The special theory of relativity and the theory of physical similarity

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We establish an analogy between the two theories: the principle of relativity & the principle of similarity, the Lorentz transformations & the similarity transformations, the Lorentz group & the similarity group. And so on.

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1. The theory of physical similarity, which is a natural generalization of the theory of geometric similarity, studies the dimensions of physical quantities. The notion of dimension of a physical quantity was introduced in 1822 by Jean Baptiste Joseph Fourier, the French mathematician and physicist (1768-1830). The very essence of the theory is so-called pi-theorem [1,2,3]. Apparently, this theorem was originally formulated by a French mathematician Joseph Louis François Bertrand (1822-1900) [4]. Despite the fact that the pi-theorem has been known for over 100 years and all the way there have been many attempts to create clear and rigorous proof of this theorem, for example, in [5], up to the day there are quite a number of questions concerning the theorem. Answers to some of them are to be found in [6,7,8,9].

It is in the framework of the theory of physical similarity that the dimensionless Reynolds, Mach, Froude, Nusselt, Prandtl numbers had appeared and proposed a developed turbulence model (A.N.Kolmogorov, 1941).

The theory of physical similarity includes the principle of similarity, meaning equality of all physical systems of units of physical quantities, and the group of similarity. The similarity transformation is the transition from one system of units to another, when each basic unit is multiplied by a coefficient. Group of similarity is a set of similarity transformations, connecting all pairs of systems possible. According to the principle of similarity, a mathematical representation of a physical law should not depend on the choice of units. In addition, each physical law is characterized by a certain set of physical quantities.

Pi-theorem is a theorem of invariants. Invariants of similarity are dimensionless complexes having the form of products of powers of physical variables with rational exponents.

2. Two affine groups theories. Since the basic provisions of the special theory of relativity are well known, we won't speak about them now.

Widely speaking, Lorentz transformations are Poincare transformations (space-time coordinates transformations: rotations, boosts and parallel shifts). Poincare transformations form a 10-parameter Poincare group, which is known to be an affine group. Note also that boosts, by themselves, do not form a group. But together with the rotations boosts form a 6-parameter Lorentz group (the proper Lorentz group) – a subgroup of the Poincare group.

Widely speaking, the group of units transformation is also an affine group, and similarity group being a subgroup.

3. Some analogies between the special theory of relativity and the physical similarity theory:

The inertial frames of reference	The systems of units
The principle of relativity	The principle of similarity
The principle of the constancy of light velocity and invariance of interval	The invariance of dimensionless complexes
The Lorentz transformations	The similarity transformations
The Lorentz group	The similarity group
The Poincare group	The affine group of transformations of units systems

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