

## **Calibration of high precision surfaces of gravitational wave telescope optics**

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We analyzed limitations of the two-flat-test absolute calibration method connected with the measurement noise. The results showed that reconstruction of the surface topography for spatial frequencies from  $1.5 \times 10^{-2}$  to  $1.3 \times 10^{-1} \text{ mm}^{-1}$  is possible for translation T values from 1 to 7 mm.

Keywords: interferometry, absolute calibration, three flat test, power spectral density, noise.

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### **Introduction**

For the purposes of the experimental detection of gravitational waves of cosmic origin the gravitational waves telescopes are applied. There are several international projects designed to detect and accurately measure gravitational waves: the American-Australian project LIGO [1], the German-British GEO600, the Japanese TAMA-300 and the Franco-Italian VIRGO and European LISA [2]. Telescopes include optical flat components with high-precision surfaces, about 0.2nm rms (root mean square value of the surface height distribution) and aperture diameter more than 250mm.

The best known methods used to measure large-aperture optical surfaces are interferometric methods in conjunction with calibration methods. In case when the quality of the test flat surface is comparable with the quality of the reference flat surface, absolute calibration methods should be applied.

In previous study [3] we suggested the absolute calibration method following the two-flat-test method [4]. The method is based on the translation of the test flat with respect to the reference flat on a distance T along the vertical and horizontal axes.

The numerical results of that study showed that the reconstruction of the surface topography is possible for a spatial frequency region from  $1.67 \times 10^{-3}$  to  $3.0 \times 10^{-2} \text{ mm}^{-1}$  for the aperture 600 mm. However, the realization of the real experiment requires to carry out the research of possible range of translation T value, which limits the reconstruction of the spatial frequency range.

In this paper we define the range of the translation T value that guarantees the reconstruction of the test flat by the two-flat-test method across a wide spatial frequency range without loss of the accuracy.

### Noise estimation

As mentioned, the range of the translation  $T$  value should be carefully chosen in order to retrieve the largest possible range of spatial frequencies. Figure 1 illustrates the filtering function depending on the different translation  $T$  values. The figure shows that the smallest translations of the test flat should be performed for the reconstruction of higher frequencies. Nevertheless, it is necessary to estimate the noise level during the measurement before choosing the translation length, because the measurement noise can effect on the reconstruction accuracy.

Using algorithm of the noise analysis [4] and considering the surface height distribution of the test flat as a signal, Signal-to-noise ratio of the measurement can be defined as follows:

$$SNR_{Diff_x}(\nu, T) = \sqrt{2} \cdot SNR_{m1}(\nu) \cdot |\sin(\pi\nu T)|, \quad (1)$$

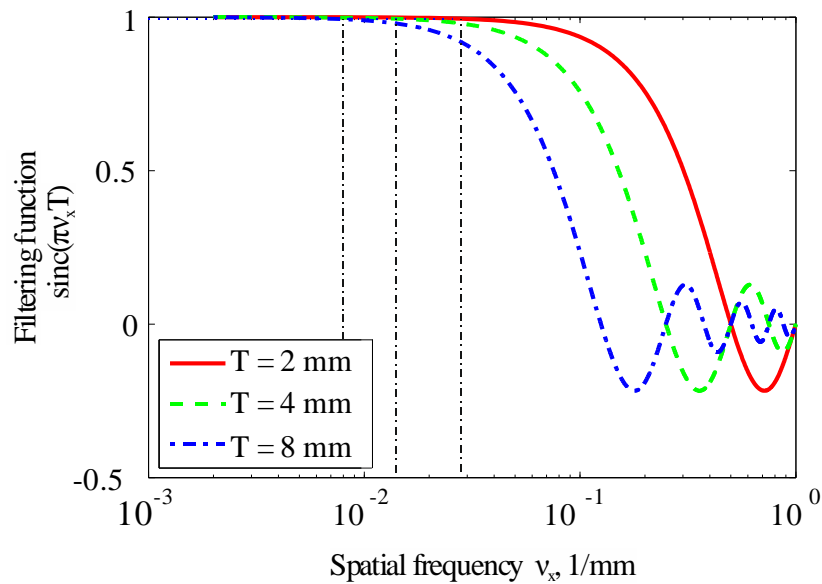


Fig. 1. Filtering function  $\text{sinc}(\pi\nu_x T)$ .

where  $SNR_{m1}(\nu)$  is a Signal-to-noise ratio for one measurement,  $\nu$  is a spatial frequency. The Signal-to-noise ratio for one measurement calculated as:

$$SNR_{m1}(v) = \frac{PSD_{m1}(v)}{\sqrt{2PSD_{noise}(v)}}, \quad (2)$$

where  $PSD_{m1}(v)$  is a power spectral density (PSD) for one measurement and  $PSD_{noise}(v)$  is a PSD of the measurement noise.

### Experimental results

In order to analyze the noise, two sequential measurements of the test flat were carry out by using the interferometer Intellium H2000 based on the Fizeau scheme [5]. The diameter of the test flat was 100mm. Translation of the test flat was performed by fine adjustment screws controlled by the engineer's dial gauge with 10um accuracy. Measurements had a 120um spatial resolution. Figure 2A and Figure 2B shows the surface profiles of these two measurements  $m_1(x,y)$  and  $m_2(x,y)$ .

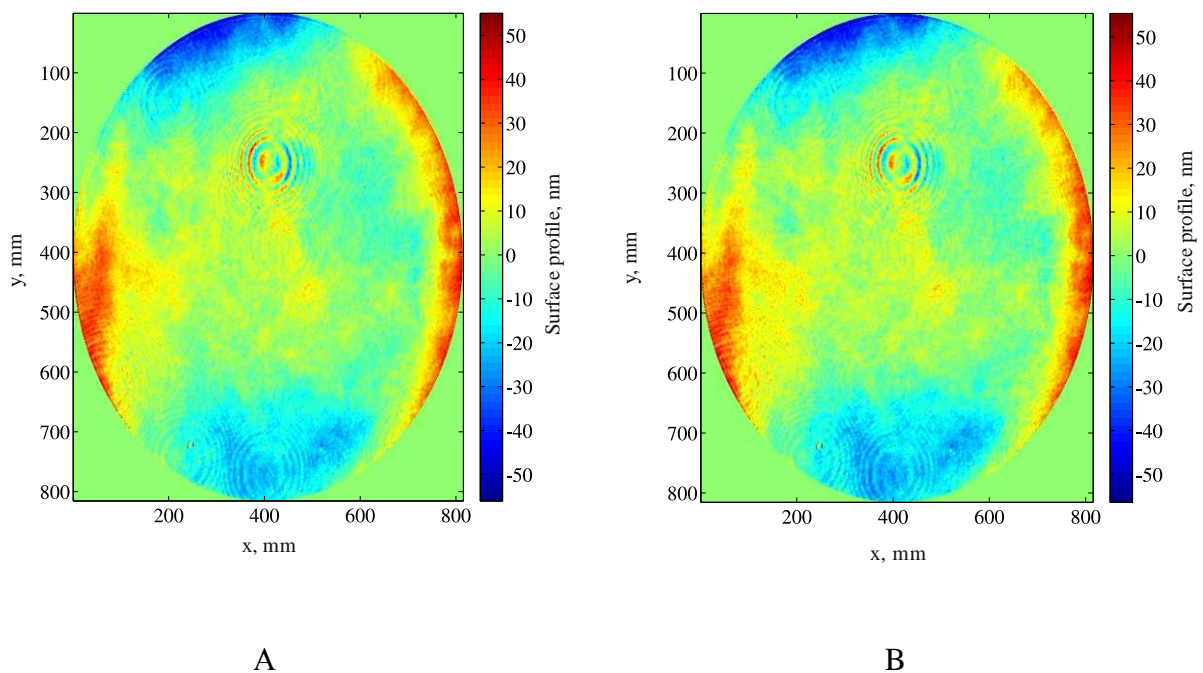


Fig. 2. A – Surface profile of the measurement  $m_1(x,y)$ ; B – Surface profile of the measurement  $m_2(x,y)$ .

The functions  $PSD_{m1}(v)$  and  $PSD_{noise}(v)$  were calculated for obtained surface profiles. Power spectral density for surface height distribution according to [6], we defined as follows:

$$PSD(v_x, v_y) = \frac{1}{A} \left| \mathfrak{F} \left[ h_{test}(x, y) \right] \Delta x \Delta y \right|^2, \quad (3)$$

where  $A$  is an area of analysis, mm,  $h_{test}(x, y)$  is a surface height distribution on the test flat,  $\Delta x \times \Delta y$  is a square size of the pixel,  $\mathfrak{F}$  is a two-dimensional Fourier transform. The power spectral density of the measurement noise  $PSD_{noise}(v)$  was calculated as:

$$PSD(v_x, v_y) = \frac{1}{A} \left| \mathfrak{F} \left[ m_1(x, y) - m_2(x, y) \right] \Delta x \Delta y \right|^2, \quad (4)$$

where  $m_1(x, y)$  and  $m_2(x, y)$  are two sequential measurements. Figure 3 shows functions of the PSD of one measurement of the test flat (dashed line), PSD of the estimated measurement noise (dotted line) and Signal-to-noise ratio for one measurement  $SNR_{m1}(v)$  (solid line). Easy to see that reconstruction of the high spatial frequencies will be impossible.

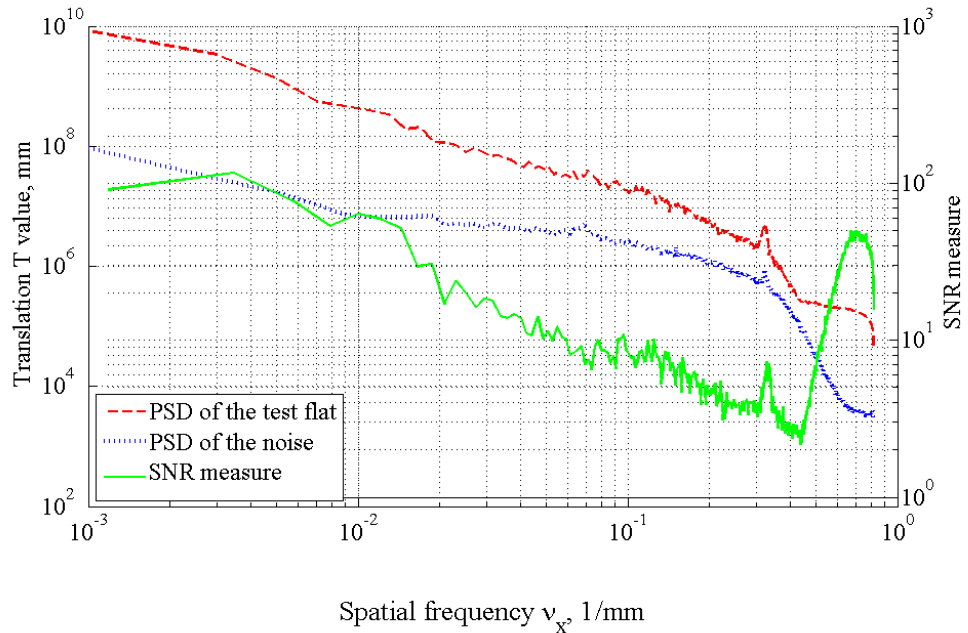


Fig. 3. PSD of surface height distribution, PSD of the noise and Signal-to-noise ratio for one measurement.

The Signal-to-noise ratio for two sequential measurements of the test flat was calculated using equation (1) for the range of the translation  $T$  value from 0 to 50mm, for the range of spatial frequencies from  $1.0 \times 10^{-2}$  to  $4.0 \times 10^0 \text{ mm}^{-1}$  and represented on Figure 4.

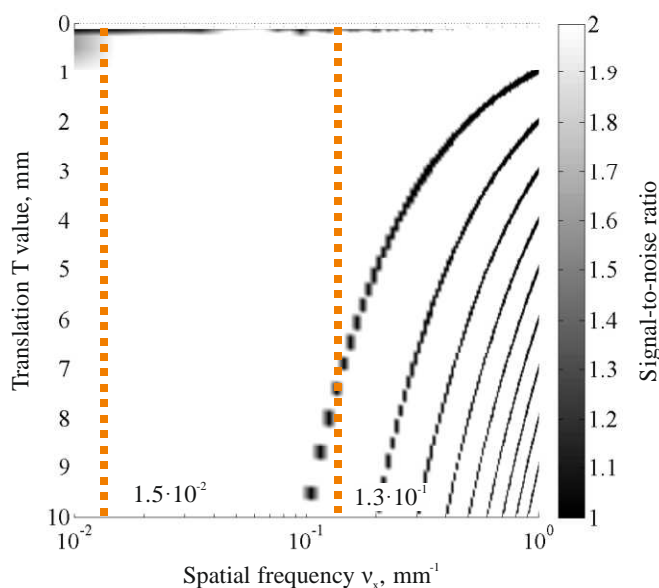


Fig. 4. Signal-to-noise ratio  $SNR_{Diffx}(\nu, T)$  as a function of translation  $T$  value and spatial frequencies.

The results of this study showed that the presence of the noise in the measurements strongly affect the quality of the reconstruction of a surface profile. The chosen threshold of the  $SNR_{Diffx}(\nu, T)$  on the Figure 4 is equal 2. The white areas on the figure represent spatial frequencies that will be successfully reconstructed while frequencies of the dark areas wouldn't be reconstructed. Orange dashed lines on the figure represent the range of the spatial frequencies from  $1.5 \times 10^{-2}$  to  $1.3 \times 10^{-1} \text{ mm}^{-1}$ , the area of our interest. As we used test flat with aperture 100mm for our measurements, we recalculated desired spatial frequencies ( $1.67 \times 10^{-3} - 3.0 \times 10^{-2} \text{ mm}^{-1}$ ) to the size of our aperture by using harmonics. The desired frequency range is equal to the range of harmonics from 1 to 18 orders for the 600mm aperture of the test flat. Accordingly, the range of harmonics from 1 to 18 orders corresponds to the spatial frequency range from  $1.5 \times 10^{-2}$  to  $1.3 \times 10^{-1} \text{ mm}^{-1}$  for the 100mm aperture of the test flat. The result shows that the reconstruction of the frequency range is possible with range of the translation  $T$  value from 1 to 7 mm.

## Conclusion

This work is devoted to the definition of the limits of the two-flat-test absolute calibration method for measurement of the accuracy of the large aperture optical details. The quality of the

reconstruction directly depends on choosing of the range of the translation T value. For the definition of this range the measurement noise analysis was performed. According to the results, the reconstruction of the test flat surface profile for spatial frequencies from  $1.5 \times 10^{-2}$  to  $1.3 \times 10^{-1}$   $\text{mm}^{-1}$  would be successful for translations of the test flat from 1 to 7 mm.

This study allows us to proceed to the next important stage of the work, to carry out a real experiment of two-flat-test absolute calibration on the optical industry.

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