The mathematical derivation or speculation of $E=\Delta mc^2$ in Einstein's September 1905 paper, and some peculiar experiments

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In his paper Einstein derived $\Delta L=\Delta mc^2$ (light energy –mass equation). It has not been completely studied; it is only valid under special conditions of the parameters involved e.g. number of light waves, magnitude of light energy, angles at which waves are emitted and relative velocity v. Einstein considered just two light waves of equal energy, emitted in opposite directions and the relative velocity v uniform. There are numerous possibilities for the parameters which were not considered in Einstein's derivation. $\Delta E=\Delta mc^2$ is obtained from $\Delta L=\Delta mc^2$ by simply replacing L by E (all energy) without derivation. Fadner correctly pointed out that Einstein neither mentioned E or $\Delta E=\Delta mc^2$ in the derivation. Herein additional results are critically analyzed, taking all possible variables into account. Under some valid conditions of parameters $\Delta L=\Delta mc^2$ is not obtained e.g. sometimes the result is $M_a = M_b$ or no equation is derivable. If all values of valid parameters are taken into account then the same derivation also gives $L \propto \Delta mc^2$ or $L=A\ \Delta mc^2$, where A is a coefficient of proportionality or $\Delta E=Ac^2\Delta m$ is also possible. On 11 December 1951, in Nobel Lecture Sir Cockcroft quoted some experimental data and stated that $\Delta E=\Delta mc^2$ is confirmed within reasonable accuracy. However simple calculations reveal that in original experiments , there is inadequacy of 16.52 % and 2.49% in the quoted data. By current standards this inadequacy from $\Delta E=\Delta mc^2$ is 9.768 %. Similar intrigues exist in calculations of energy emitted in nuclear chain reaction where secondary neutrons have velocity 2MeV or 1.954×10^7 m/s i.e. ~ 7% that of light. This velocity is in relativistic limits thus mass of neutron must be taken as relativistic mass 1.01080879u, whereas as neglecting this in calculation of energy emitted the mass of neutron is regarded as 1.0086649u (rest mass). Thus energy emitted is over estimated by 5.99MeV (173.271MeV -167.29MeV). It may be one reason for lower efficiency of reactors. In Large Hadron Collider, at energy 7 TeV the speed of proton had been measured as 0.99999997 time that speed that of light , but in the interpretation mass is regarded as 1.0086649u. Under this condition the relativistic mass of proton must be 4,082.4841 times rest mass of proton i.e. 938.272046 MeV/c^2. Thus relativistic variation of mass is completely neglected, which is not justified as not consistent with various observations of relativistic variation of mass. The speed of protons, hence relativistic mass in current run of LHC must be precisely measured when energy of protons would be 13-14MeV.

Keywords: Light energy , Cockcroft, deviation, generalized equation, relativistic mass.

DOI: 10.18698/2309-7604-2015-1-7-19

1.0. Description and critical analysis of Einstein's Thought Experiment

Newton stated in 1704 in Opticks [1] *"Gross bodies and light are convertible into one another...",* Even before Einstein directly or indirectly many scientists tried to give mathematical equation to above perception, but here Einstein's Sep. 1905 paper is being critically analyzed. In Einstein's derivation basic equation is

$$l^* = l\frac{\left(1-\frac{v}{c}cos\phi\right)}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

where ℓ is light energy emitted by body in frame (x,y,z) and ℓ^* is light energy measured in system (ξ, η, ζ), and v is uniform velocity with which the frame or system (ξ, η, ζ) is moving. This is equation for Doppler principle for light for any velocity whatever, and stated in June 1905 paper [2] in terms of frequency. The energy is a scalar quantity but in eq.(1) it has dependence on angle. The eq.(1) is based upon constancy of speed of light and c is maximum limit for speed of any particle.

Let a system of plane waves of light, referred to the system of coordinates (x, y, z), possesses the energy l; let the direction of the ray (the wave-normal) makes an angle ϕ with the axis x of the system [3].

If we introduce a new system of co-ordinates (ξ, η, ζ) moving in uniform parallel translation with respect to the system (x, y, z), and having its origin of coordinates in motion along the axis of x with the velocity v. Thus v is the relative velocity between system (x, y, z) and system (ξ, η, ζ) . The body which emits light energy is considered stationary in the system (x,y,z) and also remains stationary after emission of light energy in the system (x,y,z) in Einstein's perception. Let E_0 and H_0 are energies in coordinate system (x, y, z) and system (ξ, η, ζ) before emission of light energy, further E_1 and H_1 are the energies of body in the both systems after it emits light energy. E_i and H_i include all the energies possessed by body in two systems. Finally Einstein derived equation, $\Delta L = \Delta mc^2$.

1.1. Genuine cases neglected in Einstein's derivation

Einstein considered a luminous body emitting light energy in system (x,y,z) and energy is measured in the system (ξ, η, ζ). There are four variables in the derivation number of light waves emitted, magnitude of light energy, angle ϕ at which light energy is emitted and velocity (uniform) of system (ξ, η, ζ). Although eq.(1) is relativistic in nature yet Einstein used it in calculations of $\Delta L = \Delta mc^2$ under classical condition (v<<c) and applied Binomial Theorem. Einstein had taken super *special or handpicked* values of parameters. Thus for complete analysis the derivation can be repeated with all possible values of parameters.

(i) The body can emit numerous light waves but Einstein has taken only **TWO light waves emitted by luminous body.**

Why one or n light energy waves are neglected?

(ii) The energy of two emitted light waves may have different magnitudes but **Einstein has taken two light waves of EQUAL magnitudes (0.5L each).**

Why numerous other magnitudes (0.500001L and 0.499999L etc.) are neglected by Einstein?

(iii) Body may emit large number of light waves of different magnitudes of energy making different angles **(other than 0º and 180º as assumed by Einstein).**

Why other angles (such as 0º and 180.001º, 0.9999 º and 180º etc.) are neglected by Einstein?

Thus body needs to be specially fabricated; other forms of energy such as invisible energy are not taken in account. The conservation laws hold good for all form of energy under all conditions.

(iv) Einstein has taken velocity in classical region (v<<c and applied binomial theorem at the end) and has not at all used velocity in relativistic region. If velocity is regarded as in relativistic region (v is comparable with c), then equation for relativistic variation of mass with velocity i.e.

$$M_{rel} = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{2}$$

has to be taken in account. It must be noted that before Einstein's work this equation was given by Lorentz [4-5] and firstly confirmed by Kaufman [6] and afterwards more convincingly by Bucherer [7] . Einstein on June 19, 1948 wrote a letter to Lincoln Barnett [8] and advocated abandoning relativistic mass and suggested that it is better to use the expression for the momentum and energy of a body in motion, instead of relativistic mass. If final result is to be taken under classical conditions, then first equation can be taken in classical form.

(v) Further derivation is based upon assumption that body remains at rest in the system (ξ, η, ζ). However this condition imposed by Einstein is not obeyed in many mass energy inter-conversion phenomena. The nuclear fission is caused by the thermal neutrons which have velocity 2,185m/s. The uranium atom also moves as it is split up in barium and krypton, and emit energy. When a gamma ray photon of energy at least 1.02MeV, moves near the field of nucleus it is split up in electron and positron pair. The gamma ray photon is in motion and so is the state of electron and positron pair.

(vi) If system (ξ, η, ζ) is at rest i.e. v =0 then equation $L = \Delta mc^2$ is not derivable. So this derivation is not applicable universally.

Table I. The values of various parameters considered by Einstein and neglected by Einstein in the derivation of $L = \Delta mc^2$.

Sr No	Parameters	Einstein considered	Einstein neglected (No reason was given by Einstein why parameters are neglected).
1	No. of light waves	Two Light Waves	One, three, fouror n waves
2	Energy of light wave	Equal 0.5L and 0.5L each	Energies of the order of 0.500001L and 0.499999L are also possible. There are numerous such possibilities, which need to be probed. Bodies can emit more than two waves. The invisible waves of energy are not taken in account.
3	Angle	0° and 180°	The angles can be 0° and 180.001° or 0.9999° and 180° are also possible. There are numerous such possibilities which need to be probed.

4	Velocity	Classical region	The velocity can be in relativistic region. The velocity v can also be zero i.e. v = 0 v~c mass increases

Deductions: Einstein has taken only super-special values of parameters, and neglected many realistic values of variables. If all possible parameters are considered then result is not $L = \Delta mc^2$ but $L \ \alpha \ \Delta mc^2$.

1.2. Einstein obtained result, $L = \Delta mc^2$ under special conditions

Then Einstein [3] concluded that body emits two light waves of energy 0.5L each in system (x,y,z) where energy is E_0. Thus,

Energy before Emission = Energy after emission +0.5L + 0.5L

$$E_0 = E_1 + 0.5L + 0.5L = E_1 + L \tag{3}$$

Energy of body in system (ξ, η, ζ)

$$H_0 = H_1 + 0.5\beta L\left\{\left(1 - \frac{v}{c}cos\phi\right) + \left(1 + \frac{v}{c}cos\phi\right)\right\} \tag{4}$$

$$K_0 - K_1 = L\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right) \tag{5}$$

Applying binomial theorem (v<<c), thus neglecting magnitudes of fourth (v^4/c^4) and higher(v^6/c^6, v^8/c^8 …….) orders, we may place.

$$K_0 - K_1 = L\frac{v^2}{2c^2} \tag{6}$$

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = L\frac{v^2}{2c^2}$$

The amount of light energy L (ΔL) emitted by body when its mass decreases ($M_b - M_a$), we get

$$L = (M_b - M_a)c^2 = \Delta mc^2 \tag{7}$$

or Mass of body after emission (M_a) = Mass of body before emission

$$(M_b) - \frac{L}{c^2} \tag{8}$$

1.3. Theoretical or speculative deduction of $E=\Delta mc^2$ from $L=\Delta mc^2$

Fadner [9] has pointed out that Einstein never wrote E or ΔE in the paper, and assumed $\Delta L=\Delta mc^2$ is true for every energy as $\Delta E=\Delta mc^2$. Thus Einstein put forth equation for light energy as described by eq.(1), but regarded it true for every energy e.g. (i) sound energy, The speed of sound is 332m/s.

(ii) heat energy There is no equation like eq.(1) which relates variation of heat energy. The similar is the case of other types of energies (iii) chemical energy (iv) nuclear energy (v) magnetic energy (vi) electrical energy (vii) energy emitted in form of invisible radiations (viii) attractive binding energy of nucleus (xi) energy emitted in cosmological and astrophysical phenomena (x) energy emitted in volcanic reactions (xi) energies co-existing in various forms etc. etc. Thus Einstein simply replaced L (light energy) by every energy (E , as mentioned above) and obtained by $\Delta E=\Delta mc^2$. Further Einstein has considered that body emits light energy in visible region. But energy can also be emitted in the invisible region and Einstein did not mention at all about heat and sound energies (emitted along with light energy).

1.4. If general cases are considered, then L α Δmc^2 or L =A Δmc^2 is equally feasible

If light energy is emitted under unsymmetrical conditions [10,11] e.g. two light waves of energy (0.500001L and 0.499999L) at angles 0 and 180. If the recoil velocity of body is determined by applying law of conservation of momentum then it turns out to be 5×10^{-33} m/s, it does not affect eq.(1), as $v + 5\times10^{-33}$ m/s = v, hence eq.(1). Thus body remains at rest, even if calculations are done taking velocity $v + 5\times10^{-33}$ m/s according to definition. If body recoils (recoil velocity is calculated by law of conservation of energy) then net velocity varies and eq.(1) becomes

$$l^{*} = l\frac{\left(1-\frac{v+V_R}{c}cos\phi\right)}{\sqrt{1-\frac{\left(v+V_R\right)^2}{c^2}}} = l\frac{\left(1-\frac{v}{c}cos\phi\right)}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (9)$$

Let in case the luminous body emits two light waves of energy 0.499999L and 0.500001L in system (x,y,z) emitted in opposite directions [10-11] . Proceeding in the same way as Einstein did (except changing the magnitude of energy of light waves , 0.5L each previously), now assume slightly different energies i.e. 0.499999L and 0.500001L we get,

$$E_0 = E_1 + L \qquad (10)$$

$$H_0 = H_1 + 0.499999\beta L\left(1-\frac{v}{c}cos0^\circ\right) + 0.500001\beta L\left(1-\frac{v}{c}cos180^\circ\right) \qquad (11)$$

$$\left(H_0 - E_0\right) = \left(H_1 - E_1\right) + \beta L + 0.000002\beta L\frac{v}{c} - L \qquad (12)$$

$$\left(H_0 - E_0\right) - \left(H_1 - E_1\right) = \beta L + 0.000002\beta L\frac{v}{c} - L \tag{13}$$

$$= L\left[1 + 0.000002v/c + v^2/2c^2 - 1\right] \tag{14}$$

$$K_b - K_a = L(1 + 0.000002v/c + v^2/(2c^2) - 1) \tag{15}$$

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = L\left(0.000002\frac{v}{c} + \frac{v^2}{2c^2} - 1\right) \tag{16}$$

$$M_b - M_a = L\left(\frac{0.000004}{cv} + \frac{1}{c^2}\right) \tag{17}$$

$$\Delta mc^2 = L\left(\frac{0.000004c}{v} + 1\right)$$

$$L = \frac{\Delta mc^2}{0.000004\frac{c}{v} + 1} = \frac{\Delta mc^2}{1200 + 1} = \frac{\Delta mc^2}{1201} \tag{18}$$

$$L \alpha mc^2 \quad or\ L = A \vartriangle mc^2 \tag{19}$$

So Einstein’s derivation does not give fixed value of energy corresponding to mass annihilated. Thus if Einstein’s derivation is critically analyzed then general result is

$$L \alpha mc^2 \quad or\ L = A \vartriangle mc^2 \tag{20}$$

where A is coefficient of proportionality (there are numerous coefficients in the existing literature). Thus Einstein’s derivation of $\Delta L = \Delta mc^2$ permits different values of light energy corresponding to annihilation of mass as in eq.(20). As $\Delta E = \Delta mc^2$ is deduced from $\Delta L = \Delta mc^2$, likewise $\Delta E = A\ \Delta mc^2$ follows from $\Delta L = A\ \Delta mc^2$. If all other parameters remain same as assumed by Einstein but angle of one wave is regarded as 180.001 instead of 180, then similar equation follows. The same is true for other values of parameters as well. Prior to Einstein such equations existed in literature [12-14], thus here Einstein's derivation is completely analyzed and results are indicative of past and futuristic. Thus Einstein's derivation is extended and elaborated results are obtained. Such theoretical results may be checked in view of experimental findings.

2.0. Cockcroft and Walton experiment in 1932 does not confirm $\Delta E=\Delta mc^2$ as assumed

Cockcroft and Walton (1932) are routinely credited with the first experimental verification of mass-energy equivalence. Cockcroft and Walton examined a variety of reactions where different atomic nuclei are bombarded by protons. They focused their attention primarily on the bombardment of ^{7}Li by protons **[15-19]**

$$p + Li^7 \rightarrow \alpha + \alpha + 17.2\text{Me}$$

Let us analyze the energy considerations of reaction in view of $E=mc^2$ taking in account the atomic masses of reactants and products existing at time of Cockcroft's experiment [15-19].

^{7}Li =7.010 4u (Costa) , ^{1}H = 1.0072u , Mass of reactants = 8.0176u

Mass of products= $2\times$ ^{4}He = 8.002 2u, Mass decrease =8.0176u -8.002 2u = 0.0154 u

According to Einstein's $\Delta E=\Delta mc^2$, the mass is converted to energy (1u = 931.49MeV) . So energy equivalent to 0.0154 u, is given by 14.3449 MeV. Further Cockcroft and Walton had measured energy emitted by precise measurements and taken as 17.2MeV.

Further difference in theoretical (based on $\Delta E=\Delta mc^2$) and experimental values of energy is 2.8551MeV (17.2MeV-14.3449 MeV) . Now from theoretical and experimental values of energy we have, %age difference = 16.594

2.1. Cockcroft's Nobel Lecture in 1951

Cockcroft and Walton has taken value of mass of Li^7 equal to 7.010 4u . However Bainbridge improved precision of measurement of mass of Li^7 as 7.0130u.

^1H =1.0072u, Li^7= 7.0130u, Mass of reactants = ^1H +Li^7 = 8.0202 u

Mass of products = ^4He +^4He = 8.0022 u

Difference between masses of reactants and products =0.0180 u

Energy emitted in reaction (1u =931.49MeV) = 16.76682 MeV

Now difference in theoretically predicted and experimentally observed values of mass is (17.2MeV- 16.76682 MeV) 0.43318MeV or %age difference = 2.491

Thus it is significant deviation from $\Delta E=\Delta mc^2$, even if mass of Li^7 is taken as 1.0130u. But about this observation in Nobel Lecture (13 December, 1951) , Cockcroft stated [19]

"A little later Bainbridge re determined the mass of ^7Li to be 7.0130**.** This changed the mass decrease to 0.0180 mass units, in very good agreement with the observed figure." (pp. 177 of Nobel Lecture) . The deviation equal to 2.492 % is quiet significant, was not quoted by Cockcroft, however the mass difference (0.0180u) was mentioned.

Thus it is prudent to repeat the Cockcroft's and Walton's classic experiment with the most sensitive and sophisticated experiments measuring all parameters. Then both the equations, $\Delta E= \Delta mc^2$ and $\Delta E=A\ c^2\Delta m$ can be tested, as there are both theoretical and experimental basis for interpretation. There are also existing experimental observations which indicate that some specific experiments regarding $E =mc^2$ are absolutely necessary.

In laboratory experiments using thermal neutrons **[20-23],** for instance, it is usually found that the total kinetic energy (TKE) of the fragments that result from the fission of either U^{235} or Pu^{239} is 20-60MeV less than the Q-value of the reaction predicted by the quantity $E= \Delta mc^2$. It is typically assumed that the difference between the Q-value and the TKE of the fragments is lost into unobservable effects, such as additional excitation energy of the fragments. But energy can be lost into unobservable effects in those reactions when $E =\Delta mc^2$ is regarded as confirmed. This prediction is nearly 40 years old in the existing physics. Scientists have given traditional explanation for energy [**20-23]** and Bohr Wheelers model of nucleus have been extended. But now an alternate equation, $\Delta E=A\ c^2\Delta m$ is available, and even results inconsistent to $\Delta E= \Delta mc^2$ can be accommodated.

Further Serber [24] indicated that the efficiency of the "Little Boy" weapon that was used against Hiroshima in 1945 was about 2%. Usually, it is assumed that most of the atoms of the fissionable material simply do not undergo fission in the chain reaction and are thus "wasted" in the process. But there are no quantitative measurements. Thus there is ample scope for newer experiments, observations and deductions from existing literature.

3.0. Fast moving particles in nuclear reactions and effect of their relativistic masses

The relativistic variation of mass is well established in physics, as described in eq.(2). For simplicity, we have the following typical nuclear fission reaction,

$$_{92}U^{235} + {}_{0}n^{1} \rightarrow \left({}_{92}U^{236}\right)^{*} \rightarrow {}_{56}Ba^{144} + {}_{36}U^{89} + {}_{30}n^{1} \tag{21}$$

The range of multiplicity of neutron varies from zero to almost 10 (on average it is 2.5 for the fragment chosen). In this case the mass number is conserved if the number of neutrons is taken as 3. The above reaction also proceeds in different way [25],

$$_{92}U^{235} + {}_{0}n^{1} \rightarrow \left({}_{92}U^{236}\right)^{*} \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + {}_{0}n^{1} + {}_{0}n^{1} \tag{21a}$$

$$Mass\ of\ reactants = \left(235.0439299 + 1.0086649156\right)u = 236.0525948u \tag{22}$$

$$Mass\ of\ products = \left(143.922953 + 88.917630 + 3.02599473\right)u = 235.866577u$$

$$Mass\ of\ annihilated = \Delta m = 236.052594u - 235.866577u = 0.18601714u \tag{23}$$

$$Energy\ released\left(Q - Value\right) = 173.271MeV \tag{24}$$

3.1. Relativistic mass of Neutrons is taken in account

The neutrons which are emitted in the fission are fast (secondary) neutrons, having energy nearly equal to 2MeV (3.2×10^{-13} J); with this energy a neutron moves with relativistic speed, i.e. 1.954×10^{7}m/s (~7% that of light). The mass of a neutron moving with relativistic speed is 1.010808793u (whereas its rest mass is 1.0086649156u). If energy is calculated under these conditions (taking relativistic mass of neutron), then,

$$mass\ of\ reactants\ = \left(235.0439299 + 1.0086649156\right)u = 236.0525948u$$

$$mass\ of\ products\ = \left(143.922953 + 88.917630 + 3.032426394\right)u = 235.8730094u$$

$$Thus\ mass\ annihilated\ Dm = 0.1795854u\ and\ Energy\ released\left(Q - Value\right)\ = $$
$$= 167.28 MeV$$

Thus energy emitted is overestimated , as relativistic mass is not taken in account.

The exact measurements of relativistic masses and energies are required for assessment. The precise, specific, and independent experimental measurements of energy emitted in a single fission event will be helpful in this regard. It would be better if this value is used as the standard for understanding such reactions. It is observed that different energies are quoted for the same reaction in different references. At Large Hadron Collider [25] the experiments were conducted at energy level of 7 TeV, then proton attained speed equal to 0.99999997 times that speed that of light and temperature was $10^{16}\,^{\circ}$C (temperature more than 1 billion times greater than prevailing at the centre of the Sun). Under this condition the relativistic mass of proton must be 4,082.4841 times rest mass of proton and should be taken in account in further interpretation. Thus mass of proton would increase considerably at this stage and need to properly assessed in view of eq.(2) before drawing conclusions. In new experiments the energies are increased to 13-14TeV, thus both speed and mass of protons (particles) would be further higher. Such experiments at higher energies can be a test whether speed of lighter particles exceeds speed of light or not.

Acknowledgements

The author is highly indebted to Prof. Robert Stanley, Dr Stephen Crothers and S Hajra for discussions and encouragements at different stages of the work. Also financial assistance from Department of Science and Technology, Shimla is gratefully acknowledged.

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