Electromagnetic radiation in the moving medium with a velocity gradient

Gladyshev V.O., Tereshin A.A., Bazleva D.D., Gladysheva T.M.

Bauman Moscow State University, Moscow, Russia;

E-mail: Gladyshev <vgladyshev@mail.ru>;

We obtain the expression for the curvature of the trajectory of the light beam in a moving medium. Trajectory curvature k and the angular deviation α depend on the velocity gradient and also on the angle between the wave vector and the fluid velocity vector.

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Curvature of the trajectory of the electromagnetic waves caused by the gravitational fields of massive space objects is studied well enough as this refers to one of the three predictions of the general theory of relativity. Recently, this topic is related to the much-discussed hypothesis of the existence of the dark matter and energy [1]. At the same time the moving interstellar and intergalactic medium can also bend the path of light rays that is a source of additional amendments in determining the radius of the orbit of the star.

The movement of the medium leads to the amendments in the phase velocity of the electromagnetic radiation, in the propagation time of the electromagnetic radiation between the transmitter and the receiver, and it can also lead to the angular velocity aberration radiation.

In that case, if the space velocity of the medium is inhomogeneous and the speed alteration therein corresponds to a gradient, bending of the electromagnetic radiation trajectory in the medium arises.

Let us consider the propagation of the plane monochromatic wave in a moving medium in a geometrical approximation, when the characteristic size of the inhomogeneities of the medium is much greater than the wavelength of the radiation. At the same time we assume the very environment as a uniform one, while non-uniformity caused by the gradient of the velocity of its movement. The most interesting is the two-dimensional case and the three-dimensional generalization can be easily obtained by using the principle of superposition.

Let us assume that in the plane XOZ propagates the electromagnetic wave in the optically transparent medium with a velocity gradient.

The process of propagation of the radiation can be represented as a series of the beam refractions at the boundary of layers and within the layer the velocity variation of the medium can be ignored.

Let an electromagnetic wave falls at the point 0 on the boundary between the medium with a refractive index n_1 at an angle ϑ_0 and than refracted at an angle ϑ_2 in the medium with a refractive index n_2. Each i-th point of the trajectory corresponds to the very radius vector $\vec{r_i}$. The increment of the refraction angle is denoted by $d\vartheta_2$. Let $\vec{u}$ be the element of the displacement of the wave vector in the moving medium (ray vector) $\vec{u} = \frac{d\vec{r}}{ds}$. Assume that at some point O_k is placed the center of the propagation trajectory curvature of the radiation in the medium. Then the value ρ_k is a radius of the trajectory curvature.

Let us obtain an analytical expression for the curvature of the trajectory of the radiation within the movable medium, taking into account relativistic terms.

By definition, the expression for the curvature of the trajectory of the radiation in the medium can be written as

$$k = \frac{1}{\rho_k} = \left| \frac{d\vartheta_2}{dS} \right|. \qquad (1)$$

During the propagation of the beam from point 0 to point 1, the wave vector will turn at an angle $d\vartheta_2 = d^2 r / ds$.

To use this expression we use the dependence of the wave vector of the refracted wave from the coordinates.

The wave vector while shifting from point 0 to point 1 will vary by the value of $d\vec{k}_2 = \vec{k}_2(1) - \vec{k}_2(0)$. The rotation of the wave vector $\vec{k}_2$ at an angle $d\vartheta_2$ depends from the angle of refraction ϑ_2

$$d\vartheta_2 = -\frac{\sin(\vartheta_2)}{k_2} dk_2. \qquad (2)$$

The variation of the wave vector $\vec{k}_2$ occurs only due to the changes in its projection on the axis OZ, because k_{2z} is a tangential invariant, so the expression for the curvature takes the following form:

$$k = \frac{\sin(\vartheta_2)}{k_2}\frac{dk_{2z}}{dS}. \tag{3}$$

We take into account that the derivative of the scalar function in the direction of $\vec{u}$ is determined according to the formula

$$\frac{dk}{dS} = \frac{\partial k}{\partial x}\cos\alpha_x + \frac{\partial k}{\partial y}\cos\alpha_y + \frac{\partial k}{\partial z}\cos\alpha_z. \tag{4}$$

In this case, the first two derivatives are equal to zero $\frac{\partial k_{2z}}{\partial x} = \frac{\partial k_{2z}}{\partial y} = 0$.

That is why the expression for the curvature will be the following:

$$k = \frac{\cos\vartheta_2 \sin\vartheta_2}{k_2}\frac{\partial k_{2z}}{\partial z} = \frac{k_{2x}k_{2z}}{k_2^3}\frac{\partial k_{2z}}{\partial z}. \tag{5}$$

To determine the projection of the wave vector $\vec{k}_2$ we use the solution of the dispersion equation obtained in [2]

$$k_{2x} = \frac{\omega_0}{c}\sin\vartheta_0, \tag{6}$$

$$k_{2z} = -\frac{I_1}{c}\frac{\left[\left(\beta + \kappa\gamma_2^2\left(\beta - \beta_{2z}\right)\right)\left(1 - \left(\vec{\beta}_{2x}, \vec{d}\right)\right)\right] \pm \sqrt{Q_2}}{\left(1 - \beta^2\right) - \kappa\gamma_2^2\left(\beta - \beta_{2z}\right)^2}, \tag{7}$$

where

$$Q_2 = 1 + \kappa_2^2\gamma_2^2\left(1-\beta_{2z}^2\right) - d^2\left[1-\beta^2-\kappa_2\gamma_2^2\left(\beta-\beta_{2z}\right)^2\right] - \kappa_2\gamma_2^2\vec{d}\vec{\beta}_{2x}\left[2\left(1-\beta\beta_{2z}\right)-\left(1-\beta^2\right)\vec{d}\vec{\beta}_{2x}\right]$$

$$\kappa = n_2^2 - 1;\quad \gamma_2^{-2} = 1-\beta^2;\quad \vec{d} = c\frac{\vec{I}_t}{I_1}.$$

Assuming that the normal velocity of the boundary is absent, and $\beta = 0,\quad I_1 = -\omega_0 = -\omega_2$, so let us rewright the expressions for $\vec{d}$ and $\left(\vec{\beta}_{2t},\vec{d}\right)$ as

$$d = -\sin\left(\vartheta_0\right);\quad \left(\vec{\beta}_{2t},\vec{d}\right) = -\beta_{2t}\sin\left(\vartheta_0\right).$$

Then the expression for the normal projection of the wave vector k_{2z} will be simplified as

$$k_{2z} = -\frac{\omega_0}{c}\frac{\kappa_2\gamma_2^2\beta_{2z}\left(1-\beta_{2x}\sin\vartheta_0\right)\pm\sqrt{Q_2}}{1-\kappa_2\gamma_2^2\beta_{2z}^2}. \tag{8}$$

$$Q_2 = \cos^2\vartheta_0\left(1-\kappa\gamma_2^2\beta_{2z}^2\right) + \kappa_2\gamma_2^2\left(1-\beta_{2x}\sin\vartheta_0\right)^2. \tag{9}$$

The derivative of k_{2z} with respect to z will have the form

$$\hat{A} = \frac{-\kappa_2\beta_{2z}\gamma_2^4\left(1-\beta_{2x}\sin\vartheta_0\right)\dfrac{d}{dz}\left(\beta_{2x}^2+\beta_{2z}^2\right)}{1-\kappa_2\gamma_2^2\beta_{2z}^2} -$$

$$-\frac{\kappa_2\gamma_2^2\dfrac{\partial\beta_{2z}}{\partial z}\left(1-\beta_{2x}\sin\vartheta_0\right)}{1-\kappa_2\gamma_2^2\beta_{2z}^2} + \frac{\kappa\beta_{2z}\gamma_2^2\dfrac{\partial\beta_{2x}}{\partial z}\sin\vartheta_0}{1-\kappa_2\gamma_2^2\beta_{2z}^2};$$

$$\hat{B} = \frac{\kappa_2\beta_{2z}\gamma_2^2\left(1-\beta_{2x}\sin\vartheta_0\right)}{\left(1-\kappa_2\gamma_2^2\beta_{2z}^2\right)^2}\left[-\kappa_2\beta_{2z}^2\gamma_2^4\frac{d}{dz}\left(\beta_{2x}^2+\beta_{2z}^2\right) - 2\kappa_2\beta_{2z}\gamma_2^2\frac{\partial\beta_{2z}}{\partial z}\right];$$

$$\hat{C}=\frac{1}{2}\frac{1}{\sqrt{\cos^2\vartheta_0\left(1-\kappa_2\beta_{2z}^2\gamma_2^2\right)+\kappa_2\gamma_2^2\left(1-\beta_{2x}\sin\vartheta_0\right)^2}\left(1-\kappa_2\beta_{2z}^2\gamma_2^2\right)}\times$$
$$\times\left[\left(\cos^2\vartheta_0\left(-\kappa_2\gamma_2^4\beta_{2z}^2\frac{d}{dz}\left(\beta_{2x}^2+\beta_{2z}^2\right)-2\kappa_2\gamma_2^2\beta_{2z}\frac{\partial\beta_{2z}}{\partial z}\right)+\right.\right.$$
$$+2\gamma_2^4\kappa_2\left(1-\beta_{2x}\sin\vartheta_0\right)^2\frac{d}{dz}\left(\beta_{2x}^2+\beta_{2z}^2\right)-$$
$$\left.\left.-2\kappa_2\gamma_2^2\left(1-\beta_{2x}\sin\vartheta_0\right)\sin\vartheta_0\frac{\partial\beta_{2x}}{\partial z}\right)\right];$$

$$\hat{D}=\frac{1}{\left(1-\kappa_2\beta_{2z}^2\gamma_2^2\right)^2}\sqrt{\cos^2\vartheta_0\left(1-\kappa_2\beta_{2z}^2\gamma_2^2\right)+\kappa_2\gamma_2^2\left(1-\beta_{2x}\sin^2\vartheta_0\right)^2}\times$$
$$\times\left(-\kappa_2\gamma_2^4\beta_{2z}^2\frac{d}{dz}\left(\beta_{2x}^2+\beta_{2z}^2\right)-2\gamma_2^2\kappa_2\beta_{2z}\frac{\partial\beta_{2z}}{\partial z}\right);$$

For the trajectory curvature of the radiation we can obtain

$$k=\frac{\sin\vartheta_0\kappa_2\gamma_2^2\beta_{2z}^2\eta_2^{-1}\left(\xi_2+\sqrt{Q_2}\right)\left(\hat{A}+\hat{B}+\hat{C}-\hat{D}\right)}{\left(\sin^2\vartheta_0+\kappa_2^2\gamma_2^4\beta_{2z}^4\eta_2^{-2}\left(\xi_2+\sqrt{Q_2}\right)^2\right)^{3/2}}. \tag{11}$$

Using these approximations $\beta_{2x}>>\beta_{2x}^2;\ \beta_{2z}>>\beta_{2z}^2$, we can wright

$$\frac{\partial k_{2z}}{\partial z}=-\frac{\omega_0}{c}\frac{\kappa_2\frac{\partial\beta_{2x}}{\partial z}\sin\vartheta_0}{\sqrt{n_2^2-\sin^2\vartheta_0}}. \tag{12}$$

$$k_{2x}=\frac{\omega_0}{c}\cos\vartheta_0;\ k_{2z}=\frac{\omega_0}{c}\sqrt{n_2^2-\sin^2\vartheta_0}. \tag{13}$$

And the expression for the curvature will be the following

$$k = \left| \frac{\kappa_2 \cos\vartheta_0 \sin\vartheta_0}{\left(n_2^2 + \cos^2\vartheta_0 - \sin^2\vartheta_0\right)^{\frac{3}{2}}} \frac{\partial\beta_{2x}}{\partial z} \right|. \tag{14}$$

Note that the curvature of the trajectory has a nonzero value in the absence of rotary motion, such as shear flow.

In the particular case of rotational motion of the medium

$$\beta_{2x} = \frac{\omega}{c}\left(r_0 - z\right); \quad \beta_{2z} = \frac{\omega}{c} x;$$
$$\frac{\partial\beta_{2x}}{\partial z} = -\frac{\omega}{c}; \quad \frac{\partial\beta_{2z}}{\partial x} = \frac{\omega}{c}; \quad \frac{\partial\beta_{2x}}{\partial x} = 0; \quad \frac{\partial\beta_{2z}}{\partial z} = 0.$$

Then for the curvature of the trajectory we obtain

$$k = \frac{n_2^2 - 1}{n_2^3} \frac{\omega}{c} \sin^2\vartheta_0 . \tag{15}$$

For the angular deflection of the beam on the track with length S, we write

$$d\alpha = \frac{n_2^2 - 1}{n_2} \frac{\omega}{c} \sin^2\vartheta_2 ds. \tag{16}$$

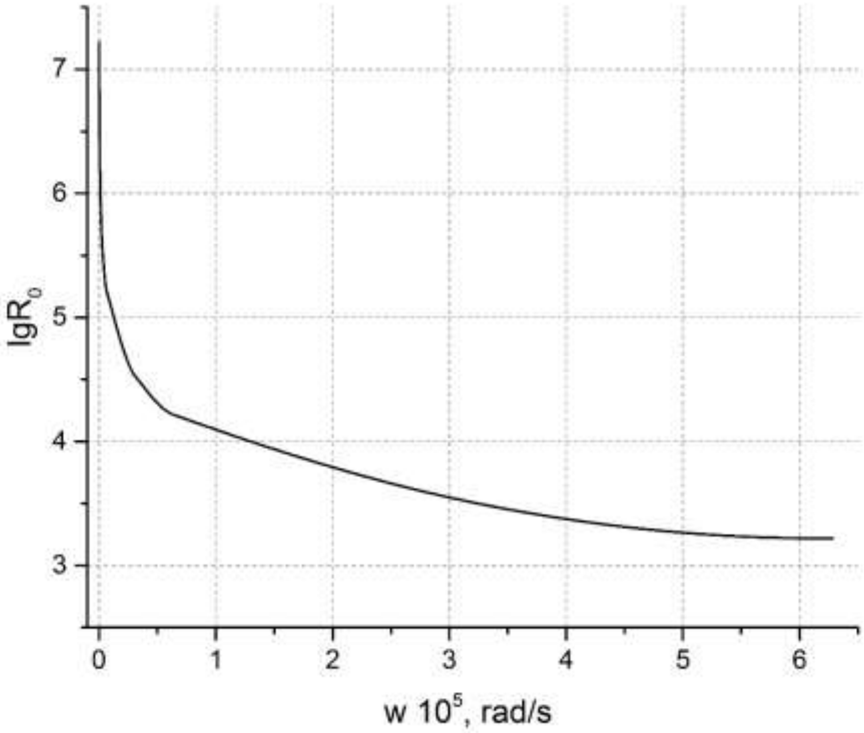

Fig. 1. The dependence of the curvature radius of the radiation trajectory in a rotating medium from the frequency of rotation

The expression obtained in the first approximation corresponds to the work [3]. The main difference is that the curvature of the trajectory and the angular deviation depends on the velocity gradient.

Fig. 1 shows the dependence of the curvature radius on the rotational speed of the medium on the basis of numerical calculation using the formulas (9) - (11). The calculations were performed with the following parameters $n_2 = 1.7643$; $\vartheta_0 = 60^\circ$; $\lambda = 532$ nm; $r_0 = 0.1$ m, allowing comparison with direct numerical calculation [4, 5].

At low speeds, the radius of curvature tends to infinity, while at large - there is a significant non-linearity $\rho_k(\omega)$, which reflects the importance of the relativistic terms. It follows from the calculations we made that the effect of the curvature of the trajectory in the medium with a velocity gradient is the effect of the first order with respect to u/c.

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