to matter domination over the field at late times.

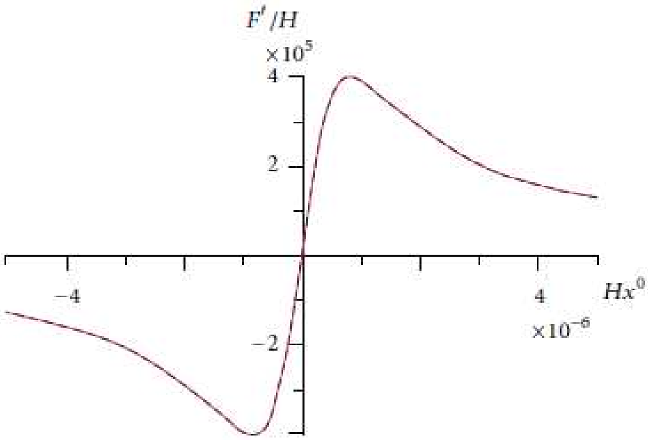

Fig. 5. F'/H in the vicinity of the turning point. Blue dashed curve is the numerical result for F_0 =–10, m/H = 10, Ω = 0.06. It coincides with the red solid curve – analytical solution (10).

Regular oscillating solutions ($\tilde{\Lambda} > 0$)

Without ordinary matter regular solutions describing a contraction-to-expansion transition exist only if $\tilde{\Lambda} < 0$. If the ordinary matter is taken into account, then there appears a possibility for regular solutions with $\tilde{\Lambda} > 0$. If Λ changes sign, then H becomes imaginary. The equations (7),(8) are invariant against $H \to iH$, but the initial conditions differ from (9) :

$$\frac{\varkappa|\alpha|}{\tilde{\Lambda}}\left[\phi'^2_0(0)+m^2\phi_0^2(0)\right]=-1+\Omega e^{-3F_0},\ \mathrm{F}'(0)=0,\ F(0)=F_0,\ \tilde{\Lambda}>0,\ a<0. \quad (11)$$

A necessary condition for regular solutions with $\tilde{\Lambda} > 0$ follows from the initial conditions (11). Regular solutions with $\tilde{\Lambda} > 0$ exist if there is an extremum moment ($F'(0) = 0$) with the energy density of ordinary matter exceeding the kinetic energy of expansion:

$$\Omega e^{-3F_0} = \frac{\varkappa\varepsilon(0)}{\tilde{\Lambda}} > 1,\ F'(0)=0,\ \tilde{\Lambda}>0.$$

In the case $m \ll H$, $\tilde{\Lambda} > 0$ the symmetric analytical solution of the equations (7),(8) is expressed in terms of trigonometric functions. The scale factor $R(x^0)$ and the rate of evolution $\boldsymbol{F}'(x^0)$,

$$R(x^0) = e^{F_0}\left[(1-\frac{1}{2}\Omega e^{-3F_0})\cos(3Hx^0) + \frac{1}{2}\Omega e^{-3F_0}\right]^{1/3}. \tag{12}$$

$$F'(\mathrm{x}^0) = H\frac{\sin(3Hx^0)}{(1-(2/\Omega e^{-3F_0}))^{-1} - \cos(3Hx^0)}, \tag{13}$$

are periodic functions with no singularity, see red curves in Figures 6 a,b. In the case $\tilde{\Lambda} > 0$ the origin $x^0 = 0$ is a moment when the scale factor $R(x^0)$ reaches its maximum. The points of minimum (where $\cos(3Hx^0) = -1$) are

$$x^0 = x^0_n = \frac{\pi}{3H}\left(1+2n\right),\ n = 0, \pm 1, \pm 2, \dots .$$

For the values of the parameters $m/H = 0.02$, $\Omega\exp(-3F_0) = 1.032$ (barely exceeding the boundary $\Omega\exp(-3F_0) = 1$) there is no difference in Figures 6 a,b between the curves found numerically and analytically. Without a massive field ($\boldsymbol{\phi_0 = 0}$) the solutions with positive $\boldsymbol{\tilde{\Lambda}}$ are fine-tuned ($\Omega\ \exp(-3F_0) = 1$) and have a periodic singularity at $\boldsymbol{x^0 = x^0_n}$. In the vicinity of each singular point $\boldsymbol{x^0_n}$, as well as at $H \to 0$, the Hubble constant H drops out, and the scale factor (12) reduces to the one of the Friedman-Robertson-Walker cosmology. Dark matter, described by a longitudinal time-like vector field $\boldsymbol{\phi_0 \neq 0}$, removes a singularity and rules out the problem of fine tuning.

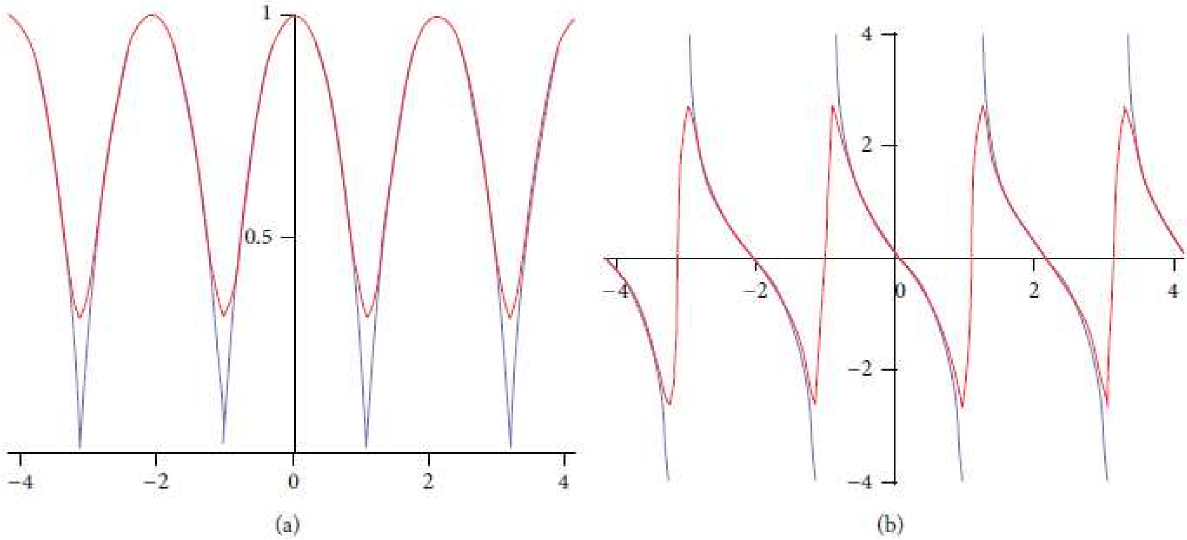

Fig. 6. Regular oscillating solutions: scale factor $R/R(0)$ (a) and rate of evolution F'/H (b). The horizontal axis is "time" Hx^0. Red curves – numerical solution coinciding with (12), (13) for $m/H = 0.02$, $\Omega\exp(-3F_0) = 1.032$. Blue curves with a periodic singularity are fine-tuned solutions at the lower boundary of the domain of regular oscillating solutions.

Conclusion

As simple a Lagrangian as possible (1) turns out an appropriate tool for macroscopic description of dark sector by vector fields. The dark substance is described via the covariant vector field equations (2) and the energy-momentum tensor (3). So far, it no longer needs to invent its own model of dark matter for understanding each observed astrophysical phenomenon.

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