On one Hawking's Hypothesis

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The Hawking's hypothesis about Euclidean nature of space-time is discussed using well known correspondence between pseudo-Riemannian metrics, Riemannian metrics and unit vector field. It is shown that in the framework of the Hawking's hypothesis this correspondence may be considered as a symmetry breaking and leads naturally to polymetric space-time theories, dark matter appearance and possible existence of particles which may propagate with super light speeds and whose possible existence is periodically discussed in literature.

Keywords: space-time, the Hawking's hypothesis, general relativity, polymetric space-time theories, dark matter

DOI: 10.18698/2309-7604-2015-1-213-219

Introduction

For the improvement of the continual integrals convergence in quantum field theory and quantum gravity the so called Wick rotation $t \rightarrow it$ ($x^0 \rightarrow ix^0$) is often used. As a result of such rotation the signature of space-time metric changes from the Lorentzian one $(-, +, +, +)$ to the Euclidean one $(+, +, +, +)$. Because of high efficiency of this procedure Hawking has made supposition that both quantum and classical space-time is really Euclidean (i.e. has Euclidean signature) and the interpretation of classical space-time as pseudo-Riemannian (Lorentzian) is a result of peculiarities of our perception [1].

In classical theory Wick rotation is impossible. Nevertheless complex time is not necessary to determined correspondence between Riemannian and pseudo-Riemannian spaces. It makes possible to discuss application of the Hawking's supposition to the classical space-time models and consider some corollaries.

On the correspondence between Riemannian and pseudo-Riemannian spaces

The correspondence between Riemannian and pseudo-Riemannian spaces without use of complex time is well known [2-4]. Let $G_{\alpha\beta}$ is Riemannian metric with signature $(+, +, +, +)$ on some smooth four-dimensional manifold M^4 . Let also u_α is some unit vector field on the same manifold, i.e. $G^{\alpha\beta}u_\alpha u_\beta = 1$. Then the equation

$$g_{\alpha\beta} = 2u_{\alpha}u_{\beta} - G_{\alpha\beta} \quad (1)$$

defines on M^4 pseudo-Riemannian metrics with Lorentzian signature $(+, -, -, -)$. Moreover u_{α} is unit time-like vector field in metrics $g_{\alpha\beta}$. It is easy to see that the correspondence between contravariant tensors $G^{\alpha\beta}$ and $g^{\alpha\beta}$ is similar to equation (1)

$$g^{\alpha\beta} = 2u^{\alpha}u^{\beta} - G^{\alpha\beta}, \quad (1a)$$

and $g^{\alpha\beta}u^{\alpha}u^{\beta} = G^{\alpha\beta}u_{\alpha}u_{\beta} = 1$.

Conversely, if $g_{\alpha\beta}$ is some pseudo-Riemannian metrics with Lorentzian signature on M^4 and u_{α} is arbitrary unit vector field then equation

$$G_{\alpha\beta} = 2u_{\alpha}u_{\beta} - g_{\alpha\beta} \quad (2)$$

defines on M^4 Riemannian metrics with signature $(+, +, +, +)$.

It must be pointed out here that regular Riemannian metrics exist on arbitrary smooth manifold M^n whereas the existing of nonzero vector field is necessary for existing of nonsingular pseudo-Riemannian metrics with Lorentzian signature. For the existence of such field the Euler characteristics of the manifold must be zero [5].

In the case of arbitrary (non unit) vector field equations (1) – (2) may be rewritten in the form

$$g_{\alpha\beta} = 2 \frac{u_{\alpha}u_{\beta}}{G^{\mu\nu}u_{\mu}u_{\nu}} - G_{\alpha\beta}, \quad G_{\alpha\beta} = 2 \frac{u_{\alpha}u_{\beta}}{g^{\mu\nu}u_{\mu}u_{\nu}} - g_{\alpha\beta}. \quad (3)$$

Outside the zeros of vector field u_{α} equalities (1) – (2) and (3) are completely equivalent.

The Hawking's hypothesis in classical physics

In accordance with the Hawking's hypothesis classical space-time must be considered as a Riemannian space, i.e. as a pair $(M^4, G_{\alpha\beta})$ where M^4 is a four-dimensional smooth manifold and

$G_{\alpha\beta}$ is a Riemannian metrics with signature $(+, +, +, +)$ on M^4 . An action integral may be written in the following general form

$$S = \int (\kappa R_G + \mathcal{L}_m) \sqrt{G} d^4 x, \quad (4)$$

where κ is some constant, R_G is the Ricci scalar corresponding to the metrics $G_{\alpha\beta}$, $G = \det \|G_{\alpha\beta}\|$ and $\mathcal{L}_m$ is a matter (source fields) Lagrangian. By force of (1) $\det \|G_{\alpha\beta}\| = |\det \|g_{\alpha\beta}\|| = -g$.

The Hawking's supposition that the interpretation of classical space-time as pseudo-Riemannian (Lorentzian) is a result of peculiarities of our perception means that the kinetic term of the matter Lagrangian $\mathcal{L}_m$ is defined by Lorentzian metrics $g_{\alpha\beta}$ which is related with Riemannian metrics $G_{\alpha\beta}$ by equations (2) or (3). In the simplest case when a scalar field φ is the only matter field $\mathcal{L}_m$ has the form

$$\mathcal{L}_m = \mathcal{L}(\varphi, \varphi_{,\alpha}) = \frac{1}{2} g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} + V(\varphi), \quad (5)$$

where $V(\varphi)$ is a potential of φ . Representation of $G_{\alpha\beta}$ in the form (2) or (3) transforms action (4) to

$$S = \int (\kappa_1 R_g + \frac{1}{2} g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} + V(\varphi) + F(u_\alpha, u_{\alpha,\beta})) \sqrt{-g} d^4 x, \quad (6)$$

With

$$(\kappa_1 R_g + F(u_\alpha, u_{\alpha,\beta})) \sqrt{-g} = \kappa R_G \sqrt{G}. \quad (7)$$

By force of the last equality the actions integrals (4) and (6) are completely equivalent.

Action (6) is an action integral of Einsteinian general relativity for the gravitational field $g_{\alpha\beta}$ and the source fields φ and u_α . It must be noted that the field u_α may be one of the source fields in (4) in general case.

Formally, action (6) defines classical space-time is generated by the fields φ and u_α with minimal coupling. In difference with usual models with minimal coupling the absence of coupling between fields φ and u_α is exact. By this reason the term $F(u_\alpha, u_{\alpha,\beta})$ in action (6) must be considered as a part of dark matter Lagrangian.

Thus the existence of dark matter is the first direct consequence of the Hawking's hypothesis.

Polymetric space-time

Vector field u_α on Riemannian manifold $(M^4, G_{\alpha\beta})$ does not preferred among others unit vector fields. It make possible to suppose coexistence of several Lorentz structures $(M^4, g_{\alpha\beta}^{(i)})$ on the same Riemannian manifold $(M^4, G_{\alpha\beta})$ which are generated by the same Riemannian metrics $G_{\alpha\beta}$ and different unit vector fields $w_\alpha^{(i)}$, where $i=1,2,...,n$. Each Lorentz structures $(M^4, g_{\alpha\beta}^{(i)})$ may corresponds to some class of matter. For the first time such supposition were made by author in 1985 [4] and were repeatedly made recently by Geroch [5].

Lorentzian metrics $g_{\alpha\beta}^{(i)}$, $i=1,2,...,n$, are defines by equations which is similar to (1):

$$g_{\alpha\beta}^{(i)} = (c_i^2 + 1) w_\alpha^{(i)} w_\beta^{(i)} - G_{\alpha\beta}, \quad (8)$$

where coefficients $c_i > 0$ have sense of maximal signal velocity in space-time with metrics $g_{\alpha\beta}^{(i)}$ and must satisfy to the following conditions which exclude coincidence of metrics $g_{\alpha\beta}$, $g_{\alpha\beta}^{(i)}$ and $g_{\alpha\beta}^{(j)}$: $c_{(i)} \neq 1$ if $w_\alpha^{(i)} = u_\alpha$ and $c_{(i)} \neq c_{(j)}$ if $w_\alpha^{(i)} = w_\alpha^{(j)}$.

The contravariant components of $g_{\alpha\beta}^{(i)}$ are defined by

$$g_{(i)}^{\alpha\beta} = \frac{c_1^2 + 1}{c_{(i)}^2} w_{(i)}^\alpha w_{(i)}^\beta - G^{\alpha\beta},$$

where $w_{(i)}^\alpha = G^{\alpha\beta} w_\beta^{(i)}$. After excluding of $G_{\alpha\beta}$ the above equations take the form

$$g_{\alpha\beta}^{(i)} = g_{\alpha\beta} - 2u_\alpha u_\beta + (c_{(i)}^2 + 1)w_\alpha^{(i)}w_\beta^{(i)}, \quad g_{(i)}^{\alpha\beta} = g^{\alpha\beta} - 2u^\alpha u^\beta + \frac{1 + c_{(i)}^2}{c_{(i)}^2}k_{(i)}^{\alpha\beta}, \quad (9)$$

Where

$$k_i^{\alpha\beta} = 4u^\alpha u^\beta (u^\rho w_\rho^i)^2 - 2(u^\alpha w_{(i)g}^\beta + w_{(i)g}^\alpha u^\beta)(u^\rho w_\rho^i) + w_{(i)g}^\alpha w_{(i)g}^\beta.$$

Here $w_{(i)g}^\alpha = g^{\alpha\beta} w_\beta^{(i)}$.

If Riemannian space $(M^4, G_{\alpha\beta})$ is generated by $n+1$ kinds of matter then the action functional may be written in the following form

$$S = \int \left(\kappa R_G + \sum_{i=0}^n \mathcal{L}_{mi} \right) \sqrt{G} d^4 x, \quad (10)$$

where $\mathcal{L}_{mi}$ is Lagrangian of i kind of matter and we assume that for the usual matter which moves in space-time $(M^4, g_{\alpha\beta})$ of the classical general relativity $i=0$, i.e. $w_\alpha^{(0)} = u_\alpha$.

In Lorentzian space-time $(M^4, g_{\alpha\beta})$ action integral (10) takes the form

$$S = \int \{ \kappa R_g + \mathcal{L}_m + F(u_\alpha, u_{\alpha,\beta}) + \sum_{i=1}^n [F_i(u_\alpha, u_{\alpha,\beta}, w_\alpha^{(i)}, w_{\alpha,\beta}^{(i)}) + \mathcal{L}_{mig}] \} \sqrt{-g} d^4 x, \quad (11)$$

where $F_i(u_\alpha, u_{\alpha,\beta}, w_\alpha^{(i)}, w_{\alpha,\beta}^{(i)}) + \mathcal{L}_{mig}$ is a result of substitution metrics $G_{\alpha\beta}$ and $g_{\alpha\beta}^{(i)}$ in the forms (3) and (9) to the Lagrangian $\mathcal{L}_{mi}$.

It is naturally suppose that the terms

$$F(u_\alpha, u_{\alpha,\beta}) + \sum_{i=1}^n [F_i(u_\alpha, u_{\alpha,\beta}, w_\alpha^{(i)}, w_{\alpha,\beta}^{(i)}) + \mathcal{L}_{mig}] \quad (12)$$

in equation (11) describe dark matter (dark energy).

Concluding remarks

So, the classical version of the Hawking's hypothesis about Euclidean nature of space-time leads to the existence of dark matter and to the polymetric space-time models with exotic matter whose maximal speed surpass the light speed in vacuum. Some comments must be made in connection with these statements.

First, the dark matter which appears in the framework of the Hawking's hypothesis and polymetric models does not excludes another forms of dark matter and dark energy which are discussed in literature.

Second, the superlight speed of signal may be possible in polymetric space-time models. According to general opinion such possibility is unavoidable leads to violation of causality (see for example [7]). Nevertheless the direct calculation shows that if the superlight motion is described by general covariant equations then the causality violation is possible iff the future cone of superlight particles or signals intersects with the past light cone of usual matter [8]. Moreover the non minimal couple between usual matter and the exotic superluminal matter is also necessary for causal violation.

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