Multidimensional conformal-flat space-times and a linear equation of state

Baranov A.M.¹, Saveljev E.V.²

¹ Krasnoyarsk State Pedagogical University named after V.P.Astaf'ev, Krasnoyarsk, Russia;

² OOO "PROFILL - 2S", Moscow, Russia;

E-mail: Baranov <alex_m_bar@mail.ru>;

The finding problem of conformal-flat cosmological models as exact solutions of the equations of gravitation for different equations of state with linear connection between pressure and energy density is demonstrated within the limits of multidimensional space-time with one time-like direction. It turns out that such approach leads to an identification of some discrete set of equations of state for which conformal factors are connected with the harmonic functions as solutions of the Laplace equations in multidimensional Euclidean spaces of an integer dimensionality. Dimensionality of these spaces, in turn, is defined by a concrete equation of state. For four-dimensional space-times the corresponding table is constructed.

Keywords: Multidimensional space-times, linear equation of state, multidimensional the Laplace equation, conformal-flat space-times, open cosmological models, exact solutions of gravitational equations.

DOI: 10.18698/2309-7604-2015-1-81-100

Introduction

At research of conformal-flat spaces on the basis of General Relativity the deriving and the physical analysis of corresponding cosmological models is often carried out in a synchronous frame of reference. Such way is not always good from the point of view of finding of exact analytical solutions of the Einstein equations because of a number of mathematical difficulties. For example, in such frame of reference it is impossible to find the solution for open cosmological model with an incoherent dust and radiation in a final analytical form. However this problem can be solved with use of not accompanying frame of reference ([1]-[4]) in which the 4-space-time metric conformal-flat and the conformal factor depends only on one variable playing a role of distance in a 4-Minkowski universes (see an approach of Fock [4]-[5]).

At the same time, it turns out that this description is equivalent to the introduction of a kinematic reference frame (see, for example, [6]-[8]) instead of the synchronous frame of reference. In addition, the use of the synchronous reference frame not fully corresponds to the symmetry of the cosmological problem, where metric depends on the one variable (see, for example, [4]).

An exact analytical solution with an arbitrary function of state defined as the ratio of the pressure to energy density, is found in article [10] at the generalization of the Friedman model of the open Universe [11] in the Fock form ([4]-[5]). The generalization of the Friedman solution to the open Universe in case of a substance, and an equilibrium lightlike radiation (such as

electromagnetic) with nonzero pressure and without the introduction of a particular equation of state was found in the paper [1]. Here we go directly to the problem of the equation of state of matter in cosmological models. In our opinion, not enough attention is given to this problem, although the equation of state is an integral part of the Einstein equations in solving the cosmological problems. Moreover, the successes of the inflationary approach at solving some problems of the Friedman cosmology convince us in the realization of the most various states of matter in the Universe (both with positive, and with the negative pressure).

In our opinion, the description of the Universe evolution from the point of view of changes in the equation of state can be a very special approach connecting as a physical aspect (thermodynamics, kinetics) so and geometric aspect (curvature, dimensionality of the space-time) of the theory of evolution of the Universe. Successes of geometric multidimensional approach to the unification of fundamental interactions leads to the idea that the dimensionality of space-time should be expressed through physical quantities characterizing the evolution of the Universe.

In the present work the conformal-flat cosmological models are considered as exact solutions of the cosmological Einstein equations in the framework of the multidimensional space-time with one time-like direction and with different from zero pressure for some equations of state in the presence of a linear relationship between the energy density and pressure. It turns out that this approach leads to the identification of some discrete set of equations of state for which conformal multipliers are closely related to harmonic functions, which are solutions of the Laplace equations in multidimensional Euclidean spaces with integer dimensionalities. The dimensionality of these spaces, in turn, is determined by the concrete equation of state.

We will continue to use further the Fock approach, in which the metric of the 4-dimensional and multidimensional space-times conformal to the Minkowski space-time and conformal factor is a function of one variable.

Multidimensional Einstein equations for cosmological models

We will take up the question: what consequences we will have after change of the space-time dimensionality in the framework of conformal-flat models and Fock's approach for the equation of state of matter which fills the Universe? To answer this question we will try in the frameworks of the Kaluza-Klein type approach to use the method named by an inductive method (see [7]). In other words, we will increase the number of space-like dimensionalities, leaving one dimensionality as time-like.

Now we will write down a metric in that form as in [1]

$$ds^2 = \exp(2\sigma)\delta_{\mu\nu}dx^\mu dx^\nu \quad (1)$$

where $\exp(2\sigma)$ is a conformal factor; $\sigma = \sigma(S)$; $S^2 = \delta_{\mu\nu}x^\mu x^\nu$; $\delta_{\mu\nu} = \text{diag}(1; -1; -1; \dots; -1)$ is a metric tensor of the Minkowski type of multidimensional space-time; the velocity of light and Newton's gravitational constant are equal to unit, and the Einstein gravitational constant is equal here $\kappa = 8\pi$. And now the Greek indices run through the values $\mu, \nu = 1, 2, \dots, N$, where N is the number of the space-like coordinates.

The gravitational field equations are postulated in the form of a system of Einstein's equations (without cosmological constant)

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa T_{\mu\nu}, \quad (2)$$

where $G_{\mu\nu}$, $R_{\mu\nu}$ and R are the Einstein tensor, the Ricci tensor and the scalar curvature, respectively, constructed from the metric (1) similarly as in the 4-dimensional case. The right part of equations (2) is a multivariate analogue of the energy-momentum tensor (EMT) for the Pascal perfect fluid describing a multidimensional "liquid":

$$T_{\mu\nu} = \varepsilon u_\mu u_\nu + p b_{\mu\nu}, \quad (3)$$

where ε is an analogue of a energy density; p is analogue of pressure; $(N+1)$ -velocity $u_\mu = \exp(\sigma) \cdot b_\mu$ is proportional to the gradient of the variable S as a function of the coordinates x^μ : $b_\mu = S_{,\mu}$; $u_\mu u^\mu = 1$ is the normalization condition for $(N+1)$ -velocity; $b_{\mu\nu} = u_\mu u_\nu - g_{\mu\nu}$ is a N -projector on the N -subspace, which plays the role of the metric tensor for N -subspace. In this case the condition of the N -subspace orthogonality and the time-like congruence u^μ is executable: $b_{\mu\nu}u^\mu = 0$.

In fact, if we take $(N+1)$ -velocity u^μ as the analog of the monad in a multidimensional space-time by analogy with 4-dimensional space-time (see, e.g., [7]), we can assume that EMT (3) is written down according to the monadic approach.

On the other side the left part of the system (2) can be written down as

$$G_{\mu\nu} = (N+1) \left(\left[\sigma'' - \frac{\sigma'}{S} - (\sigma')^2 \right] \cdot b_\mu b_\nu - \left[\sigma'' + \frac{N-1}{S} \sigma' + \frac{N-2}{S} (\sigma')^2 \right] \cdot \delta_{\mu\nu} \right), \quad (4)$$

where a prime denotes the full derivative d/dS .

Carrying out the procedure of $(1+N)$ -splitting as it is done in 4-dimensional space-time (see, e.g., [7]), we obtain the following system of equations that define the "energy density" and "pressure"

$$N(N-1) \left[\frac{\sigma'}{S} - \frac{(\sigma')^2}{2} \right] = \kappa \varepsilon \exp(2\sigma), \quad (5)$$

$$(N-1) \left[\sigma'' + \frac{N-1}{S} \sigma' + \frac{N-2}{S} (\sigma')^2 \right] = -\kappa p \exp(2\sigma). \quad (6)$$

A special case of this system, of course, are the equations ([1], (1.4)-(1.5)). Quotes we have put because, strictly speaking, the meaning of these concepts in the case of multidimensional space-time requires a completely separate consideration.

How it is known, an open model of Friedman [11] is described by the equation of state of incoherent dust $p = 0$. This equation is a special case of a linear equation of state, in many cases, used in physics,

$$p(S) = \beta_0 \varepsilon(S), \quad (7)$$

where $|\beta_0| \leq 1$; $\beta_0 = \text{const}$.

A set of physically interpretable states of matter in 4-dimensional space-time is described by the relation $p \propto \varepsilon$. There are physical vacuum, incoherent dust, the relativistic gas, etc. These states of matter correspond to specific discrete values of the parameter β_0 .

The equation of state for multidimensional space-time, i.e. the relationship between "pressure" and "energy density", is postulated in its simplest form of equation (7), which however includes the physically relevant cases.

In the future we won't use quotes because the meaning of the notions of the energy density and pressure will be clear from the context.

The system of equations (5) and (6) after taking into account equation (7) and of an exclusion of the energy density, can be reduced to one equation

$$\sigma'' + [(N-1) + \beta_0 N] \frac{\sigma'}{S} + [(N-2) + \beta_0 N] \frac{(\sigma')^2}{2} = 0 \quad (8)$$

which, in turn, by replacements

$$\sigma(S) = \frac{2}{(N-2) + N\beta_0} \ln y(S) \quad (9)$$

for $\beta_0 \neq (2-N)/N$ and $\sigma(S) = y(S)$ for $\beta_0 = (2-N)/N$ it will easily be converted into equation

$$y'' + \left(\frac{N(1+\beta_0)-1}{S} \right) y' = 0. \quad (10)$$

Although this equation integrates trivially, let us try to extract some other information. It is easy to notice that when coefficient at the first derivative of function $y(S)$ is integer then the equation (10) is a radial part of the Laplace spherical symmetric equation in a multidimensional Euclidean space of dimensionality $k = N(1 + \beta_0)$

$$y'' + \frac{(k-1)}{S} y' = \frac{1}{S^{(k-1)}} \frac{\partial}{\partial S} (S^{(k-1)} \frac{\partial y}{\partial S}) = 0, \quad (11)$$

where the variable S plays the role of radial variable. The value of S is the same in the transitions between the different Euclidean spaces by the same way as in space, for example, with three dimensions when a radius of a circle is equal to radius of the sphere, constructed on this circle. The requirement of integrality of the parameter k is equivalent to the requirement of discreteness of the values of $\beta_0 = (k/N) - 1$, the number of which for each of fixed N is equal to $(2N+1)$, and these values are rational (if the dimension of space-time is a natural number). In this case the conformal factor associates with the function $y(S)$ as

$$\exp(2\sigma(S)) = y^{\frac{4}{k-2}} \quad (12)$$

Thus, the solutions of equation (11) for the integer k are fundamental harmonic functions of order k (see, e.g. [12])

$$y_k = (B_k + \frac{A_k}{S^{k-2}}), \quad k \neq 2; \quad y_2 = (B_2 + A_2 \ln(\alpha S)), \quad k = 2, \quad (13)$$

where a choice of constants is connected with a specific physical asymptotical conformal factor.

Generally speaking the spaces of dimensionality k in which these functions are the harmonic functions will not necessarily be Euclidean spaces. The satisfiability of equation (11) with integer k can be realized in conformal-Euclidean space by requirement of vanishing the scalar curvature of such space. Really, if to consider the space of dimensionality n with conformally-Euclidean metric [13]

$$g_{ab} = \exp(2\sigma) \delta_{ab}, \quad (14)$$

where $a, b = 1, 2, \dots, n$; $\delta_{ab} = (+1, +1, \dots, +1)$, then the scalar curvature of such space will be written in form

$$R^{(n)} = 4 \left(\frac{n-1}{n+1} \right) [\exp(-\frac{n+2}{2}\sigma)] \Delta_n y, \quad (15)$$

where

$$\Delta_n y \equiv y'' + \frac{n-1}{S} y'.$$

So, in our approach it can be said that in the conformal-flat models, filled with a substance with equation of state (7), each class of the substance (i.e., each concrete value β_0 of physically interpretable discrete set of values), determined by the values of N and k , corresponds to "his" Euclidean (or conformal-Euclidean) space. The fundamental harmonic function of this space determines the gravitational field (the conformal factor) created by this substance. Thus, each type of substance is adequate to "its" multi-dimensional space-time by virtue of a uniqueness of the time-like directions.

It can see that such interpretation makes sense. We can write down the energy-momentum tensor law of conservation for our case. Once this is done, we get the ratio

$$\varepsilon = \frac{const}{[S \exp(\sigma)]^n} \quad (16)$$

where $n = N(1 + \beta_0)$ is exactly equal to the parameter k , introduced earlier. The value of $S \exp(\sigma(S)) = a(S)$ is the curvature radius of the space-like hypersurface in a synchronous frame of reference. On the other hand (see e.g. [14]), the right side can be interpreted as a value inversely proportional to the spatial volume of the system. Then n will be the dimensionality of this space volume.

Here are two examples. When $N = 3$, $\beta_0 = 0$ (incoherent dust) we have $n = 3$, i.e. the energy density is inversely proportional to the three-dimensional volume. When $N = 3$, $\beta_0 = 1/3$ (ultra-relativistic gas) we have $n = 4$ and now the energy density is inversely proportional to the space-like four-dimensional volume. In other words, the state of matter defines the "specific" space for a given substance in which the energy density decreases inversely proportionally to the volume. Moreover, it is easy to see that the only state, for which the energy density will have finite value on the cone $S = 0$, is the physical vacuum ($\beta_0 = -1$), regardless of the dimensionality N .

Four-dimensional space-time

Now we concretize the value of the dimensionality N . The first of all, we are interested by the manifold describing our Universe. We will take $N = 3$ and will consider, what values of β_0 will be distinctive for this case (in the sense of our interpretation).

The dimensionality of the Laplacian will in this case be determined by the formula

$$n = 3(1 + \beta_0), \quad (17)$$

i.e., Euclidean (conformal-Euclidean) characteristic space will have the dimensionality n .

In our case a discrete set of values b will contain seven values, starting with zero. We must remind about restriction by the condition $|\beta_0| \leq 1$. In other words, seven Euclidean (conformal-Euclidean) spaces are the subspaces of the conformal-flat 4-world with the various BB connected with integers n ($n = 0, 1, 2, \dots, 6$).

Let us write down for all values of n ($0 \leq n \leq 6$) the conformal factors, the pressure, the four-dimensional and three-dimensional scalar curvatures, the proper time as functions of the variable S , and the metric in the synchronous reference frame.

Case 1. $n = 0, \quad \beta_0 = -1$

The state of matter: physical vacuum (the open d'Sitter solution).

Then the conformal factor is

$$\exp(2\sigma) = (B + AS^2)^{-2}, \quad (18)$$

where A and B are constants. It is obvious, that everywhere we can put $B = 1$ because this constant spots a gauge select.

Now easily we easily find the pressure as the function of variable S , knowing the conformal factor

$$\kappa p = 3\beta_0 \left(\frac{2\sigma'}{S} - (\sigma')^2 \right) \exp(-2\sigma), \quad (19)$$

We will get expected effect when have done simple evaluations: pressure is constant and also depends only on constants

$$\kappa p = 12A, \quad (20)$$

which shows that the constant A is negative (energy density should be positive).

The four-dimensional scalar curvature it is discovered under the formula

$${}^4R = 6\exp(-2\sigma)\left(\sigma'' + \frac{3\sigma'}{S} + (\sigma')^2\right) = -48A > 0. \quad (21)$$

The three-dimensional scalar curvature in case with the metric (1) is calculated by means of introduction of a 3-projector $b_{ik} = u_i u_k - g_{ik}$ ($u^i = b^i \exp(-\sigma)$ -- 4-velocity; $b_i = S_{,i}$) in non comoving reference frame and equal to

$${}^3\hat{R} = -\frac{3\sigma'}{S}\exp(-2\sigma). \quad (22)$$

This curvature becomes for a physical vacuum

$${}^3\hat{R} = 6A(1 + AS^2) \quad (23)$$

At $S \rightarrow 0$ ${}^3\hat{R}$ tends to a finite limit equal to $6A$.

The solution with the metric (1) is transformed in the comoving synchronous frame of reference by transformation $t = S \cosh R$; $r = S \sinh R$, where a proper time is determined as

$$\tau = \int u_\mu dx^\mu = \int \exp(\sigma(S)) dS, \quad (24)$$

here R is a radial variable in the synchronous reference frame; u^μ is 4-velocity. This proper time is not expressed by the elementary functions. This makes difficulties in the synchronous reference frame.

Then the metric can be rewritten in the synchronous reference frame as

$$ds^2 = d\tau^2 - a^2(\tau)(dR^2 + \sinh^2 R d\Omega^2), \quad (25)$$

where $a(\tau) = S \exp(\sigma)$ is a scale factor.

In our case the proper time is easily found and equal to

$$\tau = \frac{1}{|A|} \ln \left| \frac{1 + \sqrt{|A|}S}{1 - \sqrt{|A|}S} \right| = \frac{1}{\sqrt{|A|}} \arctan h(\sqrt{|A|}S). \quad (26)$$

Choosing the domain of the change τ from 0 to ∞ , we get the domain of the change S : $0 \leq S \leq (1/\sqrt{|A|})$.

In the commoving synchronous reference frame we obtain corresponding form of the scalar 3-curvature

$${}^3R = -\frac{1}{S^2} (1 - |A|S^2)^2 = -4|A| \sinh^2(2\sqrt{|A|}\tau), \quad (27)$$

and the 4-interval can be written in the standard form

$$ds^2 = d\tau^2 - \left(\frac{1}{4\sqrt{|A|}} \right) \sinh^2(2\sqrt{|A|}\tau) dl^2, \quad (28)$$

Case 2. $n=1$, $\beta_0 = -2/3$

The state of matter: domain walls.

The conformal factor is

$$\exp(2\sigma) = (1 + AS)^{-4}. \quad (29)$$

The pressure is

$$\kappa p = \frac{8A}{S} (1 + AS^2) \quad (30)$$

and $A < 0$.

The four-dimensional scalar curvature is

$${}^4R = -\frac{36}{S} A(1 + AS^2)^2 \quad (31)$$

The three-dimensional scalar curvature is

$${}^3\hat{R} = \frac{6A}{S} (1 + AS)^3. \quad (32)$$

The proper time is

$$\tau = (|A|(1 + AS))^{-1}. \quad (33)$$

The metric in the synchronous reference frame is

$$ds^2 = d\tau^2 - A^2 \tau^4 dl^2, \quad (34)$$

Case 3. $n = 2$, $\beta_0 = -1/3$

The state of matter: relativistic strings.

The conformal factor is

$$\exp(2\sigma) = S^{2A}. \quad (35)$$

The pressure is

$$\kappa p = -A(2 + A)S^{-2(1+A)}. \quad (36)$$

The four-dimensional scalar curvature is

$${}^4R = \frac{6A(2 + A)}{S^{2(1+A)}}. \quad (37)$$

The three-dimensional scalar curvature is

$${}^3\hat{R} = -\frac{3A}{S^{2(1+A)}}. \quad (38)$$

The proper time is

$$\tau = \frac{1}{(1+A)} S^{(1+A)}. \quad (39)$$

The metric in the synchronous reference frame is

$$ds^2 = d\tau^2 - [(A+1)\tau]^{\left(\frac{2A}{1+A}\right)} dl^2. \quad (40)$$

It should be noted that this case is special from the point of view of the physical content of the found models. We can get both singular and non-singular models, depending on the sign and magnitude of the constant A .

Case 4. $n = 3$, $\beta_0 = 0$

The state of matter: incoherent dust (Friedman's Universe).

The conformal factor is

$$\exp(2\sigma) = \left(1 + \frac{A}{S}\right)^4. \quad (41)$$

The pressure is

$$p = 0; \quad \kappa\epsilon = -\frac{12A}{S^3(1+A/S)^6}. \quad (42)$$

From (42) we see that the constant A is negative.

The four-dimensional scalar curvature in non comoving reference frame is

$${}^4R = \frac{12A}{S^3(1+A/S)^6}. \quad (43)$$

The three-dimensional scalar curvature is

$${}^3\hat{R} = -\frac{6A}{S^3(1+A/S)^5} \quad (44)$$

The proper time is

$$\tau = S - 2A \ln \left| \frac{S}{A} \right| - \frac{A^2}{S}. \quad (45)$$

The scale factor can be not written as an elementary function via the proper time in the synchronous reference frame. The same we have and in standard approach (see, for example [14]).

Case 5. $n = 4$, $\beta_0 = +1/3$

The state of matter: ultra-relativistic gas.

The conformal factor is

$$\exp(2\sigma) = (1 + A/S^2)^2. \quad (46)$$

The pressure is

$$\kappa p = -\frac{A}{S^4(1+A/S^2)^4} \quad (47)$$

The four-dimensional scalar curvature ${}^4R = 0$. It is a result of the energy-momentum tensor traceless in this case.

The three-dimensional scalar curvature in non comoving reference frame is

$${}^3\hat{R} = -\frac{6A}{S^4(1+A/S^2)^4}. \quad (48)$$

The proper time is

$$\tau = S + \frac{|A|}{S} - \left(\sqrt{|A|} + \frac{1}{\sqrt{|A|}} \right). \quad (49)$$

The metric in the synchronous reference frame is

$$ds^2 = d\tau^2 - \left(\hat{\tau} - \frac{4|A|}{\hat{\tau} - \frac{4|A|}{\sqrt{\hat{\tau} - 4|A|}}} \right) dl^2, \quad \hat{\tau} = \tau + \left(\sqrt{A} + \frac{1}{\sqrt{A}} \right). \quad (50)$$

Case 6. $n = 5$, $\beta_0 = +2/3$

The state of matter: non relativistic degenerate gas.

The conformal factor is

$$\exp(2\sigma) = (1 + A/S^3)^{4/3}. \quad (51)$$

The pressure is

$$\kappa p = \frac{8A}{S^5(1+A/S^3)^{10/3}}. \quad (52)$$

The four-dimensional scalar curvature is

$${}^4R = \frac{12A}{S^5(1+A/S^3)^{10/3}}. \quad (53)$$

The three-dimensional scalar curvature in non comoving reference frame is

$${}^3\hat{R} = -\frac{6A}{S^5(1+A/S^3)^{7/3}}. \quad (54)$$

The proper time as a function of S can be not written as an elementary function.

Case 7. $n = 6$, $\beta_0 = +1$

The state of matter: super-rigid state.

The conformal factor is

$$\exp(2\sigma) = (1 + A/S^4). \quad (55)$$

The pressure is

$$\kappa p = -\frac{12A}{S^6(1+A/S^4)^3}. \quad (56)$$

The four-dimensional scalar curvature is

$${}^4R = \frac{24A}{S^6(1+A/S^4)^3}. \quad (57)$$

The three-dimensional scalar curvature in non comoving reference frame is

$${}^3\hat{R} = -\frac{6A}{S^6(1+A/S^4)^2}. \quad (58)$$

So, in the case of four-dimensional space-time we have seven physically the interpreted values β_0 or seven equations of state of matter that fills space-time. These values of β_0 allow for the description of the generated gravitational fields (conformal factors of the 4-metric) in the

language of the fundamental harmonic functions which correspond to the Euclidean spaces. Basic data obtained above for the four-dimensional space-time are summarized in Tab.1 for comparison.

Table 1.

n	$n-1$	β_0	State of matter	$\exp(2\sigma(S))$
0	-1	-1	physical vacuum	$(B + A \cdot S^2)^{-2}$
1	0	-2/3	domain walls	$(B + A \cdot S)^{-4}$
2	1	-1/3	relativistic strings	$A \cdot S^{-2B}$
3	2	0	incoherent dust	$(B + A/S)^4$
4	3	+1/3	ultra-relativistic gas	$(B + A/S^2)^2$
5	4	+2/3	non relativistic degenerate gas	$(B + A/S^3)^{4/3}$
6	5	+1	super-rigid	$(B + A/S^4)$

Table of the state of matter in multidimensional space-time

Now again we go back to the link between parameter β_0 , the dimensionality $(N+1)$ of the "basic" space-time and the order n of the fundamental harmonic function. So in the "basic" manifold, we must have at least one time-like direction and one space-like for the conservation of the concept of space-time. It is therefore natural to begin consideration of models with $N-1$. It should be noted that for each of fixed N the numbers of the associated Euclidean (conformal-Euclidean) space (or the orders of fundamental harmonic functions) are limited and are equal to $(2N+1)$.

The full picture is also clearly reflected in the form of a Tab.2. It is easy to see that only three "states of matter" are present in all dimensionalities of space-time: "physical vacuum", "incoherent dust" and "super-rigid state". These terms are in quotes because the physical interpretations of the specific meaning of these terms in the spaces with different dimensionalities requires separate consideration. In this regard there is an interesting fact that in the four-dimensional world (the third column) all possible values of the parameter β_0 have a clear physical interpretation and include all the essential states of matter that are used to find cosmological solutions.

Table 2.

N/n	1	2	3	4	5	6	7	8	9	10	11
0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	0	-1/2	-2/3	-3/4	-4/5	-5/6	-6/7	-7/8	-8/9	-9/10	-10/11
2	+1	0	-1/3	-2/4	-3/5	-5/6	-5/7	-6/8	-7/9	-8/10	-9/11
3		+1/2	0	-1/4	-2/5	-5/6	-4/7	-5/8	-6/9	-7/10	-8/11
4		+1	+1/3	0	-1/5	-5/6	-3/7	-4/8	-5/9	-6/10	-7/11
5			+2/3	+1/4	0	-5/6	-2/7	-3/8	-4/9	-5/10	-6/11
6			+1	+2/4	+1/5	0	-1/7	-2/8	-3/9	-4/10	-5/11
7				+3/4	+2/5	+1/6	0	-1/8	-2/9	-3/10	-4/11
8				+1	+3/5	+2/6	+1/7	0	-1/9	-2/10	-3/11
9					+4/5	+3/6	+2/7	+1/8	0	-1/10	-2/11
10					+1	+4/6	+3/7	+2/8	+1/9	0	-1/11
11						+5/6	+4/7	+3/8	+2/9	+1/10	0

It is necessary to indicate that the harmonic functions of the same order (in other words, with the same conformal factor) correspond to different values of β_0 in spaces of different dimensionality. In this case, can speak about an “unification” of types of interactions because for any b (except physical vacuum) there is such a space-time in which the corresponding conformal factor will describe the model, filled with matter of the type of incoherent dust with $\beta_0 = 0$.

An interesting conclusion can be reached coming back again to the 4-dimensional space-time: for the transition to a 5-dimensional space-time we can speak about the "unification" the electromagnetic interaction (this is corresponds to the Kaluza-Klein approach). For the transition to the 6-dimensional space-time we can speak about the "unification" of the state of matter with $\beta_0 = +2/3$, which is true for all the ideal Boltzmann systems, and also for bosons' and fermions' systems (see, e.g., [15]). We are talking about electroweak interactions. The substance in 4-dimensional space-time with the super-rigid equation of state “turns to incoherent dust” in the 7-dimensional space-time, i.e. when there is the strong interaction in 4-dimensional space-time. This is consistent with the results of Yu. S. Vladimirov [16], which showed that in the framework of the Kaluza-Klein approach the unification of gravitational and electroweak interactions enough 6-dimensional space-time, and for the unification of the gravity-electro-strong interaction enough 7-dimensional space-time with one time-like direction.

On the other hand, the same value of b may correspond to different orders of magnitude of fundamental harmonic functions, that is, different conformal factors depending on the dimensionality of space-time. For example, for $\beta_0 = +1/3$ we have a description via the harmonic function of the fourth order in 4-dimensional space-time and in 7-dimensional space-time we have the harmonic function of the eighth order. In 10-dimensional space-time we will have harmonic function of the twelfth order, and so on. Of course, it must be remembered that the concrete value of b responds to the different physical conditions in worlds with different dimensionalities.

The essential point of the approach considered here, in our opinion, is the opportunity of describing models of conformal-flat Universe filled with matter with arbitrary β_0 .

Conclusion

The finding problem of conformal-flat cosmological models as exact solutions of the equations of gravitation for different equations of state with linear connection between pressure and energy density is demonstrated within the limits of multidimensional space-time with one time-like direction. In this case the energy-momentum tensor (EMT) is taken as generalization of EMT in an approach of the perfect Pascal fluid for space-time with four dimensionalities. The special case is EMT for an incoherent dust with zero pressure is related to the open model of Friedman's Universe. It turns out that such approach leads to an identification of some discrete set of equations of state for which conformal factors are connected with the harmonic functions as solutions of the Laplace equations in multidimensional Euclidean spaces of an integer dimensionality. Dimensionality of these spaces, in turn, is defined by a concrete equation of state. For four-dimensional space-times the corresponding table is constructed. This table allows to trace connection between a discrete set of linear equations of state and dimensionality of the auxiliary Euclidean spaces and also the functional expression of conformal factors of the open cosmological models related to potential functions, which are solutions of the Laplace equations in these auxiliary Euclidean spaces. Thus it can be seen that three dimensional spatial-like manifold restricts a selection of discrete physically interpreted equations of state for the finding of exact solutions of the gravitation equations related to potential functions. Therefore, on the one hand, any linear equation of state can be approximated with any accuracy by any rational fraction. On the other hand, the exact solution of the many-dimensional equations of Einstein can be found only related via to potential functions when the spatial extension of space-time will be made up to necessary multidimensionality. Such possibility appears for any linear equation of state with a rational constant of proportionality at growth of the space dimensionality N ($N > 3$). For such

space-times the similar table is constructed, but without fixing of dimensionality of a spatial hypersurface. Here each value of spatial dimensionality N corresponds to $2N+1$ of linear equations of state. This table demonstrates the possibilities for each such equation of state with a rational constant of proportionality between pressure and density of energy under construction for any open cosmological model with the conformal-flat metric, but in corresponding space-time with dimensionality more than four.

References

1. Baranov A.M., Saveljev E.V. (2014). Exact solutions of the conformal-flat Universe. I. The evolution of model as the problem about a particle movement in a force field. *Space, Time and Fundamental Interactions (STFI)*, No.1. 37-46.
2. Baranov A.M., Saveljev E.V. (1984). Sfericheski-simmetrichnoe svetopodobnoe izluchenie i konformno-ploskie prostranstva-vremena [Spherically symmetric lightlike radiation and conformal-flat space-times]. *Izv. vuz.(Fizika) [News of higher educational institutions]*, No.7, 32-35.
3. Baranov A.M., Saveljev E.V. (1984). Spherically symmetric lightlike radiation and conformal-flat space-times, *Russ. Phys. J.*, V. 27., No 7, 569-572.
4. Fock V.A. (1964). *The Theory of Space, Time and Gravitation*. New York: Pergamon, U.S.A., 2nd edition.
5. Mitskievich N.V. (1969). *Fizicheskie polja v obshhej teorii otnositel'nosti [Physical Fields in General Relativity]*. Moscow: Nauka [Science].
6. Zelmanov A.L. (1973). Kinematic Invariants and Their Relation to Chronometric Invariants of Einstein Theory Of Gravity. *DAN USSR*, Vol. 209. No.4, 822-825.
7. Vladimirov Yu.S. (1982). *Sistemy otscheta v teorii gravitacii [Reference Frames in the Gravitation Theory]*. Moscow: Energoizdat.
8. Mitskievich N.V. (1972). Sistemy otscheta i konstruktivnyj podhod k nabljudаемым величинам в обshhej teorii otnositel'nosti [Reference frames and the constructional approach to observed magnitudes in general relativity]. Einstein collected book, Moscow: Nauka [Science], 67-87.
9. Baranov A.M. (2013). Conformally Galilean 4-metric and Kinematic Reference Frames. *Space, Time and Fundamental Interactions (STFI)*, No.1, 37-43.
10. Baranov A.M., Saveljev E.V. (2013). Conformal-flat model of the open Universe with an arbitrary function of state. *Space, Time and Fundamental Interactions (STFI)*, No.1, 22-27.

11. Friedman A.A. (1924). Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes. *Z. Phys.*, Vol. 21, No. 1, 326-333.
12. Timan A.F., Trofimov V.N. (1968). Vvedenie v teoriju garmonicheskikh funkciij [Introduction in theory of harmonic functions.], Moscow: Nauka [Science], 207.
13. Baranov A.M., Saveljev E.V. (1988). On one way of description of conformal flat world. *Gravitation and electromagnetism: collected articles*, Minsk: University Press, 26-29.
14. Landau L.D., Lifshitz E.M. (1988). *Klassicheskaja toerija polej [The classical Theory of Fields]*. Moskow: Nauka [Science].
15. Balescu R. (1975). *Equilibrium and nonequilibrium statistical mechanics*. New York-London-Sydney-Toronto.
16. Vladimirov Yu.S. (1987). *Razmernost' fizicheskogo prostranstva-vremeni i ob#edinenie vzaimodejstvij. [Dimensionality of physical space-time and unification of interactions]*. Moscow: Moscow State University Press.