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Department of Physics
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United Physical Society of Russian Federation
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Physical Interpretations of Relativity Theory

Proceedings of XIII International Scientific Meeting PIRT-2007

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Editid by M.C. Duffy, V.O. Gladyshev, A.N. Morozov, P. Rowlands

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This volume contains papers which accepted for inclusion in the programme of lectures of meeting "Physical Interpretation of Relativity Theory" which is organized by the Bauman Moscow State Technical University, School of Computing and Technology, University of Sunderland, Liverpool University and British Society for Philosophy of Science.

The meeting is dedicated to the 175th Anniversary of Department of Physics BMSTU.

The most important single objective of the meeting in Summer 2007 is including the advantages of the various physical, astrophysical, geometrical and mathematical interpretations of the formal structure of Relativity Theory; and to examine the philosophical and epistemological questions associated with the various interpretations of the accepted mathematical expression of the Relativity Principle and its development.

The conference is called to examine the various interpretations of the (mathematical) formal structure of Relativity Theory, and the several kinds of physical and mathematical models which accompany these interpretations.

The programme timetable, giving authors and titles of papers as presented and other details of the Moscow Meeting "Physical Interpretation of Relativity Theory" are given on the web site maintained by the Bauman Moscow State Technical University

http://fn.bmstu.ru/phys/nov/konf/pirt2007/pirt2007_en.html.

The meeting is intended to be of interest to physicists, mathematicians, engineers, philosophers and historians of science, post-graduate students.

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Contents

Low-frequency optical resonance in multi-beams Fabry-Perot resonator and problem of gravitational waves detection

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Fabry-Perot interferometer has wide-band character of response that gives considerable advantage when it is used in the interference gravitational antenna [1].

Investigation of a standing electromagnetic wave in the Fabry-Perot cavity (FPC) leads to the conclusion on arising of low-frequency optical resonance, which depends on length between mirrors L_0 and phase tuning δ of the cavity, and also on number of reflections in it [2].

Intensity of optical radiation, going out the FPC, depends on interferometer tuning and characteristics of optical radiation before an entry. Time variations of a distance L_0 between mirrors, an amplitude E_0 or phase φ_0 of electromagnetic wave before an entry of FPC lead to variations of intensities for passed I_T and reflected I_R beams.

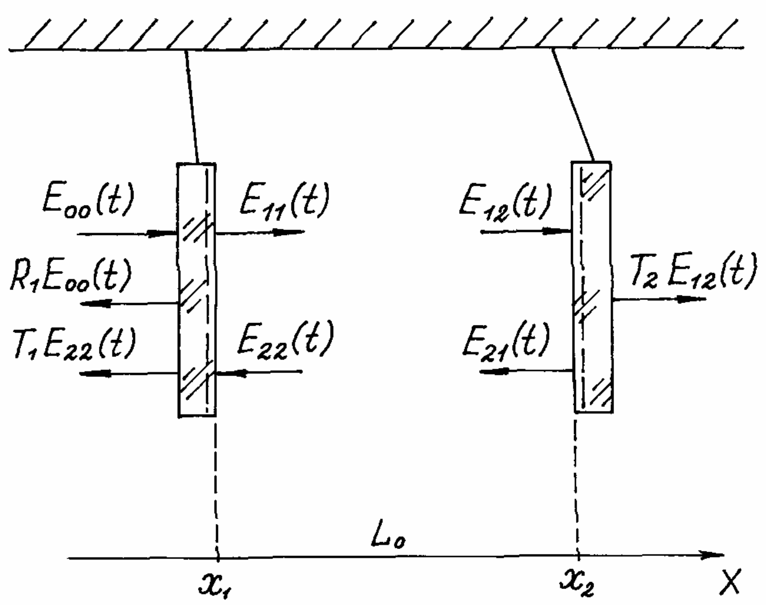

Fig.1. Free-mass multi-beam Fabry-Perot interferometer

Let us consider the Free-mass multi-beam Fabry-Perot cavity, for which a source of radiation is a coherent optical emitter with frequency $\nu_e = \omega_e / 2\pi$ and amplitude of incident light wave $E_0(t)$. Let us the mirror S_1 has the coordinate x_1 and amplitude coefficients of reflection, transmission and absorption: R_1, T_1 and respectively, and the mirror S_2 has the coordinate x_2 and amplitude coefficients of reflection, transmission and absorption R_2, T_2, B_2 (fig.1). Value $L_0 = x_2 - x_1$ is a nonperturbed length of FPC. We can notice that mirrors under action of pressure force of electromagnetic wave must be shifted; as a result the length between mirrors L_0 will depend on optical pumping.

Let us introduce a value $Z(t)$, which is proportional to the sum of amplitudes of all interfering beams [2, 3]. For $Z(t)$ we can present the equation

$$\ddot{Z} + 2\gamma\dot{Z} + \omega_0^2 Z = \dot{E}_0(t) + rE_0(t), \tag{1}$$

where

$$\gamma = -\frac{\ln\left(R_1R_2\right)}{2t_0} - \frac{\ddot{\Phi}(t)}{2\dot{\Phi}(t)}, \tag{2}$$

$$\omega_0^2 = \frac{\ln^2\left(R_1R_2\right)}{4t_0^2} + \dot{\Phi}^2(t) + \frac{\ln\left(R_1R_2\right)}{2t_0}\frac{\ddot{\Phi}(t)}{\dot{\Phi}(t)}, \tag{3}$$

$$r = -\frac{\ln\left(R_1R_2\right)}{2t_0} - \frac{\ddot{\Phi}(t)}{\dot{\Phi}(t)}. \tag{4}$$

$$\dot{\Phi}(t) = \frac{\delta(t)}{2t_0} + \frac{k_e}{t_0}x(t) + \dot{\varphi}_0(t), \tag{5}$$

From the equation (1) it follows that $Z(t)$ obeys an equation for an oscillation with one degree of freedom. If dependences of $x(t)$, $E_0(t)$ or $\varphi_0(t)$ have a view of a harmonic signal with a frequency, coinciding with ω_0, it is arising a low-frequency optical resonance (LFOR), its quality Q is inversely proportional to losses on mirrors $\overline{\Delta}$ [2,3].

Let us consider a case when only the dependence $x(t)$, $\left(E_0(t) = E_0,\ \ \varphi_0(t) = \varphi_0\right)$ is a harmonic that, and conditions $\left|k_e x(t)\right| < \overline{\Delta}$ and $\overline{\Delta} << 1$ fulfill when small movements are measured. Then from equations (2), (3) and (5) for fundamental frequency, damping factor and LFOR quality we can obtain

$$\omega_0 = \frac{\delta}{2t_0}. \qquad \gamma = \frac{\overline{\Delta}}{2t_0}, \qquad Q = \frac{\delta}{2\overline{\Delta}}. \tag{6}$$

For the real parameters δ, $\overline{\Delta}$ and t_0, which are used in multi-beams FPC [1], it can be obtained: $\omega_0 = 10^{-1}...10^{6}$ Hz. LFOR quality, when mirrors with losses of orders $\overline{\Delta} = 10^{-3}...10^{-5}$ are used, can reach $10...10^4$.

Physical reasons of rising LFOR are connected with the fact that electromagnetic waves after n refractions in FPC where $n = \delta / 2\pi$ has a phase coinciding with the initial phase of a wave.

Therefore, we can increase sensibility of FPC to signals $x(t)$, $E_0(t)$ or $\varphi_0(t)$ in the resonance region in 2–3 orders.

To make precise calculation of resonance parameters we should use precise expressions (2)–(5), which have the first and second derivatives of electromagnetic wave phase and the first derivative of mirror shift function.

The equations (1)–(5) were found in the works [2], [3]. The particularly approximate expressions (6) were obtained for optical interferometers with FPC in the works [4]-[6]. In these works equations, in which perturbations of electromagnetic wave phase arose due to altering cavity length when a gravitational wave is harmonical and amplitude and initial phase of electromagnetic wave were constant, are analyzed. Therefore LFOR was not found as to $E_0(t)$ or $\varphi_0(t)$ in these works. More over, additional optical elements were input in the interferometer with FPC in these works. The obtained equation (1) and results of numerical experiments demonstrate that LFOR appears in FPR without additional elements which essentially complicate the interferometer.

The approximation expressions (6) don't contain information about the first and second derivatives of electromagnetic wave phase inside the cavity and also about the first derivative of mirror shift function that is very interesting.

Let us estimate contribution of the third term in (3). If the second and third terms in (3) have the same sign and order of value, we can write

$$\frac{\ln R_1 R_2}{2t_0}\ddot{\Phi} - \dot{\Phi}^3 = 0. \tag{7}$$

After integrating we can obtain

$$\dot{\Phi}^2 = \frac{\ln R_1 R_2}{4t_0\left(t - \hat{t}_0\right)}, \quad t \in \left(0,\, \hat{t}_0 - \Delta\hat{t}_0\right), \quad \Delta t_0 > 0. \tag{8}$$

Here $\hat{t}_0$ – is a constant, $\Delta\hat{t}_0$ – is a small parameter defined from the condition when $\dot{\Phi}$ is maximal in physics sense. Boundaries for the variable t point out influence of additional terms for short and possibly quasi-periodical signals.

For $\dot{\Phi}$ it is valid (5), so with account (8) we can get

$$\frac{1}{2}\sqrt{\frac{\ln R_1 R_2}{t_0\left(t - \hat{t}_0\right)}} = \frac{\delta(t)}{2t_0} + \frac{k_e}{t_0}x(t) + \dot{\varphi}_0(t). \tag{9}$$

This expression indicates that time drift influence on at least one of variables in the right side.

Let us take your attention that $k_e x(t)/t_0 << \delta/2t_0$ for mirror shifts when GW arrives, but the terms have the same value order for amplitude of seismic oscillations $\propto 1\mathring{A}$.

We could assume that seismic oscillations have enough amplitude and appropriate form during some time. Hence, we need to notice that such oscillations can be eliminated by the active service system and in (9) only the first and third terms remain to be essential.

Drift of the terms is also possible, as phase tuning and initial phase are not stable magnitues.

By solving (9) relative to δ, x, φ_0, we get

$$\delta(t) = \sqrt{\frac{t_0}{t - \hat{t}_0}\ln R_1 R_2}, \quad x(t) = \frac{1}{2k_e}\sqrt{\frac{t_0}{t - \hat{t}_0}\ln R_1 R_2}, \quad \varphi_0(t) = \hat{\varphi}_0 - \sqrt{\frac{t_0}{t - \hat{t}_0}\ln R_1 R_2}, \tag{10}$$

where $\hat{\varphi}_0$ is a constant.

If we suppose that one of the obtained dependences is determined in the interval $t \in \left(0,\, \hat{t}_0 - \Delta t_0\right)$, in which a signal is detected, so the second and third terms in (9) have the same sign and order of value.

The result shows that it is possible contribution of additional terms in the real magnitude ω_0. Analogically we can make such analysis for the attenuation coefficient. Hence, a precise solution of the differential equation can be obtained with account variable character of coefficients.

More over, we need account the Doppler effect when we describe free-mass FPC.

Let us suppose that FPC mirrors oscillate according to some law. As a result the first beam after reflection on the mirror S_2 will has accordingly (8) the frequency

$$\omega_1 = \omega_e \frac{1+\beta_2}{1-\beta_2}, \tag{11}$$

where $\beta_2 = v_2 / c$, v_2 is velocity of the mirror S_2.

Motion of mirrors is low-frequency and for a period of n refractions the mirrors stay to be practically immovable.

After n reflections the beam passed FPC will have the frequency

$$\omega_n = \omega_e \left(\frac{1+\beta_1}{1-\beta_1} \frac{1+\beta_2}{1-\beta_2} \right)^n. \tag{12}$$

Additional phase difference, arising due to drift of electromagnetic wave frequency after n reflection cycles, can be calculated as

$$\Delta\varphi_\omega = \omega_e t_0 \sum_{i=1}^{n} \left(\frac{1+\beta_1}{1-\beta_1} \frac{1+\beta_2}{1-\beta_2} \right)^i - \omega_e t, \tag{13}$$

where $t = nt_0$.

Let us $\beta_1 \cong \beta_2 = \beta << 1$, then

$$\Delta\varphi_\omega = 2(n+1)\omega_e \beta t. \tag{14}$$

Phase difference, caused mirror shift, is estimated by the formula

$$\Delta\varphi_x = 2nk_e \Delta x, \tag{15}$$

where Δx is an amplitude of mirror motion.

The relation of $\Delta\varphi_\omega$ to $\Delta\varphi_x$ with account that β is on the level $\beta^{\max} \approx \frac{\Delta x}{Tc}$ and $n >> 1$ is

$$\frac{\Delta\varphi_\omega}{\Delta\varphi_x} = \frac{t}{T}, \tag{16}$$

where T is a period of repeating mirror motions under influence of outer signal.

If the period is compared with time of beam propagation in FPC like in projects of laser interference gravitational antennas, the phase shift due to changing electromagnetic wave frequency when they reflect from mirrors has the same order value as the phase shift due to shift of mirrors.

Therefore we need to have a dependence of electromagnetic wave frequency on velocity of mirror motions.

The phenomenon allows creating an alternative method for GW detection. Really, as phase shift depends on velocity of mirrors motion, so we can increase sizes of FPC. Namely, the cavity with periodic structures as reflecting mirrors is suggested in the works [7], [8].

Increasing effectiveness of measurements of super-small shifts, when GW are detected, is also possible by using LFOR in small size FPR with amplitude modulating of optical pumping [2], [3].

During numerical investigations of the FPC mathematical model it was established that LFOR has influence on interferometer sensibility in low-frequency region and increases possibility of weak signal detection. If frequencies of outer signals are $\omega_{GW} > \omega_0$, amplitude of an response decreases and response envelope has oscillating nature, and the maximum of an optical response can anticipate the maximum of GW signal. The effect with interference nature must be taken into account when different time of gravitational and neutrino splashes are considered [9],[10].

For the reached parameters of the system laser-resonator in the interferometers LIGO and VIRGO with length of shoulders 10^3 m applying LFOR allow increase sensibility of traditional antennae in the factor by 10^3.

For less sizes of the cavity (antenna TAMA with $L_0 = 10^2$ m) low-frequency optical resonance arises and can be used for higher frequences.

Large values L_0, which are planned in space experiments [11], allow to use LFOR for very low frequencies, which correspond to high amplitudes of GW from binary stars.

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What classical differential geometry has to say about the confinement and the stability of motion in the neighbourhood of branes

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In this lecture we study the classical geodesic motions of nonzero rest mass test particles and photons in the neighbourhood of submanifolds embedded in higher-dimensional warped product spaces. We show that it is possible to obtain a general picture of these motions, using the natural decoupling that occurs in such spaces between the motions in the extra dimension and the motion in the hypersurfaces. This splitting allows the use dynamical system techniques in order to investigate the possible confinement of particles and photons to hypersurfaces in five-dimensional and other higher-dimensional warped product spaces. Using such analysis, we find a novel form of quasi-confinement which is oscillatory and neutrally stable. The importance of such kind of confinement is that it is purely due to the classical gravitational effects, without requiring the presence of brane-type confinement mechanisms. We also extend our analysis to Weyl manifolds and show how non-Riemannian fields can provide confinement mechanisms. In a way, these fields may be viewed as the classical counterpart of the quantum scalar field of the braneworld scenario.

Hamiltonian Unification of General Relativity and Standard Model

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The Hamiltonian approach to General Relativity (GR) and Standard Model (SM) is studied in the context of its consistency with the Higgs effect, Newton law, Hubble cosmological evolution, and Cosmic Microwave Background (CMB) radiation physics.

The Hamiltonian formulation admits the dynamic version of the Higgs potential, where its constant parameter is replaced by the dynamic zero Fourier harmonic of the very Higgs field. In this case, the zero mode equation is a new sum-rule that predicts mass of the Higgs field ~310 GeV. The Hamiltonian formulation leads to static interactions playing the crucial role in the off-mass-shell phenomena of the type of bound state and a kaon - pion transition in the weak nonleptonic decays.

We show that the GR&SM theory can be treated as a conformal relativistic brane, where measurable distances are defined as the Weyl ratios of two intervals. The zero mode sector of the GR&SM theory leads to the Conformal Cosmology compatible with the Super Novae luminosity-distance red shift relation. We show that there are initial data of quantum creation of matter at z_I+1= 10^{15}/3. and a value of the Higgs-metric mixing "angle" ~10^{-17} is in agreement with the present-day energy budget of the Universe.

The theory of a laser interferometer receiver of the gravitational waves

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The formalism of calculation of laser interferometric receivers of a gravitational radiation, based on using of the Maxwell equations is advanced. It is shown, that at transition from the Maxwell equations been written in a general covariant form to the equations in terms of three-dimensional intensities and inductions the electric and magnetic permeability tensors are appeared in these equations:. $\varepsilon^{\alpha\beta} = \mu^{\alpha\beta} = -g^{\alpha\beta} = \delta^{\alpha\beta} + h_{\alpha\beta}$, where $h_{\alpha\beta}$ is the metrics of a gravitational wave. In linear approximation on the metrics $h_{\alpha\beta}$ the Maxwell equations are reduced into the wave equations:

$$\Box \boldsymbol{E} = \boldsymbol{F}_{\mathrm{E}}, \quad \Box \boldsymbol{H} = \boldsymbol{F}_{\mathrm{H}}, \tag{1}$$

where

$$\begin{aligned} \boldsymbol{F}_E &= -\frac{\partial^2}{\partial x^{02}}\left(\hat{h}\boldsymbol{E}\right) - \frac{\partial}{\partial x^0}\operatorname{rot}\left(\hat{h}\boldsymbol{H}\right) + \operatorname{grad}\left(\operatorname{div}\left(\hat{h}\boldsymbol{E}\right)\right), \qquad \hat{h} = \{h_{\alpha\beta}\}, \quad \alpha,\beta = 1,2,3. \\ \boldsymbol{F}_H &= -\frac{\partial^2}{\partial x^{02}}\left(\hat{h}\boldsymbol{H}\right) + \frac{\partial}{\partial x^0}\operatorname{rot}\left(\hat{h}\boldsymbol{E}\right) + \operatorname{grad}\left(\operatorname{div}\left(\hat{h}\boldsymbol{H}\right)\right) \end{aligned} \tag{2}$$

In fields of a laser beam with frequency щo and the gravitational wave with a frequency щ the sources $\boldsymbol{F}_{\mathrm{E}}$ and $\boldsymbol{F}_{\mathrm{H}}$ induce a doublet of electromagnetic waves щo ± щ with amplitude proportional to the metric of the gravitational wave. The decision of the equations (1) and (2) is obtained in linear approximation on the two small parameters - the amplitude of the gravitational wave $h_{\alpha\beta}$ and the quantity щ/щo

Two variants of the gravitational wave receivers using the scheme of the Michelson interferometer are considered. In the first variant the optical delay lines (ODL) are inserted into the each arm of an interferometer, in the second variant – the Fabry-Perot (FP) resonators.

The amplitudes of induced waves on outputs of the each arm are obtained for the ODL-receiver:

$$E^- = -r_1^{N-1} r_2^N (1/2) e^{i\omega_o t}\left[\boldsymbol{h}\left(\hat{D}\boldsymbol{A}\right)\right] + c.c., \text{ where } \boldsymbol{h} = \left(1, (1/2)h_{11}e^{i\omega t}, (1/2)h_{11}e^{-i\omega t}\right), \boldsymbol{A} = (E_{\mathrm{o}}, 0, 0),$$

$$\hat{D} = e^{-i\mu}\begin{pmatrix} 1 & 0 & 0 \\ (\omega_{\mathrm{o}}/2\omega)K_x & e^{-i\varepsilon_1} & 0 \\ -(\omega_{\mathrm{o}}/2\omega)K_x^* & 0 & e^{-i\varepsilon_1} \end{pmatrix}, \text{ in which } K_x = \frac{1}{1+\alpha} + \frac{2\alpha}{1-\alpha}e^{-i(1+\alpha)\varepsilon_1/2} - \frac{1}{1-\alpha}e^{-i\varepsilon_1}$$

м1 = kп2L_1 = щo2L_1/c, e1 = k2L_1 = щ2L_1/c, r_1, r_2 are reflectances of a near and a distant mirrors, L_1 is the distance between the mirrors, б = sinиcпsц is the directing cosine of the falling gravitational wave with an axis *x* (directed along one of the interferometer arm). Similar expression for an electric intensity (with shift of a phase of the gravitational wave on p) is obtained for the second arm. The given result generalizes the expression *Vinet* (1986) obtained for orthogonal falling of the gravitational wave.

Rather simply the more bulky expressions for the beams reflected by the Fabry-Perot resonators is obtained.

In all receivers the two beams from outputs of the interferometer arms pass through a beam splitter and interfere on a photodetector in an antiphase (-), or in a phase (+) depending on interferometer adjustment. Intensity measured by the photodetector is $I_{\pm} = \overline{(\boldsymbol{E}_x \pm \boldsymbol{E}_y)(\boldsymbol{E}^*_x \pm \boldsymbol{E}^*_y)} =$ $\overline{\boldsymbol{E}_x \boldsymbol{E}^*_x} + \overline{\boldsymbol{E}_y \boldsymbol{E}^*_y} \pm (\overline{\boldsymbol{E}^*_x \boldsymbol{E}_y} + \overline{\boldsymbol{E}_x \boldsymbol{E}^*_y})$. It is possible to work on the basic (-) or additional (+) beam (on an output from the beam splitter). In the ODL-interferometer the basic and the additional beams are actually equivalent and one convert in another at phase shift equal p. For the output light intensity (the signal on the output of the receiver) in the both types receivers we get

$$I_{\pm} = I_0\left(T_{\pm}(\Delta\mu) + T'_{\pm} h(\omega) H_{ODL}(i\omega) e^{i\omega t} + c.c.\right). \tag{3}$$

In the ODL-receiver Дм = м2 - м1 = $k_o 2\,(L_2 - L_1)$. The first term in (3) - $T_{\pm}$(Дм) is a background intensity which can be removed by adjusting of the interferometer on a dark fringe. At the such adjustment the output signal of the gravitational receiver is $I_{\pm}(t) = I_0 T'_{\pm} \int_{-\infty}^{\infty} h(\omega) H(i\omega) e^{i\omega t} d\omega$. At orthogonal falling a gravitational wave on a interferometer plane the frequency characteristic of the OGL-receiver is $H(\omega)_{ODL} = -i(\mu/2)[\sin(\omega NL/c)]/(\omega NL/c) e^{-i(\omega NL/c)}$. The impulse function of the OGL-receiver is differing from zero only during time when the beam stays in the interferometer: the ODL-receiver has no "memory".

In the FP-receiver the intensity of the interfering beams is also described by the formula (3). For adjustment on the dark fringe the parameters Дм1 and Дм2, equal differences between м1 both м2 and the nearest number multiple 2p, are used. Two modes of adjustment on the dark fringe are possible (1)- resonant adjustment, when the condition Дм1 = Дм2 = Дм– = 0 is satisfied, and
(2)- nonresonant adjustment when are satisfied the two conditions: Дм1 = Дм2 = Дм+ and спsДм+ = $\frac{r_1(1+\sigma r_2^2)}{r_2(1+\sigma r_1^2)}$, у = $t_1^2 + r_1^2$, where t_1 and r_1 are a transparence and a reflectance of the near mirror.
Frequency characteristics of the FP-receivers are some bulky and have no visualization. But the kind of the impulse function is of interest. It is the sum of two similar expressions concerning two interferometer arms. For the basic beam

$$t < 0,\ F_x^{(-)}(t) = 0, \tag{4}$$

$$t > 0,\ F_x^{(-)}(t) = -\frac{\mu c}{16L}(1 - r_1 r_2)(r_1 r_2)^n \cos(\chi + \varphi_x) \begin{cases} \dfrac{1}{1-\alpha} & t/t_0 < n + (1+\alpha)/2 \\ \dfrac{1}{1+\alpha} & t/t_0 > n + (1+\alpha)/2 \end{cases} \tag{5}$$

Here, $t_0 = 2L/c$, $n = [t/t_0]$ is an integral part of the ratio t/t_0, $(r_1 r_2)^n = \exp(-t/\tau)$, $\tau = t_0 \ln(1/r_1 r_2)$, $\cos(\chi + \varphi_x) = h_{11}/(h_+^2 + h_\times^2)^{1/2}(1 - \alpha^2)$. The impulse function has step dependence on time with duration of each "step" $t_0 = 2L/c$. Moreover in limits of one "step" at the moment of time $t_0(1+\alpha)/2$ from the beginning of a pulse the impulse function $F_x^{(-)}(t)$ varies jump on a size proportional 2б. . The impulse function of the additional beam has a sinusoid factor with the frequency $\omega_+ = \Delta\mu_+ / t_0$.

The Spherical Nonstatic solution in Relativistic Theory of Gravitation

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We investigate the spherical gravitational wave type solution in the framework of nonlinear tensor theory of gravity in Minkowski space-time, so called relativistic theory of gravity (RTG) [1]. In this approach the field equations are the Einstein ones for effective metric, which is the sum of Minkowski metric and the tensor gravitational potential h^{ik}. This field equations are added the conditions, restricting the spin states of the tensor field: $D_i(\sqrt{-g}\, g^{ik}) = 0$, where D_i – the covariant derivative in Minkowsky space and g =det g_{ik}. This conditions play the significant role, removing the gauge arbitrariness of Einstein equations and they coincide with harmonic conditions in Galilean coordinates.

In General Relativity according Birkhoff theorem any spherical gravitational field in vacuum is static. The proof of this theorem is ground on the transformation of certain spherically-symmetric metric to the coordinates in which it has a static form. But in Minkowski space such transformation is the transfer from the usual spherical coordinates to some "nonstatic" coordinates. The Birkhoff theorem means that in the case of spherical symmetry the coordinate system in which the vacuum metric depends on one coordinate only always exists, but it not means that the field was static in the starting coordinates [2]. Hence the task of the investigation of nonstatic spherical-symmetric solutions in vacuum arises. In this paper such wave type solution is found in implicit form.

We use Birkhoff theorem and present a nonstatic spherical vacuum solution in certain coordinates in the form of Schwarzschild metric. To find the solution in spherical coordinates we make the coordinate transformation and transformation coefficients will be found from harmonic conditions. We search for the partial solution of this equations, which reduce the harmonic conditions to ordinary differential second order equation. With help this transformation we receive the wave type solution with metric $g_{ik} = g_{ik}(r,u)$, where u is retarded argument [3].

The problem concerning positive definite of gravitational energy density is not trivial, because in RTG the expression t^{00} don't possesses square-law structure relative the field functions and its derivatives and it contains second derivatives too. We analyse the gravitational energy density, which contains the first derivatives only, and which is tensor continuation of known Landau-Lifshits-Fock pseudotensor of General Relativity and which we receive by variational manner.

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Correspondence principle and anisotropic propagation of light in special relativity

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The correspondence principle was taken as the guiding principle to discoveries in the old quantum theory but it has not been properly used as a heuristic principle in special relativity theory. In the present work, we apply the correspondence principle to special relativity (correspondence to classical mechanics in the limit of small velocities is meant) to derive the coordinate transformations between different inertial frames using the Lie group theory apparatus.

It enables us to derive the Lorentz transformations without assumption of linearity. It also provides a basis for discovering some unknown consequences of postulating an anisotropic propagation of light ("non-conventional synchronization") in the theoretical context of special relativity -- the topic that is quite widely discussed in the literature (see, e.g., \cite{R}). In particular, applying our approach together with an assumption of anisotropic propagation of light yields nonlinear coordinate transformations between different frames.

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Dark energy in the Universe - geometrical and physical interpretations

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Recent numerous observational data obtained from such independent sources as angular anisotropies of the temperature of the cosmic microwave background radiation, large-scale gravitational clustering of galaxies and their clusters and observations of supernova explosions at high redshifts prove convincingly that the Universe expands with a positive acceleration at the present time while it was decelerating in the past for redshifts z larger than about 0.7. If interpreted in terms of the Einstein general theory of relativity, this means that about 70\% of the total energy density of matter in the present Universe is due to a new kind of matter in the Universe ("dark energy") which is non-baryonic, has a negative pressure which modulus is very close to the dark energy density and remains unclustered at all scales where clustering of baryons and dust-like cold dark matter is seen. According to this conventional, though consistent definition, dark energy may be both of physical and geometrical origin, with an exact cosmological constant being the limiting case of both types. The latter, simplest possibility provides a good fit to all existing observational data. However, the reconstruction approach to dark energy investigation, that consists in the direct determination of its properties from observational data without assuming a specific model for it, shows that more complicated behaviour including breaking of the weak energy condition for dark energy for $z < 0.5$ combined with some increase of its energy density with redshift for larger z (i.e., the transient phantom behaviour) is not excluded, too. Such behaviour is not possible for physical dark energy in the absence of ghosts and requires some geometrical model of it. The simplest model admitting phantom behaviour which does not have ghosts and instabilities is based on scalar-tensor gravity.

PROBLEMS OF GRAVITATION, COSMOLOGY AND FUNDAMENTAL CONSTANTS

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Gravitation as a fundamental interaction that governs all phenomena at large and very small scales, but still not well understood at a quantum level, is a missing cardinal link to unification of all physical interactions.Discovery of present acceleration of the Universe, dark matter and dark energy problems are also a great challenge to modern physics, which may bring to a new revolution in it. Integrable multidimensional models of gravitation and cosmology make up one of the proper approaches to study basic issues and strong field objects, the Early and present Universe and black hole physics in particular [1, 2, 3]. Our main results within this approach are described both for cosmology and for BH physics. Problems of the absolute G measurements and its possible time and range variations are reflections of the unification problem. A need for further absolute measurements of G, its possible range and time variations is pointed out [4, 5, 6, 7, 8, 9].

1. Introduction

When we think about the most important lines of future developments in physics, we may believe that gravity will be essential not only by itself, but as a missing cardinal link of some theory, unifying all existing physical interactions: week, strong, electromagnetic and gravitational ones. In experimental activities some crucial next generation gravitational experiments verifying predictions of unified schemes will be important. Among them are: STEP - testing the corner stone Equivalence Principle, SEE - testing the inverse square law (or new non-newtonian interactions), EP, possible variations of the newtonian constant G with time, absolute value of G with unprecedented accuracy [10, 11]. Of course, gravitational waves problem, verification of torsional, rotational (GPB), 2nd order and strong field effects remain important also.

We may predict as well that thorough study of gravity itself and within the unified models will give in the next century and millennium even more applications for our everyday life as electromagnetic theory gave us in the 20th century after very abstract fundamental investigations of Faraday, Maxwell, Poincare, Einstein and others, which never dreamed about such enormous applications of their works.

Other very important feature, which may be envisaged, is an increasing role of fundamental physics studies, gravitation, cosmology and astrophysics in particular, in space experiments [12]. Unique microgravity environments and modern technology outbreak give nearly ideal place for gravitational experiments which suffer a lot on Earth from its relatively strong gravitational field and gravitational fields of nearby objects due to the fact that there is no ways of screening gravity.

In the development of relativistic gravitation and dynamical cosmology after A. Einstein and A. Friedmann, we may notice three distinct stages: first, investigation of models with matter sources in the form of a perfect fluid, as was originally done by Einstein and Friedmann. Second, studies of models with sources as different physical fields, starting from electromagnetic and scalar ones, both in classical and quantum cases (see [4]). And third, which is really topical now, application of ideas and results of unified models for treating fundamental problems of cosmology and black hole physics, especially in high energy regimes and for explanation of the greatest challenge to modern physics explaining the present acceleration of the Universe, dark matter and dark energy problems. Multidimensional gravitational models play an essential role in the latter approach [12].

The necessity of studying multidimensional models of gravitation and cosmology [1, 2, 3] is motivated by several reasons. First, the main trend of modern physics is the unification of all known fundamental physical interactions: electromagnetic, weak, strong and gravitational ones. During the recent decades there has been a significant progress in unifying weak and electromagnetic interactions, some more modest achievements in GUT, supersymmetric, string and superstring theories [13].

Now, theories with membranes, p-branes and M-theory are being created and studied. Having no definite successful theory of unification now, it is desirable to study the common features of these theories and their applications to solving basic problems of modern gravity and cosmology. Moreover, if we really believe in unified theories, the early stages of the Universe evolution and black hole physics, as unique superhigh energy regions and possibly even low energy stage, where we observe the present acceleration, are the most proper and natural arena for them.

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Second, multidimensional gravitational models, as well as scalar-tensor theories of gravity, are theoretical frameworks for describing possible temporal and range variations of fundamental physical constants [4, 5, 6, 7, 12]. These ideas have originated from the earlier papers of E. Milne (1935) and P. Dirac (1937) on relations between the phenomena of micro- and macro-worlds, and up till now they are under thorough study both theoretically and experimentally. The possible discovery of the fine structure constant variations is now at a critical further investigation.

Lastly, applying multidimensional gravitational models to basic problems of modern cosmology and black hole physics, we hope to find answers to such long-standing problems as singular or nonsingular initial states, creation of the Universe, creation of matter and its entropy, cosmological constant, coincidence problem, origin of inflation and specific scalar fields which may be necessary for its realization, isotropization and graceful exit problems, stability and nature of fundamental constants [5, 12, 14], possible number of extra dimensions, their stable compactification, new revolutionary data on present acceleration of the Universe, dark matter and dark energy etc.

Bearing in mind that multidimensional gravitational models are certain generalizations of general relativity which is tested reliably for weak fields up to 0.0001 and partially in strong fields (binary pulsars), it is quite natural to inquire about their possible observational or experimental windows. From what we already know, among these windows are:

– possible deviations from the Newton and Coulomb laws, or new interactions,

– possible variations of the effective gravitational constant with a time rate smaller than the Hubble one,

– possible existence of monopole modes in gravitational waves,

– different behavior of strong field objects, such as multidimensional black holes, wormholes, pulsars, QSO, AGN etc.

– standard cosmological tests,

– possible non-conservation of energy in strong field objects and accelerators, if brane-world ideas about gravity in the bulk turn out to be true etc.

Since modern cosmology has already become a unique laboratory for testing standard unified models of physical interactions at energies that are far beyond the level of existing and future man-made accelerators and other installations on Earth, there exists a possibility of using cosmological and astrophysical data for discriminating between future unified schemes. Data on possible time variations or possible deviations from the Newton law should also contribute to the unified theory choice and, of course became a test of cosmological models [15].

As no accepted unified model exists, in our approach [1, 2, 16, 17] we adopted simple (but general from the point of view of number of dimensions) models, based on multidimensional Einstein equations with or without sources of different nature: cosmological constant, perfect and viscous fluids, scalar and electromagnetic fields, their possible interactions, dilaton and moduli fields, fields of antisymmetric forms (related to p-branes) etc.

Our program's main objective was and is to obtain exact self-consistent solutions (integrable models) for these models and then to analyze them in cosmological, spherically and axially symmetric cases. In our view this is a natural and most reliable way to study highly nonlinear systems. In many cases we tried to single out models, which do not contradict available experimental or observational data on variations of G.

As our model [1, 2] we use n Einstein spaces of constant curvature with sources as (m+1)-component perfect fluid, (or scalar and electromagnetic fields or form-fields), cosmological or spherically symmetric metric, manifold as a direct product of factor-spaces of arbitrary dimensions. Then, in harmonic time gauge we show that Einstein multidimensional equations are equivalent to Lagrange equations with non-diagonal in general mini-superspace metric and some exponential potential. After diagonalization of this metric we perform reduction to sigma-model and Toda-like systems, further to Liouville, Abel, generalized Emden-Fowler equations etc. and try to find exact solutions. We suppose that behavior of extra spaces is the following: they are constant, or dynamically compactified, or like torus, or large, but with barriers, walls etc.

So, we realized our program in arbitrary dimensions (from 1988) [1, 2, 3, 16, 17].

In cosmology:

We obtained and studied exact general solutions of multidimensional Einstein equations with sources:

- Λ , Λ + scalar field (e.g. nonsingular, dynamically compactified, inflationary);

- perfect fluid, PF + scalar field (e.g. nonsingular, inflationary solutions);

- viscous fluid (e.g. nonsingular, generation of mass and entropy, quintessence and coincidence in 2-component model);

- stochastic behavior near the singularity, billiards in Lobachevsky space (D=11 is critical, φ destroys billiards (1994)).

For all above cases Ricci-flat solutions above are obtained for any n, also with curvature in one factor-space for arbitrary n; with curvatures in 2 factor-spaces only for total N=10, 11;

- fields: scalar, dilatons, forms of arbitrary rank (1998) - inflationary, Λ generation by forms (p-branes) [18];

- first billiards for dilaton-forms (p-branes) interaction (1999);

- quantum variants (solutions of WDW-equation [19]) for all above cases where classical solutions were obtained;
- dilatonic fields with potentials, billiard behavior for them also.

For many of these integrable models we calculated also the variation with time of the effective gravitational constant and comparison with present experimental bounds allowed to choose particular models or single out some classes of solutions.

Solutions depending on r in any dimensions:

- generalized Schwarzchild, generalized Tangerlini (BH's are singled out), also with minimal scalar field (no BH's);
- generalized Reissner-Nordstrom (BH's also are singled out), the same plus φ (no BH's);
- multi-temporal;
- for dilaton-like interaction of φ and e.-m. fields (BH's exist only for a special case);
- stability studies (stable solutions only for BH's case above);
- the same for dilaton-forms (p-branes) interaction, stability found only in some cases, e.g. for one form in particular.

PPN-parameters for most of the models were calculated.

2. Multidimensional Models

In all these theories, 4-dimensional gravitational models with extra fields were obtained from some multidimensional model by dimensional reduction based on the decomposition of the manifold

$$M = M^4 \times M_{\rm int}, \tag{1}$$

where M^4 is a 4-dimensional manifold and $M_{\rm int}$ is some internal manifold (mostly considered to be compact).

The earlier papers on multidimensional gravity and cosmology dealt with multidimensional Einstein equations and with a block-diagonal cosmological or spherically symmetric metric defined on the manifold $M = R \times M_0 \times \ldots \times M_n$ of the form

$$g = -dt \otimes dt + \sum_{r=0}^{n} a_r^2(t) g^r \tag{2}$$

where (M_r, g^r) are Einstein spaces, $r = 0, \ldots, n$. In some of them a cosmological constant and simple scalar fields were also used [19].

Such models are usually reduced to pseudo-Euclidean Toda-like systems with the Lagrangian

$$L = \frac{1}{2} G_{ij} \dot{x}^i \dot{x}^j - \sum_{k=1}^{m} A_k e^{u_i^k x^i} \tag{3}$$

and the zero-energy constraint $E = 0$ [20].

Cosmological solutions are closely related to solutions with spherical symmetry [21]. Moreover, the scheme of obtaining the latter is very similar to the cosmological approach [1, 22].

At present there exists a special interest to the so-called M- and F-theories etc. These theories are “supermembrane" analogues of the superstring models in $D = 11, 12$ etc. The low-energy limit of these theories leads to multidimensional models with p-branes.

In our papers several classes of exact solutions for the multidimensional gravitational model governed by the Lagrangian

$$\mathcal{L} = R[g] - 2\Lambda - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_a \frac{1}{n_a!} \exp(2\lambda_{a\alpha}\varphi^\alpha)(F^a)^2, \tag{4}$$

were considered. Here g is metric, $F^a = dA^a$ are forms of ranks n_a and φ^α are scalar fields and Λ is a cosmological constant (the matrix $h_{\alpha\beta}$ is invertible).

Cosmological models in diverse dimensions

Scalar fields play an essential role in modern cosmology. They are attributed to inflation models of the early universe and the models describing the present stage of the accelerated expansion as well. There is no unique candidate for the potential of the minimally coupled scalar field. Typically a potential is a sum of exponents. Such potentials appear quite generically in a large class of theories: multidimensional, Kaluza-Klein models, supergravity and string/M - theories.

Single exponential potential was extensively studied within Friedmann model containing both a minimally coupled scalar field and a perfect fluid with the linear barotropic equation of state . The attention was mainly focussed on the qualitative behavior of solutions, stability of the exceptional solutions to curvature and shear perturbations and their possible applications within the known cosmological scenario such as inflation and scaling ("tracking") . In particular, it was found by a phase plane analysis that for "flat" positive potentials there exists an unique late-time attractor in the form of the scalar dominated solution. It is stable within homogeneous and isotropic models with non-zero spatial curvature with respect to spatial curvature perturbations and provides the power-law inflation. For "intermediate" positive potentials an unique late-time attractor is the scaling solution, where the scalar field "mimics" the perfect fluid, adopting its equation of state. The energy-density of the scalar field scales with that of the perfect fluid. Using our methods for multidimensional cosmology the problem of integrability by quadratures of the model in 4-dimensions was also studied. Four classes of general solutions, when the parameter characterizing the steepness of the potential and the barotropic parameter obey some relations, were found [39]. For the case of multiple exponential potential of the scalar field and dust integrable model in 4D was obtained in [40]

As to scalar fields with the multiple exponential potential in any dimensions, it's not studied well yet although a wide class of exact solutions was obtained in our papers [29, 42]. In our recent work [31] a behavior of this system near the singularity was studied using a billiard approach suggested earlier in our papers [30, 28]. A number of S-brane solutions were found in [32, 33].

Details for 2-component D-dimensional integrable models see in [43, 37, 38]).Quite different model with dilaton, branes and cosmological constant and static internal spaces was investigated in [18], where possible generation of the effective cosmological constant by branes was demonstrated. Model with variable equations of state see in [41] with acceleration of our space and compactification of internal spaces.

Cosmological models with time variations of G.

As we mentioned before cosmological models in scalar-tensor and multidimensional theories are the framework for describing possible variations with time of fundamental physical constants due to scalar fields present in STT or generated by extra dimensions in multidimensional approach. In [44] we obtained solutions for the system of conformal scalar and gravitational fields in 4D and calculated the present possible relative variation of G at the level of less than $10^{-12} year^{-1}$. Later in the frames of a multidimensional model with a perfect fluid and 2 factor spaces (our 3D space of Friedmann open, closed and flat models) and internal 6D Ricci-flat one we obtained the same limit for such variation of G [9].

We estimated also the possible variations of the gravitational constant G in the framework of a generalized (Bergmann-Wagoner-Nordtvedt) scalar-tensor theory of gravity on the basis of field equations without using their special solutions. Specific estimates were essentially related to the values of other cosmological parameters (the Hubble and acceleration parameters, the dark matter density etc.), but the values of G-dot$/G$ compatible with modern observations do not exceeded 10^{-12} per year [54].

In [53] we continued the studies of models in arbitrary dimensions and obtained the relations for G-dot in multidimensional model with Ricci-flat internal space and multicomponent perfect fluid. A two-component example: dust + 5-brane, was considered. It was shown that G-dot$/G$ is less than $10^{-12} year^{-1}$. Expressions for G-dot were considered also in a multidimensional model with an Einstein internal space and a multicomponent perfect fluid [56]. In the case of two factor-spaces with non-zero curvatures without matter, a mechanism for prediction of small G-dot was suggested. The result was compared with exact (1+3+6)-dimensional solutions, obtained by us earlier.

Multidimensional cosmological model describing the dynamics of $n+1$ Ricci-flat factor-spaces M_i in the presence of a one-component anisotropic fluid was considered in [77]. The pressures in all spaces were supposed to be proportional to the density: $p_i = w_i \rho$, i = 0,...,n. Solutions with accelerated power-law expansion of our 3-space M_0 and small enough variation of the gravitational constant G were found. These solutions exist for two branches of the equation of state parameter w_0. The first branch describes the super-stiff matter with $w_0 > 1$, the second one may contain a phantom matter with $w_0 < -1$, e.g., when G grows with time, so this branch may describe not present, but earlier stages only.

Similar exact solutions, but nonsingular and with an exponential behavior of the scale factors were considered in [78] for the same multidimensional cosmological model describing the dynamics of $n+1$ Ricci-flat factor spaces M_i in the presence of a one-component perfect fluid. Solutions with accelerated exponential expansion of our 3-space M_0 and small variation of the gravitational constant G were found also.

Exact S-brane solutions with 2 electric branes and 2 phantom scalar fields were obtained and studied in [79]. We got the asymptotic accelerated expansion of our 3-dimensional factor space and variations, obeying the present experimental constraint of G-dot$/G$ equal or less than $10^{-12} year^{-1}$.

Some specific models in classical and quantum multidimensional cases with p-branes were analyzed in spherical symmetry [1, 2, 3, 16, 17]. Exact solutions for the system of scalar fields and fields of forms with a dilatonic type interactions for *generalized intersection rules* were studied in [26], where the PPN parameters were also calculated. Other problems connected with observations were studied in [34, 36] and general properties of BH's in a braneworld in [35].

Also, a *stability* analysis for solutions with p-branes was carried out [27, 50]. It was shown there that for some simple p-brane systems multidimensional black branes are stable under monopole perturbations while other (non-BH) sperically symmetric solutions turned out to be unstable.

Below we dwell mainly upon some problems of fundamental physical constants, the gravitational constant in particular, upon the SEE and laboratory projects to measure G and its possible variations shortly.

3. Fundamental physical constants

1. In any physical theory we meet constants which characterize the stability properties of different types of matter: of objects, processes, classes of processes and so on. These constants are important because they cannot be calculated via other constants, arise independently in different situations and have the same value, at any rate within accuracies we have gained nowadays. That is why they are called fundamental physical constants (FPC) [4, 12]. It is impossible to define strictly this notion. It is because the constants, mainly dimensional, are present in definite physical theories. In the process of scientific progress some theories are replaced by more general ones with their own constants, some relations between old and new constants arise. So, we may talk not about an absolute choice of FPC, but only about a choice corresponding to the present state of physical sciences.

Now, when the theory of electroweak interactions has a firm experimental basis and we have some good models of strong interactions, a more preferable choice is as follows:

$$\hbar,\ (c),\ e,\ m_e,\ \theta_w,\ G_F,\ \theta_c,\ \Lambda_{QCD},\ G,\ H,\ \rho,\ \Lambda,\ k,\ I \tag{5}$$

and, possibly, three angles of Kobayashi-Maskawa — θ_2, θ_3 and δ. Here α, G_F and G are constants of electromagnetic, weak, strong and gravitational interactions, H, ρ and Λ are cosmological parameters (the Hubble constant, mean density of the Universe and cosmological constant), k and I are the Boltzmann constant and the mechanical equivalent of heat, which play the role of conversion factors between temperature on the one hand, energy and mechanical units on the other, θ_w is the Weinberg angle, θ_c is the Cabibbo angle and Λ_{QCD} is the cut-off parameter of quantum chromodynamics. Of course, if a theory of four known now interactions will be created (M-, F- or other), then we will probably have another choice. From the point of view of these unified models all above mentioned ones are low energy constants.

FPC are known with different *accuracies*. The most precisely defined constant was and remain the speed of light c: its accuracy was 10^{-10} and now it is defined with the null accuracy. Atomic constants, e, $\hbar$, m and others are determined with errors $10^{-6} \div 10^{-8}$, G up to 10^{-4} or even worse, θ_w — up to 10^{-3}; the accuracy of H is about several percents. Other cosmological parameters (FPC): mean density estimations vary also within 2 percent; for Λ we have now data that its corresponding energy density exceeds the matter density (0.7 and 0.3 of the total universe mass correspondingly).

As to the *nature* of the FPC, we may mention several approaches. One of the first hypotheses belongs to J.A. Wheeler: in each cycle of the Universe evolution the FPC arise anew along with physical laws which govern this evolution. Thus, the nature of the FPC and physical laws are connected with the origin and evolution of our Universe.

A less global approach to the nature of dimensional constants suggests that they are needed to make physical relations dimensionless or they are measures of asymptotic states. Really, the speed of light appears in relativistic theories in factors like v/c, at the same time velocities of usual bodies are smaller than c, so it plays also the role of an asymptotic limit. The same sense have some other FPC: $\hbar$ is the minimal quantum of action, e is the minimal observable charge (if we do not take into account quarks which are not observable in a free state) etc.

Finally, FPC or their combinations may be considered as natural scales determining the basic units. While the earlier basic units were chosen more or less arbitrarily, i.e., the second, meter and kilogram, now the first two are based on stable (quantum) phenomena. Their stability is believed to be ensured by physical laws which include FPC. There appeared similar suggestions for a new reproducible realization of a kg, fixing values of N_A or other constants, e.g. $\hbar$ (Metrologia, 2005).

An exact knowledge of FPC and precision measurements are necessary for testing main physical theories, extension of our knowledge of nature and, in the long run, for practical applications of fundamental theories. Within this, such theoretical problems arise:

1) development of models for confrontation of theory with experiment in critical situations (i.e. for verification of GR, QED, QCD, GUT or other unified models);

2) setting limits for spacial and temporal variations of FPC. It is becoming especially important now with the idea to introduce new basic units of International System of Units (SI), based completely on fundamental physical constants.

As to a *classification* of FPC, we may set them now into four groups according to their generality:

1) Universal constants such as $\hbar$, which divides all phenomena into quantum and non-quantum ones (micro- and macro-worlds) and to a certain extent c, which divides all motions into relativistic and non-relativistic ones;

2) constants of interactions like α, θ_w, Λ_{QCD} and G;

3) constants of elementary constituencies of matter like m_e, m_w, m_x, etc., and

4) transformation multipliers such as k, I and partially c (conversion from the second to the meter). Soon there may be more after modernization of SI in 2011 - values of $\hbar$ and N_A may be fixed with zero uncertainty.

Of course, this division into classes is not absolute. Many constants moved from one class to another. For example, e was a charge of a particular object – electron, class 3, then it became a characteristic of class 2 (electromagnetic interaction, $\alpha = \frac{e^2}{\hbar c}$ in combination with $\hbar$ and c); the speed of light c has been in nearly all classes: from 3 it moved into 1, then also into 4. Some of the constants ceased to be fundamental (i.e. densities, magnetic moments, etc.) as they are calculated via other FPC.

As to the *number* of FPC, there are two opposite tendencies: the number of "old" FPC is usually diminishing when a new, more general theory is created, but at the same time new fields of science arise, new processes are discovered in which new constants appear. So, in the long run we may come to some minimal choice which is characterized by one or several FPC, maybe connected with the so-called Planck parameters — combinations of c, $\hbar$ and G (natural, or Planck system of units [12, 14]):

$$
\begin{aligned}
L &= \left(\frac{\hbar G}{c^3}\right)^{1/2} \sim 10^{-33}\ \text{cm},\\
m_L &= (c\hbar/2G)^{1/2} \sim 10^{-5}\ \text{g},\\
\tau_L &= L/c \sim 10^{-43}\ \text{s}.
\end{aligned} \tag{6}
$$

The role of these parameters is important since m_L characterizes the energy of unification of four known fundamental interactions: strong, weak, electromagnetic and gravitational ones, and L is a scale where the classical notions of space-time loose their meaning. There are other ideas about the final number of FPC (2, 1, or none). Of course, all will depend on a future unified theory.

2. The problem of the gravitational constant G measurement and its stability is a part of a rapidly developing field, called gravitational-relativistic metrology (GRM). It has appeared due to the growth of measurement technology precision, spread of measurements over large scales and a tendency to the unification of fundamental physical interaction [7], where main problems arise and are concentrated on the gravitational interaction. The main subjects of GRM are:

- general relativistic models for: a) different astronomical scales: Earth, Solar System, galaxies, cluster of galaxies, cosmology; b) for time transfer, VLBI, space dynamics, relativistic astrometry etc.;
- development of generalized gravitational theories and unified models for testing their effects in experiments;
- fundamental physical constants, G in particular, and their stability in space and time; space projects μSCOPE, STEP, SEE,...
- fundamental cosmological parameters as fundamental constants: cosmological models studies (quintessence, k-essence, phantom, multidimensional ones), measurements and observations; PLANCK, ...
- gravitational waves (detectors, sources...); LIGO, VIRGO, TAMA, LISA, RADIOASTRON,...
- basic standards (atomic clocks) and other modern precision devices (atomic and neutron interferometry, atomic force spectroscopy etc.) in fundamental gravitational experiments, especially in space for testing GR and other theories : rotational, torsional and second order effects (need uncertainty 10^{-6}), e.g. LAGEOS, Gravity Probe B, ASTROD, LATOR etc.

We are now on the level $2.3 \cdot 10^{-5}$ in measuring PPN-parameter γ and $5 \cdot 10^{-4}$ - for β, Brans-Dicke parameter $\omega > 40000$. Proposed future space missions with aimed accuracy of γ are:

1. GP-B (geodetic precession) – 10^{-5}.
2. Bepi-Colombo (retardation) – 10^{-6}.
3. GAIA (deflection) – (10^{-5}Ц – 10^{-7}).
4. ASTROD I (W.-T. Ni) (retardation) – 10^{-7}.
5. LATOR (Turyshev et. al.) – 10^{-8}.
6. ASTROD II (W.-T. Ni) – 10^{-9}.

3. There are three problems related to G, which origin lies mainly in unified models predictions:

1) absolute G measurements, 2) possible time variations of G, 3) possible range variations of G – non-Newtonian, or new interactions.

Absolute measurements of G. There were many laboratory determinations of G with errors of the order 10^{-3} and only 4 were on the level of 10^{-4} in 80's.

The official CODATA value of 1986 was

$$G = (6,67259 \pm 0.00085) \cdot 10^{-11} \cdot m^3 \cdot kg^{-1} \cdot s^{-2}, \tag{7}$$

based on Luther and Towler determination. But after precise measurements of G by different groups the situation became more vague.

As one may see from the Cavendish conference data of 1998 [52], the results of 7 groups could agree with each other only on the level 10^{-3}. So, CODATA adopted in 1999:

$$G = (6.673 \pm 0.001) \cdot 10^{-11} \cdot m^3 \cdot kg^{-1} \cdot s^{-2} \tag{8}$$

But, from 2004 CODATA gives:

$$G = 6.6742(10) \cdot 10^{-11} \cdot m^3 \cdot kg^{-1} \cdot s^{-2}. \tag{9}$$

So, we see that we are not too far (a little more one order) from Cavendish, who obtained value of G 2 centuries ago at the level 10^{-2}. The situation with the measurement of the absolute value of G is really different from atomic constants values and their uncertainties. This means that either the limit of terrestrial accuracies of defining G has been reached or we have some new physics entering the measurement procedure [7]. The first means that, maybe we should turn to space experiments to measure G [12, 11], and and second means that a more thorough study of theories generalizing Einstein's general relativity or unified theories is necessary.

There exist also some satellite determinations of G (namely $G \cdot M_{\text{Earth}}$) on the level of 10^{-9} (so, should we know G much better, our knowledge of masses of the Earth and other planets will be much better and consequently their models).

The precise knowledge of G is necessary, first of all, as it is a FPC; next, for the evaluation of mass of the Earth, planets, their mean density and, finally, for construction of Earth models; for transition from mechanical to electromagnetic units and back; for evaluation of other constants through relations between them given by unified theories; for finding new possible types of interactions and geophysical effects; for some practical applications like increasing of modern gradiometers precision, as they demand a calibration by a gravitational field of a standard body depending on G: high accuracy of their calibration (10^{-5} - 10^{-6}) requires the same accuracy of G.

The knowledge of constants values has not only a fundamental meaning but also a metrological one. The modern system of standards is based mainly on stable physical phenomena. So, the stability of constants plays a crucial role. As all physical laws were established and tested during the last 2-3 centuries in experiments on the Earth and in the near space, i.e. at a rather short space and time intervals in comparison with the radius and age of the Universe, the possibility of slow *variations* of constants (i.e. with the rate of the evolution of the Universe or slower) cannot be excluded a priori.

Time Variations of G. The problem of variations of FPC arose with the attempts to explain the relations between micro- and macro-world phenomena. He suggested that the ratio of the gravitational to strong interaction strengths, $Gm_p^2/\hbar c \sim 10^{-40}$, is inversely proportional to the age of the Universe: $Gm_p^2/\hbar c \sim T^{-1}$.

After the original *Dirac hypothesis* some new ones appeared (Gamov, Teller, Landau, Terazawa, Staniukovich etc., see [4, 12]) and also some generalized theories of gravitation admitting the variations of an effective gravitational coupling. We may single out three stages in the development of this field:

1. Study of theories and hypotheses with variations of FPC, their predictions and confrontation with experiments (1937-1977).

2. Creation of theories admitting variations of an effective gravitational constant in a particular system of units, analysis of experimental and observational data within these theories [25, 4] (1977-present).

3. Analysis of FPC variations within unified models [7, 5, 1] (present).

In [25, 44, 45] the conception was worked out that variations of constants are not absolute but depend on the system of measurements (choice of standards, units and devices using this or that fundamental interaction). Each fundamental interaction through dynamics, described by the corresponding theory, defines the system of units and the corresponding system of basic standards, e.g. atomic and gravitational (ephemeris) seconds.

There are different astronomical, geophysical and laboratory *data* on possible variations of FPC [12]. We must point out that all astronomical and geophysical estimations are strongly model-dependent. So, of course, it is always desirable to have *laboratory tests* of variations of FPC [14]. The most strict data on variations of strong, electromagnetic and week interaction constants were obtained by A. Schlyachter in 1976 (Russia) from an analysis of the ancient natural nuclear reactor data in Gabon, Oklo, because the event took place $2 \cdot 10^9$ years ago. They are the following:

$$
\begin{aligned}
|\dot{G}_s/G_s| &< 5 \cdot 10^{-19} \text{ year}^{-1}, \\
|\dot{\alpha}/\alpha| &< 10^{-17} \text{ year}^{-1}, \\
|\dot{G}_F/G_F| &< 2 \cdot 10^{-12} \text{ year}^{-1}.
\end{aligned} \tag{10}
$$

Some studies of strong interaction constant and its dependance on momenta may be found in [13]. The recent review on variations of α see in [59].

There appeared some data on a possible variation of α on the level of 10^{-16} at some z [60]. Other groups do not support these results. Also appeared data on possible violation of m_e/m_p (Varshalovich et al.) The problem may be that even if they are correct, all these results are mean values of variations at some epoch of the evolution of the Universe (certain z interval). In essence variations may be different at different epochs (if they exist at all) and at the next stage observational data should be analyzed with the account of evolution of corresponding ("true"?)cosmological models.

Now we still have no unified theory of all four interactions. So it is possible to construct systems of measurements based on any of these four interactions. But practically it is done now on the basis of the mostly worked out theory — on electrodynamics (more precisely on QED). Of course, it may be done also on the basis of the gravitational interaction (as it was partially earlier). Then, different units of basic physical quantities arise based on dynamics of the given interaction, i.e. the atomic (electromagnetic) second, defined via frequency of atomic transitions or the gravitational second defined by the mean Earth motion around the Sun (ephemeris time).

It does not follow from anything that these two seconds are always synchronized in time and space. So, in principal they may evolve relative to each other, for example at the rate of the evolution of the Universe or at some slower rate.

That is why, in general, variations of the gravitational constant are possible in the atomic system of units (c, $\hbar$, m are constant, Jordan frame) and masses of all particles — in the gravitational system of units (G, $\hbar$, c are constant by definition, Einstein frame). Practically we can test only the first variant since the modern basic standards are defined in the atomic system of measurements. Possible variations of FPC must be tested experimentally but for this it is necessary to have the corresponding theories admitting such variations and their certain effects.

Mathematically these systems of measurement may be realized as conformally related metric forms. Arbitrary conformal transformations give us a transition to an arbitrary system of measurements.

We know that scalar-tensor and multidimensional theories are corresponding frameworks for these variations. So, one of the ways to describe variable gravitational coupling is the introduction of a *scalar field* as an additional variable of the gravitational interaction. It may be done by different means (e.g. Jordan, Brans-Dicke, Canuto and others). We have suggested earlier a variant of gravitational theory with a conformal scalar field (Higgs-type field [46, 4]), where Einstein's GR may be considered as a result of spontaneous symmetry breaking of conformal symmetry (Domokos, 1976) [4]. In our variant spontaneous symmetry breaking of the global gauge invariance leads to a nonsingular cosmology [47]. Besides, we got variations of the effective gravitational constant in the atomic system of units when m, c, $\hbar$ are constant and variations of all masses in the gravitational system of units (G, c, $\hbar$ are constant). It was done on the basis of approximate [48] and exact cosmological solutions with local inhomogeneity [49].

As to experimental or observational data on variations of G, the results are of different quality. The most reliable ones are based on lunar laser ranging (Muller et al, 1993, Williams et al, 1996, Nordtvedt, 2003). They are not better than 10^{-12} per year. Here, once more we see that there is a need for corresponding theoretical and experimental studies. Probably, future space missions like Earth SEE-satellite [10, 11, 12, 14] or missions to other planets and lunar laser ranging will be a decisive step in solving the problem of temporal variations of G and determining the fates of different theories which predict them, since the greater is the time interval between successive measurements and, of course, the more precise they are, the more stringent results will be obtained.

Different theoretical schemes lead to temporal variations of the effective gravitational constant. As was shown in [5, 51, 1] temporal variations of FPC may be connected with each other in *multidimensional models* of unification of interactions. So, experimental tests on $\dot{\alpha}/\alpha$ may at the same time be used for estimation of $\dot{G}/G$ and vice versa. Moreover, variations of G are related also to the cosmological parameters ρ, Ω and q which gives opportunities of raising the precision of their determination.

As variations of FPC are closely connected with the behavior of internal scale factors, it is also a direct probe of properties of extra dimensions [8, 9, 1]. From this point of view it is an additional test of not only gravity and cosmology, but unified theories of physical interactions as well.

Non-Newtonian interactions, or range variations of G. Nearly all modified theories of gravity and unified theories predict some deviations from the Newton law (inverse square law, ISL) or composition-dependent violations of the

Equivalence Principle (EP) due to appearance of new possible massive particles (partners) [5]. Experimental data exclude the existence of these particles on a very good level at nearly all ranges except less than *millimeter* and also at *meters and hundreds of meters* ranges. Our recent analysis of experimental bounds and new limits on possible ISL violation using the new method and modern precession data from satellites, planets, binary pulsar and LLR data were obtained in [57].

In the Einstein theory G is a true constant. But, if we think that G may vary with time, then, from a relativistic point of view, it may vary with distance as well. In GR massless gravitons are mediators of the gravitational interaction, they obey second-order differential equations and interact with matter with a constant strength G. If any of these requirements is violated, we come in general to deviations from the Newton law with range (or to generalization of GR).

In [6] we analyzed several classes of such theories:

1. Theories with massive gravitons like bimetric ones or theories with a Λ-term. 2. Theories with an effective gravitational constant like the general scalar-tensor ones. 3. Theories with torsion. 4. Theories with higher derivatives (4th-order equations etc.), where massive modes appear leading to short-range additional forces. 5. More elaborated theories with other mediators besides gravitons (partners), like supergravity, superstrings, M-theory etc. 6. Theories with nonlinearities induced by any known physical interactions (Born-Infeld etc.) 7. Phenomenological models where the detailed mechanism of deviation is not known (fifth or other force).

In all these theories some effective or real masses appear leading to Yukawa-type (or power-law) deviations from the Newton law, characterized by strength α and range λ.

There exist some model-dependant estimations of these forces [12]. The most well-known one belongs to Scherk (1979) from supergravity where the graviton is accompanied by a spin-1 partner (graviphoton) leading to an additional repulsion.

Some p-brane models (ADD, branewolds) also predict non-Newtonian additional interactions of both Yukawa or power-law, in particular in the less than mm range, what is intensively discussed nowadays [14, 58]. About PPN parameters for multidimensional models with p-branes see [12].

4. Pioneer anomaly.

Some evidence on a possible violation of Newton's Law has come to us from space, namely, from data processing on the motion of the spacecrafts Pioneer 10 and 11, at length ranges of the order of or exceeding the size of the Solar system. The discovered anomalous (additional) acceleration is [63]

$$(8.60 \pm 1.34)\cdot 10^{-8}\ \mathrm{cm/s^2},$$

it acts on the spacecrafts and is directed towards the Sun. This acceleration is not explained by any known effects, bodies or influences related to the design of the spacecrafts themselves (leakage etc.), as was confirmed by independent calculations.

Many different approaches have been analyzed both in the framework of standard theories and invoking new physics, but none of them now seems to be sufficiently convincing and generally accepted. There are the following approaches using standard physics:

— an unknown mass distribution in the Solar system (Kuiper's belt), interplanetary or interstellar dust, local effects due to the Universe expansion [62];

— employing the Schwarzschild solution with an expanding boundary [63, 64] etc.

Among the approaches using new physics one can mention:

— a variable cosmological constant [65];

a variable gravitational constant [68];

— a new PPN-theory connecting local scales with the cosmological expansion [66];

— the five-dimensional Kaluza-Klein (KK) theory with a time-variable fifth dimension and varying fundamental physical constants [67];

— Moffat's [69] non-symmetric gravitational theory;

— Milgrom's [70, 71] modified Newtonian dynamics (MOND);

— special scalar-tensor theories of gravity [72];

— approaches using some ideas of multidimensional theories [73];

— modified general relativity with a generalized stress-energy tensor [74] etc.

This Pioneer anomaly has caused new proposals of space missions with more precise experiments and a wide spectrum of research at the Solar system length range and beyond:

— Cosmic Vision 2015-2025, suggested by the European Space Agency, and

— Pioneer Anomaly Explorer, suggested by NASA [75]. So, we hope they contribute a lot to our knowledge of gravity and unified models.

7. SEE and laboratory projects.

We saw that there are three problems connected with G. There is a promising new multi-purpose space experiment project SEE - Satellite Energy Exchange [10, 11], which addresses all these problems and may be more effective in solving them than other laboratory or space experiments.

We studied many aspects of the SEE-project [11, 12] and the general conclusion is that realization of the SEE-project may improve our knowledge of G, G-dot and $G(r)$ by 3-4 orders.

Another (laboratory) variant was suggested in our paper [76] to test possible range variations of G. It is the experiment on possible detection of new forces, or test of the inverse square law, parameterized by Yukawa-type potential with strength α and range λ. It was shown that the sensitivity of the method suggested may be on the level of α - 10^{-10} in the range of λ - $(0.1 - 10^7)m$ in the space of Yukawa parameters (α, λ).

Some new estimations of temporal variations of G within multidimensional models, showing present acceleration of the Universe were done in [77, 78, 79].

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Cosmological model

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Geometry and composition of the visible Universe are discussed. Physics stays on the verge of new discoveries that will change the essence of our world-view. The matter concerns origin of cosmological initial conditions and nature of dark matter.

1. Introduction

The progress in cosmology is ensured by observations. Discoveries made by observational cosmology have led us to a new understanding of the Universe. Today we know the model at large scales. After many years of hypotheses and markets of models we now have the standard cosmological model, yet separated from what we have at small scales – the standard model of elementary particles. Both models progressively converge and interact with each other leading us to a joint physical model of the World we are a part of.

Astronomers see structures unknown to physicists. They cannot touch or test them, they can learn only general properties of observed matters assuming some theoretical extrapolations (General Relativity, atomic physics, etc.). On the contrary, physicists need experiment to judge things. To understand what astronomers see, physicists are looking in labs for what is unknown to them, since there is not enough information about the target. In this way the problem of identification arises.

What do astronomers see?

They observe structures made of invisible matter, the *dark matter*. DM does not interact with light, generally - with luminous matter, or baryons. How is DM observed then? Through its gravitational influence on visible matter.

Fortunately, light is there where DM concentrations are. Fig. 1 shows a region of sky in the direction of one of DM halos, the non-linear DM concentration gravitationally bound in all three directions with total mass $\sim 10^{14} M_{\odot}$. We see optical galaxies captured by gravitational field of this concentration, X-ray gas residing at the bottom of the gravitational well, and a multi-image of one of the background galaxies that happened to be on the line of sight of the DM halo.

We study spatial distribution of DM systems analyzing galaxy catalogs and quasar absorption lines. Besides, the DM surface mass density can be reconstructed via its gravitational week lensing action on numerous background galaxies. Hence, there is more than enough independent probes of dark mass inside and beyond DM halos. We can state that the mean contrast of DM density field is larger than unity at small scale (<10 Mpc) still remaining less than unity at large scale (≥ 10 Mpc). Accordingly, we do not find DM halos exceeding $10^{15} M_{\odot}$.

Thus, we know the current DM density field. Also, we have a map of much younger matter density field using CMB anisotropy. That time ($z \sim 1000$) the mean density contrast was $\sim 10^{-5}$, and no halos had formed yet. Having these two pictures of cosmic matter distribution at different epochs of its evolution and assuming that only gravity is responsible for such evolution, we can obtain the DM energy-momentum tensor.

What are DM properties?

Actually, they are simple. DM is *weakly interacting massive particles* with cosmological density five times higher than that of baryons. WIMPs should be cold (non-relativistic) long before the equality epoch to be able to form galactic structures that we observe today. Con-

Figure 1: HST photo of a sky region in the direction of cluster of galaxies 0024 + 1654

temporary physics does not know particles with DM properties. It is necessary to go beyond the standard model. But how and in which direction? What should we look for?

Owing to such simple properties, DM has straightforwardly affected the development of the Universe gravitational potential. The DM density contrast was increasing in time due to gravitational instability. Baryons, after they decoupled from radiation, were captured into gravitational wells of DM concentrations. That is why light is there where DM is, although DM particles do not interact with light. Thanks to this remarkable feature of gravitational instability it is possible to study amount, state and distribution of DM in observations ranging from radio to X-ray bands.

The analysis of large scale structure in the Universe has revealed that the amount of non-relativistic DM entering structure is small. The overall mass density of all particles which have been involved in the process of gravitational instability, does not exceed 30% of the critical density. At the same time the characteristics of CMB anisotropy have evidenced the flat spatial geometry of our Universe. It means that the rest 70% of the critical density should be in the form that takes no part in gravitational clustering. What are the properties of such a stable medium which is not perturbed by gravitational potential of the structure and remains essentially non-clustered?

Theory gives a clear answer to this question – the pressure-to-energy ratio of this medium, $w \equiv p_{\rm DE}/\epsilon_{\rm DE}$, should satisfy the following condition:

$$|1 + w| << 1 \ . \tag{1.1}$$

Only under this inequality the medium remains Lorentz-invariant and invariable both in space and time. We call it *dark energy*. This is all we know about DE.

It is crucial that the process of gravitational instability could be launched in the Friedmann

Universe only if the seed density perturbations were present since the very beginning. The existence of primordial cosmological perturbations has nothing to do with DM or any other particles. These are the total density perturbations that were produced by the Big Bang physics. Thus, other important problem arises, the problem of origin of the seed density perturbations which have developed dynamically into DM structures.

These hot topics – searching for unknown matter and determining the initial conditions for structure formation – display new physics and are expected to be solved in near future. In this short review we dwell upon them.

2. Geometry and composition of the Universe

What we see today is a product of start conditions and evolution. Available observational data made it possible to determine characteristics of cosmological density field at different epochs of its development. It allowed us to separate information about the initial conditions and development conditions, thus giving rise to independent investigations of the early and late Universe physics.

In modern cosmology the term "early Universe" stands for the final period of the inflationary *Big Bang stage* with subsequent transition to hot period of cosmological expansion. Currently we have no model of the early Universe as we do not know BBS parameters (there are only upper bounds, see eq. (5.11)). However, we have a well-developed theory of quantum-gravitational generation of the cosmological perturbations. Using this theory, we can derive the spectra of primordial density perturbations and cosmic gravitational waves as functions of cosmological parameters, and constrain the latter if the spectra are known.

Our knowledge of the late Universe is quite opposite. We have rather precise model – we know the main matter components and cosmological parameters, the evolution of the Universe and theory of structure formation. But we do not understand how the matter components have originated.

The known properties of the visible Universe allow us to describe the geometry of both, late and early Universe, in the framework of the perturbation theory as there is a small parameter here $\sim 10^{-5}$, the amplitude of cosmological perturbations.

The main tool of geometry is metric tensor. To zeroth order the Universe is Friedmannian and described with only one function of time – the scale factor $a(t)$. The first order is a bit more complicated. The metrics perturbations are the sum of three independent modes –the scalar one $S(k)$, the vector one $V(k)$, and the tensor one $T(k)$, each of them being described by its spectrum, the function of the wave number k. The scalar mode describes the cosmological density perturbations, the vector mode is responsible for vortical matter motions and the tensor mode presents gravitational waves. If the first order fields are Gaussian then the entire geometry of our Universe is described with only four positively defined functions, $a(t)$, $S(k)$, $T(k)$ and $V(k)$. Currently we know the first two of them in some ranges of definition.

BBS was a catastrophic process of rapid expansion accompanied by intensive time varying gravitational field. Under this gravitational action the real cosmological perturbations of metric and density were being parametrically born from vacuum fluctuations. It is very general and fundamental effect of creation of any massless degree of freedom in external coupled non-stationary field.

Observational data confirm the quantum-gravitational origin of seed density perturbations responsible for structure formation in the Universe. The basic properties of the perturbation fields generated according to this mechanism are the following: the Gaussian statistics (random distribution in space), the preferred time phase ("growing" branch of evolution), the absence of characteristic scales in a wide range of wavelengths, a non-zero amplitude of the gravitational

Table 1: Basic cosmological parameters

Hubble parameter	$h = 0.7$
CMB temperature	$T = 2.725K$
3-space curvature	$\Omega_\kappa = 0$
cosmological density of baryons	$\Omega_{\rm b} = 0.05$
cosmological density of dark matter	$\Omega_{\rm DM} = 0.23$
cosmological density of dark energy	$\Omega_\Lambda = 0.72$
power-spectrum index	$n_S = 0.96$

waves. The latter is crucial for building-up the BBS model as gravitational waves couple the simplest way to the background scale factor.

Evolution of S-mode has resulted in formation of galaxies and other astronomical objects. The CMB anisotropy and polarization have emerged long before under the joint action of all three perturbation modes (S, T and V) on the photon distribution. Analysis of the observational data on galaxy distribution and the CMB anisotropy allowed us to relate S and $T+V$ modes. Making use of the fact that the sum $S+T+V \simeq 10^{-10}$ is known from the CMB anisotropy, we obtain the upper bound for the vortical and tensor perturbation modes in the visible Universe:

$$\frac{T+V}{S} < 0.2 \tag{2.1}$$

In case the latter inequality were violated the density perturbation value would not be sufficient to form the observed structure. The detection of T and/or V (e.g. cosmological magnetic field) will become possible only with further increase of observational precision.

3. From Big Bang to dark energy

Let us consider zero order geometry more detailed.

Table 1 presents average values of the cosmological parameters obtained from astronomical observations (with 10% accuracy). With these parameters, we obtain from the Friedmann equations the Hubble function, $H \equiv \dot{a}/a$, and its time derivative, $\gamma \equiv -\dot{H}/H^2$:

$$\frac{H}{H_0} = 10^{61}\frac{H}{M_P} = \left(\frac{10^{-4}}{a^4} + \frac{0.3}{a^3} + 0.7\right)^{1/2}, \tag{3.1}$$

$$\gamma = -\frac{d\ln(H\ell_P)}{d\ln a} = \frac{3(\epsilon+p)}{2\epsilon} = \frac{2\cdot 10^{-4} + 0.4a}{10^{-4} + 0.3a + 0.7a^4}, \tag{3.2}$$

where $H_0^{-1} = 14\text{Gyr} = 10^{33}\text{eV}^{-1}$ is the inverse Hubble constant, $M_P = \ell_P^{-1} = 10^{19}\text{GeV}= 10^{33}\text{cm}^{-1}$ is the Planck mass or inverse Planck scale (hereafter $c = \hbar = 1$). γ-function relates the Hubble size of the Universe with redshift, $z + 1 \equiv a^{-1}$.

Eqs.(3.1) and (3.2) evidence that all transitions from radiation to matter and to DE dominated expansions occurred at small energies pretty well known to atomic physics ($T_{rad} = 2.5\cdot 10^{-4}/a$ eV). When extrapolating eqs.(3.1) and (3.2) to earlier times (or higher energies) we learn the following properties of our Universe:

- The Universe is large, $(H_0\ell_P)^{-1} \sim 10^{61}$. At the beginning of the expansion (3.1) and (3.2) the physical size of the Universe was a factor 10^{30} higher than Planckian size ($a/H_0 >\sim$ the current length of relic quanta). Such a big factor can be explained by a pre-existed short inflationary stage with $\gamma < 1$ (BBS).

- The cosmological perturbations are acausal (scales enter horizon at $\gamma > 1$). Eqs.(3.1) and (3.2) describe decay of γ from 2 to 0.4. To explain acausality, one has to admit a pre-existed period of cosmological expansion with γ rising from values smaller than unity (BBS).

Within 14 billion years the Universe was at least once at radiation dominated state, once at matter dominated stage, and twice in state of inflation ($\gamma < 1$, by the definition): at BBS and DE stage.

4. In search for dark matter particles

DM are WIMPs which were non-relativistic long before the structure formation in the Universe (back to $T_{rad} > 10$ keV). We do not know weather WIMPs have decoupled from the thermal bath of particles or never been in equilibrium with other particles at all. There are several hypotheses on the origin of DM, but none of them has been confirmed so far.

There are messages from observational cosmology indicating that DM mystery is related with baryon asymmetry in the Universe. Two of them are the most appealing:

- The energy densities of both non-relativistic components, baryons and DM, are close to each other since the moment of their generation
- The characteristic scales of spatial distributions of baryon and DM are identical in the early Universe (the cosmological horizon of equal densities of radiation and matter = the sound horizon of hydrogen recombination)

The two matter components knew something about each other at the moments of generation.

Where is dark matter?

We know that luminous constituent of matter is observed as stars residing in galaxies of different masses and in the form of X-ray gas in clusters of galaxies. However, a greater amount of ordinary matter is contained in rarefied intergalactic gas with temperatures from several to hundred eV and also in MACHO-objects which are the compact remnants of star evolution and in the objects of small masses. Since these structures mostly have low luminosity they are traditionally called *dark baryons*.

Several scientific groups (MACHO, EROS and others) carried out the investigation of the number and distribution of compact dark objects in the halo of our Galaxy, which was based on micro-lensing events. The combined analysis resulted in an important bound – no more than 20% of the entire halo mass is contained in MACHO-objects of masses ranging from the Moon to star masses. The rest of the halo DM consists of unknown particles.

Where else is non-baryonic DM hidden?

The development of high technologies in observational astronomy of the 20th century allowed us to get a clear-cut answer to this question – non-baryonic DM is contained in gravitationally bound systems (DM halos). Unlike baryons, DM particles do not dissipate whereas baryons are radiationally cooled and settle near the halo centers attaining rotational equilibrium. DM stays distributed around the visible matter of galaxies with characteristic scale ~ 200 kpc. For example, in the Local Group of galaxies more than a half of all DM belongs to Andromeda and Milky Way.

Particles with required properties are absent in the standard model of particle physics. An important parameter that cannot be determined from observations due to the Equivalence Principle is the mass of particle. The main candidates are listed in Table 2 in ascending order of their rest masses.

Table 2: Candidates for non-baryonic dark matter particles

candidate	mass
gravitons	10^{-21}eV
axions	10^{-5}eV
"sterile" neutrino	10 keV
mirror matter	1 GeV
neutralino	100 GeV
super-massive particles	10^{13}GeV
monopoles and defects	10^{19}GeV
primordial black holes	$10^{-16} - 10^{-7} M_{\odot}$

One of the versions on agenda – the neutralino hypothesis – rises from minimal supersymmetry. This hypothesis can be verified in CERN at LHC that will run in 2008. The expected mass of these particles is $\sim 100\ GeV$, and their density in our Galaxy is a particle per cup of coffee.

DM particles are being searched in many experiments all over the world. Interestingly, the neutralino hypothesis can be independently verified both in underground experiments on elastic scattering and by indirect data on neutralino annihilation in Galaxy. So far the positive signal has been found only in one of the underground detectors (DAMA), where a season signal of unknown origin has been observed for several years now. But the range of masses and cross-sections associated with this experiment has not been confirmed in other experiments, which makes reliability and meaning of the results quite questionable.

Neutralino give an important possibility of indirect detection by their annihilation gamma-ray flux. During the process of hierarchic clustering these particles could form mini-halos of small masses with sizes comparable to that of the Solar System. Some of these mini-halos could stay intact till now. With high probability the Earth itself is inside one of these halos where the particle density is as much as tens of times higher than the mean halo density. Hence, the probability of both direct and indirect detection of DM gets higher. Availability of so different search techniques gives a solid hope that the physical nature of at least one version of DM will soon be verified.

5. In the beginning was sound

Let us consider the first order geometry more detailed.

The effect of the quantum-gravitational generation of massless fields is well-studied. Matter particles can be created with this effect (see [1, 2] etc.) (although the background radiation photons emerged as a result of the BBS proto-matter decay in the early Universe). The gravitational waves [3] and the density perturbations [4] are generated in the same way since they are massless fields and their creation is not suppressed by the threshold energy condition. The problem of the vortical perturbation creation is waiting for its researchers.

The theory of the S and T perturbation modes in the Friedmann Universe reduces to a quantum-mechanical problem of independent oscillators $q_k(\eta)$ in the external parametrical field $\alpha(\eta)$ in Minkovski space-time with the time coordinate $\eta = \int dt/a$. The action and the Lagrangian of the elementary oscillators depend on their spatial frequency $k \in (0, \infty)$:

$$S_k = \int L_k d\eta, \quad L_k = \frac{\alpha^2}{2k^3}\left(q'^2 - \omega^2 q^2\right). \tag{5.1}$$

A prime denotes derivative with respect to time η, $\omega = \beta k$ is the oscillator frequency, β is the speed of the perturbation propagation in the vacuum-speed-of-light units (henceforth, the sub-index k for q is omitted). In the case of the T mode $q \equiv q_T$ is a transversal and traceless component of the metric tensor,

$$\alpha_T^2 = \frac{a^2}{8\pi G}, \qquad \beta = 1. \tag{5.2}$$

In the case of the S mode $q \equiv q_S$ is a linear superposition of the longitudinal gravitational potential (the scale factor perturbation) and the potential of the 3-velocity of medium times the Hubble parameter [4]:

$$q_S = A + Hv, \qquad \alpha_S^2 = \frac{a^2\gamma}{4\pi G\beta^2}, \tag{5.3}$$

where $A \equiv \delta a/a$, and $v \equiv \delta\phi/\dot{\phi}$ is the potential of the 3-velocity of medium (see eq. (5.4)).

As it is seen from eq. (5.2), the field q_T is minimally coupled with background metrics and does not depend on matter properties (in General Relativity the speed of gravitational waves is equal to the speed of light). On the contrary, the relation between q_S and the external field (5.3) is more complicated: it includes both derivatives of the scale factor and some matter characteristics (e.g. the speed of perturbation propagation in the medium). We know nothing about proto-matter in the Early Universe. There are only general suggestions concerning this problem.

Commonly, ideal medium is considered with the energy-momentum tensor depending on the energy density ϵ, the pressure p, and the 4-velocity u^μ. For the S mode, the 4-velocity is potential and represented as a gradient of the 4-scalar ϕ:

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu - pg_{\mu\nu}, \qquad u_\mu = \phi_{,\mu}/w, \tag{5.4}$$

where a comma denotes the coordinate derivative, and $w^2 = \phi_{,\nu}\phi_{,\mu}g^{\mu\nu}$ is a normalizing function. The speed of sound is given by "equation of state" and relates comoving perturbations of the pressure and energy density:

$$\delta p_c = \beta^2 \delta\epsilon_c, \tag{5.5}$$

where $\delta X_c \equiv \delta X - v\dot{X}$.

In the linear order of the perturbation theory the ideal medium concept is equivalent to the field concept where the Lagrangian density $L = L(w, \phi)$ is ascribed to the material field ϕ [4]- [6]. In the field approach the speed of the perturbation propagation is found from equation:

$$\beta^{-2} = \frac{\partial \ln |\partial L/\partial w|}{\partial \ln |w|}, \tag{5.6}$$

which also corresponds to eq. (5.5). To zeroth order, β is a function of time. In most models of the early Universe one usually assumes $\beta \sim 1$ (e.g. at the radiation-dominated stage $\beta = 1/\sqrt{3}$).

The evolution of the elementary oscillators is given by Klein-Gordon equation:

$$\bar{q}'' + (\omega^2 - U)\bar{q} = 0, \tag{5.7}$$

where

$$\bar{q} \equiv \alpha q, \qquad U \equiv \frac{\alpha''}{\alpha}. \tag{5.8}$$

The solution of eq.(5.7) has two asymptotics: an adiabatic one ($\omega^2 > U$) when the oscillator freely oscillates with the excitation amplitude being adiabatically damped ($|q| \sim (\alpha\sqrt{\beta})^{-1}$),

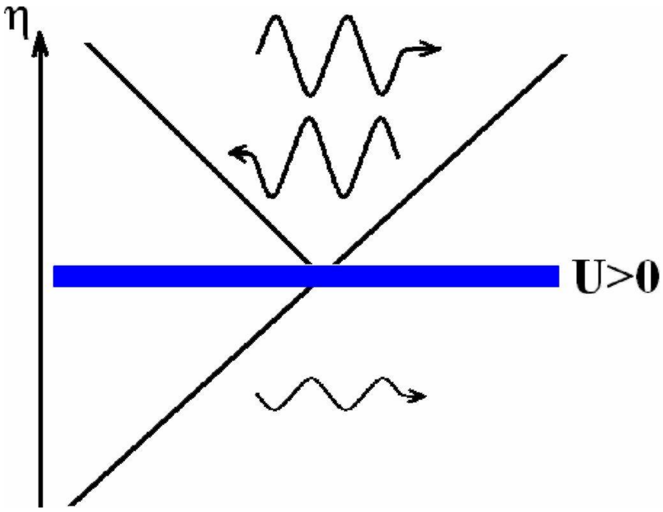

Figure 2: Illustration of solution of scattering problem for eq. (5.7)

and a parametric one ($\omega^2 < U$) when the q field freezes out ($q \rightarrow const$). The latter conditions in respect to quantum field theory implies a parametrical generation of a pair of particles from the state with an elementary excitation (see Fig. 2).

Quantitatively, the spectra of the generated perturbations depend on the initial state of the oscillators:

$$T \equiv 2\langle q_T^2 \rangle, \qquad S \equiv \langle q_S^2 \rangle, \tag{5.9}$$

where the field operators are given in the parametrical zone ($q \sim const$). The factor 2 in the tensor mode expression is due to two polarizations of gravitational waves. The state $\langle\rangle$ is considered to be a ground state, i.e. it corresponds to the minimal level of the initial oscillator excitation. This is the basic hypothesis of the Big Bang theory. In case the adiabatic zone is there, the ground (vacuum) state of the elementary oscillators is unique [7].

Thus, assuming that the function U grows from zero with time (i.e. the initial adiabatic zone is followed by the parametric one) and $\beta \sim 1$, we obtain a universal and general result for the $T(k)$ and $S(k)$ spectra:

$$T = \frac{4\pi(2-\gamma)H^2}{M_P^2}, \qquad \frac{T}{S} = 4\gamma, \tag{5.10}$$

where $k \simeq aH$ specifies the moment of creation ($\omega^2 = U$). As it is seen from eq. (5.10), the theory does not discriminate the T from S mode. It is the value of the factor γ in the creation period that matters when we relate T and S.

From the observed fact that the T mode is small in our Universe (see eq. (2.1)) we obtain the upper bound on the energetic scale of the Big Bang and on parameter γ in the early Universe:

$$H < 10^{13}\mathrm{GeV}, \qquad \gamma < 0.05. \tag{5.11}$$

The latter condition implies that BBS was just inflation ($\gamma < 1$).

We have important information on phases: the fields are generated in certain phase, only the growing evolution branch is parametrically amplified. Let us illustrate it for a scattering problem, with $U = 0$ at the *initial* (adiabatic) and *final* (radiation-dominated, $a \propto \eta$) evolution stages (see Fig. 2).

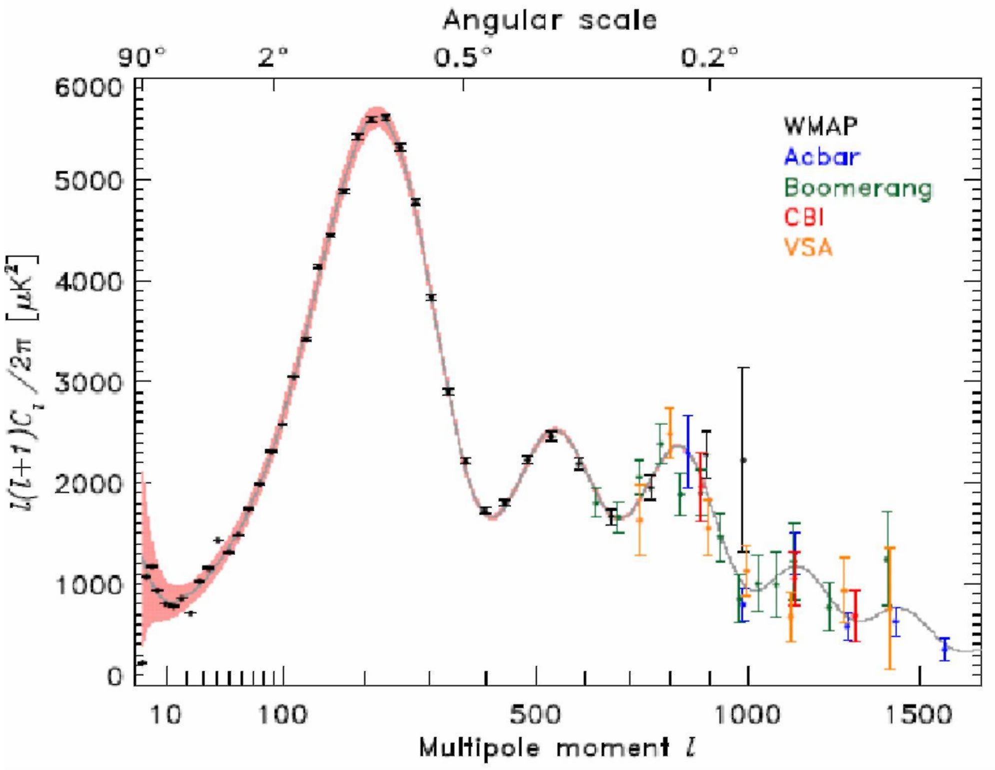

Figure 3: Manifestation of sound modulation in the CMB anisotropy spectrum

For either of the two above-mentioned stages general solution is

$$\bar{q} = C_1 \sin \omega\eta + C_2 \cos \omega\eta, \tag{5.12}$$

where the constant operators $C_{1,2}$ yield the amplitudes of the "growing" and "decaying" solutions. In the vacuum state the initial time phase is arbitrary: $\langle|C_1^{(in)}|\rangle = \langle|C_2^{(in)}|\rangle$. However, the solution of the evolution equations yields that only the growing branch of the sound perturbations takes advantage at the radiation-dominated stage: $\langle|C_1^{(fin)}|\rangle >> \langle|C_2^{(fin)}|\rangle$. This important result can be explained by the fact that only growing solution is consistent with the isotropic Friedmannian expansion from the very beginning. According to it, by the moment of matter-radiation decoupling at the recombination era, the radiation spectrum appears modulated with typical phase scales $k_n = n\pi\sqrt{3}/\eta_{rec}$, where n is a natural number.

It is these acoustic oscillations that are observed in the spectra of the CMB anisotropy (see Fig. 3, the highest peak corresponds to $n = 1$) and the density perturbations, which confirms the quantum-gravitational origin of the S mode. We see, the standard cosmological model can begin as follows. "In the beginning was sound. And the sound was of the Big Bang". It differs a bit from the scenario described in the Bible.

The sound modulation in the density perturbation spectrum is suppressed by the small factor of the baryon fraction in the entire budget of matter density. This allows one to determine this fraction independently of other cosmological tests. The oscillation scale itself is an example of the standard ruler that is used to determine cosmological parameters of the Universe.

To summarize we can say that in principle the problem of the generation of both, the primordial cosmological perturbations and the large scale structure of the Universe, is solved today. The theory of the quantum-gravitational creation of perturbations in the early Universe will be finally confirmed as soon as the T mode is discovered, which is anticipated in the nearest future. For example, the simplest BBS (power-law inflation on massive field) predicts the T mode amplitude only 5 times smaller than that of the S mode (which corresponds to $\gamma \sim 10^{-2}$) [8]. Modern devices and technologies are quite able to solve the problem of registering such small signals analyzing observational data on the CMB anisotropy and polarization.

6 Conclusion

Nowadays it became possible to separately determine properties of the early and late Universe from observational astronomical data. We understand how the primordial cosmological density perturbations that formed the structure of Universe emerged. We know crucial cosmological parameters on which the standard model of the Universe is based, and the latter has no viable rivals. However, some fundamental questions of the origin of the Big Bang and of main matter constituents remain unsolved.

Observational discovery of the tensor mode of the cosmological perturbations is a key to building-up the model of the early Universe. In this domain of our knowledge we have a clear-cut theory prediction that is already verified in the case of the S mode and can be experimentally verified for the T mode in the nearest future.

Giving a long list of hypothetical possibilities where and how to look for DM particles and DE physics theory has exhausted itself. Now it is experiment's turn. The current situation calls to mind great moments in the past history of science when quarks, W- and Z-bosons, neutrino oscillations, the CMB anisotropy and polarization were discovered.

One question is beyond the scope of this review. Why is Nature generous to us to allow us to reveal its secrets?

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Minimalising quantum mechanics

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Relativistic quantum mechanics is constructed minimally from a single creation operator with explicit energy, momentum and mass terms. The phase factor, amplitude, spinor structure and vacuum states are all automatic consequences of the initial definition. As separately-defined entities they are completely redundant. The operator can even be reduced to two terms (energy and momentum) if differentiation is defined in a discrete sense. This version of quantum mechanics is also a full quantum field theory, with an automatic incorporation of vacuum and second quantization. The fundamental interactions of particle physics are consequences of the mathematical structure alone, and do not require any additional 'physical' assumptions. This minimal construction is both simpler than other forms of quantum mechanics and at the same time more accurate and more powerful.

1. Creating a nilpotent structure

A nilpotent quantum mechanical structure[1,2] is derived most simply by first taking the classical:

$$E^2 - p^2 - m^2 = 0$$

and factorizing using noncommuting algebraic operators (multivariate 4-vector quaternions or complex double quaternions):

$$(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0. \tag{1}$$

The algebra can be described as a tensor product of quaternions and multivariate (or quaternion-like) 4-vectors (which are also equivalent to 4-vectors or complexified quaternions). The base units are:

$\boldsymbol{i}\,\boldsymbol{j}\,\boldsymbol{k}$	quaternion units	$\mathbf{i}\,\mathbf{j}\,\mathbf{k}$	vector units
1	scalar	i	pseudoscalar

The multivariate vector units **i**, **j**, **k** are effectively a complexified quaternion system, which is commutative to ***i***, ***j***, ***k***. Multivariate vectors **a** and **b** follow the product rule:

$$\mathbf{ab} = \mathbf{a.b} + i\,\mathbf{a} \times \mathbf{b}$$

There are 64 possible products of the 8 base units, which may be specified as:

$(\pm 1, \pm\, i)$	4	units
$(\pm 1, \pm\, i) \times (\mathbf{i}, \mathbf{j}, \mathbf{k})$	12	units
$(\pm 1, \pm\, i) \times (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$	12	units
$(\pm 1, \pm\, i) \times (\mathbf{i}, \mathbf{j}, \mathbf{k}) \times (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$	36	units

Together, they form a group of order 64, which requires only 5 generators for complete specification. All possible sets of generators, typically

$$i\boldsymbol{k} \qquad \mathbf{i}\boldsymbol{i}\ \mathbf{j}\boldsymbol{i}\ \mathbf{k}\boldsymbol{i} \qquad 1\boldsymbol{j}$$

follow the same overall structure, and, in the nilpotent form of quantum mechanics, they are the algebraic operators applied to the respective energy, momentum and mass terms, E, p_x, p_y, p_z, and m.

That they are isomorphic to the conventional γ matrices can be seen when we apply a canonical quantization to the left-hand bracket of (1), with the choice of an appropriate phase factor, $e^{-i(Et - \mathbf{p.r})}$, for a free particle, to give:

$$(\boldsymbol{k}\partial / \partial t \mp \mathbf{i}\boldsymbol{i}\nabla + \boldsymbol{j}m)(\pm i\boldsymbol{k}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p.r})} = 0. \qquad (2)$$

We can now describe (2) as equivalent to the Dirac equation for a free fermion, with a differential operator acting on a phase factor to produce an amplitude which is a square root of zero or nilpotent.

2. The Dirac 4-spinor

The $\pm E$ and $\pm \mathbf{p}$ in (1) and (2) represent the four simultaneous possibilities which are conventionally incorporated into a 4-component spinor ψ:

fermion / antifermion	$\pm E$
spin up / down	$\pm \mathbf{p}$

It is convenient to represent the four possible sign conventions incorporated here by the components of a row (or column) vector, for example by:

$$\begin{pmatrix} i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m \\ i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m \\ -i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m \\ -i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m \end{pmatrix}$$

leading to the four linked equations:

$$(\boldsymbol{k}\partial / \partial t - \mathbf{i}\boldsymbol{i}\nabla + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\ e^{-i(Et - \mathbf{p.r})} = 0$$
$$(\boldsymbol{k}\partial / \partial t + \mathbf{i}\boldsymbol{i}\nabla + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\ e^{-i(Et - \mathbf{p.r})} = 0$$
$$(-\boldsymbol{k}\partial / \partial t - \mathbf{i}\boldsymbol{i}\nabla + \boldsymbol{j}m)\ (-i\boldsymbol{k}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\ e^{-i(Et - \mathbf{p.r})} = 0$$
$$(-\boldsymbol{k}\partial / \partial t + \mathbf{i}\boldsymbol{i}\nabla + \boldsymbol{j}m)\ (-i\boldsymbol{k}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\ e^{-i(Et - \mathbf{p.r})} = 0$$

Significantly, all four terms in the spinor have the *same phase factor*. The sign variation applies only to operator and amplitude. In addition, for a multivariate $\mathbf{p}$,

$$\mathbf{pp} = (\boldsymbol{\sigma}.\mathbf{p})\ (\boldsymbol{\sigma}.\mathbf{p}) = pp = p^2$$

which means that we can also use **σ.p** for **p** (or **σ.∇** for ∇) in the Dirac equation, where **σ.p** is helicity, and **σ** is a pseudovector of magnitude –1.

3. The nilpotent operator as the entire source of quantum mechanics

The most important property of equation (2) is that the amplitude

$$(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

is a nilpotent, or square root of zero. The key transition from conventional quantum mechanics occurs when we assume that, even when the fermion is not a free state, the amplitude remains nilpotent and that this is the defining characteristic of the fermion state. If we assume this is *always true*, whether the particle is free or not, then we can get rid of the equation altogether by treating (± ***ik**E* ± ***i*p** + ***j**m*) as an operator. As soon as this step is taken, we depart from the conventional emphasis on the Dirac equation as the basis of relativistic quantum mechanics and privilege this operator instead. Where the fermion is no longer free, we can use covariant derivatives or include field terms to represent the operator components ***ik**E* and ***i*p**. For example, in the simplest case:

$$E \to i\frac{\partial}{\partial t} \qquad \text{becomes} \qquad E \to i\frac{\partial}{\partial t} + e\phi$$

Once the nature of the ***ik**E* and ***i*p** operators is decided, then the phase factor is uniquely determined, because it must be such that the amplitude that results squares to zero. Finding the phase factor which does this is what we mean by finding a 'solution'. So, whether the state is free or not, both the phase factor and the amplitude will be uniquely determined once the operator is defined, and hence become redundant as independent information. The same also applies to the quantum mechanical equation. We never, in fact, need to refer to an equation at all, for, although an equation can be constructed from the operator, it does not exist independently of it, and the derivation of phase factor and amplitude directly from the operator is actually more true than their derivation from any quantum mechanical equation because it does not depend on the (often incorrect) assumption that the amplitude is a constant.

However, even the operator contains redundant information, for, in the nilpotent structure, the sign variation is identical for all fermion states, and so only the first or lead term represents information. In other words, the nilpotent operator entirely removes the need for using such mysterious objects as wavefunctions and spinors. They are strictly redundant, along with phase factors, amplitudes, and quantum mechanical equations. Of course, it will often be convenient to use such terms, but, in every case, they will be constructible uniquely by a completely standard procedure as soon as the first term of the operator is defined.

But, if we take (± ***ik**E* ± ***i*p** + ***j**m*) as an operator, it is important to ask: what is it operating on? The answer has to be *vacuum* or the *rest of the universe*. Because of the way they are defined, nilpotent operators are specified with respect to the entire quantum field, they are already second quantized, and a formal second quantization process becomes unnecessary. In effect, the nilpotency condition can be taken as defining the interaction between a localized fermionic state and the unlocalized vacuum or 'rest of the universe', with which it is uniquely self-dual, and the phase becomes the mechanism through which this is accomplished. Defining a fermion, therefore, implies simultaneous definition of vacuum as 'the rest of the universe' with which it interacts.

To specify the exact state of a fermion, the ***ik**E* and ***i*p** terms will have to include its interactions with all other fermionic states. The fermion is an open system because (1) conserves energy only over the entire universe. The nilpotent structure implies energy-momentum conservation without requiring the system to be closed. The nilpotent structure is thus naturally *thermodynamic*, and provides a mathematical route to defining nonequilibrium thermodynamics. In addition the equation is clearly a statement of Pauli exclusion if we take both left- and right-hand brackets as amplitudes. It can further be taken as requiring the existence of a zero totality universe. If we extract $(\pm \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ from nothing, we are left with $-(\pm \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ and, now,

$$-(\pm \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\,(\pm \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) = 0.$$

Equation (1) – and the nilpotent condition itself – can now be seen to generate at least *five* independent meanings.

classical	special relativity
operator × operator	Klein-Gordon equation
operator × wavefunction	Dirac equation
wavefunction × wavefunction	Pauli exclusion
fermion × vacuum	nonequilibrium thermodynamics

We can now see that it is possible to define the entire structure of quantum mechanics by defining the creation operator for a single fermion $(\pm \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ as a nilpotent. Of course, there are four creation (or annihilation) operators here (or two of each):

$(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$	fermion spin up
$(\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m)$	fermion spin down
$(-\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$	antifermion spin down
$(-\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m)$	antifermion spin up

But only the first or lead term defines the real fermionic state, though the four states in the nilpotent representation could, of course, be rotated to make any of the three vacuum states into the lead term or real state. And, strictly, the spinor structure of the operator is redundant, as previously discussed, along with the phase factor, the amplitude, and the quantum mechanical equation, as it is an automatic consequence, by sign variation, from the choice of the lead term, or by respective parity (P), time-reversal (T) or charge conjugation (C) transformations, brought about by respective pre- and post-multiplication of the lead term by the appropriate quaternion operators:

P	$\boldsymbol{i}\,(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\,\boldsymbol{i} = (\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m)$
T	$\boldsymbol{k}\,(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\,\boldsymbol{k} = (-\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$
C	$-\boldsymbol{j}\,(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)\,\boldsymbol{j} = (-\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m)$

There is yet another way to look at this. If we take $(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$ and postmultiply it by any of $\boldsymbol{k}(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$, $\boldsymbol{i}(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$, or $\boldsymbol{j}(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$, the result is $(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$, multiplied by a scalar, and this can be done an indefinite number of times. So these three *idempotent* terms behave as a vacuum operators. We can even suggest specific identifications on the basis of the pseudoscalar, vector and scalar characteristics of the associated terms.

$\boldsymbol{k}\,(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$	weak vacuum	fermion creation
$\boldsymbol{i}\,(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$	strong vacuum	gluon plasma
$\boldsymbol{j}\,(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$	electric vacuum	$SU(2)$

The three additional terms in the fermion spinor then become strong, weak and electric vacuum 'reflections' of the state defined by the lead term.

We can see the three vacuum coefficients $\boldsymbol{k}$, $\boldsymbol{i}$, $\boldsymbol{j}$ as originating in (or being responsible for) the concept of discrete (point-like) charge. The operators act as a discrete partitioning of the continuous vacuum responsible for zero-point energy. In this sense, they are related to weak, strong and electric localized charges, though they are delocalized.

4. Bosonic states

The T, C and P transformations here also lead to the production of the three types of bosonic state, which are, respectively, spin 1 and spin 0 bosons, and the fermion-fermion pairing observed in Cooper pairs and other applications of the nonzero Berry phase.

spin 1 boson:	$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (-i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)$	T
spin 0 boson:	$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (-i\boldsymbol{k}E - \mathbf{ip} + \boldsymbol{j}m)$	C
Bose-Einstein (B-E) condensate / Berry phase, etc.:		
	$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \mathbf{ip} + \boldsymbol{j}m)$	P

And we can see how these three bosonic states are related to vacua produced by the three charge operators, being combinations of the fermion lead terms with the real state equivalents of the three 'vacuum' terms:

$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{k}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{k}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{k}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m) \ldots$ weak
$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (-i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (-i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m) \ldots$ spin 1

$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{j}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{j}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{j}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m) \ldots$ electric
$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (-i\boldsymbol{k}E - \mathbf{ip} + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (-i\boldsymbol{k}E - \mathbf{ip} + \boldsymbol{j}m) \ldots$ spin 0

$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{i}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{i}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ \boldsymbol{i}\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m) \ldots$ strong
$(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \mathbf{ip} + \boldsymbol{j}m)\ (i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\ (i\boldsymbol{k}E - \mathbf{ip} + \boldsymbol{j}m) \ldots$ B-E

Nilpotent operators are here seen to be *intrinsically supersymmetric*. The conversion from fermion to boson is by multiplication by an antifermionic operator; the conversion of boson to fermion is by multiplication by a fermionic operator. If we repeatedly post-multiply a fermion operator by any of the discrete idempotent vacuum operators, we will create an alternate series of antifermion and fermion vacuum states, or, equivalently, an alternate series of boson and fermion states without changing the character of the real state. We can interpret this immediately as the series of boson and fermion loops, of the same energy and momentum, required in an exact supersymmetry. Fermions and bosons become their own supersymmetric partners (with the same E, $\mathbf{p}$ and m) through the creation of these vacuum states. The mutual cancellation of the boson and fermion loops then eliminates the need for renormalization and removes the hierarchy problem altogether. Separate supersymmetric particles become superfluous.

5. Fermionic spin, helicity and zitterbewegung

Fermionic spin is a routine derivation from the $\mathbf{p}$ component of the nilpotent structure. If we mathematically define a quantity $\boldsymbol{\sigma} = -\mathbf{1}$, then

$$[\sigma, H] = [-\mathbf{1}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + i\boldsymbol{j}m] = [-\mathbf{1}, \boldsymbol{i}(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)]$$
$$= -2\boldsymbol{i}\,(\mathbf{ij}p_2 + \mathbf{ik}p_3 + \mathbf{ji}p1 + \mathbf{j}\,p_3 + \mathbf{ki}p_1 + \mathbf{kj}p_2)$$
$$= -2i\boldsymbol{i}\,(\mathbf{k}(p_2 - p_1) + \mathbf{j}(p_1 - p_3) + \mathbf{i}(p_3 - p_2))$$
$$= -2i\boldsymbol{i}\mathbf{1} \times \mathbf{p}$$

If $\mathbf{L}$ is the orbital angular momentum $\mathbf{r} \times \mathbf{p}$, then

$$[\mathbf{L}, H] = [\mathbf{r} \times \mathbf{p}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + i\boldsymbol{k}m]$$
$$= [\mathbf{r} \times \mathbf{p}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)]$$
$$= \boldsymbol{i}\,[\mathbf{r}, (\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)] \times \mathbf{p}$$

But $\quad [\mathbf{r}, (\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)]y = i\mathbf{1}\,y$.

Hence $\quad [\mathbf{L}, H] = i\boldsymbol{i}\,\mathbf{1} \times \mathbf{p}$,

and $\mathbf{L} + \boldsymbol{\sigma} / 2$ is a constant of the motion, because

$$[\mathbf{L} + \boldsymbol{\sigma} / 2, H] = 0.$$

Helicity ($\boldsymbol{\sigma}.\mathbf{p}$) is another constant of the motion because

$$[\boldsymbol{\sigma}.\mathbf{p}, H] = [-p, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + i\boldsymbol{j}m] = 0$$

For a hypothetical fermion / antifermion with zero mass,

$$(\boldsymbol{k}E + i\boldsymbol{i}\,\boldsymbol{\sigma}.\mathbf{p} + i\boldsymbol{j}m) \to (\boldsymbol{k}E - i\boldsymbol{i}p)$$
$$(-\boldsymbol{k}E + i\boldsymbol{i}\,\boldsymbol{\sigma}.\mathbf{p} + i\boldsymbol{j}m) \to (-\boldsymbol{k}E - i\boldsymbol{i}p)$$

Each of these is associated with a single sign of helicity, ($\boldsymbol{k}E + i\boldsymbol{i}p$) and ($-\boldsymbol{k}E + i\boldsymbol{i}p$) being excluded, if we choose the same sign conventions for $\mathbf{p}$. Numerically, $\pm E = p$, so we can express the allowed states as $\pm E(\boldsymbol{k} - i\boldsymbol{i})$. Multiplication from the left by the projection operator $(1 - i\boldsymbol{j}) / 2 \equiv (1 - \gamma^5) / 2$ then leaves the allowed states unchanged while zeroing the excluded ones.

Yet another significant aspect of spin emerges when we write a nilpotent Hamiltonian in the form

$$\mathcal{H} = -i\boldsymbol{j}c\,\boldsymbol{\sigma}.\mathbf{p} - i\boldsymbol{i}imc^2 = -i\boldsymbol{j}c\mathbf{1}\mathbf{p} - i\boldsymbol{i}imc^2 = \alpha c\mathbf{p} - i\boldsymbol{i}imc^2.$$

Taking $\alpha = -i\boldsymbol{j}\mathbf{1}$ as a dynamical variable, we define a velocity operator, which, for a free particle, becomes:

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt} = \frac{1}{i\hbar}\,[\mathbf{r}, \mathcal{H}] = -i\boldsymbol{j}\mathbf{1}c = c\alpha\,.$$

The equation of motion for this operator then becomes:

$$\frac{d\alpha}{dt} = \frac{1}{i\hbar}\,[\alpha, \mathcal{H}] = \frac{2}{i\hbar}(c\mathbf{p} - \mathcal{H}\alpha).$$

The standard solution for this gives the *zitterbewegung* term for the free fermion, which is interpreted as a switching between the fermion's four spin states.

6. Idempotent, nilpotent and discrete wavefunctions

Conventional relativistic quantum mechanics has been assumed to be idempotent (**AA** = **A**), rather than nilpotent (**AA** = 0), but the vacuum operators in the nilpotent theory show that idempotents are also important there. However, the nilpotent theory is a much more significant development than one based on idempotents, because it is the nilpotent nature of the theory that allows us to use constraints, based on zeroing, to remove redundant information. But there is no fundamental conflict, for we can see that the nilpotent equation actually incorporates an idempotent equation. The conventional idempotent quantum mechanics can be constructed using the same equation as nilpotent quantum mechanics, but operator and wavefunction are differently defined. The equations are precisely the same – the difference is purely one of interpretation. There isn't even a transformation required, just a redistribution of algebraic operators between differential operator and amplitude. So, we have:

IDEMPOTENT

$[(\boldsymbol{ik}\partial / \partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m)\,\boldsymbol{j}]\ [\boldsymbol{j}(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ e{-}i(Et - \mathbf{p.r})] = 0.$

operator *wavefunction*

NILPOTENT

$[(\boldsymbol{ik}\partial / \partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m)\,\boldsymbol{jj}]\ [(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ e{-}i(Et - \mathbf{p.r})] = 0.$

operator *wavefunction*

Another option available to us is to use a discrete (or, more strictly, anticommutative) differentiation process, with a correspondingly discrete wavefunction. The nilpotent operator has three terms, compartmentalised using the quaternions ***k***, ***i***, ***j***, in a similar way to real and imaginary parts. If we use discrete differentiation, we can reduce it to two. In discrete differentiation, as defined by Kauffman, to preserve the Leibniz rule,[3] and used in the previous section in our derivation of *zitterbwegung*. we take

$$\frac{\partial F}{\partial t} = [F, H] = [F, E] \quad \text{and} \quad \frac{\partial F}{\partial X_i} = [F, P_i],$$

The mass term disappears in the operator (though it has to be introduced in the amplitude). Suppose we define a nilpotent amplitude

$$\psi = \boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + \boldsymbol{j}m$$

and an operator

$$\not{D} = \boldsymbol{ik}\frac{\partial}{\partial t} - \boldsymbol{i}\mathbf{i}\frac{\partial}{\partial X_1} - \boldsymbol{i}\mathbf{j}\frac{\partial}{\partial X_2} - \boldsymbol{i}\mathbf{k}\frac{\partial}{\partial X_3},$$

with $$\frac{\partial \psi}{\partial t} = [\psi, H] = [\psi, E] \quad \text{and} \quad \frac{\partial \psi}{\partial X_i} = [\psi, P_i],$$

With some straightforward algebraic manipulation, we find that

$$\not{D}\psi = i\psi(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + \boldsymbol{j}m)$$
$$+ i(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + \boldsymbol{j}m)\psi - 2\,i(E^2 - P_1^2 - P_2^2 - P_3^2 - m^2).$$

When is ψ nilpotent, then

$$\partial\!\!\!/\, \psi = \left(\boldsymbol{k} \frac{\partial}{\partial t} + i\boldsymbol{i}\nabla \right) \psi = 0 \, .$$

This is a Dirac equation using discrete differentials. There are four remarkable things about this equation.

(1) It makes no difference whether we introduce the canonical $i\hbar$ before the differentials. The derivation is either classical or quantum, and the transition between these is completely smooth.

(2) It contains no mass term in the operator, and so annihilation and creation operators are exact negatives of each other.

(3) We can convert the differentials to covariant derivatives, and so have an operator for a distorted space-time without a mass term.

(4) Because there is no mass term, the opportunity arises to represent the appearance of mass directly in terms of an anisotropic space-time structure. For example, Bogoslovsky's geometric phase transition,[4] interpreted as a mass-creating spontaneous-symmetry breaking in the fermion-antifermion consendate (represented in nilpotent theory as a product of two nilpotent operators / amplitudes).

7. Antisymmetric wavefunctions

We may note that nilpotent wavefunctions or amplitudes, which are Pauli exclusive because nilpotent by automatic self-multiplication producing zero, are also automatically antisymmetric ($\psi_1 \psi_2 - \psi_2 \psi_1$), and so Pauli exclusive in the conventional sense as well:

$$(\pm\, i\boldsymbol{k}E_1 \pm \mathbf{i}\mathbf{p}_1 + \boldsymbol{j}m_1)\,(\pm\, i\boldsymbol{k}E_2 \pm \mathbf{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$$
$$-\,(\pm\, i\boldsymbol{k}E_2 \pm \mathbf{i}\mathbf{p}_2 + \boldsymbol{j}m_2)\,(\pm\, i\boldsymbol{k}E_1 \pm \mathbf{i}\mathbf{p}_1 + \boldsymbol{j}m_1)$$
$$= 4\mathbf{p}_1\mathbf{p}_2 - 4\mathbf{p}_2\mathbf{p}_1 = 8\, i\, \mathbf{p}_1 \times \mathbf{p}_2$$

This is a remarkable result. It implies that, instantaneously, any nilpotent wavefunction must have a **p** vector in real space (a spin 'phase') at a different orientation to any other. The wavefunctions of all nilpotent fermions instantaneously correlate because the planes of their **p** vector directions must all intersect, and the intersections actually create the *meaning* of Euclidean space, with an intrinsic spherical symmetry generated by the fermions themselves.

At the same time, the equation could also be interpreted as suggesting that each nilpotent also has a unique direction in a quaternionic phase space, in which E, **p** and m values are arranged along orthogonal axes. We may suppose here that the mass shell or real particle condition requires the coincidence between the directions in these two spaces. In addition, the **p** vector carries *all the information* available to a fermionic state, its direction also determining its E and **p** values uniquely. Three consequences of this are immediately apparent.

(1) To avoid direction duplication, one at least of the three nilpotent terms (the mass term, in fact) must have only one algebraic sign

(2) A hypothetical massless fermion and antifermion pair would require opposite helicities (say, $i\boldsymbol{k}E + \mathbf{i}\mathbf{p}$ and $-\, i\boldsymbol{k}E + \mathbf{i}\mathbf{p}$) to avoid being on the diagonal

(3) A massless fermion could not exist in practice because, since the magnitudes of E and p would always be equal in such cases, then the resultant angles would always be the same.

8. The fundamental interactions

The fermionic nilpotent operator can be used to do quantum mechanics in the conventional way by define a probability density, etc. It can also do QED, QCD and weak interaction theory, and define propagators. It solves the relativistic hydrogen atom in six lines and provides analytic solutions for spherically symmetric fields involving other potentials. More important than these, however, is the fact that its structure *demands* the existence of all the three fundamental particle physics interactions – electric, strong and weak – and their characteristic force laws and defining symmetries. Each of these is connected with one of the three terms in the operator, and all are direct consequences of nilpotent structure alone, and do not require any additional 'physical' input.

First we consider the scalar term. If we assume that the constraint of spherical symmetry exists for a point particle, then we can express the momentum term of the operator in polar coordinates, using the Dirac prescription, with an explicit spin term:

$$\boldsymbol{\sigma}.\nabla = \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \pm i\,\frac{j + ½}{r}$$

We need the spin term because the multivariate nature of the **p** term cannot be expressed in polar coordinates. The nilpotent Dirac operator now becomes:

$$\left(\pm \boldsymbol{k}E \pm \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\,\frac{j + ½}{r}\right) + \boldsymbol{ij}m\right)$$

Now, whatever phase we apply this to, we will find that we will not get a nilpotent solution unless the $1 / r$ term with coefficient $\boldsymbol{i}$ is matched by a similar $1 / r$ term with coefficient $\boldsymbol{k}$. So, simply requiring *spherical symmetry* for a point particle, requires a *Coulomb* term of the form A / r to be added to E. It is a fundamental statement of the nilpotent condition. Every nilpotent solution related to a point source requires a Coulomb term, with a $U(1)$ group symmetry, as a fundamental aspect of nilpotent structure, and it is a fundamental component of all three interactions. But, while all three terms in the nilpotent have a scalar component or magnitude, one term alone (the m term) is a pure scalar, and the existence of a pure scalar term also suggests the existence of a pure scalar interaction (the electric interaction).

The second term to be considered will be the vector term. Here, we take note of the fact that **p** is a vector, and that an explicit representation of it as a vector in the nilpotent formalism leads to a baryon structure with the precise symmetry assigned to the strong colour force. We construct an operator of the form $(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{i}p_x + \boldsymbol{j}\,m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{j}p_y + \boldsymbol{j}\,m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{k}p_z + \boldsymbol{j}\,m)$ and then observe that it has nilpotent solutions only when $\mathbf{p} = \pm\,\boldsymbol{i}\,\mathbf{i}p_x$, $\mathbf{p} = \pm\,\boldsymbol{i}\,\mathbf{j}py_y$, or $\mathbf{p} = \pm\,\boldsymbol{i}\,\mathbf{k}p_z$. These six phases, which must be nonlocally gauge invariant, may be represented:

$(\boldsymbol{ik}E + \boldsymbol{i}\,\mathbf{i}p_x + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E + \;...\; + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E + \;...\; + \boldsymbol{j}\,m)$	$+RGB$
$(\boldsymbol{ik}E - \boldsymbol{i}\,\mathbf{i}p_x + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E - \;...\; + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E - \;...\; + \boldsymbol{j}\,m)$	$-RBG$
$(\boldsymbol{ik}E + \;...\; + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E + \boldsymbol{i}\,\mathbf{j}p_y + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E + \;...\; + \boldsymbol{j}\,m)$	$+BRG$
$(\boldsymbol{ik}E - \;...\; + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E - \boldsymbol{i}\,\mathbf{j}p_y + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E - \;...\; + \boldsymbol{j}\,m)$	$-GRB$
$(\boldsymbol{ik}E + \;...\; + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E + \;...\; + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E + \boldsymbol{i}\,\mathbf{k}p_z + \boldsymbol{j}\,m)$	$+GBR$
$(\boldsymbol{ik}E - \;...\; + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E - \;...\; + \boldsymbol{j}\,m)$	$(\boldsymbol{ik}E - \boldsymbol{i}\,\mathbf{k}p_z + \boldsymbol{j}\,m)$	$-BGR$

The condition for making transitions between these phases gauge invariant requires the $SU(3)$ symmetry associated with the so-called 'colour' or strong force. (It also requires the system to have mass because of the co-existence of right- and left-handed helicity states.) The fact that the state is completely entangled and that the interaction is nonlocal means that the rate of exchange of **p** in the process is independent of any physical separation that might exist between the three components. In effect, this requires a constant force and a potential that is linear with separation. Applying a combination of Coulomb and linear potential terms produces a structure whose nilpotent solutions require the characteristic infrared slavery and asymptotic freedom associated with the strong interaction.

The final component of the nilpotent to be considered is the pseudoscalar term. Here, we invoke the third aspect of the nilpotent structure which generates a physical effect –the spinor structure and the accompanying *zitterbewgung*. Though this is only a vacuum process, it specifically requires the creation of bosonic structures of the form ($\boldsymbol{ik}E$ + $\mathbf{i}\mathbf{p}$ + $\boldsymbol{j}m$) (– $\boldsymbol{ik}E$ + $\mathbf{i}\mathbf{p}$ + $\boldsymbol{j}m$) and ($\boldsymbol{ik}E$ + $\mathbf{i}\mathbf{p}$ + $\boldsymbol{j}m$) (– $\boldsymbol{ik}E$ – $\mathbf{i}\mathbf{p}$ + $\boldsymbol{j}m$) via a harmonic oscillator mechanism, which is the characteristic defining process of the weak interaction, in real as well as vacuum states. In effect, the *zitterbewgung* ensures that a fermion is always a weak dipole in relation to its vacuum states, and the single-handedness of the weak interaction can be regarded as the result of a weak dipole moment connected with fermionic ½-integral spin. Significantly, all weak interactions between real particles require sources that are in some senses dipoles (fermion-antifermion) and so can be expected to require a dipolar potential, in addition to the Coulomb term. Any such potential combination applied to the nilpotent operator produces a series of energy levels characteristic of the harmonic oscillator. In this case, the interaction and its $SU(2)$ symmetry appears to be generated by the duality of the pseudoscalar term $\pm$ $\boldsymbol{ik}E$ in generating antifermion, as well as fermion states.

It seems to be that the *structure* of the nilpotent alone that produces the three fundamental interactions characteristic of particle physics, and that no external physical input is required. The $U(1)$ Coulomb term (electric interaction) is determined purely by spherical symmetry using polar coordinates. The $SU(3)$ group structure required for the strong interaction then comes from the vector nature of the momentum term, while the spinor structure, and the accompanying *zitterbewgung* produces the $SU(2)$ symmetry and harmonic oscillator characteristics required for the weak interaction.

So, simply by defining an operator which is a nilpotent 4-component spinor with vector properties, we necessarily imply that it is subject to electric (or other pure Coulomb), weak and strong interactions, and no other, because no other analytic nilpotent solution exists. This structure is a product of the three types of quantity (pseudoscalar, multivariate vector and scalar) which it contains, and, ultimately, these are reflections of the need for a discrete (point) source to preserve spherical symmetry and hence to conserve angular momentum. We can, in fact, identify these and their associated symmetries as being connected with the three separately conserved aspects of angular momentum: magnitude (scalar, $U(1)$, spherical symmetry does not depend on the length of the radius vector), direction (vector, $SU(3)$, spherical symmetry does not depend on the choice of axes), and handedness (pseudoscalar, $SU(2)$, spherical symmetry does not depend on whether the rotation is left- or right-handed).

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Pilot model of opto-acoustic gravitational-wave antenna (project "OGRAN")

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Abstract

The OGRAN project — a combination of laser interferometrical and acoustical bar gravitation wave antenna to be placed into underground facilities of the Baksan Neutrino Observatory is presented. General principles of the apparatus, expected sensitivity and current characteristics of the antenna prototype described; some ways of sensitivity improvement are also discussed.

1 Introduction

The modern cosmology gives "effective upper limits" on cosmic GW pulses ($10^{-19} \div 10^{-18}$) in term of metric perturbation [1]. It is interesting to note that modern optical interferometers may achieve such sensitivity without a deep cooling (only advanced versions are planned to be with "cryogenic mirrors" [2]). This is the result of the two technical ideas realized into constructions of these setups. The first is the operation at frequencies clear from mechanical resonances of "high Q suspensions" where the brownian noise is strongly suppressed. The second is using the laser optical readout with a very small back action of the photon shoot noise. By satisfying these two conditions one can transform interferometer's mirrors displacement to variations of light power with very high sensitivity. Also it is worth to mention that up to now sensitivity of super cryogenic bars are limited by noises of the SQUID readout [3].

This understanding was a basics of the Russian national project OGRAN which goal is the construction of a room temperature bar combined with FP-interferometer as a readout [4].

The OGRAN collaboration consists of three Russian Institutions:

i) Sternberg Astronomical Institute of Moscow State University,

ii) Institute for Nuclear Research RAS, (INR),

iii) Institute of Laser Physics Siberian Branch RAS (ILP).

The purpose of this project is to develop and construct the large ($\sim$ 2.5 ton) opto-acoustical GW detector to be placed in the underground camera of the Baksan Neutrino Observatory (INR) and to operate it in a duty cycle together with the world GW network at room temperature first, and then in its advanced cryogenic version.

Institute of Laser Physics keeping leading positions in Russia as a center of high frequency optical standards development. The 2 W Nd:YAG laser with special system of stabilizing the laser frequency by the external optical cavity was developed by project colaborants from this Institute.

2 The setup principles

The OGRAN setup consists of the bar with a tunnel along central symmetry axis where a high finess 3-mirrors FP interferometer attached to the bar's ends (see figure 1). The optical readout with microgap FP was developed as a low noise alternative to the SQUID read out by the Legnaro group in AURIGA setup [5]. The unique feature of OGRAN setup is implementation of the expanded optical cavity with bar's length instead of a microgap optical sensor attached at one end of the bar. This difference has a qualitative consequency: the signal responce on GW of such a combined opto-acoustical antenna is not only the acoustical exitation of the bar but it has also a contribution of an "optical part" as a result of GW-EM interaction. A theory of this antenna was considered in [6]. One of the results of this article is that in a free-mass interferometer the "optical" and "acoustical" components of signal can not be distinguished. However for the "bar-interferometer" with mirrors following nongeodesic paths difference between these two parts of signal has to be observable. One must to recognize the difference goes to zero in a "very long GW-approximation" nevertheless this new important feature of the combined antenna has to be taken into accout with an open mind.

A principal possibility to achieve the resolution $h \leq 10^{-18}$ at the bar with the optical sensor was proved in the paper [7]. Briefly the argumentation was as follows.

The realistic sensitivity of the bar antenna for GW bursts with duration τ is determined by the general formula

$$h_{min} \geq 2L^{-1}(kT_n/M\omega_\mu^2)^{1/2} \cdot (\omega_\mu \tau Q)^{-1/2} \tag{1}$$

where L, M, ω_μ, Q are the bar parameters: the length, mass, resonant frequency and quality factor; k is the Boltzmann constant. The T_n is the effective noise temperature which depends on transducer and amplifier noises and coupled with the physical temperature T through a noise factor F: $T_n = T \cdot F$. The substitution in (1) of the typical parameter's values: $L = 2$m, $M = 10^3$kg, $Q = 3 \cdot 10^5$, $\omega_\mu \simeq 10^4\text{s}^{-1}$, $\tau = 10^{-3}$s, $T = 300$K results in

$$h_{min} \geq 1.5 \cdot 10^{-19} \cdot F^{1/2} \tag{2}$$

The noise factor F is defined by the ratio of a real noise level to the thermal noise level in the antenna bandwidth $\Delta f \simeq \tau^{-1}$. For the optical interferometer readout

$$F = (2M/\tau)(G_e/G_b)^{1/2}$$

Above the spectral densities of Brownian G_b and optical G_e noises are $G_b = 2kTM\omega_\mu/Q$ and $G_e = B\omega_\mu^2(2\hbar\omega_e/\eta W)(\lambda_e/2\pi N)^2$; where ω_e, λ_e, W are the frequency, wave length, power of the optical pump; then η, N, B are the photodiode quantum efficiency, number of FP reflections and "excess noise factor" (number of times a real optical noise exceeds the short noise level). For the designed OGRAN optical parameters: $W = (1 \div 3)$W, $B \simeq (1 \div 10)$, $\lambda_e = 1.064$ μm, $\theta = 0.8$, $N = (10^3 \div 10^4)$ one can find the estimation $F \simeq 1$, so the forecasted sensitivity (2) reduces to $h \sim 10^{-19}$.

It is important to emphasize that the formula (1) supposes a whitenning of the Brownian bar noise (a cut off the bar's resonance noise region) and operation at the "wings" of the thermal noise spectrum. A response to GW exitation away from the resonance is very small but the modern optical readout is capable to register it.

3 Setup layout

The OGRAN measurement scheme (see figure 1) consists of laser with frequency locked to the 3-mirror FP-cavity of the "detector" bar, so that variations of the bar's length results in frequency variations of the output light beam which has to be measured by some discriminator based on a very stable external optical cavity ("reference"). In general this correspods to the AURIGA optical sensor [5] but in practice its more difficult to build this scheme for the long (expanded) cavity then for the micogap one.

In practice this opto-acoustical detector's FP cavity pumped with a high frequency stability laser. A simple direct injection of the beam into bar's FP resonator would requier the unrealistic frequency stability according to $\Delta\omega/\omega = h \sim 10^{-19} Hz^{-1/2}$. For this reason the practical scheme has to be composed as a "differential bridge" with automatic compensation of slow frequency drift. Fom the experimental optics two types of such "bridges" are known: first is the Michelson interferometer (that was chosen for the interferometric GW antennae); second is called a "comparator of optical standards" in which one narrow frequency EM source refers to the similar one and slow drift of both might be corrected. This type of scheme was used by the Legnaro group [5] and it had choosed also as a "preferable technique" for the OGRAN set up.

External reference cavity spacer body for the laser frequency discrimination made of low-expantion glass Sitall CO-115M to minimise internal thermal noise oscillation. This discriminator has the same geometry with longitudial symmetrical hole as the main antenna. Like the main detector it has FP interferometer composed of the three mirrors attached to the sides, but one of the mirrors is attached through PZT to drive it's optical resonant frequency. To avoid acoustical noise and to achive high mecanical Q-factor discriminator is placed in vacuum chamber. For more frequency stabilization we use improved mounting configuration of reference cavity with vertical geometry [8]. The discriminator cylindrical body is mounted vertically with central ring support system to implement as high degree of symmetry as possible. An acceleration along the cavity axis results in deformation of each half of the cavity on either side of the central mounting plane. However the decrease of length on one side of the mounting is mostly compensated by an increase of length on the other.

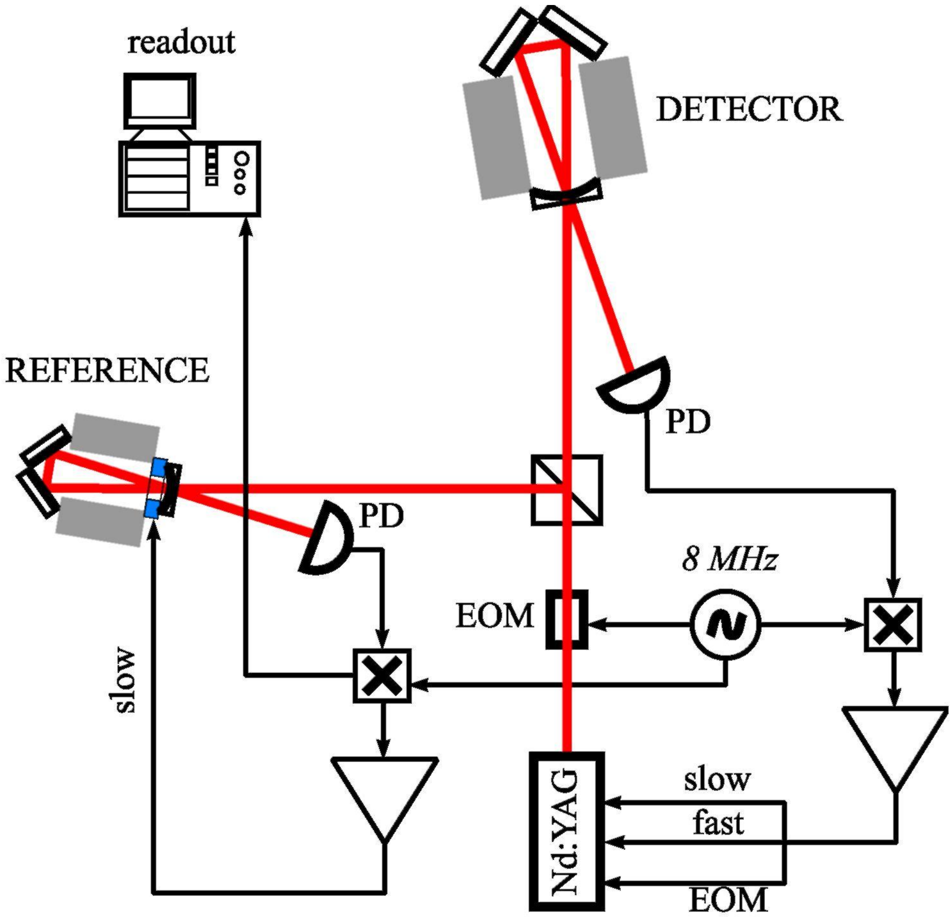

Figure 1: Principle scheme of the OGRAN project: EOM — electro-optical phase modulator; PD — photodetector.

Both detector and discriminator interferometers use three-mirrors PF cavity. The three mirrors had chosen to avoid laser beam back-scattering. Due to vast range of laser power variation in incident beam it is not possible to use one photodiode in all power ranges with good sensitivity. In the OGRAN setup the beam was splitted with the beam-splitters to 16 equal parts and 16 photodetectors are used in both channels. It gives additional profit of technical signal-to-noise ratio increasing by factor 4.

4 Current status of project and discussion

A small pilot model of the bar detector is now operational with parameters: $M \simeq$ 50kg, L = 50cm, $\omega_\mu = 2\pi \times$ 5kHz. The finess of FP-cavities of the bar and discriminator are $F \simeq 1000$. The new mirrors have been ordered from VIRGO scientific group with high reflectivity $R = 0.9997$ and small losses $\sim$ 50ppm. The estimated finess using these mirrors is about 10000.

Experiments with calibrated electrostatic excitation of detector's acoustic modes had shown sensitivity of $\sim (1 \div 2) \times 10^{-14}\,\mathrm{cm/Hz^{1/2}}$ for this setup while theoretical estimations was $\sim 3 \times 10^{-15}\,\mathrm{cm/Hz^{1/2}}$. The reason for this mismatch was reveled as technical noises of detection system. The new parts of detection system with satisfactory parameters are now gathered and reassembling of pilot setup is currently undergo.

The experimental setup modification with new high quality mirrors and power laser must allow to achieve the 1.5 of order improvement in the amplitude measurements for the pilot model out of the acoustical resonance. Meanwhile big components of the main setup are partly ready (vacuum chamber, bars) and partly are in construction as well as an infrastucture of the project: the underground lab in Baksan, hangar in SAI MSU for a test measurements with big bars etc. The first trial assemble of laser interferometer system on big main setup had been performed.

The plan of OGRAN project development foresees

i) an operation of the OGRAN setup with "room temperature bar" at the level of sensitivity $h \sim 3 \times 10^{-19}$ starting from 2008,

ii) development of the cryogenic version of OGRAN with the sensitivity $h \sim 3 \times 10^{-22}$.

OGRAN collaboration group hope to use a large experience and assistance of the Italian cryogenic bar group (ROG collaboration [9]) in this activity.

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Аппаратурная чувствительность оптоэлектронной схемы регистрации акустических колебаний ГА

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Проведен анализ основных сигнальных и шумовых параметров измерителя малых колебаний, построенного по схеме автоподстройки частоты излучения лазера за меняющейся собственной частотой резонатора Фабри-Перо. Рассмотрены вклады фундаментального фотонного шума и двух основных технических факторов - шума фотоприемника и флуктуаций частоты лазера. Получено выражение для минимально обнаружимого сигнала (чувствительности) измерителя. Представлены требования к параметрам узлов измерителя для приближения к фундаментальному порогу чувствительности. Определены критерии выбора глубины внутренней модуляции излучения лазера. Приведены численные оценки чувствительности, а также значений уровней сигнала и шума в контрольных точках.".

Как следует из представленных сегодня докладов группы, занимающейся разработкой гравитационной антенны, оптоэлектронная схема регистрации акустических колебаний антенны весьма сложна. Предлагается вниманию рассмотрение схемы регистрации исключительно в радиофизическом, отчасти - в радиотехническом аспекте.

К чувствительности (разрешению) измерительной схемы предъявляются требования регистрации тепловых, броуновских колебаний пробного тела. На резонансной частоте антенны-осциллятора спектральная плотность тепловых флуктуаций достаточно велика, и на данной частоте к разрешению схемы регистрации предъявляются относительно невысокие требования. Спектральная плотность теплового шума спадает с увеличением отстройки от резонанса механического осциллятора. Повышение чувствительности схемы регистрации смещений можно назвать снижением уровня электронных шумов регистрирующего устройства, упомянутых в предыдущих докладах. Это снижение позволяет расширить полосу пропускания гравитационного детектора.

Еще в начале 70-х годов Владимир Борисович Брагинский и Валентин Николаевич Руденко в своей работе показали перспективность применения оптико-электронной схемы регистрации механических колебаний [1]. Принцип данного измерения состоит в том, что частота лазера настраивается примерно на середину склона резонансной характеристики резонатора Фабри-Перо, при этом вариации собственной частоты резонатора, возникающие при акустических изменениях его длины, преобразуется в вариации мощности, падающей на фотоприемник. Для данного измерителя имеется аналитическое выражение для минимально обнаружимого смещения:

$$(X_{min})^2 \cong (\lambda/F)^2\, (h\nu/\eta P_0)\, \Delta f, \qquad (1)$$

где λ и ν – длина волны и частота излучения лазера ($\nu = c/\lambda$), $h\nu$ - энергия фотона,
P_0 – мощность излучения, прошедшего через резонатор в резонансе, η - квантовый выход фотоприемника, Δf – регистрируемая (шумовая) полоса частот, F - резкость (финесс) –

безразмерный параметр, характеризующий ширину полосы пропускания оптического резонатора.

Подстановка умеренных значений параметров в данное выражение позволяет прогнозировать возможность регистрации колебательный смещения на уровне десять в минус пятнадцатой степени сантиметра в шумовой полосе 1 Гц. Высокая потенциальная чувствительность измерителя определяется высокой крутизной склона резонатора Фабри-Перо, и малым действующим шумом – шумом фотонов, падающих на фотоприемник.

В настоящее время в проекте ОГРАН представилась возможность воплотить в реальность замысел оптоэлектронного измерителя смещений. На современном уровне технических решений схема измерения имеет усложненный, модернизированный вид, призванный устранить основные недостатки исходного концептуального варианта.

Первым недостатком исходной схемы является то, что на фотоприемник падает большая мощность излучения - примерно половина мощности лазера, прошедшей через резонатор. Такой режим засветки может нарушить работоспособность фотоприемника вплоть до её необратимой утраты.

Второй существенный недостаток состоит в том, что в исходной схеме велико влияние флуктуаций мощности лазера, спектральная плотность которых довольно велика на низких частотах вблизи рабочей частоты гравитационной антенны. Так возникла необходимость учета дополнительного, технического источника шума. Указанный фактор не позволяет реализовать высокую заявленную потенциальную чувствительность.

Модифицированная схема измерителя акустических колебаний опирается на методику Паунда – Дривера – Холла, известную в лазерной физике и используемую для стабилизации частоты лазера. В данной методике вводится внутренняя модуляция излучения лазера с последующей фазочувствительной демодуляцией, позволяющая сформировать дискриминаторную характеристику – зависимость выходного напряжения демодулятора от величины и знака рассогласования частот лазера и резонатора. Сформированный таким образом электрический сигнал, введенный в цепь управления частотой лазера, позволяет приблизить его частоту к собственной частоте оптического резонатора. Аналогичная схемотехника давно и широко используется, например, в радиолокации для стабилизации СВЧ-генераторов по опорным резонаторам.

В разрабатываемом измерителе смещений собственная частота резонатора относительно медленно меняется, осуществляется слежение за ней частоты лазера. Так формируется частотно-манипулированный сигнал в отличие от амплитудно-модулированного в исходной схеме измерителя.

Техника ПДХ позволяет устранить недостатки исходного варианта измерителя, указанные выше. Так, уменьшение постоянной засветки фотоприемника обеспечивается при использовании луча, отраженного от резонатора Фабри-Перо. Интенсивность излучения в нем на собственной частоте резонатора минимальна, тогда как интенсивность проходящего луча максимальна. Это и есть режим «темного пятна», упомянутый в предыдущем докладе группы.

Снижение влияния флуктуаций мощности лазера достигается использованием внутренней модуляции. При этом сигнальное воздействие переносится на достаточно высокочастотную поднесущую. В нашей схеме она примерно 5 МГц. На данной частоте спектральная плотность флуктуаций мощности достаточно невелика. Аналогично в радиотехнике использование схемы «модулятор-демодулятор» позволяет уйти от низкочастотных шумов типа «1/f ».

Для анализа работы схемы измерителя следует несколько подробнее рассмотреть схему Паунда-Дривера-Холла и её основные количественные характеристики.

В подробной схеме измерителя, представленной в предыдущем докладе, представлен электрооптический фазовый модулятор, через который проходит лазерный луч. К несущей частоте падающего луча, имеющей мощность P_C, модулятор добавляет две боковые частотные компоненты, имеющие мощности P_S. Согласно используемому простому модельному представлению свойств резонатора Фабри-Перо, вблизи резонанса несущая частота отражается очень мало и соответствующая мощность в отраженном луче пропорциональна отклонению частоты лазера от экстремума резонансной кривой. Боковые компоненты, не входящие в полосу пропускания резонатора, отражаются полностью, так что в рабочем режиме мощность излучения P_{PH}, падающего на фотоприемник, равна сумме мощностей боковых компонент:

$$P_{PH} \cong 2\, P_S. \qquad (2)$$

Второй характеристикой схемы ПДХ является выражение для крутизны дискриминаторной характеристики, приведенной к фотоприемнику [2]:

$$dP_{PH}/d\nu \equiv D_P = 8(P_C\, P_S)^{1/2}/\Delta\nu. \qquad (3)$$

Здесь dP_{PH} – вариация мощности, падающей на фотоприемник, D_P – соответствующий декремент, $\Delta\nu$ - ширина линии резонатора Фабри -Перо.

Для последнего параметра имеется соотношение:

$$\Delta\nu = (c\,/\,2L)\,/\,F,$$

где L -длина резонатора, величина (c /2L) есть межмодовое расстояние.

Целями представленной работы, выполненной в рамках проекта ОГРАН, являются:

1. Введение в рассмотрение основных технических источников шума, определяющих так называемую аппаратурную чувствительность устройства, с представлением численных оценок их влияния.

2. Строгое рассмотрение выбранной конкретной схемы измерения, преобразующей в вариации частоты лазера сигнальные возмущения и имеющиеся шумы, с последующим преобразованием сигнала и шумов при частотной демодуляции.

3. Определение для данной схемы предельной чувствительности, определяемой фотонным шумом, и сравнение с чувствительностью исходной концептуальной схемы, упомянутой выше.

В целом, когда создается прибор, которого ранее не существовало, следует постоянно выполнять сравнение измеряемых промежуточных параметров и уровней напряжений сигнала и шума со значениями, прогнозируемыми предварительный расчетом. Такой инженерно-физический расчет выполнен; он предлагается вниманию.

На рисунке представлена функциональная схема измерителя смещений проекта ОГРАН. При радиофизическом подходе схема может быть сведена к предельно упрощенному виду (рис.1).

Для описания схемы составлена система уравнений, образованная довольно простыми соотношениями:

$$U_{PH} = D_U\,(\delta\nu_S - \delta\nu_L),$$
$$U_{OC} = K(U_{PH} + U_N), \qquad \delta\nu_L = \delta\nu_N + \beta U_{OC}.$$

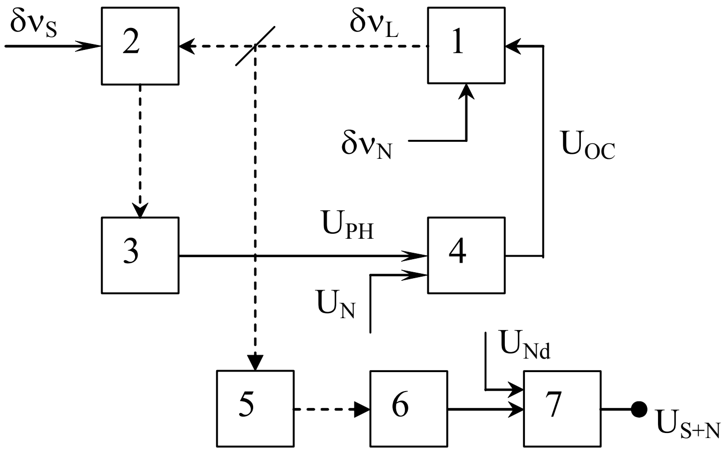

Рис.1. Функциональная схема измерителя смещений проекта ОГРАН

1. Лазер
2. Резонатор Фабри-Перо гравитационной антенны
3. Фотоприемник
4. Усилитель
5. Резонатор Фабри-Перо дискриминатора
6. Фотоприемник дискриминатора
7. Усилитель дискриминатора

Первое соотношение связывает вариацию напряжения на выходе фотоприемника с рассогласованием частот лазера и резонатора. Рассматриваются вариационные компоненты частот, соответственно, лазера $\delta\nu_L$ и резонатора $\delta\nu_S$, причем последняя является сигнальной. Декремент схемы ПДХ D_P дополнен учетом последующего преобразования сигнала вплоть до его выделения на сопротивлении нагрузки фотоприемника R:

$$D_U = R(\eta e / h\nu)\, D_P,$$

где e – заряд электрона.

Во втором соотношении, характеризующем усиление всей электронной схемы, вводится член, описывающий флуктуационный процесс, предназначенный для описания источников шумов. Это введение в систему выполняется формально так же, как стандартно вводится шум электронного усилителя, то есть приведением шума к входу. На самом деле введенный случайный процесс U_N описывает суперпозицию нескольких шумов. Из них в качестве доминирующих рассматриваются вклады от дробового шума тока, вызванного фотонами, падающими на поверхность фотоприемника, и от флуктуационного тока собственного технического шума фотоприемника.

Третье соотношение составлено так, чтобы феноменологически учесть тот эффект, что вариация частоты лазера $\delta\nu_L$ содержит не только регулярную составляющую, создаваемую управляющим напряжением U_{OC}, но и собственные флуктуации частоты, представленные случайным процессом $\delta\nu_N$. Этот шум является техническим.

Система уравнений описывает вариационные компоненты величин, характеризуемые спектрами, расположенными в частотной области, близкой к частоте сигнального воздействия. Она является следствием линейного разделения общей системы уравнений на вариационную и квазистатическую части.

Представлено решение системы уравнений для величины $\delta\nu_L$,характеризующей результирующую вариацию частоты лазера:

$$\delta\nu_L = \delta\nu_S + U_N / D_U + \delta\nu_N / K_{OC} \equiv S + N$$

Здесь параметр $K_{OC} = K\beta D_U$ есть коэффициент усиления в разомкнутой петле обратной связи.

Решение для $\delta\nu_L$ представлено в упрощенном виде, справедливом при достаточно глубокой обратной связи, при значении K_{OC} много большем единицы.

Полученное выражение содержит сигнальный член, показывающий, что вариация частоты лазера точно повторяет вариацию собственной частоты резонатора, рассматриваемой как сигнальную. Каждому из двух источников шумов, введенных в систему уравнений, соответствует флуктуационный член. Слагаемое, определяющее влияние собственных флуктуаций частоты лазера уменьшается с увеличением глубины обратной связи, что соответствует широко используемой методике ПДХ для стабилизации частоты лазера с помощью резонатора Фабри-Перо.

Поскольку предложенное исходное модельное представление измерителя предлагает описание как сигнала так и шума, то из решения следует выражение для минимально обнаружимой сигнальной вариации частоты (S=N) :

$$[\delta\nu_N]_\nu = U_N / D_U + \delta\nu_N / K_{OC} .$$

Система также имеет решение для напряжения обратной связи U_{OC}, непосредственно снимаемого с электронной схемы. Если регистрировать сигнал и шум таким образом, то не понадобилась бы дополнительная схема частотного детектирования со вторым резонатором Фабри-Перо. Однако, получаемое из него аналогичное выражение для минимально обнаружимого сигнала

$$[\delta\nu_N]_U = U_N / D_U - \delta\nu_N .$$

отличается тем, что в нем отсутствует зависимость от коэффициента K_{OC}. В этом упрощенном варианте собственные флуктуации частоты лазера не подавляются.

В решении для $\delta\nu_L$ может быть выполнен переход к спектральным представлениям рассматриваемых флуктуационных процессов. Тогда и сигнал характеризуется как спектральная плотность. Согласно стандартной методике определения классического параметра «сигнал/шум», путем приравнивания его к единице получается выражение для минимально обнаружимого сигнала измерителя механических колебаний:

$$(X_{min})^2 = (L/\nu)^2 \, [\, R^2 \, (S_{I\text{др}} + S_{IPH}) / (D_U)^2 \; + S\nu / K_{OC}^2] \Delta f \, . \qquad (4)$$

В выражении учтена связь между механическими вариациями длины резонатора и вариациями его частоты и раскрыто выражение для спектральной плотности обобщенного шумового напряжения U_N. Его спектральная плотность равна:

$$S_U = R^2 \, (S_{I\text{др}} + S_{IPH}) \, .$$

Здесь приведены обозначения спектральных плотностей дробового током фотонов и собственного шумового тока фотоприемника.

В полученной рабочей формуле для чувствительности измерителя первое слагаемое определяет вклад фундаментального фотонного шума, второе определяет вклад шума фотоприемника и третье - представляет вклад шума флуктуаций частоты лазера. Таким образом, научно-техническая задача прогноза чувствительности сведена к расчетному соотношению.

При независимом определении такого параметра лазера, как спектральная плотность флуктуаций его частоты, вклад данного источника шума, в принципе, может быть сделан относительно малым за счет выбора надлежащего значения глубины обратной связи. Что касается указанной спектральной плотности, рассмотренная задача имеет отличие от стандартных задач стабилизации частоты. Здесь имеет значение спектральной плотности флуктуаций частоты лазера не в окрестности нуля частоты, как в системах стабилизации частоты, а в области, близкой к частоте гравитационной антенны, составляющей несколько более одного килогерца. Соответственно, значение параметра обратной связи K_{OC} должно определяться на килогерцовой частоте, что влечет зав собой определенные трудности, связанные с обеспечением устойчивости к самовозбуждению при глубокой обратной связи.

Представленная формула получена для схемы автоподстройки частоты. Далее сигнал преобразуется в частотно демодуляторе в напряжение низкой частоты посредством почти аналогичной схемы ПДХ. Однако, грамотное построение схемы, использование качественного опорного оптического резонатора-дискриминатора, как показывают дальнейшие выкладки и оценки, практически ничего не меняет в приведенном выше выражении для чувствительности. Таким образом, представленное выражение справедливо для всего оптико-электронного измерителя в целом.

Первый член данной расчетной формулы (4) описывает вклад фотонного шума. Рассматривая его отдельно, используя приведенные выражения для декрементов D_U и D_P, получаем выражение для предельной чувствительности.

$$X_{min} = (1/8)\,(\lambda/F)\,(h\nu/\,\eta\,P_C)^{1/2}\Delta f^{1/2}. \qquad (5)$$

Данное выражение очень похоже на выражение (1), представленное выше для исходной схемы датчика с установкой рабочей точки на середине склона резонансной частотной характеристики. Отличие этих двух выражений состоит в том, что в последнем в знаменателе имеется восьмерка. Это значит, что модифицированная схема на основе методики ПДХ обладает существенно более высокой потенциальной чувствительностью. Как показал анализ, это является следствием того, что крутизна преобразования частоты в мощность $dP_{PH}/d\nu$ в схеме ПДХ примерно в 4 раза больше, чем у исходной схемы с модуляцией мощности на середине склона АЧХ (строго при $P_{PH} = P_C/2$).

Представим численные оценки предельной чувствительности. Для значений параметров

$$\lambda = 1{,}06\ \text{мкм},\ P_C = 300\ \text{мВт},\ \eta = 0{,}5,\ F = 1000$$

потенциальная чувствительность имеет значение

$$X_{min} = 1{,}5 \cdot 10^{-17}\ \text{см Гц}^{-1/2}.$$

Условием достижения потенциальной чувствительности является доминирование первого члена в выражении (4), из которого видно, что дробовый шум фотонов должен превысить собственный токовый шум фотоприемника. $S_{Iдр} > S_{IPH}$

Выражение для дробового шума с учетом (2) имеет вид:

$$S_{Iдр} = 2\eta e^2\,(2P_S)/\,h\nu. \qquad (6)$$

В выражение для потенциальной чувствительности (5) входит параметр P_C, и не входит значение мощности боковой (sideband) P_S, тогда как эта величина входит в выражение для крутизны преобразования сигнала (2). Этот эффект является следствием того явления, что крутизна преобразования (5) и величина дробового шума (6) в данном простом модельном представлении одинаково зависят от величины P_S, так что отношение «сигнал/шум» не зависит от этого параметра. Это значит, что чувствительность оказывается формально независимой от глубины модуляции, создаваемой электрооптическим модулятором:

$$n = P_S / P_C . \qquad (7)$$

Реально, глубина модуляции не может быть нулевой. Данное представление справедливо только в той области больших значений P_S, где дробовый ток доминирует над шумом фотоприемника, что дополнительно показано неравенством:

$$2\eta e^2 (2P_S)/ h\nu > S_{IPH} .$$

В данной области значений параметра P_S этот эффект предоставляет некоторую свободу в выборе значения данного параметра и, в соответствии с (2), установки оптимального режима засветки фотоприемника P_{PH} , в котором фотоприемник еще не входит в насыщение.

Выбранное значение мощности засветки P_{PH} определяет глубину модуляции n (7) и P_S. Это есть критерий выбора глубины модуляции.

Приведем реальные численные оценки:

$$P_C = 300 \text{ мВт}, \; P_{PH} = 30 \text{ мВт}, \; P_S = 15 \text{ мВт}, \; n = 0{,}05.$$

Таким образом, тогда как на резонатор падает довольно большая суммарная мощность, на фотоприемник попадает относительно малая его часть. Это и есть режим работы «в темном пятне».

При использованных значения параметров P_{PH}, η, $h\nu$ в фотоприемнике и его нагрузочном сопротивлении R протекает постоянный ток 12,5 мА. Соответствующий дробовый ток имеет спектральную плотность $S_{Iдр} = 4{,}0 \cdot 10^{-21}$ $А^2$/Гц . Этот лруктуационный ток создает на нагрузке R = 300 Ом шумовое напряжение $(S_U)^{1/2} = 2{,}0 \cdot 10^{-8}$ В·Гц$^{-1/2}$. Полученная величина шумового напряжения на порядок выше приведенного входного шума стандартных малошумящих электронных предварительных усилителей. Последние численные оценки обосновывают исключение данного технического источника шума из перечня источников шума, формирующих аппаратурную чувствительность измерителя.

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On propagation of electromagnetic waves nearby rotating astrophysical objects and in interstellar medium in cosmological scales

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1. Introduction

Time of propagation of electromagnetic radiation depends on a result of superposition of a primary electromagnetic wave and secondary waves arising when the primary wave interacts with atoms of a moving medium. As experimental confirmation for influence of the interstellar medium measuring of dispersion of radio-waves from pulsars which allows define electron concentration in the interstellar medium.

At exposition of process of propagation of electromagnetic radiation in the expanding Universe frequently use the relativistic law permitting to calculate a velocity of a motion of astrophysical object on magnitude of red shift of radiation, emitted by him.

For a determination of distance up to remote astrophysical object use the law of Hubble. As the obtained in such a way results based on the data of radiation, adopted in a current instant, distance up to object is determined on the moment of an emission of radiation.

By analyzing data of distances to some astrophysical objects we can conclude that they were beyond bounds of Universe at the moment of emitting and receded with velocities close to light that. So the source 3C427.1 has red shift $z = 1{,}175$, from this it follows that at the moment of emitting its velocity was equal to 0.71 c (c is light velocity).

According to idea of expansion of space the Doppler shift of frequency of radiation of the remote object is bound not to a motion of the object, and with a motion of field of space, in which the object is located.

The velocity of object concerning expanding space can be insignificant. From this approach the possibility of a motion of remote astrophysical objects concerning the Earth with velocities exceeding speed of light in vacuum follows that explains a cosmological paradox.

Also it is possible to show, that at the analysis of metrology procedures the underlying definitions of distance up to astrophysical object should be taken into account effects of an electrodynamics of motioning media.

As an example we shall show, that the time of propagation of electromagnetic radiation from a source to the observer is influenced by effect of a delay of time of propagation owing to interaction of an electromagnetic wave with atoms of an interstellar medium of expanding fields of the Universe.

The equations of an electrodynamics are noted concerning vacuum in some selected inertial system of a reference (IR).

Relating the current IR with the observer located on the Earth, we shall come to an inference, that the red shift of a spectrum of radiation is stipulated by a motion of astrophysical object concerning the current IR irrespective of, with what velocity the space is dilated, and with what velocity the object in this space goes.

It follows from the fact that one velocity - velocity of removal of astrophysical object concerning a selected IR will enter besides a velocity of a medium only in a solution of the equations of an electrodynamics of motioning mediums.

Generally at description of transformation of electromagnetic radiation in the expanding Universe we should know distribution of coordinates and velocities of particles of an interstellar medium along a trajectory of a wave vector, resulting of a velocity of a source formed by a velocity of object in expanding space and velocity of expansion of space, and also variables of rotation.

In neglect by effects stipulated by rotation, the radiation propagation through motioning substance will be accompanied by longitudinal effect of Fizeau, which can render influence on red shift of a spectrum of radiation.

But this influence is essential only in that field of space, where a density of an interstellar medium and its velocity of a motion are great, since after an exit of radiation from this field the electromagnetic wave gets a phase displacement, but its frequency remains constant.

These parameters can be high in frontier fields of the Universe, which are removed enough from the ground observer; therefore influence of effect of Fizeau on a wave length of radiation in the field of the ground observer can be neglected.

The much greater influence the motion of an interstellar medium can render on time of propagation of electromagnetic radiation in the field of space, where the velocity of a motion of atoms of an interstellar medium is rather high.

And, the velocity of a motion of a medium concerning a selected IR will include in a wave equation, in which there is an observer.

2. Equation for electromagnetic wave propagation with account interstellar medium motion

Let us properties of spacetime continuum are given with the Friedman-Robertson-Walker metric

$$d\tau^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right\}. \tag{1}$$

Here r, θ, φ, t are coordinates, $R(t)$ is a cosmological scale factor, k is spatial curvature.

The parameter R with $k=1$ can be characterized as the radius of the Universe. At the moment $t=0$ the radius $R=0$, therefore current time t_0 is time which is counted from the singularity and it can be named as Universe age.

An equation for propagation of electromagnetic or gravitation-wave signal along a radial direction can be got from (1) when $d\theta = d\varphi = 0$

$$d\tau^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2}\right\} = 0. \tag{2}$$

An expression for invariant eigentime has a view

$$d\tau^2 = \left(1-\beta_e^2\right)dt^2, \tag{3}$$

where $\beta_e = v/c$, v is group velocity of electromagnetic wave, c is light velocity in vacuum.

Let equate the expression to the length element of Friedman-Robertson-Walker (2):

$$\beta_e dt = R(t)\frac{dr}{\sqrt{1-kr^2}}. \tag{4}$$

Supposing that velocity of electromagnetic wave depends on velocity of an interstellar medium in IR of observer, we can write

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_0^{r_1} \frac{1}{\beta_e(r,t)} \frac{dr}{\sqrt{1-kr^2}}. \tag{5}$$

Here t_1, t_0 is moments of emission and detection which are counted from singular state, r_1 is dimensionless distance to space source of radiating in earth IR.

As cosmological extension can decelerate in time, so group velocity of electromagnetic wave depends on t.

We shall consider a solution of a wave equation at lack of a demarcation of media, in neglect by dispersion, at lack of a tangential component velocity of a motion of a medium, when the motion of a medium is guided against a wave vector of an electromagnetic wave.

For the module of the $i-$th wave vector of a stratum of a medium we shall receive expression

$$k_{in} = \frac{\omega_0}{c} \frac{-\kappa_i \gamma^2{}_i \beta_{in} + \sqrt{1+\kappa_i}}{1-\kappa_i \gamma_i^2 \beta_{in}^2} \tag{1}$$

Here $\kappa_i = \varepsilon_i \mu_i - 1$, $\beta_{in} = \dfrac{u_{in}}{c}$, $\gamma_i^{-2} = 1 - \beta_{in}^2$, the magnitudes $u_{in}, \varepsilon_i, \mu_i$ characterize normal component velocities, dielectric and magnetic conductivities of an i-th layer of a medium in a IR of the observer, c - velocity of a plane monochromatic electromagnetic wave in vacuum, ω_0 - frequency of an electromagnetic wave emitted from distant astrophysical object, in a IR of the observer.

This expression characterizes propagation of electromagnetic radiation without a space-time curvature and without effect of a deformation of a trajectory of a wave vector, bound with rotation.

For velocity of an instellar medium along a trajectory of light propagation relative to earth observer we can write

$$\beta_n(r,t) = \frac{\dot{R}(t)}{cR(t)}(r - r_1), \tag{7}$$

where r is a current radial coordinate.

Removal velocity of a radiating astrophysical object without account of object velocity in a local region of extending space is equal

$$\beta(t) = -r_1 \frac{\dot{R}(t)}{cR(t)}. \tag{8}$$

Relative to the function $R(t)$ we can noticed that in the extending Universe any view of $R(t)$ during extending will increase the difference $t_0 - t_1$.

To estimate influence of interstellar medium let consider integral properties from the right in (5), here we will consider that $R(t) = R_0/\mathrm{c}$, $t = t_0 - t_1$.

Velocity of a radiation propagation in $i-$th a motioning layer of a medium for earth observer will pay off under the formula

$$v_i = \frac{\omega_0}{k_{in}} = c \frac{1-\kappa_i \gamma_i^2 \beta_{in}^2}{-\kappa_i \gamma_i^2 \beta_{in} + \sqrt{1+\kappa_i}}. \tag{9}$$

From (5) we get

$$t = \frac{R_0}{c} \int_0^{r_1} \frac{1}{\beta_e(r,t)} \frac{dr}{\sqrt{1-kr^2}} = \frac{R_0}{c} \int_0^{r_1} \frac{n\left(1-\beta_n^2(r,t)\right) - \left(n^2-1\right)\beta_n(r,t)}{1-n^2\beta_n^2(r,t)} \frac{dr}{\sqrt{1-kr^2}}. \tag{10}$$

Here $n = n(r,t)$ is index of refraction of an interstellar medium or radiating field of an astrophysical object. Index of refraction depends on t, as medium density changes in time.

As interaction of gravitational radiation with a medium is very slight, propagation time of gravitational wave signal will be defined from (10), but if $\beta_e(r,t)=1$. Detection of cosmological relict gravitational waves could allow to define dependence of the scale factor on time $R(t)$.

3. Propagation time for radiation in the case when refraction index is constant

Let consider the case, when $n = Const$, $k = 0$. From (11) we have

$$t=\frac{R_0}{c}\int_0^{r_1}\frac{\tilde{\beta}\left(r-r_1\right)+n}{1+n\tilde{\beta}\left(r-r_1\right)}dr. \tag{12}$$

Here $\tilde{\beta}=\frac{\dot{R}(t)}{cR(t)}=\frac{H}{c}$, where H - Hubble constant.

Integration gives

$$t=\frac{R_0}{c}\left\{\frac{r_1}{n}-\frac{1}{\tilde{\beta}}\frac{n^2-1}{n^2}\ln\left|1-r_1n\tilde{\beta}\right|\right\}. \tag{13}$$

In the case when $\tilde{\beta}$ is found from the expression

$$\tilde{\beta}=\frac{1}{r_1n\left(1+\alpha\right)}, \tag{15}$$

from (13) we have

$$t=\frac{R_0r_1}{cn}\left\{1+\left(n^2-1\right)\left(1+\alpha\right)\ln\left|\frac{1+\alpha}{\alpha}\right|\right\}. \tag{16}$$

For delay of passing light beam from a source to a receiver by the factor $(1+|\rho|)$, the next equation should be fulfilled

$$\left(n^2-1\right)\left(1+\alpha\right)\ln\left|\frac{1+\alpha}{\alpha}\right|=(1+|\rho|)n^2-1. \tag{17}$$

When $\alpha << 1$ we can obtain the relation for ρ,α,n

$$n\cong\sqrt{1-\frac{|\rho|}{1+\ln\alpha}}. \tag{18}$$

For index of refraction of interstellar medium we have

$$n^2=1+\frac{4\pi n_{e0}e^2}{m_e(\omega_{0e}^2-\omega^2)}, \tag{19}$$

where $n_{e0}=1{,}2\times10^{-5}\ sm^{-3}$ - is concentration of electrons in ionizated gas at present, e, m_e, ω_{0e} are charge, rest time and the Longmuir frequency of an electron, ω_e is a circular frequency of an electromagnetic wave.

Excepting n, we will obtain expression for delay factor

$$\rho=-\frac{4\pi n_{e0}e^2}{m_e(\omega_{0e}^2-\omega_e^2)}\left(1+\ln\alpha\right). \tag{20}$$

Therefore, if ω_e is close to ω_{0e}, light can have considerable delay in the field where removal velocity of interstellar medium from an earth observer is equal to $\frac{c}{n\left(1+\alpha\right)}$.

In cosmological scale time delay of incoming electromagnetic wave in comparison with gravitational that leads to the important consequence. Light splash, which is according to gravitational wave detected by the earth observer, can reach the observer with delay which multiply exceeds time of observer life.

From estimation calculations it follows that defining distance should be carried out with account the effect of delay of light propagation in a moving medium. Let us take as an example the solution for constant index of refraction (14). Relative distance to source of radiation will determined by the expression

$$r_1 = \frac{1}{R_0}\frac{tcn}{1+\left(n^2-1\right)\left(1+\alpha\right)\ln\left|\frac{1+\alpha}{\alpha}\right|}. \tag{21}$$

As we can noticed from (21) when $\alpha \to 0$ the distance, passed by light beam for time t, tends to zero.

4. The case of refraction index of hyperbolical type

Let refraction index in radiating field has hyperbolical dependence on distance to an earth observer

$$n = n_0 + \frac{n_1}{pr+1}, \tag{22}$$

where n_0, n_1, p are constant values $n_0 \geq 1,\ \ n_1, p \geq 0$.

The given type of dependence can take place in the radiating field of astrophysical object. Parameters n_1, p can provide the dependence $n(r)$ which is close to real that.

In the case we have for a kernel of the equation

$$t = \frac{R_0}{c}\int_0^{r_1}\frac{1}{n_0}\left(1+\frac{\alpha_1 y+\alpha_2}{\gamma_0 y^2+\gamma_1 y+\gamma_2}\right)dr, \tag{23}$$

$$y = r - r_1,\ \alpha_1 = \tilde{\beta}\left(b-a\right),\ \alpha_2 = n_0 a - \frac{b}{n_0},$$

$$\gamma_0 = \tilde{\beta}p,\ \gamma_1 = \tilde{\beta}a + \frac{p}{n_0},\ \gamma_2 = \frac{b}{n_0}.$$

The solution has a view

$$t = \frac{R_0}{cn}\left\{r_1 + \left(\rho_1 + \frac{\rho_2}{\sqrt{D}}\right)\ln\left|\frac{b}{1+\sigma\tilde{\beta}}\right| + \frac{2\rho_2}{\sqrt{D}}\ln\left|\frac{2\gamma_0 y_1+\gamma_1+\sqrt{D}}{\gamma_1+\sqrt{D}}\right|\right\}. \tag{26}$$

$$\rho_1 = \frac{\alpha_1}{2\gamma_0},\ \rho_2 = \alpha_2 - \frac{\gamma_1\alpha_1}{2\gamma_0},\ \sigma = r_1^2 p - r_1 a,\ D = \gamma_1^2 - 4\gamma_0\gamma_2.$$

In the limit $\tilde{\beta} \to 0$ (26) looks

$$t = \frac{R_0}{c}\left\{n_0 + n_1\ln\left|1+R_0 p\right|^{\frac{1}{R_0 p}}\right\}. \tag{27}$$

In the limit $p \to 0$, we have

$$t = \frac{R_0 r_1}{c}\left(n_0+n_1\right). \tag{28}$$

5. Influence of cosmological scale factor and spatial curvature

Influence of extension of Universe may be account, if the integral in the left side of (5) will be found.

Let consider the dependence $R(t)$ of the next view

$$R(t) = R_0 \exp\left(\frac{1}{2} g(t)\right). \tag{29}$$

If $g(t)$ can be presented as the polynomial $g(t) = 2\left(Ht + lt^2 + mt^3 + ...\right) \approx 2Ht$, we get

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \frac{R_0}{H}\left(\exp\left(-Ht_0\right) - \exp\left(-Ht_1\right)\right). \tag{31}$$

If time of receiving a signal t_0, and duration of propagation t are known, moment of radiating is define as

$$t_1 = -\frac{1}{H}\ln\left(e^{-Ht_0} + Ht\right). \tag{32}$$

Solution features point out to the dependence the moment t_1 on two values t_0 and t, but not on their difference. For later stages of Universe evolution the difference $t_0 - t_1$ considerably differs from t.

Ability of spatial curvature also has to lead to altering propagation time of electromagnetic and gravitational signals in extending Universe.

Let consider the equation (10) when $n = Const$:

$$t = \frac{R_0}{c}\int_0^{r_1} \frac{\beta_n(r) + n}{1 + n\beta_n(r)} \frac{dr}{\sqrt{1 - kr^2}}. \tag{33}$$

The given equation can be reduced to

$$t = \frac{R_0}{cn_0}\left\{I_1 + \left(\sigma_1 - \sigma_2\right) I_2\right\}, \tag{34}$$

$$I_1 = \int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}}, \qquad I_2 = \int_0^{r_1} \frac{dr}{\left(r + \sigma_2\right)\sqrt{1 - kr^2}} \qquad \sigma_1 = \frac{n}{\tilde{\beta}} - r_1, \quad \sigma_2 = \frac{1 - nr_1\tilde{\beta}}{n\tilde{\beta}}$$

The solution has a view

$$t = \frac{R_0}{cn}\begin{cases} \arcsin r_1 + \dfrac{\sigma_1 - \sigma_2}{\sqrt{1 - \sigma_2^2}} \ln\left|\dfrac{r_1 + \sigma_2}{\sigma_2} \dfrac{1 + \sqrt{1 - \sigma_2^2}}{1 + \sigma_2 r_1 + \sqrt{1 - \sigma_2^2}\sqrt{1 - r_1^2}}\right|, & k = 1, \\ r_1 + \left(\sigma_1 - \sigma_2\right)\ln\left|\dfrac{r_1 + \sigma_2}{\sigma_2}\right|, & k = 0, \\ \operatorname{Arsh} r_1 + \dfrac{\sigma_1 - \sigma_2}{\sqrt{1 + \sigma_2^2}}\left(\operatorname{Arsh}\dfrac{1}{\sigma_2} - \operatorname{Arsh}\dfrac{1 - \sigma_2 r_1}{r_1 + \sigma_2}\right), & k = -1. \end{cases} \tag{37}$$

As it is not hard to noticed the curvature parameter k can have considerable influence on results of calculations when $\alpha << 1$. If $r_1 << 1$ the expressions for t with $k = 1$ and $k = 0$ are practically coincident, hence, if $r_1 \to 1$ and $\alpha << 1$, propagation time of electromagnetic and gravitational signal in the Universe with $k = 1$ by the factor 2 exceeds the same time in the Universe with $k = 0$.

6. General solution

Let consider the equation (5) with taking into account motion of interstellar medium and radial motion of radiation source from a ground observer. For a wave vector we obtain

$$k_n(r)=\frac{\omega_0(1+\beta)}{c}\frac{\left(\beta+n(r)\right)\left(1-\beta_n^2(r)\right)+\kappa\left(\beta-\beta_n(r)\right)}{\left(1-\beta^2\right)\left(1-\beta_n^2(r)\right)-\kappa\left(\beta-\beta_n(r)\right)^2}. \tag{38}$$

As $\beta(r)=\dfrac{\omega_0}{ck_n(r)}$, so the equation (5) will take a view

$$\int_{t_1}^{t_0}\frac{dt}{R(t)}=\left(1+\beta\right)\int_0^{r_1}\frac{\left(\beta+n(r)\right)\left(1-\beta_n^2(r)\right)+\kappa\left(\beta-\beta_n(r)\right)}{\left(1-\beta^2\right)\left(1-\beta_n^2(r)\right)-\kappa\left(\beta-\beta_n(r)\right)^2}\frac{dr}{\sqrt{1-kr^2}}. \tag{39}$$

Let us $n=const$. Equation (39) can be reduced to

$$\int_{t_1}^{t_0}\frac{dt}{R(t)}=\rho\int_0^{r_1}\frac{\beta_n^2(r)+d\,\beta_n(r)+e}{\beta_n^2(r)+a\beta_n(r)+b}\frac{dr}{\sqrt{1-kr^2}}. \tag{40}$$

$$\rho=-\frac{1+\beta}{n-\beta},\ a=-\frac{2\kappa\beta}{n^2-\beta^2},\ b=-\frac{1-n^2\beta^2}{n^2-\beta^2},\ d=-\frac{\kappa}{n+\beta},\ e=1+\frac{\kappa\beta}{n+\beta}.$$

General solution has a view

$$\int_{t_1}^{t_0}\frac{dt}{R(t)}=\rho\left\{r_1-\frac{\lambda_1}{\tilde{\beta}}I_1-\frac{\lambda_2}{\tilde{\beta}}I_2\right\}, \tag{41}$$

$$I_1=\int_0^{r_1}\frac{dr}{\left(r+q_1\right)\sqrt{1-kr^2}},\qquad q_1=-r_1-\frac{x_1}{\tilde{\beta}},\ x_1=-\frac{1-n\beta}{n-\beta},$$

$$I_2=\int_0^{r_1}\frac{dr}{\left(r-q_2\right)\sqrt{1-kr^2}},\qquad q_2=r_1+\frac{x_2}{\tilde{\beta}},\quad x_2=\frac{1+n\beta}{n+\beta},$$

$$\lambda_1=\frac{\left(d-a\right)x_1+e-b}{x_1+x_2},\qquad \lambda_2=-\frac{\left(d-a\right)x_2+e-b}{x_1+x_2}.$$

Let us take into account that $q_1^2<1,\, q_2^2>1$. So integrals $I_1,\ I_2$ for different values of spatial curvature are equal to

$$I_1=\begin{cases}-\dfrac{1}{\sqrt{1-q_1^2}}\left(\ln\left|\dfrac{1+q_1r_1+\sqrt{\left(1-q_1^2\right)\left(1-r_1^2\right)}}{r_1+q_1}\right|-\ln\left|\dfrac{1+\sqrt{1-q_1^2}}{q_1}\right|\right), & k=+1,\\[2ex] \dfrac{1}{\sqrt{1-q_1^2}}\left(\ln\left|\dfrac{1-q_1r_1-\sqrt{\left(1-q_1^2\right)\left(1-r_1^2\right)}}{r_1+q_1}\right|-\ln\left|\dfrac{1-\sqrt{1-q_1^2}}{q_1}\right|\right), & k=-1,\\[2ex] \ln\left|\dfrac{r_1+q_1}{q_1}\right|, & k=0.\end{cases} \tag{42}$$

$$
I_2 = \begin{cases} \dfrac{1}{\sqrt{q_2^2 - 1}} \left(\operatorname{arctg} \dfrac{\sqrt{\left(\mathrm{q}_2^2 - 1\right)\left(1 - r_1^2\right)}}{1 + q_2 r_1} - \operatorname{arctg} \sqrt{\mathrm{q}_2^2 - 1} \right), & k = +1, \\ \dfrac{1}{\sqrt{q_2^2 + 1}} \left(\ln \left| \dfrac{1 + q_2 r_1 - \sqrt{\left(1 + q_2^2\right)\left(1 - r_1^2\right)}}{r_1 + q_2} \right| - \ln \left| \dfrac{1 - \sqrt{q_2^2 + 1}}{-q_2} \right| \right), & k = -1, \\ \ln \left| \dfrac{r_1 - q_2}{-q_2} \right|, & k = 0. \end{cases} \tag{43}
$$

The found solution allows calculating propagation time of electromagnetic wave in interstellar medium with account medium motion and astrophysical source, which radiated a wave, for different magnitudes of spatial curvature and arbitrary dependence of cosmological scale factor on time.

Analysis of obtained solutions shows that delay of propagation time of light signals to earth observer from removal astrophysical objects for certain velocities and dielectric constant of interstellar medium can be essential, hence its magnitude depends on dispersion properties of a medium and can be different in different cosmological models.

7. Conclusion

In the work the solution for propagation time of electromagnetic waves in extending Universe is obtained for dispersion less approximation.

As preliminary estimations show, propagation time of electromagnetic waves depends on velocity of interstellar or intergalactic media.

In the domain where the interstellar medium or the medium of the radiating field of an astrophysical object have close to light velocity in IR of observer, that is in near boundary and more distant domains of the Universe, electromagnetic waves stronger depend on medium motion. Namely in the more distant domains of the Universe there are objects of the astronavigation basis, therefore, the effect can considerably influence on solution of tasks of astronavigation and manning of space apparatuses.

As gravitation radiation weakly interacts with matter, retarding of electromagnetic wave propagation has to lead to different time of propagation of electromagnetic and gravitational waves.

Detection of gravitational waves could allow to solve the reverse task – to define characteristics of radiating field of an astrophysical object according to difference between time of electromagnetic signal and time of gravitation wave that.

The foundations of spacetime physics in the light of recent developments in geometry

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Group property considerations of flat spacetime structures consistent with the existence of a preferred frame (in particular with a flat spacetime with absolute relation of simultaneity) lead one to a seven-dimensional manifold described by time, space and velocity coordinates. That manifold, to be referred to as phase-space-time (PST), is precisely the base space of bundles defined by Finsler connections, i.e. Finsler bundles. This geometry is, therefore, in a special relation with preferred frame theories, relation, which is further suggested by the fact that the Lorentzian signature is the canonical signature of Finslerian refibrations (i.e. over PST) of the usual bundles of pseudo-Euclidean connections.

At the level of curved spaces, consideration of preferred frames leads to teleparallelism, or zero affine curvature. This is so by virtue of the fact that a preferred frame field leads to the metric-compatible affine connection which is defined as being zero in that preferred frame field. That remark is to be seen in the context that the Levi-Civita connection (LCC) is from 1917, thus younger than GR and not, therefore, a part of its original version. Finslerian teleparallelism leads to a canonical Kaluza-Klein type of geometry that constitutes a theory of both moving frames and moving particles, rather than just a theory of moving frames. To conclude, absolute simultaneity prompts one to approach physics from the perspective of the recent developments in geometry just mentioned.

1. Introduction

In 1905 and 1915, Einstein implemented his spacetime insights using the best geometric tools then available. In retrospect, the perspective provided by progress in differential geometry and related subjects leads one to realize that the relativities suffer from unnecessary continuing use of those old tools. Our starting point for such realization is the interference of the thesis of conventionality of synchronizations with the understanding of simultaneity relations. Assume the existence of a frame $\Sigma\ (T,X,Y,Z)$ of isotropy. Let transformations to a system S' of velocity $(v,0,0)$ be

$$x' = \gamma \cdot (X - vT), \quad y' = Y, \quad z' = Z, \quad t' = \gamma^{-1} T, \tag{1.1}$$

(c=1; $\gamma = (1-v^2)^{-1/2}$). Like the Lorentz Transformations (LTs), they are compatible with the independent experiments of Michelson-Morley (MM), Kennedy-Thorndike (KT) and Ives-Stilwell (IS), but kinematic effects are here absolute. If, in such para-Lorentzian world (PLW), one synchronizes clocks in the primed frame using t'' given by

$$t'' = t' - vx', \tag{1.2}$$

one readily gets

$$t'' = \gamma \cdot (T - vX), \tag{1.3}$$

which together with the first three equations (1.1) yields the LTs from (T, X, Y, Z) to (t'', x', y', z'). To complicate matters, the standard value of time dilation makes slow clock transport synchronizat-ion equivalent to Einstein's (or ab initio equality of to and fro speeds of light), as is generally known among researchers in the foundations of special relativity (SR). Let $\varepsilon << v << c$. If a clock moves with speed $v+\varepsilon$ with respect to Σ, it will read the time

$$t'' = T\{1 - [(v+\varepsilon)^2/2]\}^{1/2} \approx T(1 - v^2/2 - v\varepsilon). \tag{1.4}$$

On the other hand, clocks in S' will read, if synchronized so that the last equation (1.1) is satisfied,

$$t' \approx T\,(1 - v^2/2). \tag{1.5}$$

From these two equations, we get,

$$t'' = t' - v\varepsilon T \approx t' - vx'. \tag{1.6}$$

In the last step, we have used that, in the lowest order, we can replace T with t', and that $\varepsilon t'$ is the distance x' traveled by the clock moving with velocity ε relative to S'. This means that, using slow clock transport to synchronize clocks in S', replaces the true time t' of PLW with a "relativistic time" t'' which is as foreign to PLW as a synchronization procedure not coincident with Einstein's would be in SR. Since slow clock transport is the practical default synchronization, the LTs apply also in the PLW world if nothing moves too fast. This does not impede, however, that an absolute simultaneity relation may have implications that the default synchronization does not remove.

From an experimental perspective, the finding of the true relation of simultaneity would consist in measuring the one-way speed of light after synchronizing clocks by a method which, unlike slow clock transport, is neutral with respect to different possible simultaneity relations. Synchronizations through a rigid shaft, to name an example, have been proposed in the past as being neutral. However, proponents of the thesis of conventionality of synchronizations would claim without further argument that the behavior of practically rigid shafts would defeat any attempt at measuring with their help the one-way speed of light. In any case, rigid shaft synchronization has practical limitations too obvious to be considered.

A better way of using practical rigidity consists in measuring high order frequency shifts in centrifuges, as these appear to distinguish SR's relation of simultaneity from PLW's [1], [2]. The so-called Ehrenfest's paradox is, however, bound to interfere with the experiment's interpretation. In general, a dynamics that is to the PLW what the present dynamics is to SR would also have to be used for the analysis of experiments trying to discriminate between those relations of simultaneity. Theoretical work exists which, without developing a full PLW dynamics, appears to show that PLW provides a better formulation for relativistic quantum mechanics [3].

In his work on the subject of the testing of SR, this author has been led in a different though related direction, owing to the realization that mathematics was late to provide the structures that physics required. Thus, for example, flat spacetime is better viewed as an elementary Finsler space rather than as a pseudo-Euclidean one, whether or not we maintain the standard flat metric [4]. That could not be understood in 1905. Also, GR adopted the LCC by default, at a time when no reasonable alternative affine connections were yet known. More importantly, the mathematics of relativistic quantum mechanics and of its non-relativistic limit emerged only in 1960-62 [5]-[7]. It is constituted by Kähler's extension of Cartan's exterior calculus of differential forms to an exterior-interior calculus, appropriate for both quantum mechanics and GR/differential-geometry. It allows one to specify the torsion from the alternative perspective of a quantum mechanical equation, giving rise to a different picture of the world [8].

2. Simultaneity relations versus synchronization procedures

In 1905, Einstein actually went beyond reconciling dynamics with electrodynamics since, in the process, he made the one-way speed of light a universal constant as a matter of convention, even though conventions are not in principle the stuff of which universal constants are made. The constancy of the two-way speed, which need not be questioned, suffices to make c a universal constant. Though the LTs are inescapable from at least a practical perspective, the argument for them is different in PLW from what it is in SR. If we assume the first principle of SR (equivalence of inertial frames, equivalently group property), we are led to only three options: the LTs, the Galilean transformations and the usual rotations even in spacetime planes. Among these options, only the first one is reasonable nowadays. See, however, [9]-[10] for an option claimed to be based on equivalence, but incorporating a groupoid rather than a group. If, on the other hand, we adopt absolute simultaneity and the behavior of rulers and clocks demanded by those three experiments, we obtain the LTs because of the effect of slow clock transport. The issue then is to find non-trivial effects of an absolute simultaneity relation.

At this point, it is worth considering that very advanced exterior-interior calculus of differential forms [5]-[7]. Components of quantities in that calculus have three series of indices:

$$u = u_{\mu\mu'\ldots}{}^{\nu\nu'\ldots}{}_{\lambda\lambda'\ldots} \tag{2.1}$$

(Greek and Latin indices run from 0 to 3, and 1 to 3. In (2.1), one series of subscripts corresponds to multilinear functions of vectors and the other to functions of hypersurfaces. In the absence of the subscripts, (2.1) represents tangent tensors. We have the example of the affine curvature,

$$d(d\boldsymbol{e}_\mu) = d(\omega_\mu{}^\nu \boldsymbol{e}_\nu) = (d\omega_\mu{}^\nu - \omega_\mu{}^\lambda \wedge \omega_\lambda{}^\nu)\boldsymbol{e}_\nu = R_\mu{}^\nu{}_{\lambda\lambda'}\, \omega^\lambda \wedge \omega^{\lambda'}\, \boldsymbol{e}_\nu, \tag{2.2}$$

where $d\boldsymbol{e}_\mu$ is not exact, but is some given vector-valued differential 1-form. We still write $d\boldsymbol{e}_\mu$ since it becomes exact when the affine curvature is zero. A similar approach to the affine curvature is

$$d(d\boldsymbol{u}) = R_\mu{}^\nu{}_{\lambda\lambda'}\omega^\lambda \wedge \omega^{\lambda'}\, u^\mu\, \boldsymbol{e}_\nu \;\; (= \Omega_\mu{}^\nu\, u^\mu\, \boldsymbol{e}_\nu) \tag{2.3}$$

where the u^μ constitute a basis of linear forms [11]. In terms of coordinate bases, we have

$$\Omega_\mu{}^\nu = R_\mu{}^\nu{}_{\lambda\lambda'} dx^\lambda \wedge dx^{\lambda'}. \tag{2.4}$$

Kähler's use of three series of indices implies that three algebras have to be considered.

With this new perspective, we return to the issue of simultaneity relations versus synchronization procedures. We mean that, under the same synchronization (say, Einstein's), we may still have different underlying simultaneity relations, their difference possibly showing in indirect ways. Our purpose at this point is to show a source for indirect ways. Since we are dealing with competing linear transformations, namely (1.1) and the LTs (with generalization in both cases of the direction of the velocity between frames and incorporation of the rotations and translations), the issue consists in discriminating subsets of affine transformations. In affine space, a position vector,

$$\boldsymbol{P} = t\, \boldsymbol{a}_0 + x\, \boldsymbol{a}_1 + y\, \boldsymbol{a}_2 + z\, \boldsymbol{a}_3, \tag{2.5}$$

is defined. Differentiating it, we have:

$$d\boldsymbol{P} = dt\, \boldsymbol{a}_0 + dx\, \boldsymbol{a}_1 + dy\, \boldsymbol{a}_2 + dz\, \boldsymbol{a}_3, \tag{2.6}$$

which can further be written as

$$d\boldsymbol{P} = \eta_\lambda{}^\nu\, dx^\lambda\, \boldsymbol{a}_\nu . \tag{2.7}$$

Equation (7) shows the presence of two algebras in d**P**, namely an algebra of tangent vectors (bold face) and an algebra of cochains, i.e. integrands (*d**P*** is integrated on curves). ***P*** is a grade zero cochain. The subtleties of synchronizations affect the algebra of cochains, not the one of tangent vectors. In other words, absolute simultaneity may still require LTs, but have consequences at variance with the principle of SR in the tangent algebra, which is not affected by synchronization procedures.

3. Kinematical versus dynamical testing

Although the "classification" of testings of the flat spacetime structure into kinematical and dynamical that will now follow is not exact, it is an effective starting point to get deep into the general subject of testing the flat spacetime structure. We use the term kinematical testing to refer to those experiments and their analyses where one does not build a dynamical theory for the task; it appears that virtually all testing done so far is kinematical. The term dynamical testing, on the other hand, is used to refer to analyses that rely on dynamical theories built for the explicit purpose or analyzing experiments. More precisely, under the umbrella of dynamical testing one would be considering families of dynamics consistent with the different flat spacetime structures among which one wishes to discriminate.

Speaking of dynamics, the issue arises of what would the fundamental interactions look like in a PLW. This, however, has limited interest for experiments, as we not only need to have different

electrodynamics consistent with the different kinematical structures among which one wishes to discriminate, but also different theories of elasticity and/or fluids (Rembielinski, however, appears to have found a specific situation where one can simplify the dynamical complications that one might expect in principle [12]. Let us start by considering the simplest case, the construction of a family of electrodynamics consistent with some family of flat spacetime structures.
In principle, different families of rectilinear coordinate transformations may be viewed through their dual tangent bases as being representative of different flat spacetime structures, thus of different restrictions of the set of all bases of spacetime vectors. Because of the properties of the LTs, one associates with them the invariant T^2-X^2-Y^2-Z^2. Vice versa, one views the bundle of pseudo-orthonormal bases as representing the world of SR. Then the bases that correspond to the absolute relation of simultaneity are determined by comparison of (1.1) with the LTs through equation (1.2), which, for arbitrary direction of the velocity, becomes

$$t'' = t' - v_i\, x'. \qquad (3.1)$$

Those bases, $\boldsymbol{e}'_\mu$, are such that

$$x'^{\mu}\, \boldsymbol{e}'_{\mu} = x''^{\mu}\, \boldsymbol{e}''_{\mu}, \qquad (3.2)$$

where $x''^{i} \equiv x'^{i}$, and where the $\boldsymbol{e}''_\mu$ are the pseudo-orthonormal bases. Upon computing, one sees that even the $\boldsymbol{e}'_i$ are not in general orthonormal to each other. Let us then try to apply this idea to the obtaining of an alternative to Maxwell-Lorentz electrodynamics.
Maxwell's equations can be expressed in terms of any coordinate system. It is clear, however, that their usual form in terms of explicit derivatives of the fields is consistent with the LTs. The issue now is whether this association affects just the algebra of cochains (thus, in flat spacetime, the validation of the LTs), or whether it also affects the tangent algebra. In the former case, Maxwell's equations do not permit one to distinguish SR from PLW since the de facto synchronization is Einstein's in both cases. In the latter case, the PLW restriction of vector bases would give rise to an alternative to the LTs. The answer comes from an unexpected authoritative source.
E. Cartan asked himself the question of generalizing Maxwell's equations to general spacetime manifolds (The standard answer that one writes the equations in tensor form to achieve covariance is largely empty, since there are connection-dependent and connection-independent ways of doing so; exterior differentiation of scalar-valued differential forms is a connection-independent covariant differentiation). Although we are not interested here in Cartan's question per se, we are, however, interested in his argument because of the related issue he considered of whether the generalization depends on the connection of spacetime. The answer to this question in turn depends on whether the content of Maxwell's equations is best represented by its integral or point forms. Cartan sided with the integral form, which corresponds to Maxwell's equations being about scalar-valued differential forms, if these are understood as being 2-cochains (i.e. functions of surfaces in spacetime). In that case, therefore, electrodynamics will look in PLW like it does in SR.
The issue is more complicated, however, for two reasons, one of them having been mentioned by Cartan himself. He pointed out that there is more to electrodynamics than Maxwell's equations, and we must certainly be interested in all of electrodynamics and not just those equations. There is the issue of the energy-momentum relations, and these have to do with vector-valuedness, and thus with connections. There may be a difference at the level of energy-momentum relations between choosing one or another restriction of vector bases, corresponding respectively to SR and PLW. The question immediately arises of why the energy relations depend on the connection of spacetime if the Maxwell's equations do not. Until one has an answer to this question, one cannot safely use electrodynamics for discriminating between SR and PLW.
Another complication has to do with the following. The independence on connection of Maxwell's equations is clear if we view those equations as pertaining to two different differential 2-forms: F for the fields E and B, and G for the inductions D and H. The equations would look like $dF = 0$ and

$dG = j$, where F and G are two independent differential forms. J is a differential 3-form. But we know from the charges-in-vacuum case that the components and G constitute a rearrangement of the components of F, which is to say that those two differential forms are not independent; one is the Hodge dual of the other. Hence, $dG = j$ is actually $d^*F = j$ or, equivalently, $^*d^*F = {}^*j$. This last equation can be written as $\delta F = {}^*j$, where δ is the interior derivative with respect to the LCC (unlike the exterior derivative, the interior derivative of scalar-valued differential forms depends on connection). Hence Maxwell's equations might depend on restriction after all.
This author sought to resolve this issue in geometric terms. We shall not enter here the why so. We shall briefly report on the line of thinking that led serendipitously to new discoveries. After reporting on them, we shall proceed with a more economical path to results based on progress in differential geometry, calculus of differential forms and applied group theory. In the title and for brevity reasons, we have referred only to geometry, to which group theory and calculus are related.
This author's argument went as follows. He assumed that a family of electrodynamics compatible with a family of flat spacetime structures (among which one wishes to discriminate and containing SR and PLW) had to be of geometric nature. The question then arises of where is the geometry in the relativistic member of the family of electrodynamics, i.e. in Maxwell-Lorentz electrodynamics. If the latter can be represented in terms of some connection pertaining to classical differential geometry (as opposed to gauge geometry), one would try to adapt that representation to the different flat spacetime structures. But, as soon as contact was made between the torsion of spacetime and the electromagnetic field through the equations of the motion of charged particles, the research took a life of its own. Instead of obtaining a dynamical test theory of SR, some sort of unified geometric field theory started to emerge. One retrospectively finds that if one had first developed the relevant mathematics independently of any physical motivation, one would have encountered in any case the unified field theory, given the fact that the equations of structure of the emerging geometry look remarkably like the field equations of the physics. In the remainder of this paper, we shall show how developments in geometry suggest an alternative view of the foundations of the physics. We start by considering developments that have to do with the relation of simultaneity.

4. Group property and the required type of Finsler geometry

We proceed to consider the group property of the general PLW transformations, not considered here for reasons of space. The issue was first solved by Rembielinski, with his realizations of the Lorentz group [12]. His solution is also found in the present author's study of the group property of families of transformations generated by the assumption of "non-null results" for one or more of the MM, KT and IS experiments, or by a non-SR relation of simultaneity. The group property for those families arises from the recognition that the transformations to be studied are not those such as (1.1) (or their generalization to arbitrary velocity and inclusion of rotations). All such transformations relate Σ and S'. One must relate frames of type S'**,** moving with velocities $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ with respect to Σ [13]. Among the groups found for the aforementioned families, there were those already found by Rembielinski [12], which correspond to the absolute relation of simultaneity.
Of interest here is that (x'^{μ}_1, V'^{i}_1) goes to (x'^{μ}_2, V'^{i}_2) by virtue of the boosts pertaining to those families, parameterized by $\boldsymbol{v}$. Those boosts may be viewed as referring to the passage from point to point, or as giving the coordinates of a point relative to two bases. However, the passage from (x'^{μ}_1, V'^{i}_1) to (x'^{μ}_2, V'^{i}_2) may be viewed only in the second way; since the velocities $\boldsymbol{V}$ are velocities of frames, not of the particles whose motion is referred to those frames. Hence, frames corresponding to alternatives to SR must be viewed as lying not over the spacetime manifold, but over the bundle of directions of spacetime (viewed as just a topological space, which is then considered as base space for other bundles). Even for the SR case, the total space of its Finsler bundle is the original set of bases, labeled by the four coordinates of the point where each basis is tangent, and six more

coordinates determining the velocity and orientation of the basis. That set is fibrated now over the topological bundle of directions. The base space having become larger (seven dimensions instead of four), the fibers have become smaller (dimension three). The reason for calling this a Finsler structure is that, in general, the products $\boldsymbol{e}'_\mu \cdot \boldsymbol{e}'_\nu$ depend on the velocities.

5. The Cartan-Klein-Clifton view of Finsler geometry

Cartan used the term false spaces of Riemann to refer to the Riemannian spaces of before the advent of the birth in 1917 of the LCC; they had to do simply with metrics as expressions for distance, at a time when a concept of affine connections did not exist (much less a general theory thereof). In Cartan's view, the requirement for a manifold to qualify as a space was the presence of a concept of geometric equality, or a succedaneum for it, like the LCC. Even to this day, it appears that only one book on differential geometry [14] formally approaches Riemannian geometry from the Cartan perspective of generalizing the so called Klein or elementary geometries. Fittingly, the subtitle of [14] is "Cartan's Generalization of Klein's Erlangen Program".

Standard Finsler theory is based on a concept of distance more general than Riemann's, often with total disregard for the connection, which provides a concept of equality of vectors and geometric figures at nearby points. Cartan would thus qualify the standard Finsler spaces as "false spaces of Finsler", if he had been confronted with such an issue. There is very little redemption in adjoining to the false Finsler space the canonical connection of the distance since, in general and like in prior geometry, connections should not have to be canonically determined by a distance.

Our inclusion of the name of Clifton in characterizing the type of Finsler geometry that concerns us here responds to the fact that he formalized the theory of affine-Finsler geometry, but did not publish (See [15] for historical detail). Let us first give the essence of what Clifton's metric-Finsler connections on spacetime were before he considered the simply affine-Finsler ones.

On the seven-dimensional phase-spacetime with coordinates (x^i, u^i, t), we define connection forms (ω^0, ω^i, $\omega_0^{\ i}$, $\omega_i^{\ j}$), independent except for satisfying the constraints

$$\omega_0^{\ i} = \omega_i^{\ 0}, \qquad\qquad \omega_i^{\ j} = -\omega_j^{\ i}. \tag{5.1}$$

The differential equations

$$d\boldsymbol{P} = \omega^0\boldsymbol{e}_0 + \omega^i\boldsymbol{e}_i\,, \quad d\boldsymbol{e}_0 = \omega_0^{\ i}\boldsymbol{e}_i, \qquad d\boldsymbol{e}_i = \omega_i^{\ j}\boldsymbol{e}_j, \tag{5.2}$$

can be integrated upon curves on phase-spacetime, giving rise to (inhomogeneous in general) LTs. For this purpose, one considers on phase-spacetime only the curves, called natural liftings. They satisfy the condition $u^i = dx^i/dt$ at every point, i.e. where the three 1-forms $dx^i - u^i dt$ are zero. Since the ω^i's are of the form $A_j^{\ i}(dx^j - u^j dt)$, the system $\omega^i = 0$ for all three values of i are the equations for the natural liftings. The system

$$\omega^i = 0, \qquad\qquad \omega_0^{\ i} = 0, \tag{5.3}$$

determines a family of natural liftings through each point of spacetime. They constitute autoparallels or lines of constant direction, in the sense that the tangent vector is constant along the curve. In the absence of a formal presentation here, suffice to say that, on Finsler bundles over phase-spacetime, $\boldsymbol{e}_0$ becomes the 4-velocity of the frames. Thus the equation $d\boldsymbol{u} = \boldsymbol{0}$ for autoparallels becomes $d\boldsymbol{u} = d\boldsymbol{e}_0 = \omega_0^{\ i}\boldsymbol{e}_i = \boldsymbol{0}$, and Eqs. (5.3) follow. On the other hand, the equations

$$\omega_0^{\ i} = 0, \qquad\qquad \delta\textstyle\int(\omega^0, \text{modulo } \omega^i) = 0, \tag{5.4}$$

represent the natural liftings that are stationary curves. It is clear that ω^0 can always be written as a linear combination of dt and ω^i. Hence, the equations (5.4) are equivalent to

$$\omega_0^{\ i} = 0, \qquad\qquad 0 = \delta\textstyle\int ds, \tag{5.5}$$

where ds is, of course

$$ds = [(\omega^0/d\lambda)^2 - \Sigma(\omega^i/d\lambda)^2]^{1/2} d\lambda = (\omega^0\,,\, \text{mod } \omega^i)\,. \tag{5.6}$$

We have taken into account that, on curves, all differential 1-forms are multiples of just one, which can be chosen to be the parameter λ, thus justifying the ratios in (5.6). In general, $(\omega^0)^2 - \Sigma(\omega^i)^2$ is velocity-dependent. The parameter λ can be taken to be time itself. The connection with the Lagrangian formalism is then obvious: ω^o, *modulo* ω^i plays the role of Ldt. There is the issue of the signature, related to the signature of the Hessian, $\partial^2 L/\partial u^i\partial u^j$. We chose the Lorentzian signature. It is worth emphasizing, as if it were not obvious, that, on sections of the (re)fibrations over phase-spacetime, the $\omega_0{}^i$ are of the form

$$\omega_0{}^i = E^i{}_j du^j + F^i{}_j (dx^j - u^j dt) + G^i dt. \tag{5.7}$$

Similarly, the torsion and curvatures (metric and affine) also have terms of types additional to the traditional ones. All that is clear in the formalization that we proceed to motivate.

Cartan's revolutionary idea in differential geometry was to replace the groups in elementary (equivalently flat, nowadays called Klein) geometries with the structure of frame bundle, which are then assigned the same adjective (affine, Euclidean, etc) as the main group in the corresponding elementary geometry. The question then is: what is the flat geometry that Finslerian connections generalize? Before dealing with this issue, we need deal with a related one. Cartan viewed Euclidean connections as particular cases of affine connections, the manifolds on which the latter connections live not being endowed necessarily with a concept of distance or of a metric. The metric (in pre-Finsler geometry) amounts to a restriction of the tangent vector bases to those that satisfy the condition $\boldsymbol{e}_\mu . \boldsymbol{e}_\nu = \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is a principal diagonal matrix composed of 1's and –1's. This in turn implies by differentiation that

$$\omega_{\mu\nu} + \omega_{\nu\mu} = 0. \tag{5.8}$$

One may still "extend" the Euclidean or Lorentzian connection to an affine connection by the action of the linear group, in which case Eq. (5.8) becomes

$$\omega_{\mu\nu} + \omega_{\nu\mu} = dg_{\mu\nu}. \tag{5.9}$$

The difference between a general affine connection and an extension of a (pseudo)-Euclidean connection is that, in the first case, all n^2 differential forms $\omega_\mu{}^\nu$ are independent and in the second case they are not. We should in principle approach Finsler geometry with the same perspective. Our question thus becomes: what is the Finslerian generalization of affine connections, to which we shall refer as affine-Finsler geometry?

Upon this author's prodding, Clifton undertook the task of going beyond his aforementioned approach to Finsler geometry and proceeded to formulate affine-Finsler geometry independently of metric considerations. The alternative approach focuses on connections, thus on some form of geometric equality, a Kleinean guiding idea. The equations of structure are defined at the same time as the affine-Finsler connection itself. The pre-Finslerian affine connections result from a specialization of this definition [15].

Clifton's formalization of this version of Finsler geometry [15] stopped short of developing a formal theory of metric-Finsler connections, except for adopting the well known restrictions $\boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu = \eta_{\mu\nu}$ or $\boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu = \delta_{\mu\nu}$, which lead to Eq. (5.8). The concept of distance, defined as $\int(\omega^o$, *modulo* $\omega^i)$ becomes an invariant under that restriction. The concept of metric remains

$$ds^2 \equiv d\boldsymbol{P} \cdot d\boldsymbol{P} = \boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu\, \omega^\mu \omega^\nu, \tag{5.10}$$

or better yet as

$$ds^2 \equiv \boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu\, \omega^\mu \otimes \omega^\nu = g_{\mu\nu}\, dx^\mu \otimes dx^\nu. \tag{5.11}$$

That was sufficient for solving interesting problems with metric-Finsler connections [16].

It was clear to this author from work on test theories of SR, and specifically on the PLW, that (5.8) is inappropriate in general in Finsler geometry, since non-SR flat spacetimes yield condition $\boldsymbol{e}_\mu . \boldsymbol{e}_\nu = g_{\mu\nu}(v^i)$. To be specific, consider Eqs. (3.1)-(3.2), with $x''^i = x'^i$. We readily have

$$t'\boldsymbol{e}'_0 + x'\boldsymbol{e}'_i = (t' - v_i\, x')\boldsymbol{e}''_0 + x'\boldsymbol{e}'_i. \tag{5.12}$$

Hence

$$\boldsymbol{e}'_i = \boldsymbol{e}''_i - v_i\, \boldsymbol{e}'_0. \tag{5.13}$$

Since the vectors $\boldsymbol{e}''_i$ are orthonormal, the vectors $\boldsymbol{e}'_i$ (which correspond to PLW) are neither orthogonal nor unitary. Hence, it is clearly inappropriate to associate relations 5.1 with velocity dependent metrics. It may be possible, however, to reintroduce new metrics. The relations between different restrictions of the vector space would have to be maintained, however. A general formulation of metric-Finsler connections has not yet been undertaking. From a physical perspective, this author does not see at this point the need for it; the Finslerian requirements of flat spacetime physics are easily met without them.

Clifton's formulation does not mention what the Klein (i.e.. elementary) geometry corresponding to affine-Finsler connections is. It is a trivial problem to read it from his definition of affine Finsler connections, which is of a differential topology nature. The testing of SR provides restrictions that constitute elementary metric-Finsler elementary geometries of interest ([17], Section 4).

One must be aware of the fact that, whereas in Riemannian geometry (with or without torsion) the information contained in the metric is equivalent to the information contained in the distance 1-form, the same is not the case in Finsler geometry. Giving the metric is equivalent to giving the differential forms (ω^o, ω^i), to which one often refers as the square root of the metric, which is to be compared with the (in general, but not in Riemannian geometry) more restricted contents present in (ω^o, *mod* ω^i). Consider, for instance, the SR expression $\gamma(dt-vdx)$, which we shall compare with the corresponding absolute simultaneity expression $\gamma^{-1}dt$. We have:

$$\gamma(dt-vdx) = \gamma^{-1}dt - \gamma v(dx-vdt), \tag{5.14}$$

which becomes $\gamma^{-1}dt$ *mod* dx-vdt, thus *mod* ω^1. Hence, the distance is the same for SR and the PLW (among other worlds), even if the corresponding metrics (or restrictions of the bundles of tangent bases) are not. In general, we have

$$(\omega^o, \mathit{mod}\ \omega^i) = [g_{00}(x)+2g_{0i}(x)v^i+g_{ij}(x)v^{ij}]^{1/2}\, dt \tag{5.15}$$

for (pseudo)-Riemannian metrics [18].

We conclude with the statement of another most important result: the canonical signature of the bundles of metric-Finsler geometry (i.e. metric-compatible affine-Finsler connections, avoiding the confusing term of Finslerian (pseudo)-Euclidean connections) is the Lorentzian signature, not the positive definite signature. The reason is as follows. In order to make tangent bases to the four-dimensional spacetime manifold sit over the seven dimensional PST manifold, one constructs bases of reduced tangent bases to the latter manifold, which are in isomorphic correspondence with the former bases. In the process, one has to choose a preferred direction. Suppose that the signature is Lorentzian. If the direction chosen as preferred for the construction is timelike, the group on the fibers of the refibration over PST is the rotation group in three dimensions. But, if the chosen direction is spacelike, the group in the fibers is the Lorentz group for a three-dimensional spacetime subspace of the four-dimensional spacetime. The three dimensional subspace may be said to have been artificially split by the choice of a spatial rather than a timelike preferred direction. This artificial split, made clear by comparison with the choice of a temporal direction as preferred, is at work also for any choice of direction for any signature other than the Lorentzian one, since only the latter signature provides a special type of direction. It is then clear that one should view the spacetimes of SR and PLW as Klein geometries on which Finsler connections are modeled.

6. The Lorentz force and the torsion of Finslerian connections

We shall now show that the aforementioned Cartan-Clifton brand of Finsler geometry completely changes the perspective of the problem of classical geometrization (i.e. not in the auxiliary bundles of gauge theories) of the electromagnetic field. In a pre-Finslerian context (and possibly in other branches of Finsler geometry), one may ask what is the metric or the torsion a spacetime must have in order to achieve that the autoparallels (lines of constant direction) become the equations of motion of SR with Lorentz force. Here, the Lorentz force is unavoidable. Let us be specific.

Finslerian torsions are of the type [15]

$$\Omega^{\mu} = R^{\mu}{}_{\nu\lambda}\, \omega^{\nu}\wedge \omega^{\lambda} + S^{\mu}{}_{\nu i}\, \omega^{\nu}\wedge \omega_0{}^{i} = R'^{\mu}{}_{\nu\lambda}\, dx^{\nu}\wedge dx^{\lambda} + S'^{\mu}{}_{\nu i}\, dx^{\nu}\wedge dv^{i}, \tag{6.1}$$

where the v^i are the 3-velocity coordinates (named V^i in section 4). We shall refer to the S (and S') components as properly Finslerian, to be distinguished from the more conventional R (and R'). It is to be noticed, however, that $R^{\mu}{}_{\nu\lambda}\, \omega^{\nu}\wedge \omega^{\lambda}$ *is* not to be identified with a conventional torsion. Thus $R^{0}{}_{\nu\lambda}\, \omega^{\nu}\wedge \omega^{\lambda}\, \boldsymbol{e}_0$ in the Finsler bundle is not to be identified with $R^{0}{}_{\nu\lambda}\, \omega^{\nu}\wedge \omega^{\lambda}\, \boldsymbol{e}_0$ in the usual bundle, since $\boldsymbol{e}_0$ is the four-velocity of the frames in each and every frame field of the first bundle (In the usual bundle, on the other hand, $\boldsymbol{e}_0$ transforms in a well known way from one frame field to another). Only the temporal component of the torsion contributes to the autoparallels. It does so in such a way that, other than the gravitational contribution, only the Lorentz force results if $S^{0}{}_{\nu i}=0$. It is just an issue of identification of the coefficients $R^{0}{}_{\nu\lambda}$ with components of the electromagnetic field, up to a constant C. Because of its importance, let us proceed to show this.

In anticipation of said identification, we write Ω^0 as $-CE_r dt\wedge dx^r + CB_i dx^j\wedge dx^k$ (with summation over cyclic permutations in the last term). Since we are interested in natural liftings, we compute *mod* ω^i or, equivalently, *mod* σ^i, where $\sigma^i \equiv dx^i - u^i dt$. It is then convenient to write the temporal component of the first equation of structure for that torsion as

$$-CE_r dt\wedge(\sigma^r + u^r dt) + CB_{i'}(\sigma^{j'} + u^{j'} dt)\wedge(\sigma^{k'} + u^{k'} dt) = d\omega^0 - \omega^r\wedge \omega_r{}^0, \tag{6.2}$$

where (i', j', k') constitutes like (i,j,k) a cyclic permutation of (i', j', k'). We now let i' take the values i, j and k. The indices j' and k' will then take the values that correspond to them according to the notational conventions made. The left-hand side of (6.2) then becomes

$$-CE_i dt\wedge(\sigma^i + u^i dt) - C(B_k u^j - B_j u^k)dt\wedge\sigma^i - C(B_k\sigma^j - B_j\sigma^k)\wedge\sigma^i . \tag{6.3}$$

We proceed to compute the right hand side of (6.3). We choose to write ω^0 in terms of the four independent differential forms dt and σ^r. On the other hand, the ω^r can be written as linear combinations of just the three independent 1-forms σ^r. We thus have:

$$\omega^0 = l dt + \Lambda_r \sigma^r, \qquad \omega^i = A^i{}_r \sigma^r. \tag{6.4}$$

The right hand side of equations (6.3) then reads

$$d\omega^0 - \omega^r\wedge \omega_r{}^0 = (d\Lambda_m - l_{,m} dt + \omega_r{}^0 A^r{}_m)\wedge \sigma^m + (\Lambda_m - l._m)dt \wedge du^m, \tag{6.5}$$

where we have used that $l_{,m} dx^m\wedge dt = l_{,m}\sigma^m\wedge dt$, and where $l._m = \partial l/\partial u^m$. Using (6.3) and (6.5) in (6.2), we observe that the resulting equation is of the type

$$\alpha_i\wedge \sigma^i + (\Lambda_m - l._m)dt \wedge du^m = 0, \tag{6.6}$$

$$\alpha_i \equiv d\Lambda_i - l_{,i} dt + \omega_r{}^0 A^r{}_i + C[E_i + B_k u^j - B_j u^k]dt - C(B_k\sigma^j - B_j\sigma^k). \tag{6.7}$$

This implies that both terms on the left hand side of Eq. (6.5) must be zero and, therefore,

$$\Lambda_m = l._m , \tag{6.8}$$

$$d\Lambda_i - l_{,i} dt + \omega_r{}^0 A^r{}_i + C[E_i + B_k u^j - B_j u^k]dt - C(B_k\sigma^j - B_j\sigma^k) = C_{im}\sigma^m, \tag{6.9}$$

with $C_{im} = C_{mi}$. Substituting (6.8) in (6.9) and solving for $\omega_r{}^0 A^r{}_i$, one gets:

$$\omega_r{}^0 A^r{}_i = l_{,i} dt - dl._i - C[E_i + (B_k u^j - B_j u^k)]dt, \qquad mod\ \sigma^m. \tag{6.10}$$

Metric compatibility allows us to replace $\omega_r{}^0$ with $\omega_0{}^r$. The equations of the autoparallels, $\omega_0{}^r=0$, become

$$0 = -l_{,i} dt + dl._i + C[E_i + (B_k u^j - B_j u^k)]dt, \tag{6.11}$$

If E and B are zero, we obtain the Lagrangian form of the geodesic equations (stationary curves). If the temporal component of the torsion is not zero, we get additional terms that, as we see in Eq. (6.11), take the form of the Lorentz force. Hence, the issue is not how to geometrize the equations of motion of electrodynamics but rather how does one justify ignoring the result just shown. Put in another way, we have retrospectively found that, in the Finslerian fibers of the Cartan-Klein-Clifton refibration of the usual bundles of frames, the temporal part of the torsion is the only one that contributes to the equation of the autoparallels (this result holds when there are S components). Furthermore, that equation exhibits both the gravitational contribution to the equation of the motion and a force of the Lorentz type. Hence, an in depth understanding of the geometry of the world of absolute simultaneity leads one to consider Finsler bundles. In the equation for their autoparallels, we encounter the Lorentz (type of) force law without escape.

7. Teleparallelism

The introduction of preferred frames in physics in the context of the Robertson test theory leads one to consider the Finslerian structures to which we have made reference, and hence to finding Lorentz force terms in the autoparallels. We now document another implication of preferred frame fields (a preferred frame at each point): they have canonical connections associated with them with the property, here called teleparallelism (TP), of zero affine curvature. Although the argument will be made for the usual or pre-Finslerian affine connections, it is largely valid also for affine connections. We shall not discuss why this is so, or what one would have to be vigilant about, since this is, after all, just a motivational step towards the geometric structure of the next section.

A preferred frame field Σ defines a teleparallel connection, namely the connection which is zero in Σ. The affine curvature then is obviously zero. Conversely, if the affine curvature is zero, the second equation of structure implies that the connection equations $d\boldsymbol{e}_\mu = \omega_\mu{}^\nu \boldsymbol{e}_\nu$ are integrable over whole regions. For metric compatible connections, one can always write the affine connection as the sum of the LCC, $\alpha_\mu{}^\nu$, plus a tensor $\beta_\mu{}^\nu$ (one of three indices is hidden) called contorsion. In frame fields with zero affine connection, one obtains the contorsion as the negative of the LCC:

$$\beta_\mu{}^\nu = - \alpha_\mu{}^\nu. \tag{7.1}$$

The torsion is then obtained as:

$$\Omega^\mu = - \omega^\nu \wedge \beta_\nu{}^\mu. \tag{7.2}$$

In general frame fields, the connection is not zero, but we still have

$$\omega_\nu{}^\mu = \alpha_\mu{}^\nu + \beta_\mu{}^\nu. \tag{7.3}$$

Substitution in the equation for the affine curvature and rearrangement of terms leads to

$$d\alpha_\mu{}^\nu - \alpha_\mu{}^\lambda \wedge \alpha_\lambda{}^\nu = - \beta_\mu{}^\lambda \wedge \beta_\lambda{}^\nu - (d\beta_\mu{}^\nu - \omega_\mu{}^\lambda \wedge \beta_\lambda{}^\nu - \beta_\mu{}^\lambda \wedge \omega_\lambda{}^\nu), \tag{7.4}$$

the left-hand side being of course the metric curvature, or curvature determined canonically by the metric. Since the Einstein contraction of the left hand side yields the Einstein tensor, the same contraction of the right hand side may be viewed, if we follow the GR script, as the geometric version of the energy-momentum tensor of all physics except gravitation. In other words, TP implies a geometric set of Einstein equations, which become physical if, as we started to show in the previous section, the torsion connects with physical fields.

In TP, the first equation of structure becomes, if the *S* components of Finslerian torsions are zero,

$$R^\varphi{}_{\kappa\lambda;\eta} + R^\varphi{}_{\lambda\eta;\kappa} + R^\varphi{}_{\eta\kappa;\lambda} + R^\varphi{}_{\iota\lambda} R^\iota{}_{\eta\kappa} + R^\varphi{}_{\iota\eta} R^\iota{}_{\kappa\lambda} + R^\varphi{}_{\iota\kappa} R^\iota{}_{\lambda\eta} = 0. \tag{7.5}$$

If we drop the quadratic terms (weak field approximation) and set $R^i{}_{\kappa\lambda}$ equal to zero, we obtain the homogeneous pair of Maxwell's equations for the value zero of the φ index. Since $R^i{}_{\kappa\lambda}$ is to be associated with the *SO(3)* symmetry implicit in the Finslerian refibration, it is tentatively identified with the weak interaction. In addition, if one only needs the quantities present in the metric to develop a TP connection, physics may not have made the right move when it adopted the LCC (born in 1917!). The reason is that one cannot integrate energy-momentum tensors in GR. Integrating vector-valued differential forms requires constant frame fields, which do not exist under the LCC. When cosmologists use the comoving frame of the universe, they are using a preferred frame field. Unaware of it, their integrations implicitly imply that they are assuming TP.

To summarize, absolute simultaneity is associated with a preferred frame. One such frame at each point (arranged differentiably) becomes a preferred frame field, which in turn defines teleparallel connections.

8. The Kaluza-Klein space associated with Finslerian teleparallelism

In the Finsler theory under consideration, the base space of the bundle is phase-spacetime. The differential 1-forms that span the base space are ω^o, ω^i, $\omega_0{}^i$. The ω^μ represents the translation of the frames, and the $\omega_0{}^i$ define $d\boldsymbol{e}_0$, which in turn is $d\boldsymbol{u}$, where $\boldsymbol{u}$ is what one usually refers to as the 4-velocity. This yields readily a Kaluza-Klein construction, the fifth dimension representing a generic curve. $\boldsymbol{u}$ is the tangent vector, proper time being its dual coordinate on that curve. We have used the term Kaluza-Klein (KK) space since, like in standard Kaluza-Klein theory, the additional coordinate is of a "different nature"; fields do not depend on it [19].

This new construction is relevant on several grounds, one of them being that differential geometry is a theory of moving frames, rather than a theory where the motion of particles and not only of frames enters in a formal way in the equations of structure. This is an issue raised in passing by Cartan, as documented in [4]. On the other hand, we have the following feature. Although Finsler geometry is endowed with a rich complex of differential invariants, its algebraic structure is rather poor. In particular, no Clifford structure seems to emerge in Finsler bundles. Symptomatically, the problem of the Laplacian remains unresolved in those bundles. This operator is intimately related to the pair of exterior and interior derivatives, the two pieces of the algebraically rich Kähler derivative that one defines when a Clifford structure of differential forms exists. The KK space solves those problems and others, which we do not discuss here for lack of space. It is significant that the relation between torsion and electromagnetic field that we found in Finsler geometry is preserved in the KK space. In other words, when the torsion is taken to be (up to an appropriate multiplicative constant) the product of the electromagnetic 2-form and the vector $\boldsymbol{u}$, the equations of the autoparallels remain the equations of motion of SR with Lorentz force.

The question arises of what is the connection of this KK space with the foundations of spacetime physics. The unit vector $\boldsymbol{u}$ tangent to the τ coordinate lines is not orthogonal to the elements of the SR 4-dimensional frames. It is clear that this has to be so since the products $\boldsymbol{u}\cdot\boldsymbol{e}_\mu$ have the natural interpretation of components of the 4-velocity. On the other hand, the system of equations (3.1)-(3.2) implies not only that the spacetime frames associated with absolute simultaneity are not orthogonal, but that the spatial elements of the bases themselves are not orthogonal. This results from attaching quadratic forms to the restrictions associated with the different relations of simultaneity, orthonormality being associted with the standard Lorentz-Minkowski-Einstein for SR; the metrics for other restrictions are then determined by the relations between their respective sets of frames [17]. One may in principle ignore those quadratic forms themselves and retain just the relations between frames. Metrics can then be introduced in the KK space in such a way that those relations are maintained and, at the same time, the spatial elements of the bases are orthogonal to each other regardless of, for instance, relations of simultaneity. We then have the following situation. $\boldsymbol{u}$ is orthonormal to the 3-subspace determined by the spatial elements of the PLW frames, but not those of the SR frames. We can thus redefine in the PLW the dot product in the spatial subspace, so that orthonormality follows in the four dimensional subspace that excludes time directions (This property is not shared by the frames of SR, by virtue of what has been said above. Further study is needed of the significance of these features, especially in KK spaces corresponding to SR and PLW.

Of great interest is the fact that those KK spaces have a rich double Clifford algebra, one pertaining to integrands (or differential forms in presentations such as Rudin's [20]), and another one pertaining to tangent and cotangent tensors. In the usual perspective, the obtaining of the metric from more fundamental concepts involves a tensor product, $g_{\mu\nu}\,\omega^\mu\otimes\omega^\nu$, and a dot product (since $g_{\mu\nu} \equiv \boldsymbol{e}_\mu\cdot\boldsymbol{e}_\nu$), i.e.

$$d\boldsymbol{P}(\otimes,\ \cdot)d\boldsymbol{P} = \omega^\mu\boldsymbol{e}_\mu\,(\otimes,\ \cdot)\,\omega^\nu\boldsymbol{e}_\nu = \omega^\mu\otimes\omega^\nu\boldsymbol{e}_\mu\cdot\boldsymbol{e}_\nu = g_{\mu\nu}\,\omega^\mu\otimes\omega^\nu. \qquad (8.1)$$

In the KK space, $d\boldsymbol{P}(\otimes,\ \cdot)d\boldsymbol{P}$ is replaced with $d\wp(\vee,\vee)d\wp$ ($d\wp$ is the 5-dimensional translation element, and the Clifford products $\vee$ refer to each of the two aforementioned algebras).

Limitations of space dictate that we reserve further developments on this KK space for a sequel to this paper, to be presented in the 2007 conference on Finslerian Extensions of Relativity Theory.

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The Necessity of Unifying Gravitation and Electromagnetism and the Mass-Charge Repulsive Effects in Gravity

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It is found that the Reissner-Nordstrom metric enables to show that general relativity + electrodynamics are not yet a close system. In gravity, there is a repulsive effect due to the electromagnetic energy. Analysis of this effect shows that the geodesic equation as the equation of motion is inadequate. To include the force of charge-mass repulsion, modifications of electromagnetic and gravitational theories are necessary. Moreover, it is shown that the unification within the theoretical framework of a five-dimensional theory would resolve this problem because of the additional metric elements. In the five-dimensional theory of Einstein and Pauli, those elements were disregarded as having no physical meaning. Concurrently, a limitation of the formula $E = mc^2$ is proven and experimental verifications of the new force are discussed. Thus, although the so-called "covariance principle" is proven as not generally valid, the full meaning of relativity is still emerging after 100 years of Einstein's creation.

1. Introduction

Einstein initiated the unification of electromagnetism and gravity probably because of their formal similarity. However, the need of unification of these forces has not yet been clarified although the theory of Kaluza and Klein manages to give a formal unification without any modification [1]. Maxwell showed, however, that unification is a remedy to remove the shortcomings of the theories to be unified [1]. Accordingly, Einstein's unification [2] would fail because Einstein considered general relativity is logically complete [3]. Thus, Einstein and Pauli [4] regarded all the "extra" metric elements in the five-dimensional theory as having no physical meaning.

Nevertheless, the rise of Yang-Mills theory [5] results in a great advance of unifying the weak, the strong, and the electromagnetic force in term of the standard model [6]. First, the electromagnetic and the weak forces were unified. Next the strong force can also model in terms of a Yang-Mill theory. Then, the unification with gravity is the next goal. Many claimed that the string theory would give this final unification. However, string theorists tried for more than a quarter of a century without any visible success [6]. Recently, critics started to openly question the competence of string theorists, the validity of string theory, and even the relevance of unification [6, 7].

In this paper, it will be shown that general relativity is not a closed system as Einstein claimed [3]. Thus, the problem would be independent of even whether the string theory is relevant. Apart from the difficulty in mathematics, a hidden problem is that theorists do not understand general relativity and related theories yet. Thus, the real problem is that general relativity is not yet ready for the stage of unification. For instance, the editorial the Royal Society still rejects Einstein's requirement on weak gravity [8, 9] since the "covariance principle" is proven invalid only recently [10]. Moreover, there are examples of unphysical metrics which are diffeomorphic to the Schwarzschild solution [11].

Most of those who work on the issue of unification are particle physicists or mathematicians, who are excellent essentially in their own fields. Naturally, they rely on experts of relativity. Unfortunately, those perceived "experts" actually do not understand general relativity [12, 13, 14], and Feynman [15] was aware of their inadequacy. For instance, except in Einstein's original works, there are no textbooks or reference books (including the British Encyclopedia [2006]) that stated and explained Einstein's equivalence principle correctly although this principle is stated squarely in page 57 of Einstein's book, "The Meaning of Relativity"" [2]. In addition, some of such theorists criticized Einstein without getting the facts straight first [12, 14].

About 25 year ago, we [16] believe that, as in the case of electromagnetism, the unification is due to internal inadequacy of such theories. Obviously, the radiation reaction force is missing in electrodynamics [1]. However, it was very difficult to identify inadequacy in general relativity because things are not clearly defined. Although Einstein's equivalence principle is clear [2, 17], it was very difficult to apply since the coordinates were ambiguous. It took a long time to recognize that Einstein's theory was not even self-consistent because of two reasons. First, Einstein's theory of measurement is actually inconsistent with his equivalence principle [18]. Moreover, he over-looked that his measuring instruments are in a free fall state and his method of measurement is actually not executable for the length of an extended object [20]. Second, the so-called "covariance principle" is not generally valid in physics [10]. In fact, Einstein's argument of the justification of the "covariance principle" was actually not valid (see also Appendix).

However, problems seem to be rectifiable within the theoretical framework of general relativity [10, 12-14, 18-20]. A major problem remains to be fixed is that Einstein's equation must be modified to have a dynamic solution [21, 22]. However, an equation of first order approximation, which was derived independent of the Einstein equation, would give dynamic solutions for massive sources [23]. It is interesting to note that Einstein and Rosen [22, 24] were the first who discovered the non-existence of wave solutions. However, Einstein did not explain his equivalence principle sufficiently although he objected that Pauli's version is a misinterpretation of his principle [25]. The famous formula $E = mc^2$, [1)] was "derived" in 1905 [17], but only Einstein saw the limitation of $E = mc^2$ [26]. Moreover, Einstein did not see that his formula is inconsistent with the notion that light consists of just electromagnetic waves [27].

Thus, when the Riessner-Nordstrom metric is used to show that $m = E/c^2$ is not generally valid [28], some theorists later counter with using the formula $E = mc^2$ to misinterpret the metric [29-31]. They even did not do a simple differentiation that would uncover such an interpretation as invalid (see also section 2).

Moreover, the skeptics demand for additional experimental verification on the limitation of $E = mc^2$. Then such an investigation leads to focusing attention to the Riessner-Nordstrom metric. This metric turns out to be a key to find shortcomings of the theoretical framework of relativity. In section 3, it will be shown that the geodesic equation as an equation of motion is inadequate and this cannot be fixed within the theoretical framework of general relativity + electromagnetism.

However, there are indications that such a problem can be resolved in the theoretical framework of a five-dimensional theory. Thus, the conjecture [1, 16] of more than 25 years ago that unification is due to internal inadequacy is proven. Now, it becomes obvious that any attempt to have a unification including gravity must study general relativity first. Thus, clearly the lack of progress in unification should not be blamed on string theorists alone.

2. The Reissner-Nordstrom Metric, and the Repulsive Effect

Ironically, the famous formula $E = mc^2$ is also a formula that many physicists do not understand properly [28]. Einstein himself has made clear that this formula must be understood in terms of energy conservation [26]. This formula means that there is energy related to a mass, but it does not mean that, for any type of energy, there is a related mass [26, 28, 30].

A root of misunderstanding $E = mc^2$ is related to the fact that its derivation [17] has not been completed. A crucial step is Einstein's implicit assumption of treating light as a bundle of massless particles. However, in Einstein's derivation of 1905, gravity was not considered owing to the limitation of Newtonian gravity. Consequently, it was not aware that an electromagnetic energy-stress tensor is incompatible with the energy-stress tensor of massless particles [27, 29].

However, general relativity makes it explicit that the gravity generated by mass and that by the electromagnetic energy are different, as shown by the existence of repulsive effect in the Riessner-Nordstrom metric [32-34],

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \qquad (1)$$

where q and M are the charge and mass of a particle and r is the radial distance (in terms of the Euclidean-like structure [2)] [19, 20]) from the particle center. In metric (1), the gravitational components generated by electricity have not only a very different radial coordinate dependence but a different sign that makes it a new repulsive gravity in general relativity.

In fact, it is probably that the publication of this metric in 1916 and 1918 that ended Einstein's misconception starting from 1905 [35] that any energy related to a mass $m = E/c^2$. However, such a misinterpretation [13, 14] is crucial to the unconditional universal coupling assumption [3)] for the singularity theorems of Hawking and Penrose [36]. Thus, some theorists would even ignore that the Hulse-Taylor experiment has proven the extended universal coupling is incorrect [13, 21]. Moreover, in his book Will [37] continue to use his misinterpretations $m = E/c^2$ eight years after it has been proven incorrect [28]. From his book, it is clear that such a misinterpretation was prevailing.

Some argued that the effective mass in metric (1) is M- $q^2/2r$ (in the units, the light speed $c = 1$) since the total electric energy outside a sphere of radius r is $q^2/2r$. [4)] However, they overlooked that the gravitational forces would be different. From metric (1), the gravitational force is different from the force created by the "effective mass" $M - q^2/2r$ because

$$-\frac{1}{2}\frac{\partial}{\partial r}\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) = -\left(\frac{M}{r^2} - \frac{q^2}{r^3}\right) > -\frac{1}{r^2}\left(M - \frac{q^2}{2r}\right). \qquad (2)$$

They achieved only exposing further an inadequate understanding in the theory of relativity [30-32].

Moreover, the nonequivalence between electromagnetic energy and mass can be obtained without detailed calculations. From Einstein equation $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$ [37], R is invariant with an additional electromagnetic energy-stress tensor $T(E)_{\mu\nu}$ in the source [31]. Thus, according to general relativity, the electromagnetic energy cannot be equivalent to mass.

To show the repulsive effect, one needs to consider only g_{tt} in metric (1). According to Einstein [2],

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \qquad \text{where} \quad \Gamma^\mu{}_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})g^{\mu\nu}/2 \qquad (3)$$

and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ are defined by the metric $g_{\mu\nu}$. However, we consider only the case $dx/ds = dy/ds = dz/ds = 0$. Thus,

$$\frac{d^2x^\mu}{ds^2} = -\Gamma^\mu{}_{tt}\frac{dct}{ds}\frac{dct}{ds}, \qquad \text{where} \qquad -\Gamma^\mu{}_{tt} = -\frac{1}{2}(2\frac{\partial g_{t\nu}}{\partial ct} - \frac{\partial g_{tt}}{\partial x^\nu})g^{\mu\nu} = \frac{1}{2}\frac{\partial g_{tt}}{\partial x^\nu}g^{\mu\nu} \tag{4}$$

since $g_{\mu\nu}$ is static. (One need not worry whether the gauge of the Reissner-Nordstrom metric is physically valid since the gauge affects only the second order approximation of $g_{t\,t}$ [38].) For a particle *P* with mass m at **r**, the force on *P* is

$$-m\frac{M}{r^2} + m\frac{q^2}{r^3} \tag{5}$$

in the first order approximation since $g^{r\,r} \cong -1$. Thus, the second term is a repulsive force.

3. The Interaction with a Charge and Five-dimensional Theory

If the particles are at rest, then the force acts on the charged particle *Q* has the same magnitude

$$(m\frac{M}{r^2} - m\frac{q^2}{r^3})\hat{r}, \quad \text{where} \quad \hat{r} \quad \text{is a unit vector} \tag{6}$$

since the action and reaction force is equal and in the opposite direction. However, if one considers the motion of the charged particle with mass M and calculates the metric according to the particle *P* of mass m, only the first term is obtained. Thus, the geodesic equation is inadequate for the equation of motion of the inverse problem of the Reissner-Nordstrom metric. Moreover, since the second term is proportional to q^2, it is not a Lorentz force either.[5)] In other words, it is necessary to have a repulsive force with the coupling q^2 to the charged particle *Q* in a gravitational field generated by masses. In conclusion, force (6) to particle *Q* is beyond current theoretical framework of gravitation + electromagnetism. [6)]

However, in a five-dimension theory [1], the geodesic equation would include the coupling of q^2. The geodesic is

$$\frac{d}{ds}\left(g_{ik}\frac{dx^k}{ds}\right) = \frac{1}{2}\frac{\partial g_{kl}}{\partial x^i}\frac{dx^k}{ds}\frac{dx^l}{ds} + \left(\frac{\partial g_{5k}}{\partial x^i} - \frac{\partial g_{5i}}{\partial x^k}\right)\frac{dx^5}{ds}\frac{dx^k}{ds} - \Gamma_{i,55}\frac{dx^5}{ds}\frac{dx^5}{ds} - g_{i5}\frac{d^2x^5}{ds^2} \tag{7a}$$

$$\frac{d}{ds}\left(g_{5k}\frac{dx^k}{ds} + \frac{1}{2}g_{55}\frac{dx^5}{ds}\right) = \Gamma_{k,55}\frac{dx^5}{ds}\frac{dx^k}{ds} - \frac{1}{2}g_{55}\frac{d^2x^5}{ds^2} + \frac{1}{2}\frac{\partial g_{kl}}{\partial x^5}\frac{dx^l}{ds}\frac{dx^k}{ds}, \tag{7b}$$

where $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, $\quad$ μ, ν = 0, 1, 2, 3, 5 $\quad$ ($d\tau^2 = g_{kl}dx^k dx^l$; *k, l* = 0, 1, 2, 3) .

If instead of *s*, τ is used in (8), the Lorentz force suggests

$$\frac{q}{Mc^2}\left(\frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i}\right) = \left(\frac{\partial g_{i5}}{\partial x^k} - \frac{\partial g_{k5}}{\partial x^i}\right)\frac{dx^5}{d\tau}$$

Thus,

$$\frac{dx^5}{d\tau} = \frac{q}{Mc^2}\frac{1}{K}, \qquad K\left(\frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i}\right) = \left(\frac{\partial g_{i5}}{\partial x^k} - \frac{\partial g_{k5}}{\partial x^i}\right) \qquad \text{and} \qquad \frac{d^2x^5}{d\tau^2} = 0 \tag{8}$$

where *K* is a constant. It thus follows that

$$\frac{d}{d\tau}\left(g_{ik}\frac{dx^k}{d\tau}\right) = \frac{1}{2}\frac{\partial g_{kl}}{\partial x^i}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau} + \left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}\right)\frac{q}{Mc^2}\frac{dx^k}{d\tau} - \Gamma_{i,55}\left(\frac{q}{Mc^2}\right)^2\frac{1}{K^2} \tag{9a}$$

$$\frac{d}{d\tau}\left(g_{5k}\frac{dx^k}{d\tau} + \frac{1}{2}g_{55}\frac{q}{KMc^2}\right) = \Gamma_{k,55}\frac{q}{KMc^2}\frac{dx^k}{d\tau} + \frac{1}{2}\frac{\partial g_{kl}}{\partial x^5}\frac{dx^l}{d\tau}\frac{dx^k}{d\tau}, \tag{9b}$$

One may ask what the physical meaning of the fifth dimension is. Note that although the string theorists talk about space of much higher dimensional, they have no physical reason except for mathematical validity of their speculation. They claimed that those dimensions are curl up. Our position is that the physical meaning the fifth dimension is not yet very clear [1], except some physical meaning is given in equation (8). The fifth dimension is assumed [1] as part of the physical reality, and the metric signature is (+,-,-,-,-). However, our approach is to find out the full physical meaning of the fifth dimension as our understanding gets deeper. Unlike mathematics, in Physics things are not defined right at the beginning. For example, it takes us a long time to understand the physical meaning of energy-momentum conservation.

For a static case, it follows from (9) and (6), we have the forces on the charged particle Q in the ρ-direction [6)]

$$-\frac{mM}{\rho^2} \approx \frac{Mc^2}{2}\frac{\partial g_{tt}}{\partial \rho}\frac{dct}{d\tau}\frac{dct}{d\tau}g^{\rho\rho}, \quad \text{and} \quad \frac{mq^2}{\rho^3} \approx -\Gamma_{\rho,55}\frac{1}{K^2}\frac{q^2}{Mc^2}\,g^{\rho\rho} \tag{10a}$$

and

$$\Gamma_{k,55}\frac{q}{KMc^2}\frac{dx^k}{d\tau} = 0, \quad \text{where} \quad \Gamma_{k,55} \equiv \frac{\partial g_{i5}}{\partial x^5} - \frac{1}{2}\frac{\partial g_{55}}{\partial x^k} = -\frac{1}{2}\frac{\partial g_{55}}{\partial x^k} \tag{10b}$$

in the (-*r*)-direction. The meaning of (11b) is the energy momentum conservation. It is interesting that the same force would come from different type of metric element depending on the test particle used. Thus,

$$g_{tt} = 1 - \frac{2m}{\rho c^2}, \quad \text{and} \quad g_{55} = \frac{mMc^2}{\rho^2}K^2 + \text{constant} \tag{11}$$

In other words, g_{55} is a repulsive potential plus a constant. Since g_{55} depends on M, it is a function of local property, and thus is difficult to calculate. This is different from the metric element g_{tt} that depends on a distant source of mass m.

On the other hand, since g_{55} is independent of q, $(\partial g_{55}/\partial\rho)/M$ depends only on the distant source with mass m. Thus, this force though acting on a charged particle, would penetrate electromagnetic screening. This would make such a force easier to be identified. From (12), it is possible that a charge-mass repulsive potential would exist for a metric based on the mass M of the charged particle Q. However, since P is neutral, there is no charge-mass repulsion force (from $\Gamma_{k,55}$) on P.

4. Experimental Verification of Mass-Charge Repulsive Force.

The repulsive force in (6) can be detected with a neutral mass. To see the effect of repulsive gravity, one must have

$$\frac{1}{2}\frac{\partial}{\partial r}\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) = \frac{M}{r^2} - \frac{q^2}{r^3} < 0 \tag{12}$$

Thus, repulsive gravity would be observed at $r < q^2/M$. For the electron the repulsive gravity would exist only inside the classical electron radius r_0 $(= 2.817\times10^{-13}cm)$. It would be very difficult to test a single charged particle. [7)]

However, the existence of repulsive gravity can actually be verified with a charged metal ball. (The test object should be an isolator such as glass and china. This will greatly reduce the static electrical effect.) The reason is that the attractive effect in gravity is proportional to mass related to the number of electrons, but the repulsive effect in gravity is proportional to square of charge related to the square of the number of electrons. Thus, when the electrons are numerous enough accumulated in a metal ball, the effect of repulsive gravity will be shown in a macroscopic distance.[8)]

Now, consider the charge q and mass M is consist of N electrons, i.e., $q = Ne, \quad M = Nm + M_0$, where M_0 is the mass of the metal ball, m and e are the mass and charge of an electron. To have sufficient electrons, the necessary condition is

$$N > \frac{r}{r_0}, \qquad \text{where} \qquad r_0 = \frac{e^2}{mc^2} = 2.817 \times 10^{-13}\ \text{cm}. \tag{13}$$

For example, if $r = 10\ cm$, then it requires $N > 3.550 \times 10^{13}$. Thus $q = 5.683 \times 10^{-7}\ Coulomb$. Then, one would see the attractive and repulsive additional forces change hands, although this experiment is difficult just like other small effects.

Similarly, the mass-to-charge repulsive force in (7) can be detected with a charge particle. However, since the repulsive force is very small, the interference of electricity would be comparatively large. Thus, it would be necessary to screen the electromagnetic effects out. The modern capacitor is such a piece of simple equipment that can do this screening.

When a capacitor is charged, it separates the electron from the atomic nucleus, but there is no change of mass. Thus, the capacitor would have less weight after being charged. [9)] This is a nonlinear force towards charges. This simple experiment would confirm the mass-charge repulsive force, and thus the unification in term of a five-dimensional theory.

5. Conclusions and Discussions

It has been shown that the theoretical framework of general relativity is inadequate, and modification is necessary. However, it should be noted that no new theory is compelling. All you can show is that it is a feasible way to solve the problem.

One may ask whether the force F_{cm} of charge to mass repulsion and the force F_{mc} of mass to charge repulsion are the same kind of force. Since they have different origin; according to Einstein's equation, the repulsive term in g_{tt} is due to the electromagnetic energy, where the term in g_{55} is due to mass alone. Since the electromagnetic energy is subjected to electromagnetic screening, the force F_{cm} would also be subjected to screening although the force F_{mc} would not.

It should be pointed out that the screening effect to the force F_{cm} is only a result of the current four-dimensional theory. From the viewpoint of the five-dimensional theory, the charge would create an independent field to react with the mass. To test this, one should observe whether there is a repulsive force from a charged capacitor to a mass particle. For instance, one can have a large spherical capacitor to do the testing. From the viewpoint of five-dimensional theory, an additional repulsive force on the test mass would be observed after sufficiently charging up.

In other words, the charge-mass repulsive force mq^2/r^3 is a prediction of the five-dimensional theory and is independent of the four known forces. It should be noted also that in electrodynamics the term $-\Gamma_{k,55}(dx^5/d\tau)^2$ is also necessary because it has been shown in 1981 that the terms $\partial g_{5k}/\partial x^5$ are related to the radiation reaction force [1]. Moreover, if the investigation of electric energy leads to a charge-mass repulsive force, it is expected that the magnetic energy would generate a current-mass repulsive force. However, this is beyond the scope of this paper.

Gravitation was considered as producing attractive force only, and all the coupling constants were assumed to have the same sign. Recently, it is proven that for the radiation of binary pulsars the coupling constants must have different signs [8, 20]. Now, it is shown that even the electromagnetic energy would produce repulsive forces. Thus, the physical picture provided by Newton is just too simply for a phenomenon as complicated as gravity that relates to everything.

It should be noted, however, that the five-dimensional theory is far from a theory of everything since the issues of particle creation and annihilation are not addressed. Moreover, in this paper only the static case is considered, and formula (7) is essentially derived from general relativity. This would make this calculation on a very firm ground. For the dynamic cases, a five-dimensional theory would help the necessary modification of the field equation of general relativity [1, 39].

Moreover, since many still do not understand $E = mc^2$, this manifests that misunderstandings actually started from special relativity and electromagnetism. They also ignored issues such as the conflict between the "covariance principle" and Einstein's requirement on weak gravity, 10) and they believed this invalid principle [10]. Newtonian invalid notions are still dominating, and the current theory of general relativity is not yet a self-consistent theory [10, 13, 14, 18]. Thus, it is unrealistic to expect the string theorists to perform a miracle in unification. Einstein is really a genius and the full meaning of general relativity is still emerging after 100 years of its creation. Now, it is clear that unification is a necessity.

In closing, we quote a remark by Einstein and Pauli [4], who wrote in 1943

> "When one tries to find a unified theory of gravitational and electromagnetic fields, he cannot help feeling that there is some truth in Kaluza's five-dimension theory."

It turns out that their observation would be a prophecy for the future advancement of such unification.11) Moreover, since a theory of weak interaction must be unified with electromagnetism, the necessity of unifying gravitation and electromagnetism would imply also that the goal of the string theorists is, independent of their desire, a realistic problem.

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Appendix: Invalidity of the "Covariance Principle"

The so-called "covariance principle" is a favorite among applied mathematicians, who often overlooked physical requirements. In fact, the creation of such a principle is due to Einstein's failure to identify adequately the physical meaning of the coordinates. Einstein called it the "principle of covariance" [17], "The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant)." This leads to the notion of Lorentz manifolds [37] that cannot be one-one corresponding to a four-dimensional Minkowski space. Then, for such a manifold, Einstein's requirement for weak gravity may not be applicable since a mathematical coordinate system may not relate to a physical frame of reference.

The crucial point of the covariance principle is the validity of any Gaussian coordinate system as a space-time coordinate system in physics. For this, Einstein's supporting arguments [17] are as follows:

That this requirement of general covariance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences."

Einstein's arguments, though convinced many, are actually false. Note that the meaning of measurements is crucially omitted. First, his arguments are incompatible with his earlier argument for defining time relating to local clocks [17]. Moreover, in order to predict events, one must be able to relate events of different locations in a definite manner [10]. Moreover, Zhou correctly pointed out that coordinates must have physical meaning [40].

Zhou argued [40], "When we come to solve the field equation of moving matter, we must first define the geometrical configuration of matter, the symmetry of the configuration, its density distribution, pressure, and velocity of motion in space-time. All of them have to be expressed in terms of coordinates." Note that all physical predictions, including Einstein's own three tests, must be understood in terms of the physical meaning of coordinates [10].

In view of that the "principle of covariance" is an interim assumption due to Einstein's certain ignorance of the space-time coordinates [17], it is only natural that such an invalid "principle" is a source of theoretical inconsistence in Einstein's theory [14]. For instance, although the bending of light can be derived from different metrics, these metrics give different formulas for the de Sitter precession [10]. Since the root of such covariance is due to Einstein's failure in laying down coordinates in a definite manner [17], to resolve this problem, one must identify the physical meaning of space-time coordinates [20].

ENDNOTES

1. In his 1909 Salzburg talk, Einstein strongly emphasized that inertial mass is a property of all forms of energy, and therefore electromagnetic radiation must have mass [35; p. 118-119].
2. The existence of a Euclidean-like structure is a necessary condition for a physical space [20]. Then, Einstein's condition for weak gravity can be rigorously defined [23] although the Royal Society failed to understand this [13, 8].
3. Hawking in his recently (June 2006) visit to China, still misleadingly told his audience that his theory was based on general relativity only. The root of his problem would be that he still does not understand the formula $E = mc^2$. Although Thorne [41] had asked the possibility of a different coupling sign, Hawking dismissed it as impossible.
4. The electric energy density at r is $q^2/8\pi r^4$ [32]. Thus, the total electric energy outside a radius r is

$$\frac{1}{8\pi}\int_r^\infty \rho^2 d\rho \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \frac{q^2}{\rho^4} = \int_r^\infty d\rho \frac{q^2}{2\rho^2} = \frac{q^2}{2r}.$$

   This result of Weinberg [42] is correct, since the distance in the frame of reference is decided by the Euclidean-like structure [8, 20]; whereas the metric determines only the space contractions [13, 18]. A similar calculation of total mass [36] gives a continuation from the internal to the external of the Schwarzschild solution. However, if a factor $(g_{rr})^{1/2}$ is added to the integration [2, 36], this results in a larger mass, and would lead to another inconsistency [18, 27]. Nevertheless, some theorists objected the above calculation as incorrect. They believed since 1959 that a plane-wave is not bounded [9], and thus differ from Einstein and the Wheeler School [32]. They even claimed Einstein's requirement on weak gravity were incorrect because their knowledge in general relativity was out dated [43].

5. Currently, for a charged particle under the influence of gravity, the Lorentz force and the radiative reaction force are added to the geodesic equation to form an equation of motion. However, since there is no external electromagnetic field, the Lorentz force is absent. Also, since this is a static case, the radiative reaction force is also absent [44].
6. In this approach, we calculate the field of generated by charge particle Q, then the force acting at on particle P; and the field generated by particle P, then the force acting at Q. This physical approach, which is often used in electrodynamics, is valid because the field generated by a particle, does not make itself move. For the metric generated by particle P, the metric would be $ds^2 = (1- 2m/\rho)dt^2 - (1-2m/\rho)^{-1}d\rho^2 - \rho^2 d\,\Omega'^2$, where (ρ, θ, φ') is a new coordinate system with P at the center. Thus, the force on Q in the ρ-direction would be only $-M(m/\rho^2)$. Note that the distance between P and Q is $r = \rho$, and thus there should be another term in the ρ-direction as $q^2(m/\rho^3)$.
7. According to the Riessner-Nordstrom metric, the event of horizon would be [45] at $M \pm (M^2 - q^2)^{1/2}$. If $M^2 - q^2 > 0$, one would have $(q^2/M) < M + (M^2 - q^2)^{1/2}$. However, $M^2 > q^2$ may not be valid, for instance, the electron does not have an event of horizon because $e > m_e$ ($e = 1.381\times10^{-34}$ cm, $m_e = 6.764\times10^{-56}$ cm). Nevertheless some theorists [46] claimed that (q^2/M) is always inside of an event of horizon.
8. Although a net repulsive force is difficult to observe, this method would test the repulsive effects.
9. From the Internet, one would know that experiments of weighting capacitors have been performed for many years. The experimentalist Mr. Liu thought this reduction of weight as a lost of mass. Since nobody was able to explain his experiment in terms of well-known theories, the general belief was that this reduction is due to experimental errors. Nobody thought of this having anything to do with electric charge since in a capacitor the electromagnetic force is screened. Mr. Liu currently considers his experiment challenges Newton's law of gravity and Einstein's formula E = mc^2. Moreover, he has not just one but three ``unexplainable'' experiments, and posts a reward of about $2,500 US for anybody who can explain any of these experiments to his satisfaction. Mr. Liu has a web site http://www.cqfyl.com.
10. Validity of the plane-waves of Bondi, Pirani, & Robinson [8], is based on arguing that for a manifold, Einstein's requirement for weak gravity may not be applicable since a manifold may not relate to a physical frame of reference with the Euclidean-like structure [20]. Thus, the editorial of the Royal Society [42] initiated a serious challenge to the Wheeler school [32] including Ohanian and Ruffini [47], Wald [36], Will [37], who claim to have the standard theory, as well as others such as Landau & Lifshitz [48], Straumann [49], and those who believed Einstein's requirement on weak gravity and also the so-called "covariance principle". Responding to such a challenge is particular important to Will, who has built his career essentially on the Parameterized Post-Newtonian Approximation, while claiming validity of the "covariance principle". Although the paper of Bondi et al [8] is well known, nobody responded to their challenge before. This seems to suggest unequivocally that few theorists

such as Zhou [40, 50, 51] other than the editorial of the Royal Society understand the implication of the so-called "covariance principle" in physics.

11. It has been shown that a rigorous cylindrical condition may not be compatible with Kaluza's theory [52].

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Experimental Observation of Infinitely Large Propagation Velocity of Bound Electromagnetic Fields in Near Zone

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1. Introduction

Classical electrodynamics is empirically well supported theory within its domain of application which boundary is established by quantum electrodynamics. The advent of quantum theory has not led to the collapse of classical views but, rather, has helped to highlight their limits of applicability. Moreover, in modern physics there is one basic issue (which has entirely classical origin) concerning the unique propagation rate of all fundamental physical forces with the speed of light. This basic premise of the classical standpoint seems to be sometimes at odds with observable behavior of quantum mechanical systems at very small length scales where some indications on non-locality take place. In spite of the considerable effort to place all quantum mechanical effects into the classical locality (or causality) framework, it is generally acknowledged that there is still no conceptually unproblematic consistent causal approach to observable manifestations of non-locality within domains of quantum theories.

The notions of locality and causality are central to theorizing in classical electrodynamics and they are assumed to be trustworthy for any macroscopic region, at least as large as atomic or molecular length scale, so establishing a natural boundary with quantum effects. However, the actual extrapolation of the causality and locality properties of classical EM fields to very small distances (up to quantum mechanical limits) has very scarce empirical basis, i.e. it is practically taken for granted without any serious experimental support. In fact, after Hertz's discovery of EM waves [1] in agreement with the predictions of Maxwell's theory, the fundamental task in verifying propagation characteristics of classical EM fields had been taken as definitively complete. Perhaps, it can be qualified as the main reason for persistent disinterest in providing solid empirical ground for EM field causality properties at very small macroscopic level. As a consequence, since the time of Hertz's experiments all systematic empirical analysis of retardation rates of EM fields in space regions very close to EM field sources (near zone) had been either abandoned or given already little fundamental importance.

However, there are several methodological and historical circumstances [2] as well as recent experimental data [3] that show the obvious necessity to obtain additional empirical information in order to endorse or disprove existent status of causality at very small distances.

Modern classical electrodynamics [4, 5] distinguishes velocity-dependent (bound) and acceleration-dependent (radiation) components. The ratio of the radiation to bound field strength increases with distances R from the EM field source and the radiation contribution becomes dominant at relatively large distances. In common situations, the two contributions are equal at a distance from a source $\lambda/2\pi$ (λ is the wavelength of EM radiation). The space domain $R>\lambda/2\pi$ where the radiation predominates is called the far zone. The region $R<\lambda/2\pi$ where the bound field predominates is close to the EM field source and it is regarded as the near zone.

The clear distinction between both space domains should be taken into account at constructing a consistent and rigorous approach to EM field causality at all macroscopic length scales. Nevertheless, available scientific literature shows that among the criteria for evaluating existent empirical tests of causality, the internal structure of EM field (superposition of bound and radiation components) does not play any important role. Not distinguishing carefully enough between bound and radiation components is one of the main reasons why the experimental verification of EM field propagation (causality) properties can not be taken as definitively complete [2, 3]. Any ideally rigorous test of the causal behavior of the whole EM field has necessary to be based on separate (or individual) tests for bound and radiation components. Nearly one century technical treatment of EM radiation leaves no doubt with respect to its causal properties at every macroscopic length scales (up to atomic level). Contrary, due to the lack of any detailed information on causal characteristics of bound EM fields, the motivation now is to complete this task experimenting with EM field within the near zone.

Thus, at UHF frequency ≈125 MHz, the boundary $\lambda/2\pi$ between the two domains is placed approximately at 40 cm from the EM field source. Laboratory measurements [3] were carried out at this frequency within the macroscopic region (40÷200 cm) where the ratio of the bound to radiation field strength was not too far from unity. At larger distances the EM radiation predominates and in our experiments it constituted nearly the whole signal detected at 300 cm. We used this information in order to reconstruct radiation components at different positions within the region of space 40÷200 cm. Then, we could compare it with the whole EM signal (superposition of bound and radiation components) detected at the same spatial points. Thus, the time difference between detectable signal and radiation component became available from the experiment and could be studied as a function on a distance between emitting and receiving antennas. This approach to bound fields has been regarded as zero-crossing method [3]. Interestingly, experimental data showed considerable disagreement with theoretical predictions for the case when the standard causality properties are supposed to be applicable both to bound and radiation fields. Moreover, experimental data nearly perfectly fitted the prediction calculated numerically for the case when propagation rate of bound fields in near zone highly exceeds the velocity of light.

There is, then, a perspective open for a possible disagreement between the actual status of causality (or locality) and observable behavior of bound fields in the near zone. Nevertheless, the zero-crossing method alone suggested and implemented in [3] is obviously insufficient for arriving at trustworthy quantitative conclusions since it is based on the analysis of some specific time moment at which the total composite signal (bound plus radiation terms) crosses zero level. As a result, it can be sensitive only to a notable difference between propagation characteristics of bound and radiation field components in the near zone without providing exact values of the propagation rate of bound EM fields and its possible dependence on a distance from the emitting source. This intrinsic limitation of the zero-crossing method suggests a search of an alternative and independent approach in which experimental observation of the causal propagation of bound EM fields might be considerably amended. Following this suggestion of improvement as a step forward in comparison with the zero-crossing method, we propose in this work to study bound field components separately by explicit decomposition of the detectable signal on bound and radiation contributions within the whole time interval. To fulfill this task will involve a consideration of an additional co-axial configuration between emitting and receiving antennas as well as an extension of the theoretical description given in [3]. The use of the experimental set-up implemented already in the preceding work [3] will ensure an important possibility of cross-verification and qualitative comparison of new experimental data with the previous results.

2. Theoretical background

The general approach to magnetic field structure as a superposition of bound (velocity-dependent) $\boldsymbol{B}_u$ and radiation $\boldsymbol{B}_a$ (acceleration-dependent) components generated by EM field source as well as to the actual account of retardation effects is explicable in terms of the time-varying Biot-Savart's law [6, 7], which for filamentary conducting circuits acquires the form [7]

$$\boldsymbol{B}(\boldsymbol{R},t) = \boldsymbol{B}_u + \boldsymbol{B}_a = \frac{1}{4\pi\varepsilon_0 c^2}\int\left\{\frac{[\boldsymbol{I}]_c}{R'^2} + \frac{1}{cR'}\left[\frac{\partial J}{\partial t}\right]_c\right\}\boldsymbol{k}\times\boldsymbol{n}dl\,, \tag{1}$$

where $\boldsymbol{I}$ is the conduction current; R′ is the distance between the point of observation and the source point where the volume element of integration dV is located, $n = \boldsymbol{R}'/R$, $\boldsymbol{k}$ is the unit vector in the direction of $\boldsymbol{I}$ and the corresponding quantity placed inside the square bracket is being determined at the retarded time $t\text{-}R'/c$.

Retarded integral [1] describes causal relations between EM phenomena taking place in emitting loop antenna (EA) and present-time value of magnetic field that is closely associated with the actual understanding of the causality principle.

In this sense, electromotive force (e.m.f.) $\varepsilon(t)$ induced in receiving loop antenna (RA) is also understood as retarded cause-effect relationship described by Faraday's induction law:

$$\varepsilon(t) = \frac{1}{4\pi\varepsilon_0 c^2}\frac{d}{dt}\int\int_S\left[\oint_\Gamma\left\{\frac{[\boldsymbol{I}]_c}{R'^2} + \frac{1}{cR'}\left[\frac{\partial J}{\partial t}\right]_c\right\}\boldsymbol{k}\times\boldsymbol{n}dl\right], \tag{2}$$

where S is the area of RA.

Mathematical treatment of retarded integrals frequently requires consideration of verisimilar approximations. The most commonly used one is so-called electrically small antenna. In the framework of this requirement the radii r_{EA} and r_{RA} of EA and RA loops are to be small enough in comparison with the EM radiation wavelength. The other frequently used precondition is quasi-stationary current approximation which means that conduction current I has the same phase in all angular coordinates φ of EA at some present time t, i.e. $I(t,\varphi)=I(t)f(\varphi)$. In particular, we shall use one special case of quasi-stationary current approximation when $f(\varphi)$ does not depend on φ. Finally, in order to use series expansion with respect to r_{EA}/R and r_{RA}/R, we shall restrict our analysis of EM fields to distances $R>r_{\mathrm{EA}}$, r_{RA}.

If the time-variation of $I(t)$ is close to harmonic (quasi-harmonic approximation will be fulfilled in our experimental realization), all higher order time derivatives $((\partial I)/(\partial t))$, $((\partial^2 I)/(\partial t^2))$ etc also will not depend on φ and then will have the same present-time value over the perimeter of the emitting loop. Hence one can factor $((\partial I)/(\partial t))$, $((\partial^2 I)/(\partial t^2))$ etc out from the integral sign and neglecting second order retardation effects, Eq. (2) can be presented in general compact format which will be convenient for further considerations:

$$\varepsilon = -\frac{S_{EA}S_{RA}}{4\pi\varepsilon_0 c^2}\left\{k_{b1}\frac{[\partial I/\partial t]_c}{R^3} + k_{b2}\frac{[\partial^2 I/\partial t^2]_c}{R^2} + k_{f1}\frac{[\partial^2 I/\partial t^2]_c}{R^2} + k_{f2}\frac{[\partial^3 I/\partial t^3]_c}{R}\right\}, \tag{3}$$

where $S_{EA}=\pi r^2$; $S_{RA}=\pi r^2$, and k_{b1}, k_{b2}, k_{f1} and k_{f2} are some geometric factors. One can show that the first two terms in *rhs* of Eq. (3) originate from the bound field component, while the third and firth terms – from the radiative field component.

Further on we shall consider two particular positional configurations between EA and RA. Co-planar configuration will take place if both EA and RA loops belong to the same plane. By analogy, co-axial configuration will correspond to the position of EA and RA sharing the same axis of symmetry. It is essential that in co-axial configuration, the coefficients k_{b2}, k_{f1} =0, and the e.m.f. contains only two components R^{-3} and R^{-2}:

$$\varepsilon_{ax} = -\frac{S_{EA}S_{RA}}{4\pi\varepsilon_0 c^2}\left\{k_{b1}\frac{[\partial I/\partial t]_c}{R^3} + k_{f1}\frac{[\partial^2 I/\partial t^2]_c}{R^2}\right\} \tag{4}$$

We mention that in the limit $R>>r_{EA}$, r_{RA}, $k_{b1}=k_{f1}=2$.

In co-planar configuration, $k_{f1}=0$, and

$$\varepsilon_{pl} = -\frac{S_{EA}S_{RA}}{4\pi\varepsilon_0 c^2}\left\{k_{b1}\frac{[\partial I/\partial t]_c}{R^3} + k_{b2}\frac{[\partial^2 I/\partial t^2]_c}{R^2} + k_{f2}\frac{[\partial^3 I/\partial t^3]_c}{R}\right\}, \tag{5}$$

with $k_{b1}= k_{b2}=k_{f2}=1$ in the limit $R>>r_{EA}$, r_{RA}.

To conclude our discussion of the standard approach, we note that expressions (4) and (5) for predicting the time-variation of e.m.f. in RA are in agreement with the principle of finite causal propagation at universal speed of light by attaching the same retardation rate to both bound and free radiation magnetic fields. Therefore, it can be taken as the basis for theoretical predictions to be compared with experimental observations at all length scales. If causal conditions meaningful for bound and radiation fields are distinct, then one would expect to detect observable deviations from theoretical predictions based on (4) and (5).

One also can use an obvious advantage of the co-axial configuration in which the resultant e.m.f. (4) is composed only of R^{-3} and R^{-2} terms due to bound and radiation contributions, respectively. Moreover, two different contributions can be studied separately within the space region where they are dominant. In fact, at larger distances ($R>\lambda/2\pi$) the R^{-2} radiation contribution and at smaller distances ($R<\lambda/2\pi$) the R^{-3} bound contribution will impose the retardation rate and the dependence on a distance R of the whole signal. It provides the basis for a methodologically rigorous approach to bound fields in the near zone of EM sources.

Here it is worth reminding that there is actually no explicit empirical information on bound fields as far as to their propagation properties is concerned. In view of experimental indications [3] on a possible inadequacy of standard views in the near zone, it would be reasonable to explore theoretically the type of alternative predictions when the velocity of propagation of bound fields (further denoted as v) can differ from the speed of light. Thus, if one discerns v from c, one can check that the fundamental structure of the resultant e.m.f. as a superposition of bound and radiation contributions remains unalterable. Moreover, dimensionless coefficients in Eqs. (4), (5) due to a particular configuration between EA and RA keep also unchanged:

$$\varepsilon_{ax} = -\frac{S_{EA}S_{RA}}{4\pi\varepsilon_0 c^2}\left\{k_{b1}\frac{[\partial I/\partial t]_v}{R^3} + k_{f1}\frac{[\partial^2 I/\partial t^2]_c}{R^2}\right\}, \tag{6}$$

$$\varepsilon_{pl} = -\frac{S_{EA}S_{RA}}{4\pi\varepsilon_0 c^2}\left\{k_{b1}\frac{[\partial I/\partial t]_v}{R^3} + \frac{c}{v}k_{b2}\frac{[\partial^2 I/\partial t^2]_v}{R^2} + k_{f2}\frac{[\partial^3 I/\partial t^3]_c}{R}\right\}, \tag{7}$$

where quantities $[\partial I/\partial t]_v$, $[\partial^2 I/ \partial t^2]_v$ are being determined at the retarded time $t\text{-}R/v$.

Eqs.(6), (7) are model-dependent and can be regarded as methodological analogies of standard Eqs.(4), (5), sharing the same theoretical predictions either at $v=c$ or at very large distances $R>>\lambda/2\pi$ where the R^{-3} contribution becomes irrelevant. Thus, the use of model Eqs. (6), (7) can be justified only if there are clear experimental evidences for inadequacy of Eqs.(4), (5) to describe empirical data within a finite region of space referred as the near zone where bound fields are dominant. In fact, according to mathematical properties of (6) and (7), the whole signal ε_{pl} or ε_{ax} is the most sensitive to a possible difference between v and c only in the near zone of EM sources. It reflects the general assumption that bound and free radiation fields are independent of each other in the corresponding area of their domain. Thus, within the near zone the R^{-3} contribution prevails and determines propagation characteristics of the whole signal that can be studied experimentally in order to obtain convincing evidence in favor of either $v=c$ or $v\neq c$. An appropriate experimental procedure will be defined in the next Section.

3. Experimental measurements and data processing

The experiments have been carried out with the multi-section emitting and receiving antennas with the radius 5 cm. A driving circuit of EA is constituted by a fast high-voltage (>5 kV) spark gap connected to the antenna via the blocking capacitor. The circuit generated short quasi-harmonic signals with the period approximately 8 ns. In order to avoid reflected wave interference, both EA and RA were mounted on a wooden table removing all metallic objects (with the capacity to reflect EM radiation) at distances exceeding 1.5 m in all range of variation of R. It assured no measurement interference by reflected EM waves during the period of the first 8 ns. Technical detail of experimental setup are described in [3].

To perform our measurements we set EA and RA either in co-planar or in co-axial configuration and keeping their orientations unchanged, we varied the distance R between their centers. In co-axial configuration the range of variation of R was 20÷300 cm whereas in co-planar position it was R=40÷300 cm. For the region of space $R\leq100$ cm we used small step ΔR=10 cm and for larger distances R>100 cm the step was doubled in size ΔR=20 cm. At each space position e.m.f. signal $\varepsilon(t)$ induced in RA was recorded in a digital format with the oscilloscope Tektronix TDS-3052. Its sampling rate is 5 Gsample per second or 0.2 ns per channel and expected time resolution is about 0.02 ns. The maximal voltage sensitivity available by the oscilloscope is 1 mV/div. Each signal was recorded after 128 averaging and numerically interpolated by cubic splines.

3.1. Processing of signals in co-axial configuration

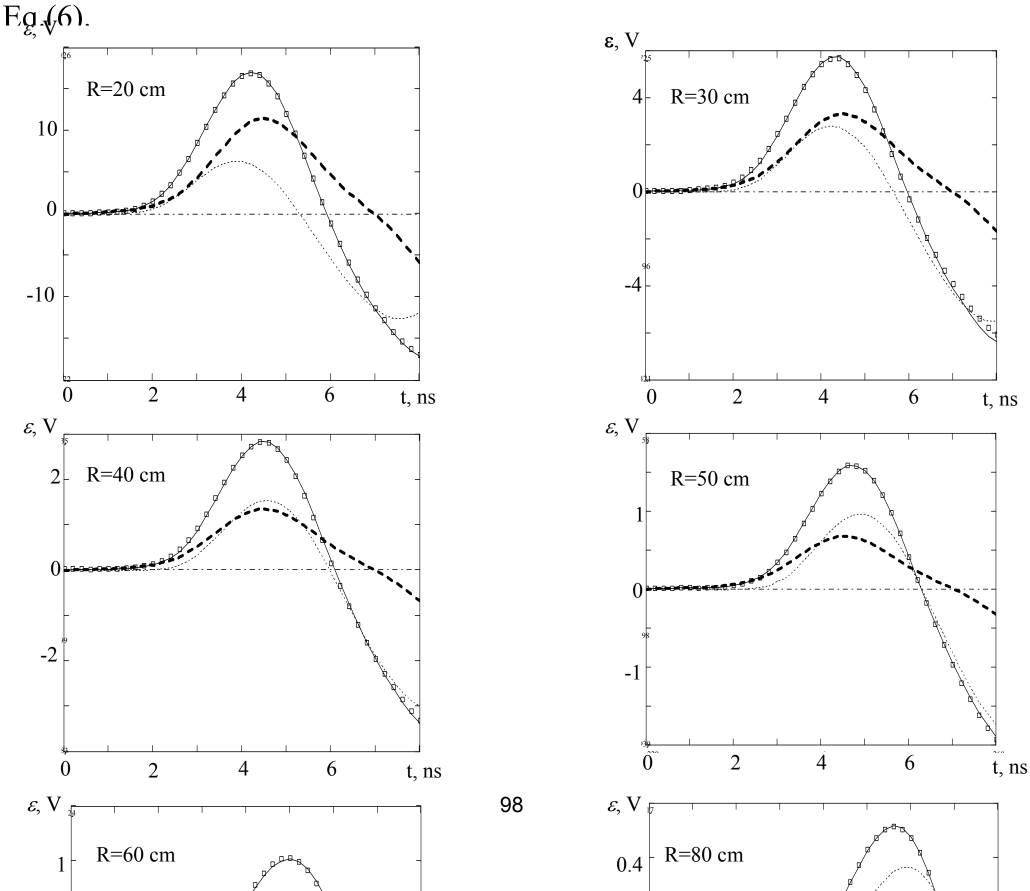
In this configuration the recorded signal consists of R^{-3} bound and R^{-2} radiation components within the whole range of variation of R=20÷300 cm. The amplitude and retardation phase relationships of each term in the resultant e.m.f. are clearly specified by the standard Eq. (4). The free radiation contribution $\varepsilon_f(t,R)$ proportional to R^{-2} is dominant at large distances $R_{max}\gg\lambda/2\pi$ and therefore, constitutes nearly the whole signal $\varepsilon_f(t,R_{max})\approx\varepsilon(t,R_{max})$. Once the signal has been recorded at R_{max} and taking into account that radiation fields propagate with the speed of light, one can reconstruct the radiation contribution $\varepsilon_f(t,R)$ as a part of the whole signal $\varepsilon(t,R)$ at each position within the space domain R=20÷240 cm. It can be implemented by a simple re-scaling of $\varepsilon(t,R_{max})$ with the factor $(R_{max}/R)^2$ as well as by corresponding time shift $(R_{max}-R)/c$. Then, according to the standard Eq. (4), the bound contribution $\varepsilon_b(t,R)$ has to be recovered at each spatial position by subtraction of the reconstructed radiation signal $\varepsilon_f(t,R)$ from the recorded signal $\varepsilon(t,R)$. Position of bound contributions obtained as a function of R is equivalent to the knowledge of corresponding time shifts $\Delta t_b(R_1,R_2)$ between bound contributions detected at different distances R_1 and R_2. Then the average propagation velocity of bound fields as a function on a distance R can be evaluated by $v((R_1+R_2)/2))=(R_1-$

$R_2)/(\Delta t_b(R_1,R_2)))$ where R_1 and R_2 is a pair of the closest spatial positions so that R_1-R_2=ΔR is the step used in our measurements.

We took R_{max}=300 cm and at this distance the radiation contribution constituted more than 90% of the whole recorded signal $\varepsilon(t,R_{max})$. As a first approximation, it is already acceptable to estimate the retardation rate of the bound component at different R. Within the limit of precision available in our measurements, we reconstructed the position of bound contributions inside the near zone and did not find any observable retardation which was expected on the base of the standard Eq. (4). Here we stress that the model Eq.(6) (where v is unknown and is to be determined from experiment data) was used only after having obtained strong disagreement with the expected retardation rate for bound EM fields within the near zone of the EA predicted by the standard Eq. (4).

Results of separation of recorded signal into R^{-3} bound and R^{-2} components are presented in Fig.1. We observed no retardation of bound components between R=20 cm, 30 cm and 40 cm. There are also clear indications on the absence of retardation of bound fields within the initial domain R=0÷20 cm. Put in other terms, bound fields appear to possess a propagation velocity highly exceeding the speed of light inside the near zone $R \leq \lambda/2\pi \approx 40$ cm. It turns out to be in line with the results of our previous work [3].

At larger distances, we observed finite time shifts Δt_b that are still considerably smaller than $\Delta t=\Delta R/c$ expected for the standard retardation rate of R^{-3} contributions. Between R=40 cm and R=50 cm the time shift Δt_b corresponded to the average value v=8.2c whereas between R=50 cm and R=60 cm Δt_b gave v=4.3c. Nevertheless, at larger distances the observed time shift tended to the value Δt=((10cm)/c)≈ 0.33 ns which is assumed if the speed of light c determines the retardation. Both dependencies of Δt_b (R) and $v(R)$ as functions on a distance can be found in Fig. 2a and Fig. 2b, respectively. One clearly notes two strong tendencies of empirical results presented in Fig. 2a: (a) zero time shifts within the near zone and (b) the approximation to the standard time shift at larger distances. Both of them can be taken as physically meaningful hav ing in mind that inside, respectively, the near and far zones the R^{-3} and R^{-2} contribution prevail and determine the propagation rate of the whole signal. However, the type of transition (between both tendencies in Fig. 2) from small to large distances can be qualified as model-dependent since it is determined by the methodological Eq.(6).



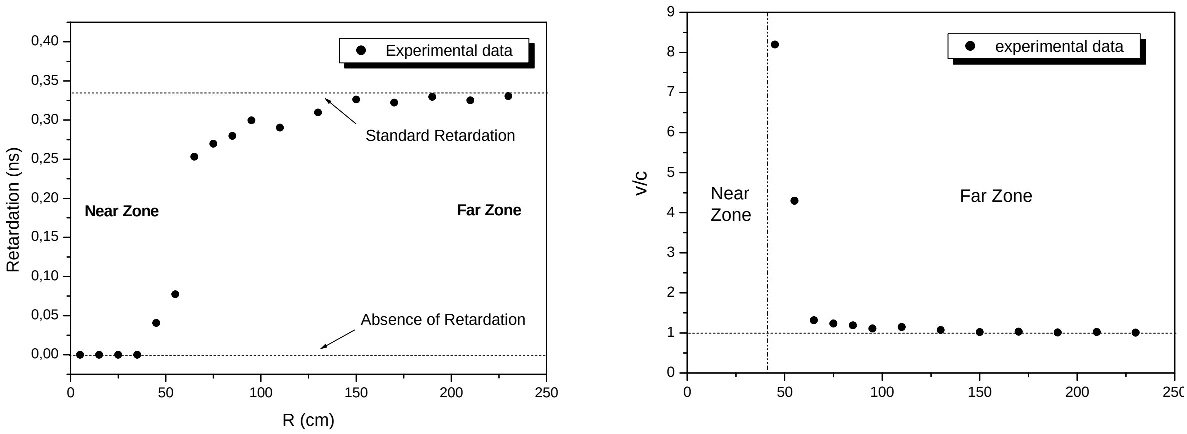


Fig. 2: (a) Retardation time shift $\Delta t_b(R)$ of bound contribution measured between two closest spatial positions. At large distances the retardation time shift tends to the standard value Δt=((10cm)/c)≈0.33 ns. (b) The propagation velocity of bound fields determined as reciprocal to $\Delta t_b(R)$-dependence.

3.2. Processing of signals in co-planar configuration

Contrarily to the previous considerations, in co-planar configuration a decomposition of e.m.f. into three independent components (one of them with the weight coefficient c/v) according to Eq. (5) or (7) can not be performed by implementing the method described above for co-axial configuration. Nevertheless, the explicit form of functions $(\partial I/\partial t)$) and $(\partial^2 I/\partial t^2)$) are numerically available already from the previous two-component analysis in co-axial configuration. Since at large distances EM radiation predominates, the lacking information on $(\partial^3 I/\partial t^3)$ which is responsible for the shape of R^{-1} radiation components, can be extracted directly from experimental measurements by recording the detectable signal at $R_{\max}$=300 cm. Having obtained the shapes of $(\partial I/\partial t)$, $(\partial^2 I/\partial t^2)$, $(\partial^3 I/\partial t^3)$ and interpolating them with cubic splines, we are in a position to numerically reconstruct the whole signal according to the general analytical expression (7), including the case v=c. This is also possible due to the fact that dimensionless coefficients $k_{b1}^{pl}(R), k_{b2}^{pl}(R)$ and $k_{f1}^{pl}(R)$ are not dependent on any particular propagation velocity of bound fields.

After having obtained exact numerical values of $k_{b1}^{pl}(R), k_{b2}^{pl}(R)$ and $k_{f1}^{pl}(R)$ as functions on a distance R we are in a position to get a quantitative comparison between numerically synthesized and recorded signals. In our analysis we decided to calculate the value of the error functional σ(R):

$$\sigma(R) = \frac{1}{A(m_2 - m_1)} \sqrt{\sum_{i=m_1}^{m_2} \left(\varepsilon(R,t_i) - \varepsilon_{syn}(R,t_i)\right)^2}\ , \qquad (8)$$

where $\varepsilon(R,t_i)$ and $\varepsilon_{syn}(R,t_i)$ are the recorded and synthesized signals in the i^{th} channel on the time scale of the digital oscilloscope; t_i is the present time correspondent to the i^{th} channel; A is the amplitude of the recorded signal at the first half-period; m_1, m_2 determine the initial and the final channels of the signal half-period.

We estimated the value of $\sigma(R)$ with respect to two basic hypotheses (used to theoretically reconstruct the whole signal $\varepsilon_{syn}(R,t_i)$ that are listed below:

1. Standard retardation condition v=c or Eq.(5).

2. Causal framework of bound fields is determined by Eq.(7) and by $v(R)$-dependence given in Fig. 2b.

For the second hypothesis the estimated deviation from the recorded signal turned out to be more than one order of magnitude smaller than that calculated in the case of the standard condition *v*=*c*. The difference can be visually appreciated in Fig.7. Importantly, the amplitude of the signal synthesized under the first hypothesis (*v*=*c*) is notably bigger than the corresponding am-

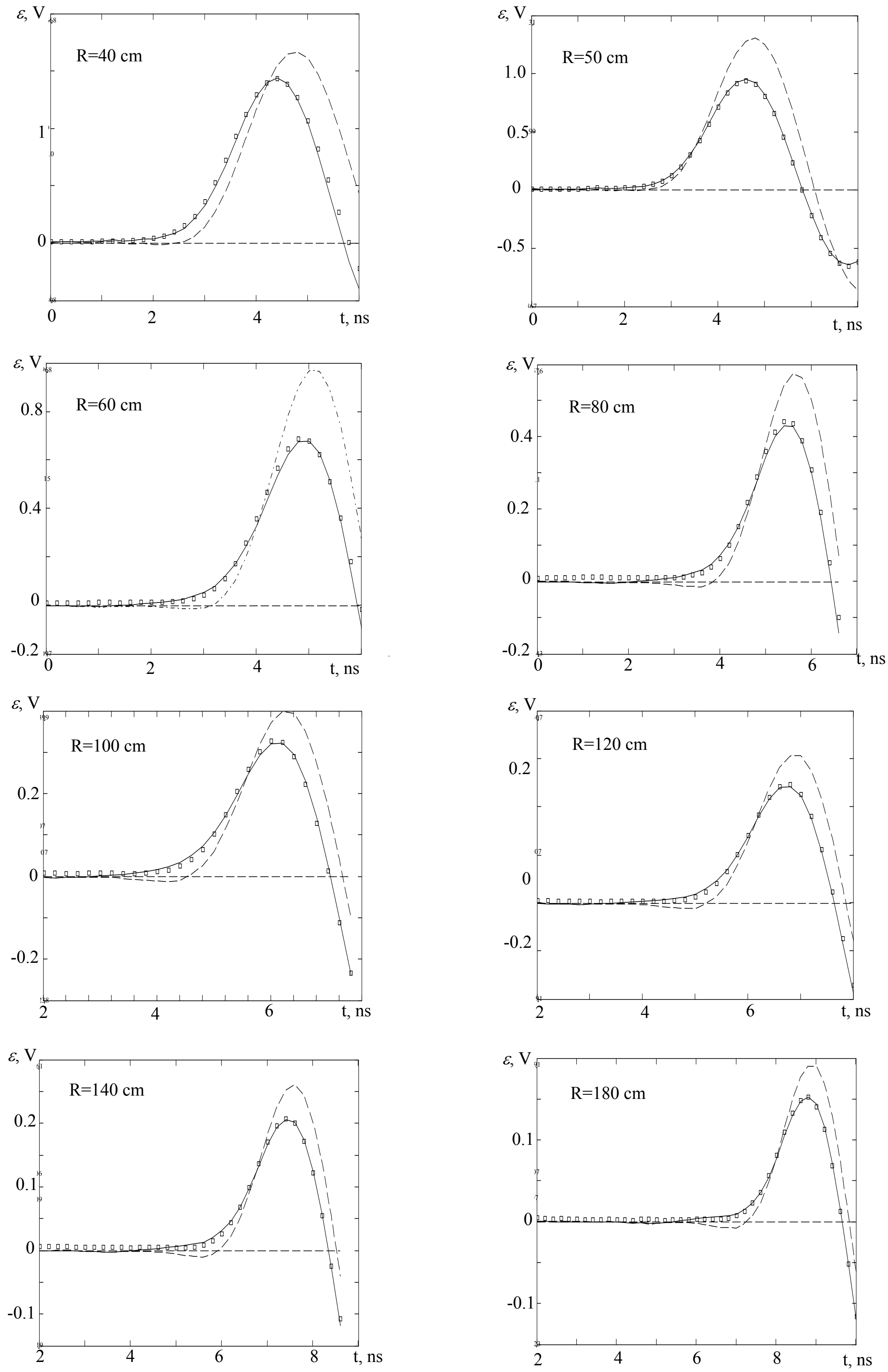


Fig. 3. Co-planar configuration: comparison of the detectable signal (hollow squares) produced in the RA at different spatial positions and numerically synthesized signals obtained under: (a) the first hypothesis in agreement with the standard condition *v*=*c* (dash lines); (b) the second hypothesis with the causal framework of bound contributions determined by Eq.(7) (continuous lines).

plitude of the recorded signal. This circumstance is due to the contribution of the second term in Eq.(7). Contrarily, if the hypothesis 2 is assumed to be valid then the second term in Eq.(7) is fully suppressed at small distances ($R<\lambda/2\pi$) since it is weighted by negligibly small numerical factor c/v in agreement with Fig. 2b. It gives an additional indication on the propagation velocity of bound fields v highly exceeding the velocity of light in the near zone of EM sources by supporting the fact that R^{-2} contribution in co-planar configuration is suppressed at $R<\lambda/2\pi$.

4. Conclusions

In order to provide a solid theoretical basis for experimental verification of EM field causality in the near zone of macroscopic EM sources, we started with the general approach based on conventional solutions to Maxwell's equations. Under electrically small antenna and quasi-stationary current approximations we derived the general expression (3) for the resultant e.m.f. induced in RA specifying the origin of every contribution. In our laboratory measurements we used co-planar and co-axial configurations between EA and RA, Eq. (3) is splitted into Eqs. (4) and (5) providing to the detectable signal a simpler representation as a superposition of a reduced number of components.

The co-axial configuration has an important advantage in giving the resultant e.m.f. as a superposition of only R^{-3} and R^{-2} terms due to bound and radiation contributions, respectively. This circumstance and the well-established fact that radiation fields propagate with the speed of light at any macroscopic length scales assured a direct and unambiguous decomposition of the detectable e.m.f. into bound and radiation contributions. The empirical information on the position of bound contributions on the time scale obtained as a function of R provided the knowledge of correspondent time shifts related to propagation (causal) characteristics of bound EM fields. According to standard views, if both positions are separated by a step ΔR then the speed of light c determines an observable time shift $\Delta t=\Delta R/c$. Nevertheless, experimentally found causal behavior of bound components in co-axial configurations showed no retardation ($\Delta t=0$) inside the near zone, tending to the value $\Delta t=\Delta R/c$ at large distances.

As a cross-verification of the results obtained in co-axial configuration, we carried out a comparison between numerically synthesized and recorded signals in co-planar configuration. Importantly, this analysis confirms that in co-planar configuration R^{-2}-term (proportional to the factor $c/v(R)$) turns out to be strongly suppressed within the near zone where $v(R)$ is much greater than c. It ought to be considered as an additional argument in favor of the developed model used in co-planar configuration since it is based on the observable amplitude relations between different signal's components.

On a qualitative level, these data keep in line with the result of our previous work [3] and come to a fundamental disagreement with the current causal interpretation of the classical EM theory. Put in other terms, within the near zone $R<\lambda/2\pi$ there is no empirical support for the validity of standard views in respect to the propagation of bound EM fields with the speed of light. Specifically, gauge-independent bound fields alone exhibit non-local properties in the region of space close to EM source where they are dominant. Strictly speaking, this result has to be distinguished from an apparent superluminality which takes place in the causal framework of the conventional EM theory: phase velocities of the signal front as a superposition of bound and radiation components (propagating with the same rate) are apparently greater than c in the near zone[8]. This circumstance highlights the importance to study causal propagation of bound and radiation fields separately by the decomposition of the detectable signal into respective contributions.

At present stage it is unrealistic and unreasonable to believe that the results reported in this work are sufficient to determine the causal framework meaningful for bound EM fields at all length scales. There is an obvious need for complementary cross-verifications based on independent methodological and experimental procedures. For instance, one might want to intend it by variation of intrinsic parameters such as wavelength of EM radiation which formally defines the frontier $\lambda/2\pi$ between the near and far zones. As a consequence, enhancing or reducing EM radiation frequency will provide a new quantitative information on causal propagation of EM bound fields in smaller or in larger near zone, respectively.

From a foundational standpoint the manifest non-locality of bound fields in regions close to EM sources might suggest a previously unknown intimate relationship between classical bound EM fields and quantum mechanical phenomena. Finally, we are tempted to think that these non-local properties exhibited by bound fields in the near zone are in agreement with Maxwell's fundamental equations and can be part of the paradigm of a causal physical theory which unifies classical and quantum descriptions. Another strong motivation for studying bound EM fields realistically appears to be a perspective of possible implications for such important areas of applied physics as near fields of radiative systems, plasma physics, thermonuclear fusion etc.

Acknowledgments

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Can (dG/dt)/G bound the local cosmological dynamics?

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One of important aspects in testing the General Relativity at interplanetary scales is imposing constraints on time variation in the gravitational constant, $(dG/dt)/G$, which can be done, for example, by using the data of lunar laser ranging (LLR) [1] or by a combined analysis of several types of optic and radiometric observations of the planets and spacecraft [2]. The best constraints achieved by now are $(4 \pm 9)\times10^{-13}$ per year from LLR and $(-2 \pm 5)\times10^{-14}$ per year from the combined analysis.

The next step undertaken by some authors is an "automatic" recalculation of the above constraints to limit a probable cosmological (Hubble) expansion at the local scales. For example, as was claimed in [1], "the $(dG/dt)/G$ uncertainty is 83 times smaller than the inverse age of the Universe, t_0 = 13.4 Gyr… Any isotropic expansion of the Earth's orbit which conserves angular momentum will mimic the effect of dG/dt on the Earth's semimajor axis, $(da/dt)/a = -(dG/dt)/G$ (where a is the scale factor of the Friedmann–Robertson–Walker metric)… There is no evidence for such local (~1 AU) scale expansion of the solar system."

Unfortunately, the above-stated equivalence between the effect of variable G and the cosmological expansion is based solely on the Newtonian arguments. The aim of our report is to present a more accurate treatment of this problem in the framework of General Relativity. Although consideration of the general case of a multi-component Universe is very difficult, it can be done quite easily for the particular case of the Universe filled only with Lambda-term (or the so-called "dark energy"), which is assumed now to be the main ingredient responsible for the cosmological dynamics. As follows from our calculations [3], manifestation of the Lambda-term in some components of the metric tensor really looks like the effect of variable G if we assume that $G = G_0 + (dG/dt)\, t$, where $dG/dt = -c\,(\Lambda/3)^{1/2}$. Unfortunately, such interpretation is not self-consistent: the Lambda-dependence of a few other components is irreducible to the variable coefficient of gravitational coupling. Therefore, the available limits on $(dG/dt)/G$, in general, cannot be reinterpreted as a constraint on the local value of Hubble constant, and the problem of cosmological dynamics at planetary scales remains open.

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Quantum cosmological solutions: their dependence on the choice of gauge conditions and physical interpretation

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In "extended phase space" approach to quantum geometrodynamics numerical solutions to Schrödinger equation corresponding to various choice of gauge conditions are obtained for the simplest isotropic model. The "extended phase space" approach belongs to those appeared in the last decade in which, as a result of fixing a reference frame, the Wheeler – DeWitt static picture of the world is replaced by evolutionary quantum geometrodynamics. Some aspects of this approach were discussed at two previous PIRT meetings. We are interested in the part of the wave function depending on physical degrees of freedom. Three gauge conditions having a clear physical meaning are considered. They are the conformal time gauge, the gauge producing the appearance of Λ-term in the Einstein equations, and the one covering the two previous cases as asymptotic limits. The interpretation and discussion of the obtained solutions is given

1. Introduction

In this paper we present solutions to quantum geometrodynamical Schrödinger equation corresponding to various choice of gauge conditions for the simplest isotropic model. It is widely accepted in quantum geometrodynamics to illustrate general ideas taking simple cosmological models as examples. The reason why physicists working in this field appeal to simple models is that now quantum geometrodynamics is just as far from being a completed theory as it was decades ago. One must confess that hitherto there is no agreement on what "first principles" this theory should be based and what is the form of master equation for a wave function of the Universe. The first version of quantum geometrodynamics, proposed by Wheeler and DeWitt [1, 2], encountered a number of fundamental problems (for discussion, see [3, 4, 5]). The main problem is the so called "frozen formalism", or the absence of time evolution. It is easy to see that the source of the problem of time consists in the application of the Dirac postulates to gravitational field, according to which not the Schrödinger equation but the constraints as conditions on a wave function play the central part in the theory. As a result of impossibility to resolve the problems of the Wheeler – DeWitt quantum geometrodynamics in its own limits, in the last decade there appear a new tendency in the development of the theory which can be called Evolutionary Quantum Gravity. This tendency may be characterized by the two features: firstly, the recognition of the fact that it is impossible to obtain the evolutionary picture of the Universe without fixing a reference frame and, secondly, the rejection of the Wheeler – DeWitt equation and the reestablishment of the role which the Schrödinger equation plays in any quantum theory.

The tendency embraces several approaches (see, for example, [6, 7], where a dust fluid is considered as a good choice to fix a reference frame in quantum gravity), to which the "extended phase space" approach belongs. Some aspects of the latter were discussed at two previous PIRT meetings [8, 9]. The approach is based on a careful analysis of peculiarities of quantization of the Universe as a whole [10, 11]. The analysis showed that quantum geometrodynamics as a mathematically consistent theory failed to be constructed in a gauge invariant way, therefore, the Wheeler – DeWitt equation, being a constraint on a state vector, loses its significance and should be replaced by a gauge dependent Schrödinger equation resulting from the Hamiltonian formulation of the theory in ex-

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tended phase space. A wave function satisfying the Schrödinger equation is determined on extended configurational space that involves gauge gravitational degrees of freedom equally as physical ones. However, we are actually interested in the part of the wave function depending on physical degrees of freedom only, since this very function defines probability distributions of physical quantities.

In Section 2 we shall describe the model and the Schrödinger equation for the physical part of wave function for the given model. Since the form of the Schrödinger equation is gauge dependent, to obtain descriptions of the Universe corresponding to various gauge conditions (in other words, to various reference frames) one has to solve, in fact, absolutely different differential equations. It naturally leads us to the question, is there any correspondence among solutions of the equations? And how should they be interpreted?

Let us note that while in [6, 7] the authors work with a certain parametrization of gravitational variables (as a rule, it is the Arnowitt – Deser – Misner parametrization [12]) and some "privileged" reference frame, our approach, though was applied to cosmological models with finite degrees of freedom, aimed at including arbitrary parametrizations and a wide enough class of gauge conditions. We shall consider three gauge conditions having a clear physical meaning: the conformal time gauge, the gauge producing the appearance of Λ-term in the Einstein equations, and the one covering the two previous cases as asymptotic limits. For a closed universe, the first and third gauges gives rise to a discrete Hamiltonian spectrum, while the second gauge leads to a continuous spectrum. From a pure methodical viewpoint, the first and third cases are much easier to be treated, and in Section 3 numerical solution for these cases will be presented, meantime the second case admits qualitative consideration only. Section 4 contains physical interpretation and conclusions.

2. The model and the Schrödinger equation for the physical part of the wave function

The action for a closed isotropic universe is

$$S = \int dt\left(-\frac{1}{2}\frac{a\dot{a}^2}{N} + \frac{1}{2}Na\right) + S_{(mat)} + S_{(gf)}, \tag{2.1}$$

$$S_{(mat)} = -\int dt N a^3 \varepsilon(a), \quad S_{(gf)} = \int dt \pi_0\left(\dot{N} - \frac{df}{da}\dot{a}\right). \tag{2.2}$$

Matter fields are described in this model phenomenologically, without a clear indication on the nature of the fields. The dependence of its energy density $\varepsilon(a)$ on the scale factor a determines its equation of state, namely, for the power dependence $\varepsilon(a) = \frac{\varepsilon_0}{a^n}$ the equation of state is known to be $p_{(mat)} = \left(\frac{n}{3} - 1\right)\varepsilon_{(mat)}$, ε_0 is a constant whose dimensionality in the Plank units is $\rho_{Pl} l_{Pl}^n$. Since we are interested in early enough stages of the Universe evolution, we shall suppose that the Universe was filled with radiation with the equation of state $p_{(mat)} = \frac{1}{3}\varepsilon_{(mat)}$, i.e.

$$\varepsilon(a) = \frac{\varepsilon_0}{a^4}, \tag{2.3}$$

$S_{(gf)}$ is a gauge-fixing part of the action, its variation giving rise to gauge dependent terms in the Einstein equations. In ordinary quantum theory this terms are to be excluded by asymptotic boundary conditions. As was argued in [10], in the case of the Universe with a non-trivial topology, which, in general, does not possess asymptotic states, making use of asymptotic boundary condition is not justified.

If so, the gauge-fixing action describes a subsystem of the Universe, some medium, whose state is determined by a chosen gauge. In (2.2) a differential form of the gauge condition

$$N - f(a) = 0 \tag{2.4}$$

is used. The equation of state for this subsystem is

$$p_{(obs)} = \frac{1}{3}\frac{f'(a)}{f(a)} a\varepsilon_{(obs)}, \tag{2.5}$$

The index (*obs*) indicates that this subsystem corresponds to an observer studying the Universe evolution in his reference frame.

The action (2.1) is a particular case of the action for a cosmological model with a finite number degrees of freedom considered in [8, 11]. The Schrödinger equation for the physical part of the wave function looks like

$$\left[-\frac{1}{2}\sqrt{\frac{N}{a}}\frac{d}{da}\left(\sqrt{\frac{N}{a}}\frac{d\Psi}{da}\right) + \frac{1}{2}Na\Psi - Na^3\varepsilon(a)\Psi\right]\Bigg|_{N=f(a)} = E\Psi, \tag{2.6}$$

From the classical point of view, E is given by

$$E = -\int \sqrt{-g}\, T^0_{0(obs)} d^3x, \tag{2.7}$$

$T^{\nu}_{\mu(obs)}$ is a quasi energy-momentum tensor obtained by variation of the gauge-fixing action; it is not a real tensor in the sense that it depends on a gauge condition. $T^{\nu}_{\mu(obs)}$ describes the subsystem of the observer in the gauged Einstein equations [8]. It can be shown that the integral (2.7) of $T^0_{0(obs)}$ taken over space is a conserved quantity for the class of gauge conditions (2.4). Thus, E characterizes the energy of the observer subsystem.

It may be said that on a phenomenological level this approach takes into account interaction between the observer subsystem and the physical Universe. The interaction causes rebuilding of energy balance of two subsystems. It is expected that at the late stage of the Universe evolution, when the Universe is well described by General Relativity, gauge effects are negligible, and the values of E must be very close, if not equal, to zero. However, at the early quantum stage E may have essentially non-zero values, and the exploration of its spectrum is the main task of this work.

Now we consider several gauge conditions.

1. The conformal time gauge $N = a$. The equation of state of the observer subsystem is the same as that of the matter: $p_{(obs)} = \frac{1}{3}\varepsilon_{(obs)}$. Substituting $N = a$ and (2.3) in (2.6), we get

$$-\frac{1}{2}\frac{d^2\Psi}{da^2} + \frac{1}{2}a^2\Psi - \varepsilon_0\Psi = E\Psi, \tag{2.8}$$

after redefinition

$$E + \varepsilon_0 \to E \tag{2.9}$$

we obtain the equation

$$-\frac{1}{2}\frac{d^2\Psi}{da^2} + \frac{1}{2}a^2\Psi = E\Psi. \tag{2.10}$$

Therefore, Eq. (2.10) describes the Universe filled with a "substance" with the equation of state $p = \frac{1}{3}\varepsilon$. Just some part of the energy of this substance may be due to a usual matter while the other part may be due to gauge, or observer, effects.

It was shown in [13] that Eq. (2.10) can be obtained in the limits of the Wheeler – DeWitt quantum geometrodynamics by rewriting of the Wheeler – DeWitt equation $H\Psi = 0$ as a Schrödinger-like equation $\tilde{H}\Psi = E\Psi$. Under additional requirements, that imply choosing a certain gauge condition and including a certain kind of matter into the model, the classical Hamiltonian constraint $H = 0$ can be presented in a new form, $\tilde{H} = E$, $H = \tilde{H} - E$, where E is a conserved quantity which appears from phenomenological consideration of this kind of matter. So, in this approach, $E = \varepsilon_0$, i.e. E is entirely due to the usual matter (radiation).

On the other side, the need for making a choice of gauge to rewrite the Wheeler – DeWitt equation in the special form $\tilde{H}\Psi = E\Psi$ witnesses to gauge noninvariance of the Wheeler – DeWitt theory. As was already emphasized above, the Wheeler – DeWitt equation loses its meaning, and it seems to be reasonable rejecting it rather trying to hold it by any means.

The effective potential $U(a) = \frac{1}{2}a^2$ is given at Fig. 1(a).

2. $Na^3 = 1$. The gauge is believed to produce the appearance of Λ-term in the Einstein equations, since it is the analog of a more general condition $\det\left\|g^{\mu\nu}\right\| = 1$. The equation of state $p_{(obs)} = -\varepsilon_{(obs)}$. The Schrödinger equation takes the form

$$-\frac{1}{2}\frac{1}{a^4}\frac{d^2\Psi}{da^2} + \frac{1}{a^5}\frac{d\Psi}{da} + \frac{1}{2a^2}\Psi - \frac{\varepsilon_0}{a^4}\Psi = E\Psi . \qquad (2.11)$$

Here ε_0 characterizes a contribution of the matter fields (radiation). If one includes into the model de Sitter false vacuum with the equation of state $p_{(vac)} = -\varepsilon_{(vac)}$ and the dependence $\varepsilon(a) = \varepsilon_0$, it does not affect the form of the equation (2.11) after redefinition (2.9). Then one could say that vacuum energy as well as gauge effects are responsible for eigenvalues of E.

The effective potential $U(a) = \frac{1}{2a^2} - \frac{\varepsilon_0}{a^4}$ depends on the parameter ε_0. According to modern cosmological notions, the Universe was created in a metastable under the barrier depicted at Fig. 1(b) and then tunneled through the barrier. The smaller the parameter ε_0 is, the higher and narrower the barrier becomes. There is a non-zero probability for arbitrary large values of the scale factor a; it means that the Universe may expand to infinity in spite of the sign "+" we have put before the second term in (2.1), which corresponds to the closed model. It demonstrates that a naïve correspondence between the kind of a cosmological model and the form of the effective potential has no grounds.

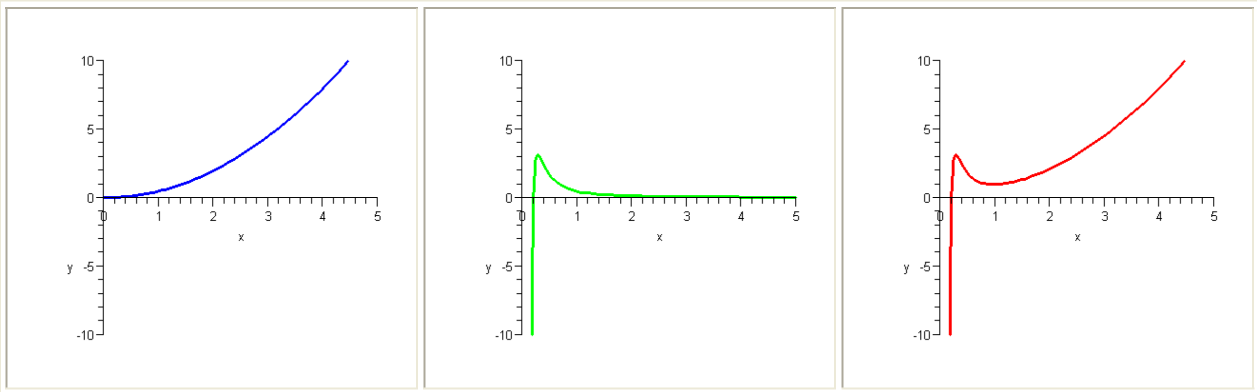
3. $N = a + \frac{1}{a^3}$. This gauge covers the two previous cases as asymptotic limits. The equation of state is

$$p_{(obs)} = \frac{1}{3}\frac{a^4 - 3}{a^4 + 1}\varepsilon_{(obs)} . \qquad (2.12)$$

At $a \to 0$ the equation gives $p_{(obs)} = -\varepsilon_{(obs)}$; at $a \to \infty$ it gives $p_{(obs)} = \frac{1}{3}\varepsilon_{(obs)}$. Again, after redefinition (2.9) the Schrödinger equation looks like following

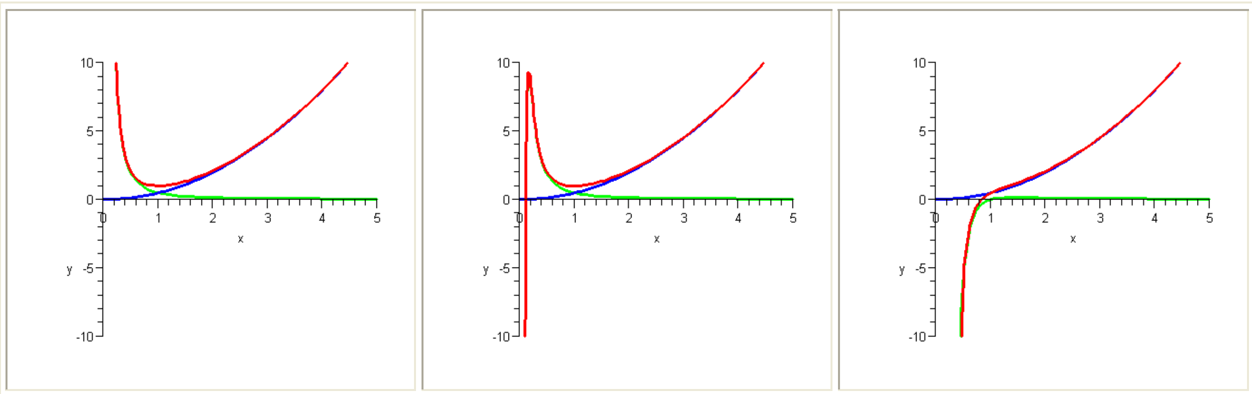
$$-\frac{1}{2}\left(1 + \frac{1}{a^4}\right)\frac{d^2\Psi}{da^2} + \frac{1}{a^5}\frac{d\Psi}{da} + \frac{1}{2}a^2\Psi + \frac{1}{2a^2}\Psi - \frac{\varepsilon_0}{a^4}\Psi = E\Psi . \qquad (2.13)$$

It is easy to check that Eqs. (2.11), (2.10) are the asymptotic limits of (2.13) at $a \to 0$ and $a \to \infty$ respectively. In this case the Universe is believed to be filled by some mixture of matter and vacuum. In consequence of the redefinition (2.9), the value of E is due to matter contribution as well as gauge effects. Like in a previous case, the effective potential $U(a) = \frac{1}{2}a^2 + \frac{1}{2a^2} - \frac{\varepsilon_0}{a^4}$ depends on the parameter ε_0 and depicted at Fig. 1(c). The barrier at small a disappears when $\varepsilon_0 = 0$ and $\varepsilon_0 \geq 0.1$. The potential for some value of ε_0 is shown at Fig. 2. One can see that the potentials of Eq. (2.11) (green graph) and of Eq. (2.10) (blue graph) are asymptotic forms of the potential of Eq. (2.13).



a) $N = a$	b) $N = \frac{1}{a^3}$, $\varepsilon_0 = \frac{1}{50}$	c) $N = a + \frac{1}{a^3}$, $\varepsilon_0 = \frac{1}{50}$

Fig. 1. The effective potentials for Eqs. (2.10), (2.11), (2.13).



a) $\varepsilon_0 = 0$	b) $\varepsilon_0 = \frac{1}{150}$	c) $\varepsilon_0 = \frac{1}{2}$

Fig. 2. The effective potentials for some values of ε_0

3. Numerical solutions

The Hamiltonian operators in Eqs. (2.10), (2.13) have a discrete spectrum, and one meets no technical difficulties to obtain numerical solutions to these equations. The operator in (2.6) is Hermitian for an arbitrary gauge (2.4) if the measure in Hilbert space of solution is taken to be

$$M(a) = \sqrt{\frac{a}{f(a)}} . \tag{3.1}$$

One can see that the measure, like the equation itself, is gauge-dependent.

The standard method of finding eigenvalues and eigenfunctions consists in the expansion onto a basis functions which are orthonormal on the interval $[0, \infty]$ with the measure (3.1):

$$\Psi(a) = \sum_n c_n \psi_n^s(a) ; \tag{3.2}$$

$$\psi_n^s(a) = \sqrt{\frac{n!}{(n+s)!}} \frac{1}{\sqrt{M(a)}} a^{\frac{s}{2}} L_n^s(a) = \sqrt{\frac{n!}{(n+s)!}} \left(\frac{f(a)}{a} \right)^{\frac{1}{4}} a^{\frac{s}{2}} L_n^s(a) ; \tag{3.3}$$

$$\int_0^\infty \psi_n^{s*}(a) \psi_m^s(a) M(a) da = \delta_{nm} , \tag{3.4}$$

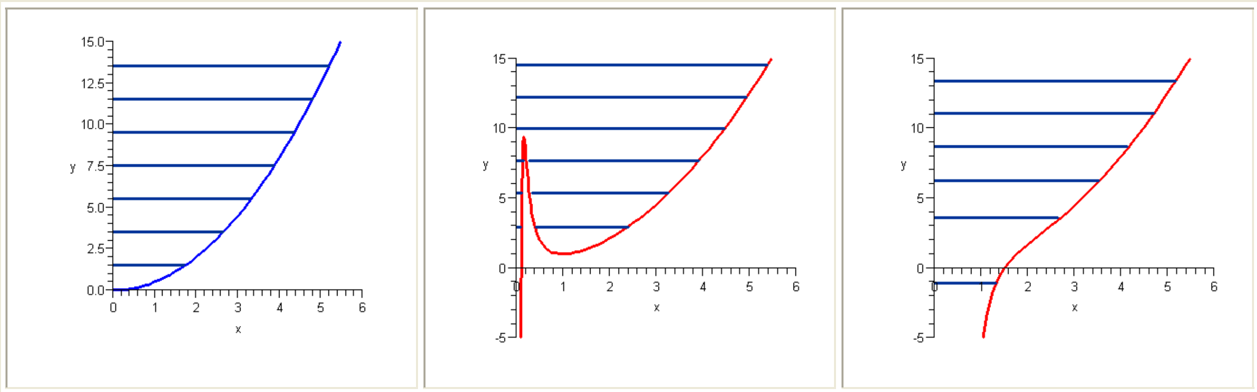
$L_n^s(a)$ are Laguerre polynomials. The problem is reduced to finding eigenvalues and eigenvectors of the Hamiltonian matrix in the basis (3.3). The more terms are held in the expansion (3.2), the higher the precision is. The results of calculations of first five eigenvalues are presented at Table 1.

Table 1.

Eq. (2.10), $N = a$		1.5	3.5	5.5	7.50001	9.50008
Eq. (2.13), $N = a + \frac{1}{a^3}$	$\varepsilon_0 = 0$	2.87886	5.32668	7.66977	9.9591	12.2175
	$\varepsilon = 1/500$	2.87846	5.32635	7.66947	9.95882	12.2173
	$\varepsilon_0 = 1/150$	2.87754	5.32558	7.66877	9.95817	12.2166
	$\varepsilon_0 = 1/50$	2.87489	5.32337	7.66677	9.9563	12.2149
	$\varepsilon_0 = 1/2$	2.77519	5.24152	7.59315	9.88783	12.1496
	$\varepsilon_0 = 1$	2.66102	5.15088	7.51266	9.81349	12.0792
	$\varepsilon_0 = 3$	2.04887	4.72486	7.14847	9.48369	11.7714
	$\varepsilon_0 = 4$	1.59368	4.47069	6.94071	9.29951	11.602
	$\varepsilon_0 = 5$	0.972188	4.1924	6.71849	9.1044	11.4236
	$\varepsilon_0 = 7$	-1.07592	3.59902	6.25063	8.69468	11.0497

One can see that for Eq. (2.10), $N = a$ the spectrum is equidistant, the difference between eigenvalues is equal to 2 in the Plank units (the deviation from this value is entirely due to calculation inaccuracy).

In the case of Eq. (2.13), $N = a + \frac{1}{a^3}$, the eigenvalues do not differ significantly for $\varepsilon_0 \le \frac{1}{50}$ and converge to limiting values at $\varepsilon_0 = 0$. For $\varepsilon_0 > \frac{1}{50}$ the spectrum levels tend to go down into the potential pit. The schematic picture of the spectrum is shown at Fig.3.



a) Eq. (2.10), $N = a$	b) Eq. (2.13), $\varepsilon_0 = \dfrac{1}{150}$	c) Eq. (2.13), $\varepsilon_0 = 7$

Fig. 3. The spectrum levels for some potentials.

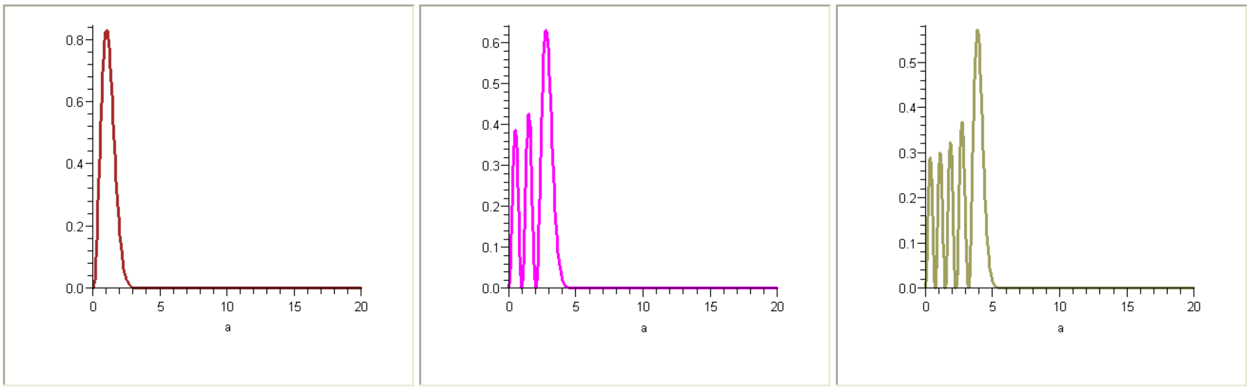
Fig. 4 pictures the probability distributions for the first (ground state), third and fifth solutions to Eq. (2.10) and Eq. (2.13) when $\varepsilon_0 = \frac{1}{150}$ and $\varepsilon_0 = 7$.One can see that at the qualitative level the probability distributions do not significantly differ. The peak of the probability distribution in the all cases tends to shift to large values of the scale factor *a* for larger eigenvalues of *E*. One could expect this result since the matter and gauge effects contribute to the value of *E*. So, when the energy of matter increases, there may be enough probability for the scale factor to reach large values.

4. Concluding remarks

We should recognize that we have considered a very simple model and the obtained results are not of high degree of generality. The present work is just a small step "to find the way".

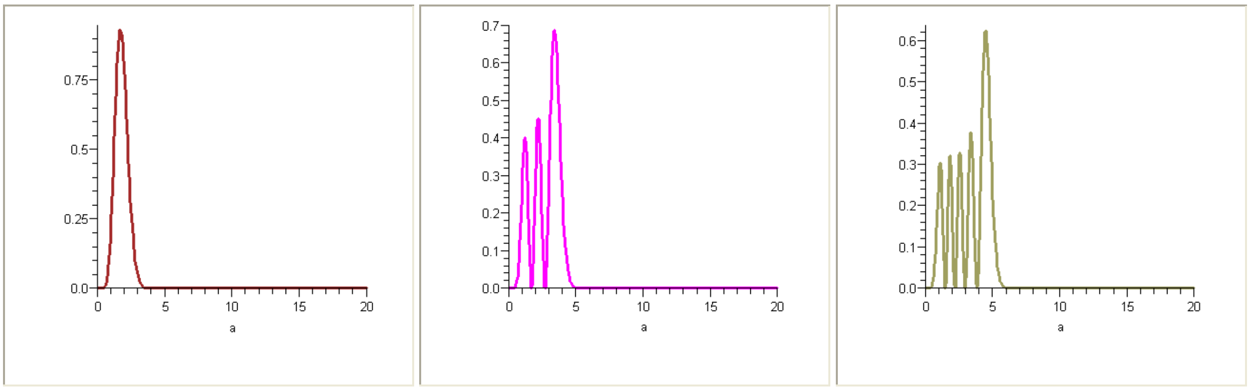
We have seen that the second gauge condition, $N = \frac{1}{a^3}$, leads to a continuous spectrum of eigenvalues of the Schrödinger equation (2.9), while the two other gauges, $N = a$ and $N = a + \frac{1}{a^3}$, leads to a discrete spectrum, in other words, the second case is substantially different. It seems that one should seek for the reason in the structure of spacetime. Indeed, the gauge $N = \frac{1}{a^3}$ corresponds to the Universe in which the interval of proper time between two subsequent spacelike hypersurfaces tends to zero as $a \to \infty$, meantime it is not the case for the two other gauges. Since any gauge condition determines the form of the effective potential, this circumstance require a more careful exploration. It would be interesting to study the gauge $N = 1 + \frac{1}{a^3}$, for which at $a \to \infty$ the reference frame becomes a synchronous one (*N*=1) and the equation of state of the observer subsystem at $a \to \infty$ is that of dust: $p_{(obs)} = 0$.

The resemblance of probability distributions for solutions to Eqs. (2.10), (2.13) also deserves our attention. It demonstrates that one can reveal some relation among solutions for certain classes of gauge conditions. Let us note that the problem, how solutions to the Wheeler – DeWitt equation are related, was discussed as soon as its parametrization noninvariance had been realized. Then Halliwell [14] proposed to restrict the class of admissible parametrizations. Since parametrization and gauge conditions have a unified interpretation [5], it implies also a restriction of the class of admissible gauge conditions, i.e. it is permissible to describe the Universe in one or several “privileged” reference frames.



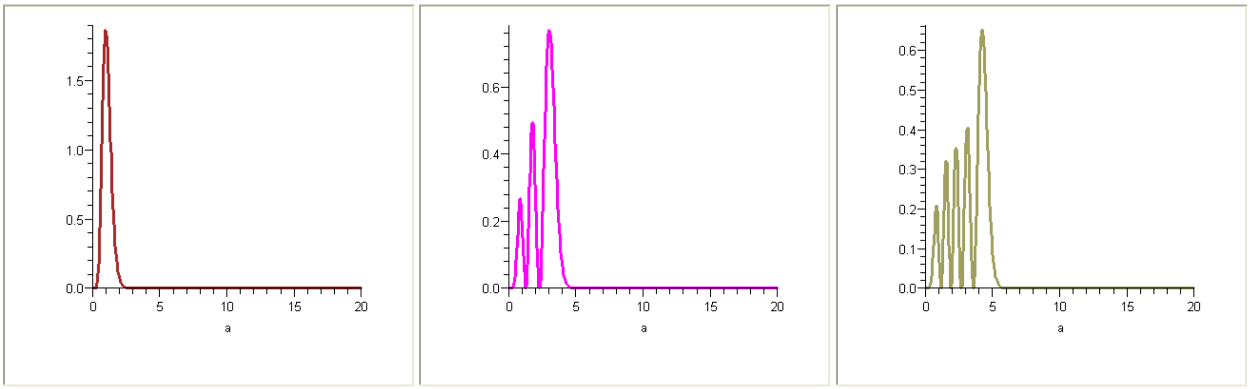
$|\Psi_0|^2$, $E_0 = 1.5$ | $|\Psi_2|^2$, $E_2 = 5.5$ | $|\Psi_4|^2$, $E_4 = 9.50008$

a) The probability distributions for Eq. (2.10), $N = a$



$|\Psi_0|^2$, $E_0 = 2.87754$ | $|\Psi_2|^2$, $E_2 = 7.66877$ | $|\Psi_4|^2$, $E_4 = 12.2166$

b) The probability distributions for Eq. (2.13), $N = a + \dfrac{1}{a^3}$, $\varepsilon_0 = \dfrac{1}{150}$



$|\Psi_0|^2$, $E_0 = -1.07592$ | $|\Psi_2|^2$, $E_2 = 6.25063$ | $|\Psi_4|^2$, $E_4 = 11.0497$

b) The probability distributions for Eq. (2.13), $N = a + \dfrac{1}{a^3}$, $\varepsilon_0 = 7$

Fig. 4. The probability distributions for some solutions.

This way seems to be artificial since we do not know for sure what reference frame is privileged. Our point of view is that we face a new problem of finding classes of gauge conditions within which solutions to the Schrödinger equation are stable enough with respect to a choice of gauges, the determination of the classes being inseparable from our understanding of spacetime structure.

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GRAVITATIONAL REPULSION AND ITS COSMOLOGICAL CONSEQUENCES

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Abstract. It is shown that the gravitational interaction can have the repulsion character in the case of gravitating systems with positive values of energy density and pressure satisfying energy dominance condition in the frame of the Poincaré gauge theory of gravity. Cosmological consequences of gravitational repulsion are considered in the case of homogeneous isotropic models with vanishing and not vanishing pseudoscalar torsion function. Cosmological inflationary solutions corresponding to regular Big Bang and regular accelerating Universe are discussed.

1 Introduction

As it is known, in the framework of the general relativity theory (GR) as well as Newton's theory of gravity the gravitational interaction in the case of gravitating systems with positive values of energy density and pressure satisfying energy dominance condition always has the character of attraction but not repulsion. As a result, the problem of gravitational singularities takes place in GR. One of the most important cases of this topic is the problem of cosmological singularity (PCS). The solution of PCS is connected with the search of gravitation theory, in the frame of which the gravitational interaction at extreme conditions (extremely high energy densities and pressures) has the repulsion character preventing the appearance of cosmological singularity. Although in the frame of GR the gravitational interaction can have the repulsion character in the case of gravitating systems with negative pressure, however, the PCS can not be solved by taking into account such systems. According to widely known opinion, the solution of PCS has to be connected with quantum gravitational effects beyond Planckian conditions, when the energy density in the Universe surpasses the Planckian one. Although a number of regular cosmological solutions obtained in the frame of candidates to quantum gravitation theory - string theory/M-theory and loop quantum gravity - are known at present, these solutions have some difficulties of physical character, in particular, in the frame of string theory the condition of energy density positivity is broken [1].

As it was shown in our papers [1-3], the gravitational interaction can have the repulsion character in the case of gravitating systems with positive values of energy density and pressure satisfying energy dominance condition in the frame of the Poincaré gauge theory of gravity (PGTG), which is the gravitation theory in 4-dimensional Riemann-Cartan space-time. The PGTG is direct generalization of GR, which is necessary, if one supposes that the Lorentz gauge field corresponding to fundamental group in physics - the Lorentz group - exists in the nature (see [1]).

This talk is devoted to discussion of cosmological consequences of gravitational repulsion in the frame of PGTG in the case of homogeneous isotropic models (HIM). The geometrical structure of HIM in general case is characterized by three functions of time: the scale factor of Robertson-Walker metrics R(t) and two torsion functions S_1 and S_2 determining not vanishing components of torsion tensor (see below); unlike S_1 the torsion function S_2 has pseudoscalar character with respect to spatial inversions. The appearance of gravitational repulsion is connected with space-time torsion essentially, and in the case of HIM the functions S_1 and S_2 play the different role. Two types of HIM with vanishing and not vanishing function S_2 are investigated in Section 3 and Section 4 respectively. In Section 2 some necessary information concerning PGTG and HIM is given.

2 PGTG and homogeneous isotropic models

At fist let us mention some general relations of the PGTG. Gravitational field variables in PGTG are the tetrad $h^i{}_\mu$ (translational gauge field) and the Lorentz connection $A^{ik}{}_\mu$ (Lorentz gauge field); corresponding field strengths are the torsion tensor $S^i{}_{\mu\nu}$ and the curvature tensor $F^{ik}{}_{\mu\nu}$ defined as

$$S^i{}_{\mu\,\nu} = \partial_{[\nu}\, h^i{}_{\mu]} - h_{k[\mu} A^{ik}{}_{\nu]}\,,$$

$$F^{ik}{}_{\mu\nu} = 2\partial_{[\mu} A^{ik}{}_{\nu]} + 2A^{il}{}_{[\mu} A^k{}_{|l\,|\nu]}\,,$$

where holonomic and anholonomic space-time coordinates are denoted by means of greek and latin indices respectively. As sources of gravitational field in PGTG are energy-momentum and spin tensors.

We will consider the PGTG based on gravitational Lagrangian given in general form containing both a scalar curvature and different invariants quadratic in the curvature and torsion tensors

$$\begin{aligned}\mathcal{L}_{\rm G} = f_0\, F + F^{\alpha\beta\mu\nu}\left(f_1\, F_{\alpha\beta\mu\nu} + f_2\, F_{\alpha\mu\beta\nu} + f_3\, F_{\mu\nu\alpha\beta}\right) + F^{\mu\nu}\left(f_4\, F_{\mu\nu} + f_5\, F_{\nu\mu}\right) + f_6\, F^2 \\ +S^{\alpha\mu\nu}\left(a_1\, S_{\alpha\mu\nu} + a_2\, S_{\nu\mu\alpha}\right) + a_3\, S^{\alpha}{}_{\mu\alpha} S_\beta{}^{\mu\beta}, \qquad (1)\end{aligned}$$

where $F_{\mu\nu} = F^{\alpha}{}_{\mu\alpha\nu}$, $F = F^{\mu}{}_{\mu}$, f_i $(i = 1, 2, \ldots, 6)$, a_k $(k = 1, 2, 3)$ are indefinite parameters, $f_0 = (16\pi G)^{-1}$, G is Newton's gravitational constant (the light speed in the vacuum $c = 1$). Gravitational equations of PGTG obtained from the action integral $I = \int (\mathcal{L}_g + \mathcal{L}_m)\, hd^4x$, where $h = \det\,(h^i{}_\mu)$ and $\mathcal{L}_m$ is the Lagrangian of matter, contain the system of 16+24 equations corresponding to gravitational variables $h^i{}_\mu$ and $A^{ik}{}_\mu$.

In the case of HIM the torsion tensor $S^\lambda{}_{\mu\nu} = -S^\lambda{}_{\nu\mu}$ can have the following non-vanishing components [4, 5]: $S^1{}_{10} = S^2{}_{20} = S^3{}_{30} = S_1(t)$, $S_{123} = S_{231} = S_{312} = S_2(t)\dfrac{R^3 r^2}{\sqrt{1-kr^2}}\sin\theta$, spatial spherical coordinates are used. By choosing the tetrad in diagonal form, we obtain the following non-vanishing tetrad components of the curvature tensor denoted by means of the sign $\hat{}$:

$$F^{\hat{0}\hat{1}}{}_{\hat{0}\hat{1}} = F^{\hat{0}\hat{2}}{}_{\hat{0}\hat{2}} = F^{\hat{0}\hat{3}}{}_{\hat{0}\hat{3}} \equiv A_1, \qquad F^{\hat{1}\hat{2}}{}_{\hat{1}\hat{2}} = F^{\hat{1}\hat{3}}{}_{\hat{1}\hat{3}} = F^{\hat{2}\hat{3}}{}_{\hat{2}\hat{3}} \equiv A_2,$$
$$F^{\hat{0}\hat{1}}{}_{\hat{2}\hat{3}} = F^{\hat{0}\hat{2}}{}_{\hat{3}\hat{1}} = F^{\hat{0}\hat{3}}{}_{\hat{1}\hat{2}} \equiv A_3, \qquad F^{\hat{3}\hat{2}}{}_{\hat{0}\hat{1}} = F^{\hat{1}\hat{3}}{}_{\hat{0}\hat{2}} = F^{\hat{2}\hat{1}}{}_{\hat{0}\hat{3}} \equiv A_4,$$

with

$$\begin{aligned} A_1 &= \dot{H} + H^2 - 2HS_1 - 2\dot{S}_1, \\ A_2 &= \frac{k}{R^2} + (H - 2S_1)^2 - S_2^2, \\ A_3 &= 2\left(H - 2S_1\right)S_2, \\ A_4 &= \dot{S}_2 + HS_2, \end{aligned} \tag{2}$$

where $H = \dot{R}/R$ is the Hubble parameter and a dot denotes the differentiation with respect to time.

The Bianchi identities in this case are reduced to two following relations:

$$\begin{aligned} \dot{A}_2 + 2H\left(A_2 - A_1\right) + 4S_1A_1 + 2S_2A_4 &= 0, \\ \dot{A}_3 + 2H\left(A_3 - A_4\right) + 4S_1A_4 - 2S_2A_1 &= 0. \end{aligned} \tag{3}$$

By using the gravitational Lagrangian (1) the system of gravitational equations for HIM takes the following form [5]

$$a\left(H - S_1\right)S_1 - 2bS_2^2 - 2f_0A_2 + 4f\left(A_1^2 - A_2^2\right) + 2q_2\left(A_3^2 - A_4^2\right) = -\frac{\rho}{3}, \tag{4}$$

$$a\left(\dot{S}_1 + 2HS_1 - S_1^2\right) - 2bS_2^2 - 2f_0\left(2A_1 + A_2\right) - 4f\left(A_1^2 - A_2^2\right) - 2q_2\left(A_3^2 - A_4^2\right) = p, \tag{5}$$

$$f\left[\dot{A}_1 + 2H\left(A_1 - A_2\right) + 4S_1A_2\right] + q_2S_2A_3 - q_1S_2A_4 + \left(f_0 + \frac{a}{8}\right)S_1 = 0, \tag{6}$$

$$q_2\left[\dot{A}_4 + 2H\left(A_4 - A_3\right) + 4S_1A_3\right] - 4f\,S_2A_2 - 2q_1S_2A_1 - \left(f_0 - b\right)S_2 = 0, \tag{7}$$

where

$$\begin{aligned} &a = 2a_1 + a_2 + 3a_3, \qquad b = a_2 - a_1, \\ &f = f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6\,, \\ &q_1 = f_2 - 2f_3 + f_4 + f_5 + 6f_6, \qquad q_2 = 2f_1 - f_2, \end{aligned}$$

ρ is the energy density, p is the pressure and the average of spin distribution of gravitating matter is supposed to be equal to zero. Equations (4)–(5) do not contain high derivatives for the scale factor R, if $a = 0$, moreover, equations (6)–(7) take more symmetric form, if

$2f = q_1 + q_2$. Then by using the Bianchi identities (3), the system of gravitational equations for HIM take the following form:

$$-2b\, S_2^2 - 2f_0 A_2 + 4f\left(A_1^2 - A_2^2\right) + 2q_2\left(A_3^2 - A_4^2\right) = -\frac{1}{3}\rho, \tag{8}$$

$$-2b\, S_2^2 - 2f_0\left(2A_1 + A_2\right) - 4f\left(A_1^2 - A_2^2\right) - 2q_2\left(A_3^2 - A_4^2\right) = p, \tag{9}$$

$$f\left[\left(\dot{A}_1 + \dot{A}_2\right) + 4S_1\left(A_1 + A_2\right)\right] + q_2\, S_2\left(A_3 + A_4\right) + f_0 S_1 = 0, \tag{10}$$

$$q_2\left[\left(\dot{A}_3 + \dot{A}_4\right) + 4S_1\left(A_3 + A_4\right)\right] - 4f\, S_2\left(A_1 + A_2\right) - \left(f_0 - b\right) S_2 = 0. \tag{11}$$

The system of equations (8)–(11) together with definition of curvature functions (2) is the base of our investigation of HIM below. Note also that the conservation law for spinless matter has usual form:

$$\dot{\rho} + 3H\left(\rho + p\right) = 0. \tag{12}$$

In order to investigate inflationary cosmological models we will consider below HIM filled with non-interacting scalar field ϕ minimally coupled with gravitation and gravitating matter with equation of state in the form $p_m = p_m(\rho_m)$ (values of gravitating matter are denoted by means of index "m"). Then the energy density ρ and the pressure p take the form

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + \rho_m \quad (\rho > 0), \quad p = \frac{1}{2}\dot{\phi}^2 - V + p_m, \tag{13}$$

where $V = V(\phi)$ is a scalar field potential. By using the scalar field equation in homogeneous isotropic space

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \tag{14}$$

we obtain from (12)-(13) the conservation law for gravitating matter

$$\dot{\rho}_m + 3H\left(\rho_m + p_m\right) = 0. \tag{15}$$

3 Generalized cosmological Friedmann equations and regular inflationary HIM

At first we will consider HIM with $S_2 = 0$, then the system of gravitational equations (8)-(11) is reduced to three equations having the following solution

$$\begin{aligned} A_1 &= -\frac{1}{12f_0}\,\frac{\rho + 3p - \alpha\left(\rho - 3p\right)^2/2}{1 + \alpha\left(\rho - 3p\right)}\,, \\ A_2 &= \frac{1}{6f_0}\,\frac{\rho + \alpha\left(\rho - 3p\right)^2/4}{1 + \alpha\left(\rho - 3p\right)}\,, \\ S_1 &= -\frac{1}{4}\frac{d}{dt}\ln\left|1 + \alpha(\rho - 3p)\right|\,, \end{aligned} \tag{16}$$

where indefinite parameter $\alpha = \dfrac{f}{3{f_0}^2}$ has inverse dimension of energy density. By using the solution (16) and definitions (2) of curvature functions A_1 and A_2, we obtain the following generalized cosmological Friedmann equations (GCFE)

$$\frac{k}{R^2} + \left\{ \frac{d}{dt} \ln \left[R\sqrt{|1+\alpha\left(\rho - 3p\right)|}\right] \right\}^2 = \frac{8\pi G}{3} \, \frac{\rho + \frac{\alpha}{4}\left(\rho - 3p\right)^2}{1+\alpha\left(\rho-3p\right)}\,, \tag{17}$$

$$R^{-1}\frac{d}{dt}\left[\frac{dR}{dt} + R\frac{d}{dt}\left(\ln\sqrt{|1+\alpha\left(\rho-3p\right)|}\right)\right] = -\frac{4\pi G}{3}\,\frac{\rho + 3p - \frac{\alpha}{2}\left(\rho-3p\right)^2}{1+\alpha\left(\rho-3p\right)}\,. \tag{18}$$

The difference of (17)–(18) from Friedmann cosmological equations of GR is connected with terms containing the parameter α. The value of $|\alpha|^{-1}$ determines the scale of extremely high energy densities. Solutions of GCFE (17)-(18) coincide practically with corresponding solutions of GR, if the energy density is small $|\alpha(\rho-3p)| \ll 1$ $(p \neq \frac{1}{3}\rho)$. The difference between GR and GTG can be essential at extremely high energy densities $|\alpha(\rho-3p)| \gtrsim 1$. Ultrarelativistic matter $(p = \frac{1}{3}\rho)$ and gravitating vacuum $(p = -\rho)$ with constant energy density are two exceptional systems, because GCFE (17)–(18) are identical to Friedmann cosmological equations of GR in these cases independently on values of energy density. By using (13)–(15) the GCFE (17)–(18) can be transformed to the following form [2]

$$\begin{aligned}
&\left\{ H\left[Z + 3\alpha\left(\dot{\phi}^2 + \frac{1}{2}Y\right)\right] + 3\alpha\frac{\partial V}{\partial\phi}\dot{\phi}\right\}^2 + \frac{k}{R^2}\,Z^2 \\
&= \frac{8\pi G}{3}\left[\rho_m + \frac{1}{2}\dot{\phi}^2 + V + \frac{1}{4}\alpha\left(4V - \dot{\phi}^2 + \rho_m - 3p_m\right)^2\right] Z,
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\dot{H}\left[Z + 3\alpha\left(\dot{\phi}^2 + \frac{1}{2}Y\right)\right] + 3H^2\left[Z - \alpha\dot{\phi}^2 + \alpha Y \right.\\
&\left. -\frac{3\alpha}{2}\left(\frac{dp_m}{d\rho_m}Y + 3\left(\rho_m + p_m\right)^2\frac{d^2p_m}{d\rho_m^2}\right)\right] + 3\alpha\left[\frac{\partial^2 V}{\partial\phi^2}\dot{\phi}^2 - \left(\frac{\partial V}{\partial\phi}\right)^2\right] \\
&= 8\pi G\left[V + \frac{1}{2}\left(\rho_m - p_m\right) + \frac{1}{4}\alpha\left(4V - \dot{\phi}^2 + \rho_m - 3p_m\right)^2\right] - \frac{2k}{R^2}Z,
\end{aligned} \tag{20}$$

where $Z \equiv 1 + \alpha\left(\rho - 3p\right) = 1 + \alpha\left(4V - \dot{\phi}^2 + \rho_m - 3p_m\right)$ and $Y = \left(\rho_m + p_m\right)\left(3\frac{dp_m}{d\rho_m} - 1\right)$. The GCFE lead to restrictions on admissible values of variables for scalar field and gravitating matter. In fact, if the energy density ρ is positive and $\alpha > 0$, from equation (19) in the case $k = +1,\,0$ follows the relation:

$$Z = 1 + \alpha\left(4V - \dot{\phi}^2 + \rho_m - 3p_m\right) \geq 0. \tag{21}$$

The condition (21) is valid not only for closed and flat models, but also for cosmological models of open type ($k = -1$) [1]. The domain of admissible values of scalar field ϕ, time derivative $\dot{\phi}$ and energy density ρ_m determined by (21) is limited in space of these variables by bound L defined as

$$Z = 0 \quad \text{or} \quad \dot{\phi} = \pm\left(4V + \alpha^{-1} + \rho_m - 3p_m\right)^{\frac{1}{2}}. \qquad (22)$$

From Eq. (19) the Hubble parameter on the bound L is equal to

$$H_L = -\frac{\dfrac{\partial V}{\partial \phi}\dot{\phi}}{\dot{\phi}^2 + \dfrac{1}{2}Y}. \qquad (23)$$

By given initial conditions for variables (ϕ, $\dot{\phi}$, ρ_m) and also in the case $k = \pm 1$ for R there are two different solutions corresponding to two values of the Hubble parameter following from (19):

$$H_{\pm} = \frac{-3\alpha\dfrac{\partial V}{\partial \phi}\dot{\phi} \pm \sqrt{D}}{Z + 3\alpha[\dot{\phi}^2 + \dfrac{1}{2}Y]}, \qquad (24)$$

where

$$D = \frac{8\pi}{3M_p^2}\left[\rho_m + \frac{1}{2}\dot{\phi}^2 + V + \frac{1}{4}\alpha\left(4V - \dot{\phi}^2 + \rho_m - 3p_m\right)^2\right] Z - \frac{k}{R^2}Z^2. \qquad (25)$$

Obviously, the expression (24) for $H_{\pm}$ will be regular, if $Y \geq 0$. This relation is valid, in particular, for all models filled by gravitating matter with $p_m \geq \frac{\rho_m}{3}$ and scalar fields with potentials applying in chaotic inflation theory. Unlike GR, the values of H_+ and H_- in GTG are sign-variable and, hence, both solutions corresponding to H_+ and H_- can describe the expansion as well as the compression in dependence on its sign. Below we will call solutions of GCFE corresponding to H_+ and H_- as H_+-solutions and H_--solutions respectively. Note that at asymptotics like GR the sign of H_- is negative and the sign of H_+ is positive. In points of bound L we have $D = 0$, $H_+ = H_-$ and the Hubble parameter is determined by (23). If initial conditions correspond to asymptotics of H_--solution, then unlike GR by compression stage the derivative $\dot{\phi}$ does not diverge and by reaching the bound L the transition from H_--solution to H_+-solution takes place.

All discussed cosmological solutions have bouncing character. In order to study cosmological models at a bounce, let us analyze extreme points for the scale factor $R(t)$: $R_0 = R(0)$, $H_0 = H(0) = 0$. (This means that in the case of H_+–solutions $H_{+0} = 0$ and in the case of H_-–solutions $H_{-0} = 0$). Denoting values of quantities at $t = 0$ by means of index "0", we obtain from (19)–(20):

$$\frac{k}{R_0^2}Z_0^2 + 9\alpha^2\left(\frac{\partial V}{\partial \phi}\right)_0^2\dot{\phi}_0^2 = \frac{8\pi}{3M_p^2}\left[\rho_{m0} + \frac{1}{2}\dot{\phi}_0^2 + V_0 + \frac{1}{4}\alpha\left(4V_0 - \dot{\phi}_0^2 + \rho_{m0} - 3p_{m0}\right)^2\right] Z_0, \quad (26)$$

$$
\begin{aligned}
\dot{H}_0 = &\left\{ \frac{8\pi}{M_p^2} \left[V_0 + \frac{1}{2}\left(\rho_{m0} - p_{m0}\right) \right.\right. \\
&\left. + \frac{1}{4}\alpha \left(4V_0 - \dot{\phi}_0^2 + \rho_{m0} - 3p_{m0} \right)^2 \right] \\
&\left. -3\alpha \left[\left(\frac{\partial^2 V}{\partial \phi^2} \right)_0 \dot{\phi}_0^2 - \left(\frac{\partial V}{\partial \phi} \right)_0^2 \right] - \frac{2k}{R_0^2} Z_0 \right\} \\
&\times \left\{ Z_0 + 3\alpha \left[\dot{\phi}_0^2 + \frac{1}{2} Y_0 \right] \right\}^{-1}. \qquad (27)
\end{aligned}
$$

We see from (27) unlike GR the presence of gravitating matter satisfying the energy dominance condition ($p_m \leq \rho_m$) does not prevent from the bounce realization. Eq. (26) determines in space of variables $(\phi, \dot{\phi}, \rho_m)$ extremum surfaces depending on the value of α and in the case of closed and open models also parametrically on the scale factor R_0. In the case of various scalar field potentials applying in inflationary cosmology the value of $\dot{H}_{+0}$ or $\dot{H}_{-0}$ is positive on the greatest part of extremum surfaces, which can be called "bounce surfaces" [1]. By giving concrete form of potential V and choosing values of R_0, ϕ_0, $\dot{\phi}_0$ and ρ_{m0} at a bounce, we can obtain numerically particular bouncing solutions of GCFE for various values of parameter α. Like GR, if initial value of scalar field at the beginning of cosmological expansion is large ($\phi \geq 1M_p$, where M_p denotes the Planck mass) [6], corresponding solution is inflationary cosmological solution containing in addition to inflationary stage also compression stage, transition stage from compression to expansion and a stage after inflation with oscillating regime for scalar field. For given scalar field potential properties of regular inflationary cosmological solutions depend on initial conditions at a bounce and parameter α. Numerical analysis of such solutions in the case of simplest scalar field potentials applying in chaotic inflation was carried out in [7]. In dependence on given conditions at a bounce, the compression stage can have quasi-de-Sitter character or not[7]. From physical point of view, regular inflationary solutions without quasi-de-Sitter compression stage have the most physical interest. Properties of inflationary cosmological models after inflation can be different in comparison with that of GR, if the scale of extremely high energy densities is essentially less than the Planckian one [7]. Because the duration of transition stage is essentially smaller than the duration of inflationary stage, discussed regular inflationary solutions describe regular Big Bang or Big Bounce.

4 HIM with pseudoscalar torsion function and accelerating Universe

In general case of HIM with not vanishing function S_2 gravitational equations (8)-(9) lead to the following cosmological equations [5]

$$k/R^2 + (H - 2S_1)^2 = \frac{1}{6f_0 Z}\left[\rho + 6\left(f_0 Z - b\right)S_2^2 + \frac{\alpha}{4}\left(\rho - 3p - 12bS_2^2\right)^2\right] - \frac{3\alpha\varepsilon f_0}{Z}\left[\left(HS_2 + \dot{S}_2\right)^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right], \quad (28)$$

$$\dot{H} + H^2 - 2HS_1 - 2\dot{S}_1 = -\frac{1}{12f_0 Z}\left[\rho + 3p - \frac{\alpha}{2}\left(\rho - 3p - 12bS_2^2\right)^2\right] - \frac{\alpha\varepsilon}{Z}\left(\rho - 3p - 12bS_2^2\right)S_2^2 + \frac{3\alpha\varepsilon f_0}{Z}\left[\left(HS_2 + \dot{S}_2\right)^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right], \quad (29)$$

where $Z \equiv 1 + \alpha\left[(\rho - 3p) - 12\left(b + \varepsilon f_0\right)S_2^2\right]$, the energy density ρ and the pressure p are determined by (13) and dimensionless parameter $\varepsilon \equiv \frac{q_2}{f}$. These equations contain the torsion functions S_1 and S_2. In accordance with (10) the torsion function S_1 is

$$S_1 = \frac{3\alpha}{4Z}\left[4\left(2b - \varepsilon f_0\right)S_2\dot{S}_2 - \frac{\partial V}{\partial\phi}\dot{\phi} - H\left(Y + 2\dot{\phi}^2\right)\right], \quad (30)$$

where

$$Y \equiv \left(\rho_m + p_m\right)\left(3\frac{dp_m}{d\rho_m} - 1\right) + 12\varepsilon f_0 S_2^2.$$

According to (11) the torsion function S_2 satisfies the following differential equation of second order:

$$\varepsilon\left(\ddot{S}_2 + 3H\dot{S}_2 + 3\dot{H}S_2 - 4\dot{S}_1 S_2 + 12HS_1S_2 - 16S_1^2S_2\right) - \frac{1}{3f_0}\left(\rho - 3p - 12bS_2^2\right)S_2 - \frac{(f_0 - b)}{f}S_2 = 0\,. \quad (31)$$

By taking into account that various parameters of HIM have to be small at asymptotics, we see from (31), that if $|\epsilon| \ll 1$, the pseudoscalar torsion function has the following asymptotics:

$$S_2^2 = \frac{f_0(f_0 - b)}{4fb} + \frac{\rho - 3p}{12b}. \quad (32)$$

Then we have at asymptotics: $Z \to (b/f_0)$, $S_1 \to 0$ and the cosmological equations (28)–(29) at asymptotics take the form of cosmological Friedmann equations with cosmological constant:

$$\frac{k}{R^2}+H^2=\frac{1}{6b}\left[\rho+\frac{3\,(f_0-b)^2}{4f}\right], \tag{33}$$

$$\dot{H}+H^2=-\frac{1}{12b}\left[\rho+3p-\frac{3\,(f_0-b)^2}{2f}\right]. \tag{34}$$

From Eqs.(33)–(34) we see, that parameter b has to be very close to f_0, but smaller than f_0. The value of b leading to observable acceleration of cosmological expansion depends on the scale of extremely high energy density defined by α^{-1}. If we take into account that the value of $\frac{3}{4}(f_0-b)^2/f=\frac{1}{4}\alpha^{-1}(1-b/f_0)^2$ is equal approximately to $0.7\rho_{cr}$ ($\rho_{cr}=6f_0H_0^2$, H_0 is the value of the Hubble parameter at present epoch), then we obtain that $b=[1-(2.8\rho_{cr}\alpha)^{1/2}]f_0$. If we suppose that the scale of extremely high energy densities is larger than the energy density for quark-gluon matter, but less than the Planckian energy density, then we obtain the corresponding estimation for b, which is extremely close to f_0.

By using the expression (30) for the torsion function S_1 and also formulas (13)-(15), we can find from cosmological equation (28) extremum surfaces in space of variables (ϕ, $\dot{\phi}, \rho_m$, $S_2, \dot{S}_2$) depending essentially on parameters α and ϵ, in points of which the Hubble parameter H vanishes. Unlike HIM with vanishing pseudoscalar torsion function discussed in Section 3, the bounce for regular inflationary solutions in this case can be realized only in limited domain of extremum surfaces, where values of S_2 are negligibly small [8]. Such solutions have bouncing character and contain at asymptotics the stage of accelerating expansion.

5 Conclusion

As it was shown above, in the frame of PGTG the gravitational repulsion effect, which takes place in the case of usual gravitating systems satisfying energy dominance condition, leads to important cosmological consequences connected with the PCS and with the problem of dark energy. All inflationary cosmological models without pseudoscalar torsion function are regular in metrics, Hubble parameter, its time derivative by virtue of gravitational repulsion effect at extreme conditions provoked by the torsion function S_1. At the same time the pseudoscalar torsion function S_2 can lead to gravitational repulsion at asymptotisc of cosmological expansion, when energy density of gravitating matter is small. The pseudoscalar torsion function plays the role similar to that of dark energy in GR. From physical point of view would be better to say that in the frame of PGTG we can build regular inflationary Big Bang scenario with accelerating stage of cosmological expansion in asymptotics without dark energy.

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Active and Passive Boost in Spacetime

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Abstract

The physical meaning of a Lorentz boost is still not appreciated 102 years after Einstein's seminal 1905 paper. An *active Lorentz boost* mixes the concept of a vector and the concept of a bivector in physical space. In special relativity, the concept of a vector and a bivector must be replaced by the observer dependent concept of a *relative vector* and a *relative bivector*. The exact relationship between an active Lorentz boost and the commonly employed passive Lorentz boost is clearly explained for the first time.

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I. THE ALGEBRA $\mathcal{C}_3$ OF COMPLEX VECTORS.

Let $\mathbf{B}$ be an orthonormal basis for a complex 3-dimensional vector space.

We define a complex scalar product: $A \circ B$

By giving $\mathcal{C}^3$ **a** *complex vector product*, **a comprehensive geometric interpretation, and extending** $\mathcal{C}^3$ **to include the complex scalars, we will arrive at the 4-dimensional** *complex vector algebra* **or** *space-time algebra* $\mathcal{C}_3$**.**

We define a complex vector product $A \otimes B$ **where** $i \in \mathcal{C}$ **is the imaginary unit with** $i^2 = -1$**.**

For real vectors a and b, the dot and cross products are $\mathbf{a} \cdot \mathbf{b}$ **and** $\mathbf{a} \times \mathbf{b}$**. We see that** $\mathbf{a} \circ \mathbf{b} = \mathbf{a} \cdot \mathbf{b}$ **for real vectors.**

We give $i(\mathbf{a} \times \mathbf{b})$ **the interpretation of the** *bivector* $\mathbf{a} \wedge \mathbf{b}$**, or** *directed area segment* **having the right-handed normal** $\mathbf{a} \times \mathbf{b}$**. In the sense of Grassmann, the bivector** $\mathbf{a} \wedge \mathbf{b}$ **is the directed area obtained by sweeping the vector a out along the vector b. See Figures 1 and 2.**

Unlike the dot and cross products, which are real linear, the complex scalar and complex vector products are complex linear.

For real vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, **We give** $(\mathbf{a} \wedge \mathbf{b}) \wedge \mathbf{c} = \mathbf{a} \wedge (\mathbf{b} \wedge \mathbf{c})$ **the geometric interpretation of the** *trivector* **or directed element of volume obtained by sweeping the bivector** $\mathbf{a} \wedge \mathbf{b}$ **out along the vector c.**

Choosing $\mathbf{a} = \mathbf{e_1}, \mathbf{b} = \mathbf{e_2}, \mathbf{c} = \mathbf{e_3}$ **in the above gives**

$$i = \mathbf{e_1} \circ (\mathbf{e_2} \otimes \mathbf{e_3}) = \mathbf{e_1} \wedge \mathbf{e_2} \wedge \mathbf{e_3},$$

so i has the geometric interpretation of a unit trivector or ***unit pseudoscalar.***

Combining the complex scalar and complex vector products of the complex vectors A and B gives the *associative geometric product*

The scalar and vector products can be defined in terms of the fundamental geometric product:

The *complex vector algebra* $\mathcal{C}_3$ **of the complex vector space** $\mathcal{C}^3$ **is defined by**

$$\mathcal{C}_3 := \mathcal{C} \oplus \mathcal{C}^3 = \{\alpha + A|\ \alpha \in \mathcal{C}, A \in \mathcal{C}^3\} \tag{1}$$

taken with the geometric product. $\mathcal{C}_3$ **is a closed complex** 4**-dimensional linear space or an** 8**-dimensional linear space over the reals.**

The complex vector algebra $\mathcal{C}_3$ **is isomorphic to the algebra of complex** 2×2 **Pauli matrices** $\mathcal{P}$**.**

Unlike the Pauli matrices, $\mathcal{C}_3$ **has a comprehensive geometric interpretation and generalizes the universally known Gibbs-Heaviside vector algebra of space to a** *space-time algebra* **for special relativity.**

II. SPECIAL RELATIVITY IN $\mathcal{C}_3$.

All *observables* **in space-time are elements of the complex vector algebra** $\mathcal{C}_3$**. The** *observer dependent* **objects are:**

- **TIME = real scalars,**
- **VECTORS,**
- **BIVECTORS**
- **TRIVECTORS = pseudoscalars.**

The basic operations in space-time are *active* and *passive* rotations and *active* and *passive velocity transformations* or *boosts*.

In contrast, in a *passive* rotation, the vector x stays fixed, but the reference frame of the observer is rotated in the opposite direction.

For a *passive* boost, the vector x stays fixed whereas the reference frame of the observer is given a speed of $v/c = \tanh\phi$ in the opposite direction.

Note for a rotation and a boost the square of a complex vector is preserved under both active rotations and boosts.

In complex vector algebra rotations and velocity transformations are on an equal footing with clear geometric interpretation.

A. Event Horizon of an inertial system.

All events in the Universe take place in the space-time algebra $\mathcal{C}_3$. The *event* $X = ct + \mathbf{x}$ occurs at the *time* t and at the *place* $\mathbf{x}$, measured by an observer in the inertial system $\mathcal{H}$, where c is the speed of light.

If $X(t)$ is the *space-time line* of an inertial observer then its space-time velocity $V(t) = \frac{dX}{dt} = c$, so the velocity of a inertial observer is always $\mathbf{v} = 0$.

The *orthonormal rest frame* of a given inertial system is defined by the algebraic properties

$$\mathbf{e_1}^2 = \mathbf{e_2}^2 = \mathbf{e_3}^2 = 1, \mathbf{e}_j\mathbf{e}_k = -\mathbf{e}_k\mathbf{e}_j, \text{ and } \mathbf{e_1}\mathbf{e_2}\mathbf{e_3} = i \tag{2}$$

The *universal mapping* relates the event horizons $\mathcal{H}$ and $\mathcal{H}'$,

Each observer measures position and time in the usual Newtonian way. The assumptions of special relativity that go beyond the Newtonian-Galilean World are specified by how a boost changes the way a given event is measured.

Galilean transformation of coordinates, reference [4].

Thus, an active boost of the event horizon $\mathcal{H}'$ in the direction of the x'-axis at the speed $-v$ is equivalent to a passive boost of the event horizon $\mathcal{H}$ in the direction of the x-axis at the speed v.

III. CLASSIFICATION OF COMPLEX VECTORS

1. A is a relative vector if $A^2 > 0$,
2. A is a nilpotent if $A^2 = 0$,
3. A is a relative bivector if $A^2 < 0$.

IV. ADDITION OF VELOCITY FORMULAS

The bivectors j and k are moving with relative velocities $\mathbf{u}_v = \mathbf{a}\tanh\phi$ and $\mathbf{u}_w = \mathbf{b}\tanh\rho$ w.r.t. the bivector i, respectively.

Note that the velocity $\mathbf{v}_w$ is incommensuate with the vectors $\mathbf{u}_v$ and $\mathbf{u}_w$ because of the bivector part $\mathbf{u}_v \wedge \mathbf{u}_w$.

The passive boost is used by Baylis in the *APS* formulation of special relativity [2], and a coordinate form of this passive approach was used by Einstein in his famous 1905 paper [8]. Whereas Hestenes in [3] employs the active Lorentz boost, in [5] he uses the passive form of the Lorentz boost. (Also, see Fock 1955.)

The distinction between ACTIVE and PASSIVE boosts is the source of much confusion in the literature [9]. Whereas an ACTIVE boost mixes vectors and bivectors of $\mathcal{C}_3$, the PASSIVE boost mixes the vectors and scalars of $\mathcal{R}^3$ in the geometric algebra $\mathcal{C}_3$.

A geometric interpretation of this result is found in Minkowski spacetime.

v. MINKOWSKI SPACETIME

We split the space-time algebra $\mathcal{C}_3$ of observables into the even subalgebra of the *Dirac algebra* of *Minkowski spacetime* $\mathcal{R}^{1,3}$, the 4-dimensional pseudoeuclidean vector space with signature $\{+,-,-,-\}$..

Hestenes in [5] used the geometric algebra $\mathcal{R}_{1,3} := gen(\mathcal{R}^{1,3})$ to construct space-time algebra. The geometric algebra $\mathcal{R}_{1,3}$ is isomorphic to the algebra of Dirac 4×4 matrices.

We assume that

- **The complex vector algebra $\mathcal{C}_3 = \mathcal{R}_{1,3}^{+}$, *i.e.*, the elements of $\mathcal{C}_3$ make up the even subalgebra of $\mathcal{R}_{1,3}$. There exist an orthonormal frame of spacetime vectors $\gamma_0, \gamma_1, \gamma_2, \gamma_3 \in \mathcal{R}_{1,3}$ such that**

$$\gamma_0^2 = 1 = -\gamma_1^2 = -\gamma_2^2 = -\gamma_3^2$$

and where $\mathbf{e}_k = \gamma_k \gamma_0$ for $k = 1, 2, 3$.

- **To each event horizon $\mathcal{H}$ there corresponds a unique *timelike* vector $\gamma_0 \in R^{1,3}$ such that $\gamma_0^2 = 1$ and such that for all elements $\mathcal{A} = \alpha + A \in \mathcal{C}_3$, $\overline{\mathcal{A}} = \gamma_0 \mathcal{A}^- \gamma_0$.**

- **For each event $X \in \mathcal{H}$ and corresponding event $X' = Xe^{\phi\theta} \in \mathcal{H}'$,**

$$x = X\gamma_0 = X'\gamma_0' \in \mathcal{M}$$

is the *unique event* in Minkowski spacetime.

The Minkowski bivector

$$\mathbf{u}_v$$

is the *relative velocity* of v w.r.t. u. The vector

$$vw,$$

gives the ACTIVE formula for the addition of velocities.

We now calculate the relative velocity $\mathbf{u}_{vw}$ of the system w with respect to v as measured in the frame of u.

The *active* boost taking the unit timelike vector $\hat{v}_\parallel$ into the unit timelike vector $\hat{w}_\parallel$ is equivalent to the passive boost in the plane of the spacetime bivector D.

CONCLUSIONS:

- **Each different system u measures *passive relative velocities* between the systems v and w differently by a boost in the plane of the Minkowski bivector $D = (w - v) \wedge u$.**

- **There is a *unique* active boost that takes the system v into w in the plane of $v \wedge w$.**

- **The concept of a passive and active boost become equivalent when $u \wedge v \wedge w = 0$, the case when $\mathbf{b} = \pm \mathbf{a}$.**

New effects of non-inertial motion in vector-quaternion version of relativity

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1. Introduction

Successful centenary of the Einstein's Special Relativity (SR) could not stop persistent search for other versions of a space-time model. Among them 3+3 space-time theories were not rear, e.g. works of M.Pavsic [1], E.A.B.Cole, S.A.Bushanan [2,3]. These 3+3-type theories are all based upon heuristic ideas reflecting intuitive belief into the universe's great symmetry. But use of the traditional SR methods within structurally more complicated space-time schemes lead to interpretational difficulties. Surprisingly a version of specific non-Abelian 3+3-relativity theory is found hidden in mathematics of quaternion (Q-) numbers discovered by William Hamilton in 1843. This report is devoted to presentation of the version with demonstration of its interrelation with SR and of its benefits for practical purposes.

2. Quaternion algebra in short

There are publications referring to quaternions as to more or less convenient instrument of theoretical physics (e.g. [4]). But Q-math itself turns out to comprise many physical features; one of them is inherited invariance of the Q-algebra under Lorentz-type transformations. Q-algebra is based upon 4 units, one scalar unite 1 (normally is omitted) and three vector units $\mathbf{q}_k \equiv \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ satisfying the multiplication rule[2]

$$1\mathbf{q}_k = \mathbf{q}_k 1 = \mathbf{q}_k, \qquad \mathbf{q}_k \mathbf{q}_l = -\delta_{kl} + \varepsilon_{klj}\mathbf{q}_j, \tag{1}$$

so that vector units are "imaginary" and dependent on each other, e.g.

$$\mathbf{q}_1^{\ 2} = -1, \ \mathbf{q}_3 = \mathbf{q}_1\mathbf{q}_2 = -\mathbf{q}_2\mathbf{q}_1 .$$

Linear combinations of the Q-units with real number coefficients produce Q-numbers; all such numbers form Q-algebra commutative and associative in addition and still associative but due to Eq. (1) non-commutative in multiplication. The Q-algebra generalizes algebra of complex numbers and is proved to be the last possible associative algebra with division. Hamilton also noted that three "imaginary" Q-units have geometrical meaning of a constant vector triad initiating Cartesian system of coordinates. The fact most important to our purposes is that the basic Eq. (1) is form-invariant under transformations of vector units [5]

$$\mathbf{q}_{k'} = U\mathbf{q}_k U^{-1} \qquad \text{or} \qquad \mathbf{q}_{k'} = O_{k'l}\mathbf{q}_l, \tag{2}$$

where full sets of the operators U and O respectively compose 2 : 1 isomorphic groups $SL(2,C)$ and $SO(3,C)$, both too isomorphic to the Lorentz group. It means that the transformations in Eqs. (2) give new sets of vector Q-units obeying the rule of Eq. (1) but in general being functions of transformation parameters, while the scalar unit is always the same and constant. An illustration: if Q-units are given by 2×2 - matrices (simple Pauli-type representation)

$$\mathbf{q}_1 = -i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \mathbf{q}_2 = -i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \mathbf{q}_3 = -i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{3}$$

and the transformation from $SO(3,C)$ is represented by a 3×3-matrix, e.g. by a matrix of simple rotation about axis No 3

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[2] δ_{kl} is 3D Kroneker symbol, ε_{jkl} is 3D Levi-Civita symbol; summation convention is assumed.

$$O_{k'l} = \begin{pmatrix} \cos\Phi & \sin\Phi & 0 \\ -\sin\Phi & \cos\Phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

then the result of the transformation gives a new set of Q-units

$$\mathbf{q}_{1'} = -i\begin{pmatrix} 0 & e^{-i\Phi} \\ e^{i\Phi} & 0 \end{pmatrix},\ \mathbf{q}_{2'} = -i\begin{pmatrix} 0 & -ie^{-i\Phi} \\ ie^{i\Phi} & 0 \end{pmatrix},\ \mathbf{q}_{3'} = -i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{5}$$

obeying Eq. (1). It is important to stress that parameter Φ in Eqs. (4), (5) is in general a complex number; hence geometric consequence of the invariance of Eq. (1) under transformations from Eq. (2) is two-folded. Case 1. A vector-quaternion (with real coefficients) of the type $\mathbf{a} = a_j \mathbf{q}_j$ is trivially shown to preserve its form-invariance under transformations from Eq. (2) with real parameters

$$\mathbf{a}' = a_{j'}\mathbf{q}_{j'} = a_k \mathbf{q}_k = \mathbf{a}; \tag{6}$$

in this case the invariance-group is reduced to $SU(2) \subset SL(2,C)$ or to $SO(3,R) \subset SO(3,C)$. These transformations turn the former constant Q-triad to Cartan's movable frame. If rotation parameters are dependent on time of observer located at the triad's origin the triad becomes a helpful tool of classical mechanics: using Eq. (6) one immediately generates dynamic equations for a particle observed from arbitrary rotating frame of reference [5]

$$\ddot{x}_n + 2\dot{x}_j\, \omega_{jn} + x_j\, \dot{\omega}_{jn} + x_m\, \omega_{mj}\, \omega_{jn} = f_n \tag{7}$$

where x_n is the particle's coordinate, f_n is resulting physical force per particle's mass and ω_{jn} is the Q-triad's time-transport connection physically being an instant angular velocity of the frame's rotation. Eq. (7) explicitly comprises all classical "non-inertial" forces but remains non-relativistic: the rotating observer needs an extra device to measure time (physical clock). Case 2. If coefficients at Q-units are complex then thus formed objects are called bi-quaternions (BQ). Whole set of BQ-numbers does not constitute algebra with division mostly due to lack of general definition of the numbers norm. But one can distinguish a family of physically significant BQ-vectors

$$\mathbf{z} = (a_k + ib_k)\mathbf{q}_k = \mathbf{a} + i\mathbf{b}, \tag{8}$$

a and **b** given in real numbers, whose "forced" normalization

$$\mathbf{z}\,\overline{\mathbf{z}} = (a_n + ib_n)(a_n + ib_n) = a^2 - b^2$$

yields for 3D vectors **a** and **b** the orthogonality condition irrespective of the frame choice

$$a_{k'}\, b_{k'} = a_n O_{nk'}\, b_m\, O_{mk'} = \delta_{mn}\, a_n\, b_m = 0\,. \tag{9}$$

Eq. (9) is automatically fulfilled for a BQ-vector from Eq. (8) whose vector **b** is aligned with e.g. $\mathbf{q}_1$ while vector **a** is in the orthogonal plane $\mathbf{q}_2 - \mathbf{q}_3$

$$\mathbf{z} = ib_1\, \mathbf{q}_1 + a_2\, \mathbf{q}_2 + a_3 \mathbf{q}_3\,. \tag{10}$$

The form-invariance condition for the BQ-vector from Eq.(10) is

$$\mathbf{z} = ib_1\, \mathbf{q}_1 + a_2\, \mathbf{q}_2 + a_3 \mathbf{q}_3 = ib_{1'}\, \mathbf{q}_{1'} + a_{2'}\, \mathbf{q}_{2'} + a_{3'} \mathbf{q}_{3'}\,. \tag{11}$$

BQ-vector **z** is proved to keep its form under reduced subgroup $SO(1,2) \subset SO(3,C)$ of simple rotations with real and imaginary parameters so that the frame's rotations with real parameters are allowed only about vector $\mathbf{q}_1$ while rotations with imaginary parameters are allowed only about directions $\mathbf{q}_2$ and $\mathbf{q}_3$. Analogous spinor subgroup $SL(1,2) \subset SL(2,C)$ can be easily constructed, using formula connecting vector and spinor representations [6].

$$U = \pm \frac{1 - O_{k'n}\, \mathbf{q}_k \mathbf{q}_n}{2\sqrt{1 + Tr\, O}}\,.$$

Thus we have a purely mathematical fact: each normalizable BQ-vector built of two reciprocally orthogonal 3D-vectors remains form-invariant under real and imaginary rotations of vector Q-units, the rotations forming groups $SO(1,2)$ or $SL(1,2)$ being too subgroups of the Lorentz group. This hints to look for physical analogue of this math in the domain of relativity.

3. Vector-quaternion model of relativity

If a particle is observed from $\mathbf{q}_k$ as from a frame of reference then the change of time may e.g. be aligned with direction $\mathbf{q}_1$; following earlier idea of Minkowsky this coordinate is regarded as imaginary: $i\,dt$. The other two directions form a plane of real coordinates reserved to measure the particle's displacement $d\mathbf{r}$ e.g. aligned with $\mathbf{q}_2$. Then an above discussed normalized BQ-vector comprising information of space-time relations

$$d\mathbf{z} = idt\,\mathbf{q}_1 + dr\,\mathbf{q}_2 \tag{12}$$

is form-invariant under arbitrary $SO(1,2)$-transformations of $\mathbf{q}_k$. Mathematically Eq. (12) can be regarded as a specific quaternion square root of Special Relativity line-element since multiplied by itself the BQ-vector gives the well-known expression

$$dz^2 = dt^2 - dr^2$$

invariant under conventional Lorents transformations. General form of the basic BQ-vector from Eq. (12) can be written as

$$d\mathbf{z} = (idt\,e_k + dx_k)\,\mathbf{q}_k = dt(i\,e_k + v_k)\,\mathbf{q}_k \tag{13}$$

with time-direction unite vector $e_k\,e_k = 1$ always orthogonal to the particle velocity $v_k = dx_k / dt$

$$e_k v_k = 0\,.$$

Eq. (13) shows that the constructed space-time model has 6 dimensions and it is a symmetric sum of two 3D spaces

$$Q_6 = R_3 \oplus T_3$$

where R_3 is the usual 3D-space of coordinate and velocity change while T_3 is also a 3D-space but imaginary with respect to R_3. In this model the observer works only with some sections of the 6D-space; but since the objects of the observations are found in real 3D-space, and imaginary time axis is distinguished, an illusion of 4 dimensions emerges. The tools of physical measurements in the Q_6-model are three spatial rulers $\mathbf{q}_k$ and built-in geometric clock represented by reciprocally imaginary "time rulers" $\mathbf{p}_k \equiv i\,\mathbf{q}_k$, the two triads being obviously co-aligned. The tool-set $\Sigma \equiv \{\mathbf{p}_k\,,\,\mathbf{q}_k\}$ with an observer in the initial point represents full physical frame of reference.

Main statement of the quaternion model of relativity

All physically sustainable frames of reference are interconnected by "Rotation Equations" (RE) in the vector-group terms[3]

$$\Sigma' = O\,\Sigma\,, \qquad O \in SO(1,2)\,; \tag{14}$$

physical sustainability implies form-invariance of basic vector from Eq. (13) under transformations of Eqs. (14). Cinematic effects of Special Relativity are readily found in the Q-relativity model.

Simple boost. Σ-observer always can align one of his spatial vectors (e.g. $\mathbf{q}_2$) with velocity of inertially moving body

$$d\mathbf{z} = dt\,\mathbf{p}_1 + dr\,\mathbf{q}_2\,. \tag{15}$$

Let frame Σ' result from Σ due to RE

[3] RE in spinor-group terms has the form $\Sigma' = U\,\Sigma\,U^{-1}$, $U \in SL(1,2)$; beautiful in structure it seems less technological in computations due to vector character of conventional cinematic magnitudes, so all physical effects are calculated below in vector-group formalism.

$$\Sigma' = O_3^{i\psi}\,\Sigma\,, \qquad O_3^{i\psi} = \begin{pmatrix} \cosh\psi & i\sinh\psi & 0 \\ -i\sinh\psi & \cosh\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}; \tag{16}$$

the mapping, physically a simple boost, obviously keeping vector of Eq. (15) form-invariant

$$dt\,\mathbf{p}_1 + dr\,\mathbf{q}_2 = dt'\mathbf{p}_{1'} + dr'\,\mathbf{q}_{2'} \tag{17}$$

too yields well known coordinate transformations

$$dt' = dt\cosh\psi + dr\sinh\psi\,, \qquad dr' = dt\sinh\psi + dr\cosh\psi$$

with respective effects of length and time segments contraction. If observed particle is the body of reference of the frame Σ' then $dr'=0$, and velocity of Σ' is found as $V = \dfrac{dr}{dt} = \tanh\psi$.

Addition of velocities. Let frames Σ' and Σ'' move relatively to Σ with velocities $V_1 = \tanh\psi_1$, $V_2 = \tanh\psi_2$, the vectors forming angle α in the spatial plane $\mathbf{q}_2$-$\mathbf{q}_3$ of Σ, and e.g. $\mathbf{V}_1 \uparrow\uparrow \mathbf{q}_2$. Then the two primed frames result from respective RE: $\Sigma' = O_3^{i\psi_1}\,\Sigma$, $\Sigma'' = O_3^{i\psi_2} O_1^{\alpha}\,\Sigma$ whose composition yields expression for Σ'' as a function of Σ': $\Sigma'' = O_3^{i\psi_2} O_1^{\alpha} O_3^{-i\psi_1}\,\Sigma'$, or explicitly

$$\begin{pmatrix} \mathbf{q}_{1''} \\ \mathbf{q}_{2''} \\ \mathbf{q}_{3''} \end{pmatrix} = \begin{pmatrix} \cosh\psi_2 & i\sinh\psi_2 & 0 \\ -i\sinh\psi_2 & \cosh\psi_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cosh\psi_1 & -i\sinh\psi_1 & 0 \\ i\sinh\psi_1 & \cosh\psi_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{q}_{1'} \\ \mathbf{q}_{2'} \\ \mathbf{q}_{3'} \end{pmatrix}.$$

The first row of this matrix equation gives the component $\mathbf{p}_{1''} \equiv i\mathbf{q}_{1''}$

$$\mathbf{p}_{1''} = (\cosh\psi_1\,\cosh\psi_2 - \cos\alpha\sinh\psi_1\sinh\psi_2)\,\mathbf{p}_{1'} +$$
$$+(\sinh\psi_1\,\cosh\psi_2 - \cos\alpha\cosh\psi_1\sinh\psi_2)\,\mathbf{q}_{2'} - \sin\alpha\sinh\psi_2\,\mathbf{q}_{3'}\,. \tag{18}$$

Since now Σ'' is observed from Σ' the basic BQ-vector takes the form

$$dt''\mathbf{p}_{1''} = dt(\mathbf{p}_{1'} + V_y\,\mathbf{q}_{2'} + V_z\,\mathbf{q}_{3'})\,, \qquad \text{or} \qquad \mathbf{p}_{1''} = \cosh\psi\,(\mathbf{p}_{1'} + V_y\,\mathbf{q}_{2'} + V_z\,\mathbf{q}_{3'})$$

with $\cosh\psi = dt'/dt''$. Comparison of the last expression with Eq. (18) gives

$$\cosh\psi = \cosh\psi_1\cosh\psi_2\,(1 - \mathbf{V}_1\cdot\mathbf{V}_2)$$

and components of velocity Σ'' relatively to Σ'

$$V_y = \frac{V_1 - V_2\cos\alpha}{1-\mathbf{V}_1\cdot\mathbf{V}_2}\,, \qquad V_z = -\frac{V_2\sin\alpha\sqrt{1-V_1^2}}{1-\mathbf{V}_1\cdot\mathbf{V}_2}\,, \quad V^2 = \frac{(V_1-V_2)^2 - (\mathbf{V}_1\times\mathbf{V}_2)^2}{(1-\mathbf{V}_1\cdot\mathbf{V}_2)^2}\,, \tag{19}$$

standard vector notations are used. If Σ'' and Σ' move relatively to Σ in opposite directions: $\alpha = \pi$ then Eq. (19) reduces to familiar "parallel velocities sum" expression $V = (V_1 + V_2)/(1 + V_1V_2)$.

4. Non-inertial frames

There are no evident obstacles to localization of complex parameters in RE; the localization leads to description of non-inertial motion of relativistic frames. This essentially widens ability of Q-relativity model compared to SR, the latter offering a limited analysis of non-inertial-motion effects only from inertial frame standpoint. Well-known examples are the hyperbolic motion and Thomas precession [7,8]. Quaternion model suggests solutions from viewpoint of any observer; algorithm of computation of any cinematic problem is illustrated by following examples.

Hyperbolic motion. First, the RE is constructed for the cinematic system. Motion is non-inertial but rectilinear, e.g. along $\mathbf{q}_2$, hence RE can be written as $\Sigma' = O_3^{i\psi(t')}\,\Sigma$, function $\psi(t')$ to be determined. Basic BQ-vector for Σ' observed from Σ follows from the RE

$$d\mathbf{z}' = dt'\,\mathbf{p}_{1'} = dt\,\mathbf{p}_1 + dr\,\mathbf{q}_2\,, \tag{20}$$

it allows to compute Σ'-observer's proper acceleration

$$\mathbf{a}' = d^2\mathbf{z}'/dt'^2 = d\,\mathbf{p}_{1'}/dt' = \dot\psi\,\mathbf{q}_2 \tag{21}$$

that is assumed constant $\dot\psi(t') = \varepsilon = const$; hence $\psi(t') = \varepsilon t'$, integration constant chosen zero. Eq. (20) yields standard relation

$$dt/dt' = \cosh\psi \tag{22}$$

used together with Eq. (21) to find Σ'-Σ time ratio

$$t(t') - t_0 = \int dt' \cosh(\varepsilon t') = \frac{1}{\varepsilon}\sinh(\varepsilon t')\ ; \qquad t'(t) = \frac{1}{\varepsilon}\ln\left[\varepsilon t + \sqrt{1+(\varepsilon t)^2}\right], \qquad t_0 = 0$$

and all other cinematic characteristics: relative velocity

$$V(t) = \tanh(\varepsilon t') = \tanh[\sinh(\varepsilon t)] = \frac{\varepsilon t}{\sqrt{1+(\varepsilon t)^2}},$$

acceleration and coordinate as functions of the observer's time

$$a(t) = \frac{dV(t)}{dt} = \frac{\varepsilon}{[1+(\varepsilon t)^2]^{3/2}}, \qquad r(t) = \int V(t)dt = \frac{1}{\varepsilon}\sqrt{1+(\varepsilon t)^2} - \frac{1}{\varepsilon}.$$

For small times the solution reduces to respective case in classical mechanics, asymptotic behavior is reasonable, and as expected the formulae exactly coincide with those given in [8]. On the contrary, the cinematic problem of Σ seen from Σ' has not been ever regarded; the new standpoint is given by inverse RE: $\Sigma = H_3^{-\psi(t')}\,\Sigma'$ with the same $\psi(t') = \varepsilon t'$; from the basic vector

$$d\mathbf{z} = dt'\,\mathbf{p}_{1'} - dr'\,\mathbf{q}_{2'} = dt\,\mathbf{p}_1$$

expected value of proper acceleration of Σ and standard time relation are straightforwardly found

$$\mathbf{a} = d^2\mathbf{z}/dt^2 = d\,\mathbf{p}_1/dt = 0, \qquad dt'/dt = \cosh\psi.$$

Characteristics of apparent motion of Σ are computed according to the above given algorithm:

$$t(t') = \int dt'/\cosh(\varepsilon t') = \frac{1}{\varepsilon}\arcsin[\tanh(\varepsilon t')],\ t_0 = 0;$$

$$V'(t') = \tanh(\varepsilon t')\ ; \qquad r'(t') = \frac{1}{\varepsilon}\ln[\cosh(\varepsilon t')],\ r_0 = 0; \qquad a'(t') = \frac{\varepsilon}{\cosh^2(\varepsilon t')}.$$

Short time behavior again is classical, but for large Σ'-time the Σ-clock tends to stop

$$t_{t'\to\infty} \to \frac{\pi c}{2\varepsilon},$$

fundamental velocity c is shown explicitly.

Thomas precession. A constant vector, e.g. directed by gyroscope axis (spin), uniformly orbiting in a plane, from viewpoint of an immobile observer rotates with a certain frequency. This effect proposed by Thomas in 1927 and cumbersome in computation in the framework of SR [9] is easily discovered in Q-model. The adequate RE construction has three stages. First, inertial frame Σ at the orbit center constantly oriented with respect to "immobile stars" is made to rotate about vector No 1 at angle $\alpha(t) = \omega t$ so that vector No 2 always chases the orbiting gyroscope. Second, relativity is switched on: the new rotating frame is "transported" from the center to orbit by hyperbolic rotation about vector No 2 at "angle" $i\psi = const$. Third, retrograde rotation at space angle $-\alpha'$ makes final frame Σ', now at the orbit, similarly oriented to the same stars. Thus RE takes the form $\Sigma' = O_1^{-\alpha'}\,O_2^{i\psi}\,O_1^{\alpha}\,\Sigma$, or explicitly

$$\begin{pmatrix}\mathbf{q}_{1'}\\ \mathbf{q}_{2'}\\ \mathbf{q}_{3'}\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos\alpha' & -\sin\alpha'\\ 0 & \sin\alpha' & \cos\alpha'\end{pmatrix}\begin{pmatrix}\cosh\psi & 0 & -i\sinh\psi\\ 0 & 1 & 0\\ i\sinh\psi & 0 & \cosh\psi\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\mathbf{q}_1\\ \mathbf{q}_2\\ \mathbf{q}_3\end{pmatrix},$$

giving projection of the gyroscope axis, e.g. space vector $\mathbf{q}_{2'}$, e.g. onto $\mathbf{q}_3$

$$\langle \mathbf{q}_{2'} \rangle_3 = \sin(\alpha - \alpha') - 2\sinh^2\frac{\psi}{2}\cos\alpha\,\sin\alpha'. \tag{23}$$

The two observers find the angle α' different. For Σ-observer $\alpha' = \omega' t$ with $\omega'(\Sigma)$ angular velocity of retrograde rotation of Σ′. For Σ′-observer obviously $\alpha' = \omega' t' = \omega t = \alpha$; use here of the Σ′-Σ time ratio $t' = t / \cosh\psi$ resulting from the first raw of the matrix RE [same as Eq. (22)] leads to Σ′-Σ angular velocity correlation $\omega'(\Sigma) = \omega \cosh\psi$. Thus Eq. (23) written for Σ-observer acquires the form

$$\langle \mathbf{q}_{2'} \rangle_3 = \cos(\omega - \omega')t - 2\sinh^2\frac{\psi}{2}\cos\omega t\,\sin\omega' t \tag{24}$$

showing that the gyroscope axis apparently rotates in retrograde direction with most noticeable part of Thomas precession

$$\omega_T \equiv \omega - \omega' = \omega(1 - \cosh\psi); \tag{25}$$

for small orbit velocity V Eq. (25) reduces to known expression: $\omega_T \cong -(\omega/2)(V/c)^2$. The second term of right hand side of Eq.24 being of the next $(V/c)^2$-order is much smaller than that of Eq. (25). Eqs. (23), (24) and (25) precisely coincide with those given in [7,9].

5. New non-inertial relativistic effects

Inverse Thomas precession. Apparent rotation of inertial (immobile) frame Σ from viewpoint of orbiting but constantly oriented frame Σ′ was never considered earlier. In Q-model the solution is readily found from inverse RE: $\Sigma = O_1^{-\alpha}\, O_2^{-i\psi}\, O_1^{\alpha'}\, \Sigma'$; the cyclic frequency ratio here is "symmetric" to that of previous case $\omega(\Sigma') = \omega'(\Sigma')\cosh\psi$, and the result of calculation of respective precession is similar to that of Eq. (25) $\omega_T \equiv \omega'(\Sigma') - \omega(\Sigma') = \omega'(1 - \cosh\psi)$. In Solar System this effect ought to incorporate into Mercury orbit perihelion shift observed from the Earth traveling around the Sun with velocity $V \cong 30\ km/s$. Calculation gives a modest value of 100-year precession angle: $\alpha_T \cong -100 \cdot \pi \cdot (V/c)^2 = -\pi \cdot 10^{-6} = -0{,}65''$, small compared to Mercury perihelion gravitational shift.

Relativistic oscillator. Another interesting problem is kinematics of a frame whose origin moves harmonically along a straight line with velocity value periodically approaching that of light relatively to an immobile frame. Let non-inertial frame Σ′ be observed from inertial frame Σ, the motion aligned with $\mathbf{q}_{2'}$, respective RE and basic BQ-vector are

$$\Sigma' = O_3^{i\psi(t')}\,\Sigma\,, \quad d\mathbf{z}' = dt'\mathbf{p}_{1'} = dt\,\mathbf{p}_1 + dx\,\mathbf{q}_2\,. \tag{26}$$

The frame Σ′ performs linear oscillations when proper acceleration of Σ′-observer computed as in Eq. (21) is a harmonic function of Σ′-time

$$\mathbf{a}' = d^2\mathbf{z}'/dt'^2 = \frac{d\psi}{dt'}\mathbf{q}_2 = \Omega'\beta\cos\Omega' t'\,\mathbf{q}_2\,, \tag{27}$$

Ω', $\beta < 1$ are some constants, acceleration is chosen maximal at $t' = 0$. Eq. (27) has solution

$$\psi(t') = \beta\sin\Omega' t'$$

taken with zero constant phase so that initially the two frames are reciprocally immobile. Eq. (26) gives Σ′-Σ time ratio

$$t = \int \cosh\psi(t')\,dt' = \int \cosh(\beta\sin\Omega' t')\,dt'\ ,$$

the integral is computed using the series

$$\mathrm{ch}\,u = 1 + \sum_{n=1}^{\infty}\frac{1}{(2n)!}u^{2n}\,, \qquad |u| < \infty\,,$$

(inequality $|u| \equiv |\beta\sin\Omega' t'| < \infty$ is satisfied since $\beta < 1$, $\sin\Omega' t' < 1$) and the table integral

$$\int \sin^{2n} y\,dy = \frac{1}{2^{2n}}\binom{2n}{n}y + \frac{(-1)^n}{2^{2n-1}}\sum_{k=0}^{n}(-1)^k\binom{2n}{k}\frac{\sin(2n-2k)y}{2n-2k}\,;$$

substitution $y \equiv \Omega' t'$ yields

$$t = t' + \sum_{n=1}^{\infty} \frac{\beta^{2n}}{(2n)!} \left[\frac{1}{2^{2n}} \binom{2n}{n} t' + \frac{(-1)^n}{2^{2n-1}} \sum_{k=0}^{n-1} (-1)^k \binom{2n}{k} \frac{\sin(2n-2k)\,\Omega' t'}{(2n-2k)\,\Omega'} \right]. \tag{28}$$

One cycle of the oscillation is completed when $\Omega' T' = 2\pi$, T' being Σ'-observer's period; this along with Eq. (28) leads to Σ'-Σ periods and cycle frequencies ratios

$$T = T' \left(1 + \sum_{n=1}^{\infty} \frac{\beta^{2n}}{(2n)!} \frac{1}{2^{2n}} \binom{2n}{n} \right), \qquad \Omega' = \Omega \left(1 + \sum_{n=1}^{\infty} \frac{\beta^{2n}}{(2n)!} \frac{1}{2^{2n}} \binom{2n}{n} \right). \tag{29}$$

Eq. (29) gives the value of Σ-observed frequency $\Omega(\Sigma)$ smaller than the proper values Ω', i.e. process of oscillation is seen slower than it is actually felt by Σ'-observer. Computation of Σ-observed acceleration, velocity and coordinate, of Σ' demands inversion of Eq. (28): $t' = t'(t)$; this can be done in the approximation $\beta = V/c << 1$

$$t' \cong t - \frac{\beta^2}{4}\left(t - \frac{1}{2\Omega} \sin 2\Omega t \right),$$

V being a constant measured in velocity units. Now all characteristics of motion can be found. Relative velocity of Σ' observed Σ from is

$$V(t) = c \tanh\left(\beta \sin \Omega' t'\right) \cong V \sin \Omega t \left(1 - \frac{1}{3}\beta^2 + \frac{7}{12}\beta^2 \cos^2 \Omega t \right); \tag{30}$$

at the beginning and at the end of period the two frames are at rest to each other $V(0,T) = 0$, while maximal value of velocity ("amplitude") is $\tilde{V} \equiv V(T/4) = V(1 - \beta^2/3)$. Thus Σ'-oscillations seen from Σ apparently have non-harmonic character. Acceleration and coordinate are found from Eq. (30) as

$$a(t) = \frac{dV(t)}{dt} \cong \Omega \tilde{V} \cos \Omega t \left(1 - \frac{7}{6}\beta^2 + \frac{7}{4}\beta^2 \cos^2 \Omega t \right),$$

$$x(t) = \int V(t) dt \cong x_0 - \frac{\tilde{V}}{\Omega} \cos \Omega t \left(1 + \frac{7}{36}\beta^2 \cos^2 \Omega t \right),$$

the integration constant

$$x_0 = \frac{\tilde{V}}{\Omega}\left(1 + \frac{7}{36}\beta^2 \right)$$

is chosen to satisfy the following conditions

$$x(0) = x(T) = 0, \quad x(T/4) = x_0, \quad x(T/2) = 2\,x_0,$$

meaning, that at the initial and final moments of period the two frames are not only relatively immobile but too are found at the same point in space. Thus the $\Sigma'(\Sigma)$-cinematic problem is solved. Solution of inverse cinematic problem $\Sigma(\Sigma')$ can be straightforwardly sought for within similar algorithm; since there is no need to invert time-time ratio, the $\Sigma(\Sigma')$ is given by exact integrals. This in particular means that the oscillator model can serve as an effective instrument for detailed analysis of the so-called twin paradox [10].

Fast satellites position shift.

Noticed above apparent drop of cycle frequency value compared to real one [Eq. (29)] prompts to search for a relativistic effect within Solar System. Idea of the effect is very simple. The Earth and a planet with fast satellites move with relative (though variable) speed, hence the Earth's observer mostly finds satellites' cycle frequency value smaller than the real one; this means that an observed satellite persistently is behind expected position on its trajectory, the shift accumulating with time from beginning of observation: the "planet's clock" is more and more late. Quaternion model helps to compute the effect. Let $\tilde{\Sigma} = (\mathbf{q}_{\tilde{1}}, \mathbf{q}_{\tilde{2}}, \mathbf{q}_{\tilde{3}})$ be a constant frame attached to the Sun, vectors $\mathbf{q}_{\tilde{2}}, \mathbf{q}_{\tilde{3}}$ forming the ecliptic plane, and $\Sigma = (\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$ is too a constant frame parallel to $\tilde{\Sigma}$ attached to the Earth. Let the Earth (Σ) and a planet (frame Σ') move along approximately circled orbits with radiuses R_E and R_P with constant linear velocities

V_E and V_P; $\Omega_E = V_E / R_E$, $\Omega_P = V_P / R_P$ are respective angular velocities, $\alpha = \Omega_E t$ and $\beta = \Omega_P t$ are angles from abscissa $\mathbf{q}_{\tilde{2}}$ to the Earth's and the planet's position on the orbit; all magnitudes are measured by Σ-observer. Coordinates of the planet in the Earth Σ-observer coordinate system (x_2, x_3) are

$$x_2 = R_P \cos\beta - R_E \cos\alpha\,, \tag{31}$$

$$x_3 = R_P \sin\beta - R_E \sin\alpha\,. \tag{32}$$

Frame Σ′ is connected with Σ by simple rotational equation $\Sigma' = O_3^{i\psi(t)}\,\Sigma$; relative Σ′-Σ velocity formula following from Eqs. (31) , (32)

$$V^2 = \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2 = V_E^2 + V_P^2 - 2V_E V_P \cos(\alpha - \beta) = \tanh^2\psi$$

determines the hyperbolic parameter further on regarded small $\psi = \psi(t) << 1$. Respective basic BQ-vector

$$d\mathbf{z} = dt'\mathbf{p}_{1'} = dt\left(\mathbf{p}_1 + \frac{dx_2}{dt}\mathbf{q}_2 + \frac{dx_3}{dt}\mathbf{q}_3\right)$$

gives the Σ′-Σ change-of-time correlation $dt = dt'\cosh\psi$. If period T of the planet's fast satellite revolution is small compared to time of observation t: $T << t$, then $T = T'\cosh\psi$: observed from the Earth period is apparently longer than it really is. Vice versa, observed frequency ω is smaller than the real one ω'

$$\omega = 2\pi / T = 2\pi / (T\cosh\psi) = \omega' / \cosh\psi\,,$$

i.e. for Σ-observer the process of satellite revolution seems slower than it is in reality. Hence the satellite orbit angle observed from the Earth

$$\varphi_{obs}(t) = \int \omega\, dt = \omega' \int dt / \cosh\psi(t) \cong \omega' t \left\{\left[1 - \frac{1}{2c^2}(V_E^2 + V_P^2)\right] + \frac{V_E V_P}{c^2}\frac{\sin(\alpha-\beta)}{\alpha-\beta}\right\}$$

is smaller than that calculated theoretically

$$\varphi_{calc}(t) = \omega' t\,,$$

and the difference is a linear function of time

$$\Delta\varphi = \varphi_{obs} - \varphi_{calc} \cong \omega' t\left[-\frac{V_E^2 + V_P^2}{2c^2} + \frac{V_E V_P}{c^2}\frac{\sin(\alpha-\beta)}{\alpha-\beta}\right].$$

The second term in square brackets tends to zero with time, so the final expression reduces to

$$\Delta\varphi \cong -\frac{V_E^2 + V_P^2}{2c^2}\omega' t\,, \tag{33}$$

for the Earth observer satellites of other Solar System planets travel apparently slower than they actually do, the effect accumulating with time as expected. Given a satellite orbit radius r, its position shift is found in units of length $\Delta l = r\,\Delta\varphi$. It is important to note that the shift formula given by Eq. (33) is an improved version of analogous formula suggested in paper [5]; previous expression was noticed to depend on initial reciprocal positions of the Earth and planet, in Eq. (33) this ambiguity is removed. The following table gives values of the effect for five fast satellites of Mars and Jupiter. Orbital linear velocities are: of the Earth V_E = 29.8 km/s, of Mars V_P = 24.1 km/s, of Jupiter V_P = 13.1 km/s; value of light velocity is c = 299 793 km/s; observation period is chosen 100 years.

Satellites	Cycle frequency ω': 1/s	Angular shift $\Delta\varphi$: ′/100 yrs	Linear shift Δl : km/100yrs	Linear size a: km
Phobos (Mars)	0.00023	20.19	55	20
Deimos (Mars)	0.00006	5.10	35	12
Metis (Jupiter)	0.00025	15.77	587	40

Adrastea (Jupiter)	0.00024	15.58	585	20
Amalthea (Jupiter)	0.00015	9.33	492	189

Phobos is a good target for observations due to its closure to the Earth and big speed: in average every month it is apparently 50 m behind its real position on the orbit, and there are indirect and direct confirmations of the effect cited from NASA and European Space Agency (ESA) information [11][4], [12][5]; but no comment or explanation was ever related to a relativistic effect. Other plausible candidates, Jupiter satellites Metis and Adrastea whose average annual shift is comparable to the moons size, are considered too small to be observed from Earth. But new methods of observation of tiny Jupiter satellites elaborated in the Institute of Astronomy, University of Hawaii [13], encourage search for discussed relativistic shift effect in Jupiter surroundings too.

Pioneer anomaly as pure relativistic effect

Recently [14] the earlier formula (from paper [5]) analogous to Eq. (33) was used to explain Pioneer anomaly, the space probes' Pioneer 10/11 Sun-ward secular acceleration $a_P = (8 \pm 3) \cdot 10^{-10}\ m/s^2$ claimed experimentally observed [15]. Combined with ether drift effect the discussed formula has given very good coincidence (up to 0.26%) with value of emission angle shift required explaining observation data of Pioneer's signal Doppler residuals. This surprisingly exact result nevertheless deserves criticism since obtained by Q-method mathematical description of a specific mechanical model cannot bear universal character and fit to arbitrary relativistic situation. Using Q-method an appropriate relativistic Earth-probe model is built and formula for pure relativistic frequency shift of probes' signal is derived

$$f = 1/T = 1/(T \cosh \psi) = f'/\cosh \psi = f'\sqrt{1-(V/c)^2}\ ,$$

T, V are period of the signal wave, and Earth-probe relative velocity value function. Experimental observation of the frequency change must lead to conclusion that there exists respective "Doppler velocity" V_D entering formula known from Special Relativity, and "Doppler acceleration". Preliminary calculations [16] (only radial components of the probe's velocity and acceleration in Newtonian gravity are taken into account using NASA data [17,18]) the apparent "Doppler deceleration" of Pioneer 10 relative to the Sun is found as $a_{SD} = (3.63 - 1/23) \cdot 10^{-10}\ m/s^2$, i.e. (45-15)% of observed Doppler residual, while for Pioneer 11 similar deceleration is $a_{SD} = (10.03 - 5.02) \cdot 10^{-10}\ m/s^2$, what is within cited above limits of error of a_P.

6. Conclusion

Given examples demonstrate ability of Q-relativity model to fast and clear solve many relativistic problems including those of non-inertial motion. We stress that all above solutions are deduced on the base essentially different from that of SR and by a short and transparent investigation free of any extra assumptions. Q-relativity theory geometrically differs from standard 4-dimensional SR model at least in two main features. First, Q-space-time has 3-space + 3-time = 6 dimensions; time direction is uniquely distinguished as orthogonal (and imaginary) to a plane formed by two generally non-collinear space-like vectors, particles' velocity and acceleration. Second, quaternion units used as a frame vectors multiply to form a specific metric object $\mathbf{g}_{kl} \equiv \mathbf{q}_k \mathbf{q}_l = -\delta_{kl} + \varepsilon_{klj}\mathbf{q}_j$ whose symmetric Cartesian metric part serves to determine vector

[4] NASA: "Phobos 2 operated nominally throughout its cruise and Mars orbital insertion phases, gathering data on the Sun, interplanetary medium, Mars, and Phobos. Shortly before the final phase of the mission, during which the spacecraft was to approach within 50 m of Phobos' surface and release two landers, one a mobile 'hopper', the other a stationary platform, contact with Phobos 2 was lost. The mission ended when the spacecraft signal failed to be successfully reacquired on 27 March 1989. The cause of the failure was determined to be a malfunction of the on-board computer".

[5] ESA: "This tiny moon is thought to be in a 'death spiral', slowly orbiting toward the surface of Mars. Here, Phobos was found to be about five kilometers ahead of its predicted orbital position. This could be an indication of an increased orbital speed associated with its secular acceleration, causing the moon to spiral in toward Mars".

norm, while skew-symmetric traceless matrix-vector part does not affect the norm value in classical physics but manifests itself in quantum mechanics as Pauli spin term generator [19]. Detailed study shows that several families of such Q-metric spaces are found within mathematics of quaternions [20] including those representing geometric base of described above relativity model. Thus naturally merging from fundamental mathematics the Q-relativity model, despite mentioned peculiarities, nonetheless leads to all results of standard SR theory and moreover provides easy solutions for non-inertial motion problems. The Q-model is to be developed in many ways [21]. In particular, transparent Q-vector structure of equations establishing cinematic ratios offers to formulate dynamic equations for interacting relativistic frames [22]; preliminary study of two relativistic bodies problem will be given elsewhere. Curved Q-spaces may serve as geometric background of specific quaternion gravity theory; too they probably are to be used to form compactified complimentary dimensions for unified standard or string theories.

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The connection of field-theory equations with the equations for material systems

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Abstract

The existing field theories are based on the properties of closed exterior forms, which correspond to conservation laws for physical fields.

In the present paper it is shown that closed exterior forms corresponding to field theories are obtained from the equations modelling conservation (balance) laws for material systems (material media).

The process of obtaining closed exterior forms demonstrates the connection between field-theory equations and the equations for material systems and points to the fact that the foundations of field theories must be conditioned by the properties of equations conservation laws for material systems.

1. Peculiarities of differential equations for material sistems

Equations for material systems are equations that describe the conservation laws for energy, linear momentum, angular momentum and mass. Such conservation laws can be named as balance ones since they establish the balance between the variation of a physical quantity and corresponding external action.

[The material system - material (continuous) medium - is a variety (infinite) of elements that have internal structure and interact among themselves. Thermodynamical, gasodynamical and cosmologic system, systems of elementary particles and others are examples of material system. (Physical vacuum can be considered as an analog of such material system.) Electrons, protons, neutrons, atoms, fluid particles and so on are examples of elements of material system.]

The equations of balance conservation laws are differential (or integral) equations that describe a variation of functions corresponding to physical quantities [1-3]. (The Navier-Stokes equations are an example [3].)

It appears that, even without a knowledge of the concrete form of these equations, one can see specific features of these equations and their solutions using skew-symmetric differential forms [4-6].

To do so it is necessary to study the conjugacy (consistency) of these equations.

The functions for equations of material systems sought are usually functions which relate to such physical quantities like a particle velocity (of elements), temperature or energy, pressure and density. Since these functions relate to one material system, it has to exist a connection between them. This connection is described by the state-function. Below it will be shown that the analysis of integrability and consistency of equations of balance conservation laws for material media reduces to a study the nonidentical relation for the state-function.

Let us analyze the equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with the material system), and the second is an accompanying one (this system is connected with the manifold built by the trajectories of the material system elements).

The energy equation in the inertial frame of reference can be reduced to the form:

$$\frac{D\psi}{Dt} = A_1$$

where D/Dt is the total derivative with respect to time, ψ is the functional of the state that specifies the material system, A_1 is the quantity that depends on specific features of the system and on external energy actions onto the system. {The action functional, entropy, wave function can be regarded as examples of the functional ψ. Thus, the equation for energy presented in terms of the action functional S has a similar form: $DS/Dt = L$, where $\psi = S$, $A_1 = L$ is the Lagrange function. In mechanics of continuous media the equation for energy of an ideal gas can be presented in the form [3]: $Ds/Dt = 0$, where s is entropy.}

In the accompanying frame of reference the total derivative with respect to time is transformed into the derivative along the trajectory. Equation of energy is now written in the form

$$\frac{\partial\psi}{\partial\xi^1} = A_1 \tag{1}$$

Here ξ^1 is the coordinate along the trajectory.

In a similar manner, in the accompanying reference system the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial\psi}{\partial\xi^\nu} = A_\nu, \quad \nu = 2, ... \tag{2}$$

where ξ^ν are the coordinates in the direction normal to the trajectory, A_ν are the quantities that depend on the specific features of material system and on external force actions.

Eqs. (1) and (2) can be convoluted into the relation

$$d\psi = A_\mu \, d\xi^\mu, \quad (\mu = 1, \nu) \tag{3}$$

where $d\psi$ is the differential expression $d\psi = (\partial\psi/\partial\xi^\mu)d\xi^\mu$.

Relation (3) can be written as

$$d\psi = \omega \tag{4}$$

here $\omega = A_\mu \, d\xi^\mu$ is the skew-symmetrical differential form of the first degree (the summation over repeated indices is implied).

Relation (4) has been obtained from the equation of the balance conservation laws for energy and linear momentum. In this relation the form ω is that of the first degree. If the equations of the balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this form will be a form of the second degree. And in combination with the equation of the balance conservation law for mass this form will be a form of degree 3. In general case the evolutionary relation can be written as

$$d\psi \,=\, \omega^p \tag{5}$$

where the form degree p takes the values $p \,=\, 0, 1, 2, 3$. (The relation for $p = 0$ is an analog to that in the differential forms of zero degree, and it was obtained from the interaction of energy and time.)

Since the balance conservation laws are evolutionary ones, the relations obtained are also evolutionary relations, and the skew-symmetric forms ω and ω^p are evolutionary ones.

Relations obtained from the equation of the balance conservation laws turn out to be nonidentical. In the left-hand side of evolutionary relation (4) there is a differential that is a closed form. This form is an invariant object. The right-hand side of relation (4) involves the differential form ω, that is not an invariant object because in real processes, as it is shown below, this form proves to be unclosed.

For the form to be closed the differential of the form or its commutator must be equal to zero. Let us consider the commutator of the form $\omega \,=\, A_\mu d\xi^\mu$. The components of the commutator of such a form can be written as follows:

$$K_{\alpha\beta} \,=\, \left(\frac{\partial A_\beta}{\partial \xi^\alpha} \,-\, \frac{\partial A_\alpha}{\partial \xi^\beta}\right) \tag{6}$$

(here the term connected with the manifold metric form has not yet been taken into account).

The coefficients A_μ of the form ω have been obtained either from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator of the form ω constructed from the derivatives of such coefficients is nonzero. This means that the differential of the form ω is nonzero as well. Thus, the form ω proves to be unclosed and is not an invariant quantity.

This means that the relation (4) involves not an invariant term. Such a relation cannot be an identical one. Hence, without the knowledge of particular expression for the form ω, one can argue that for actual processes the relation obtained from the equations corresponding to the balance conservation laws proves to be nonidentical. In similar manner it can be shown that general relation (5) is also nonidentical.

{The peculiarities of the evolutionary relation are connected with the differential form that enters into this relation. This is a skew-symmetric form with the basis, in contrast to the basis of exterior form, is a deforming (nondifferentiable) manifold. (About the properties of such skew-symmetric form one can read, for example, in paper [6]). The peculiarity of skew-symmetric forms defined on such manifold is the fact that their differential depends on the basis. The commutator of such form includes the term that is connected with a differentiating the basis. This can be demonstrated by an example of the first-degree skew-symmetric form.

Let us consider the first-degree form $\omega \,=\, a_\alpha dx^\alpha$. The differential of this form can be written as $d\omega = K_{\alpha\beta}dx^\alpha dx^\beta$, where $K_{\alpha\beta} = a_{\beta;\alpha} - a_{\alpha;\beta}$ are the components of the commutator of the form ω, and $a_{\beta;\alpha}$, $a_{\alpha;\beta}$ are the covariant derivatives. If we express the covariant derivatives in terms of the connectedness (if it is possible), then they can be written as $a_{\beta;\alpha} = \partial a_\beta/\partial x^\alpha + \Gamma^\sigma_{\beta\alpha} a_\sigma$, where the first term results from differentiating the form coefficients, and the second term results from differentiating the basis. If we substitute the expressions for covariant derivatives into the formula for the commutator components, we obtain the following expression for the commutator components of the form ω:

$$K_{\alpha\beta} = \left(\frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta}\right) + (\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta}) a_\sigma$$

Here the expressions $(\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})$ entered into the second term are just components of commutator of the first-degree metric form that specifies the manifold deformation and hence equals nonzero. (In commutator of the exterior form, which is defined on differentiable manifold the second term is not present.) [It is well-known that the metric form commutators of the first-, second- and third degrees specifies, respectively, torsion, rotation and curvature.]

The skew-symmetric form in the evolutionary relation is defined in the manifold made up by trajectories of the material system elements. Such a manifold is a deforming manifold. The commutator of skew-symmetric form defined on such manifold includes the metric form commutator being nonzero. (The commutator of unclosed metric form, which is nonzero, enters into commutator (6) of the evolutionary form $\omega = A_\mu d\xi^\mu$.) Such commutator of differential form cannot vanish. And this means that evolutionary skew-symmetric form that enters into evolutionary relation cannot be closed. The evolutionary relation cannot be an identical one. (Nonclosure of evolutionary form and an availability of additional term in this form commutator governs the properties and peculiarities of nonidentical evolutionary relation and its physical importance.)}

Nonidentity of the evolutionary relation means that initial equations of balance conservation laws are not conjugated, and hence they are not integrable. The solutions of these equations can be functional or generalized ones. In this case generalized solutions are obtained only under degenerated transformations.

The evolutionary relation obtained from equations of balance conservation laws for material systems (continuous media) carries not only mathematical but also large physical loading [6,7]. This is due to the fact that the evolutionary relation possesses the duality. On the one hand, this relation corresponds to material system, and on other, as it will be shown below, describes the mechanism of generating physical structures. This discloses the properties and peculiarities of the field-theory equations and their connection with the equations of balance conservation laws for material systems.

Physical significance of nonidentical evolutionary relation.

The evolutionary relation describes the evolutionary process in material system since this relation includes the state differential $d\psi$, which specifies the material system state. However, since this relation turns out to be not identical, from this relation one cannot get the differential $d\psi$. The absence of differential means that the system state is nonequilibrium.

The evolutionary relation possesses one more peculiarity, namely, this relation is a selfvarying relation. (The evolutionary form entering into this relation is defined on the deforming manifold made up by trajectories of the material system elements. This means that the evolutionary form basis varies. In turn, this leads to variation of the evolutionary form, and the process of intervariation of the evolutionary form and the basis is repeated.)

Selfvariation of the nonidentical evolutionary relation points to the fact that the nonequilibrium state of material system turns out to be selfvarying. (It is evident that this selfvariation proceeds under the action of internal force whose quantity is described by the commutator of the unclosed evolutionary form ω^p.) State of material system changes but remains nonequilibrium during this process.

Since the evolutionary form is unclosed, the evolutionary relation cannot be identical. This means that the nonequilibrium state of material system holds. But in this case it is possible a transition of material system to a locally equilibrium state.

This follows from one more property of nonidentical evolutionary relation. Under selfvariation of the evolutionary relation it can be realized the conditions of degenerate transformation. And under degenerate transformation from the nonidentical relation it is obtained the identical relation.

From identical relation one can define the state differential pointing to the equilibrium state of the system. However, such system state is realized only locally due to the fact that the state differential obtained is an interior one defined only on pseudostructure, that is specified by the conditions of degenerate transformation. And yet the total state of material system remains to be nonequilibrium because the evolutionary relation, which describes the material system state, remains nonidentical one.

The conditions of degenerate transformation are connected with symmetries caused by degrees of freedom of material system. These are symmetries of the metric forms commutators of the manifold. {To the degenerate transformation it must correspond a vanishing of some functional expressions, such as Jacobians, determinants, the Poisson brackets, residues and others. Vanishing of these functional expressions is the closure condition for dual form. And it should be emphasize once more that the degenerate transformation is realized as a transition from the accompanying noninertial frame of reference to the locally inertial system. The evolutionary form and nonidentical evolutionary relation are defined in the noninertial frame of reference (deforming manifold). But the closed exterior form obtained and the identical relation are obtained with respect to the locally-inertial frame of reference (pseudostructure)}.

Realization of the conditions of degenerate transformation is a vanishing of the commutator of manifold metric form, that is, a vanishing of the dual form commutator. And this leads to realization of pseudostructure and formatting the closed inexact form, whose closure conditions have the form

$$d_\pi \omega^p = 0, d_\pi{}^* \omega^p = 0 \tag{7}$$

On the pseudostructure π from evolutionary relation (5) it is obtained the relation

$$d_\pi \psi = \omega_\pi^p \tag{8}$$

which proves to be an identical relation since the closed inexact form is a differential (interior on pseudostructure).

The realization of the conditions of degenerate transformation and obtaining identical relation from nonidentical one has both mathematical and physical meaning. Firstly, this points to the fact that the solution of equations of balance conservation laws proves to be a generalized one. And secondly, from this relation one obtains the differential $d_\pi \psi$ and this points to the availability of the state-function (potential) and that the state of material system is in local equilibrium.

Relation (8) holds the duality. The left-hand side of relation (8) includes the differential, which specifies material system and whose availability points to the locally-equilibrium state of material system. And the right-hand side includes a closed inexact form, which is a characteristics of physical fields. The closure conditions (7) for exterior inexact form correspond to the conservation law, i.e. to a conservative on pseudostructure quantity, and describe a differential-geometrical structure. These are such structures (pseudostructures with conservative quantities) that are physical structures formatting physical fields[6,7].

The transition from nonidentical relation (5) obtained from equations of the balance conservation laws to identical relation (8) means the following. Firstly, an emergency of the closed (on pseudostructure) inexact exterior form (right-hand side of relation (8)) points to an origination of the physical structure. And, secondly, an existence of the state differential (left-hand side of relation (8)) points to a transition of the material system from nonequilibrium state to the locally-equilibrium state.

Thus one can see that the transition of material system from nonequilibrium state to locally-equilibrium state is accompanied by originating differential-geometrical structures, which are physical structures. Massless particles, charges, structures made up by eikonal surfaces and wave fronts, and so on are examples of physical structures.

The duality of identical relation also explains the duality of nonidentical evolutionary relation. On the one hand, evoltionary relation describes the evolutionary process in material systems, and on the other describes the process of generating physical fields.

The emergency of physical structures in the evolutionary process reveals in material system as an emergency of certain observable formations, which develop spontaneously. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, and others. It appears that structures of physical fields and the formations of material systems observed are a manifestation of the same phenomena. The light is an example of such a duality. The light manifests itself in the form of a massless particle (photon) and of a wave.

This duality also explains a distinction in studying the same phenomena in material systems and physical fields. In the physics of continuous media (material systems) the interest is expressed in generalized solutions of equations of the balance conservation laws. These are solutions that describe the formations in material media observed. The investigation of relevant physical structures is carried out using the field-theory equations.

The unique properties of nonidentical evolutionary relation, which describes the connection between physical fields and material systems, discloses the connection of evolutionary relation with the field-theory equations. In fact, all equations of existing field theories are the analog to such relation or its differential or tensor representation.

2. Specific features of field-theory equations

The field-theory equations are equations that describe physical fields. Since physical fields are formatted by physical structures, which are described by closed exterior *inexact* forms and by closed dual forms (metric forms of manifold), is obvious that the field-theory equations or solutions to these equations have to be connected with closed exterior forms. Nonidentical relations for functionals like wave-function, action functional, entropy, and others, which are obtained from the equations for material media (and from which identical relations with closed forms describing physical fields are obtained), just disclose the specific features of the field-theory equations.

The equations of mechanics, as well as the equations of continuous media physics, are partial differential equations for desired functions like a velocity of particles (elements), temperature, pressure and density, which correspond to physical quantities of material systems (continuous media). Such functions describe the character of varying physical quantities of material system. The functionals (and state-functions) like wave-function, action functional, entropy and others, which specify the state of material systems, and corresponding relations are used in mechanics and continuous media physics only for analysis of integrability of these equations. And in field theories such relations play a role of equations. Here it reveals the duality of these relations. In mechanics and continuous media physics these equations describe the state of material systems, whereas in field-theory they describe physical structures from which physical fields are formatted.

It can be shown that all equations of existing field theories are in essence relations that connect skew-symmetric forms or their analogs (differential or tensor ones). And yet the nonidentical relations are treated as equations from which it can be found identical relation with include closed forms describing physical structures desired.

Field equations (the equations of the Hamilton formalism) reduce to identical relation with exterior form of first degree, namely, to the Poincare invariant

$$ds = -H\,dt + p_j\,dq_j \tag{9}$$

The Schrődinger equation in quantum mechanics is an analog to field equation, where the conjugated coordinates are replaced by operators. The Heisenberg equation corresponds to the closure condition of dual form of zero degree. Dirac's *bra-* and *cket-* vectors made up a closed exterior form of zero degree. It is evident that the relations with skew-symmetric differential forms of zero degree correspond to quantum mechanics. The properties of skew-symmetric differential forms of the second degree lie at the basis of the electromagnetic field equations. The Maxwell equations may be written as $d\theta^2 = 0$, $d^*\theta^2 = 0$, where $\theta^2 = \frac{1}{2}F_{\mu\nu}dx^\mu dx^\nu$ (here $F_{\mu\nu}$ is the strength tensor). The Einstein equation is a relation in differential forms. This equation relates the differential of dual form of first degree (Einstein's tensor) and a closed form of second degree –the energy-momentum tensor. (It can be noted that, Einstein's equation is obtained from differential forms of third degree).

The connection the field theory equations with skew-symmetric forms of appropriate degrees shows that there exists a commonness between field theories describing physical fields of different types. This can serve as an approach to constructing the unified field theory. This connection shows that it is possible to introduce a classification of physical fields according to the degree of skew-symmetric differential forms. From relations (5) and (8) one can see that relevant degree of skew-symmetric differential forms, which can serve as a parameter of unified field theory, is connected with the degree p of evolutionary form in relation (5). It should be noted that the degree p is connected with the number of interacting balance conservation laws. {The degree of closed forms also reflects a type of interaction [7]. Zero degree is assigned to a strong interaction, the first one does to a weak interaction, the second one does to electromagnetic interactions, and the third degree is assigned to gravitational field.}

The connection of field-theory equations, which describe physical fields, with the equations for material media discloses the foundations of the general field theory. As an equation of general field theory it can serve the evolutionary relation (5), which is obtained from equations the balance conservation laws for material system and has a double meaning. On the one hand, that, being a relation, specifies the type of solutions to equations of balance conservation laws and describes the state of material system (since it includes the state differential), and, from other hand, that can play a role of equations for description of physical fields (for finding the closed inexact forms, which describe the physical structures from which physical fields are made up). It is just a double meaning that discloses the connection of physical fields with material media (which is based on the conservation laws) and allows to understand on what the general field theory has to be based.

In conclusion it should be emphasized that the study of equations of mathematical physics appears to be possible due to unique properties of skew-symmetric differential forms. In this case, beside the exterior skew-symmetric differential forms, which are defined on differentiable manifolds, the skew-symmetric differential forms, which, unlike to the exterior forms, are defined on deforming (nondifferentiable) manifolds [6], were used.

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Some physical displays of the space anisotropy relevant to the feasibility of its being detected at a laboratory

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The impact of local space anisotropy on the transverse Doppler effect is examined. Two types of laboratory experiments aimed at seeking and measuring the local space anisotropy are proposed. In terms of the conventional special relativity theory, which treats 3D space to be locally isotropic, the experiments are of the type of "null-experiments". In the first-type experiments, a feasible Doppler shift of frequency is measured by the Mössbauer effect, with the Mössbauer source and absorber being located at two identical and diametrically opposed distances from the center of a rapidly rotating rotor, while the γ-quanta are recorded by two stationary and oppositely positioned proportional counters. Either of the counters records only those γ-quanta that passed through the absorber at the moment of the passage of the latter near a counter. The second-type experiments are made using the latest radio physics techniques for generating monochromatic oscillations and for recording weak signals. The effect expected due to space anisotropy consists in frequency modulation of the harmonic oscillations coming to a receiver that rotates at a constant velocity around the monochromatic wave emitter. In this case the modulation depth proves to be proportional to the space anisotropy magnitude.

1 . Introduction

The recent rapid development of theoretical physics, astrophysics, and elementary particle physics has brought a much deeper insight into space physics. It has become clear, in particular, that the Riemannian model of locally isotropic space time can but approximately describe the properties of the real space.

As long ago as the early seventies of the former century, the researches into the extensive air showers of elementary particles induced by super high-energy cosmic ray protons yielded the pioneer experimental data indicating that the true behavior of the energy spectrum of primary cosmic protons contradicted the respective theoretical predictions [1], [2] made in terms of the locally isotropic space model. The recent studies have finally confirmed the discrepancy between the theoretical predictions and the experimental data on the primary cosmic proton spectrum (the absence of so-called GZK effect). Besides, such phenomena were discovered as, for instance, the violation of discrete space-time symmetries in weak interactions of elementary particles, the temperature anisotropy of the microwave background radiation, and the offbeat behavior of the Hubble parameter that characterizes the Universe expansion. All the phenomena cannot be described in any way consistently in terms of the theory based on the model for locally isotropic space-time.

Generally speaking, the void space, or the space filled with locally isotropic matter, can only exhibit local isotropy. As to the void space, it is merely a mathematical idealization of physical vacuum that consists of locally isotropic fluctuations of quantized fields. As to the locally isotropic matter, this can be exemplified by the Higgs condensate, i.e. the classical constant scalar field induced by spontaneous violation of gauge symmetry in the unified theory for electroweak interactions. The standard version of the theory describes quite properly the diverse effects caused by electroweak interactions. However, the magnitude of the anomalous muon magnetic moment calculated in terms of the theory differs from the respective experimental value. This fact indicates that the locally anisotropic fermion-antifermion condensate, rather than the scalar Higgs condensate, arises under spontaneous violation of gauge symmetry. Such a condensate gives rise to local anisotropy of space-time, thereby altering the geometric properties of the latter.

Contrary to the locally isotropic space-time described by the Riemannian geometry, the locally anisotropic space-time is described by the more general Finslerian geometry [3]. The pioneer viable Finslerian model for space-time was constructed in [4]–[19]. In terms of the model, the symmetry with respect to local three-dimensional rotations of space proves to be partially broken, but the relativistic symmetry (i.e. symmetry with respect to transformations that relate different locally inertial reference frames to each other) remains valid, although the transformations proper differ from the Lorentz transformations.

It should be noted finally that, in addition to the relativistically-invariant Finslerian event space with partially broken 3D isotropy, works [20]–[22] described the 3-parametric family of flat relativistically-invariant Finslerian event spaces with entirely broken 3D isotropy. As to the well-known Berwald-Moór space, it belongs just to that family. Considering that the properties of the Berwald-Moór space are studied in detail in [23], we shall only treat the event space with partially broken 3D isotropy.

2 . Doppler effect in the flat Finslerian event space with partially broken 3D isotropy

According to [4] and [6], the metric of a flat Finslerian event space with partially broken 3D isotropy is

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)\,, \tag{1}$$

where the unit vector $\boldsymbol{\nu}$ indicates a preferred direction in 3D space while the parameter r determines the magnitude of space anisotropy, characterizing the degree of deviation of the metric (1) from the Minkowski metric. Thus, the anisotropic event space (1) is a generalization of the isotropic Minkowski space of conventional special relativity theory.

The 3-parameter noncompact group of the generalized Lorentz transformations, which link the physically equivalent inertial reference frames in the anisotropic space-time (1) and leave the metric (1) to be invariant, is

$$x'^{\,i} = D(\boldsymbol{v},\boldsymbol{\nu})\, R^i_j(\boldsymbol{v},\boldsymbol{\nu})\, L^j_k(\boldsymbol{v})\, x^k\,, \tag{2}$$

where $\boldsymbol{v}$ stands for the velocities of moving (primed) inertial reference frames; the matrices $L^j_k(\boldsymbol{v})$ present the ordinary Lorentz boosts; the matrices $R^i_j(\boldsymbol{v},\boldsymbol{\nu})$ present additional rotations of the spatial axes of the moving frames around the vectors $[\boldsymbol{v}\,\boldsymbol{\nu}]$ through the angles

$$\varphi = \arccos\left\{1 - \frac{(1-\sqrt{1-\boldsymbol{v}^{\,2}/c^2})[\boldsymbol{v}\boldsymbol{\nu}]^2}{(1-\boldsymbol{v}\boldsymbol{\nu}/c)\boldsymbol{v}^{\,2}}\right\}$$

of relativistic aberration of $\boldsymbol{\nu}$; and the diagonal matrices

$$D(\boldsymbol{v},\boldsymbol{\nu}) = \left(\frac{1-\boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1-\boldsymbol{v}^{\,2}/c^2}}\right)^r I$$

present the additional dilatational transformations of the event coordinates. The structure of the transformations (2) ensures the fact that, despite of a new geometry of event space, the 3-velocity space remains to be a Lobachevski space.

To obtain an exact relativistic formula that would describe the Doppler effect in the flat locally anisotropic space (1), we must know how the components of the wave 4-vector $k^i = (\omega/c\,,\boldsymbol{k})$ are related to each other in the initial and moving reference frames. It can be readily demonstrated (see [6] for instance) that the components of the wave 4-vector are transformed as

$$k'^{\,i} = D^{-1}\, R^i_n\, L^n_j\, k^j\,; \tag{3}$$

whence it follows that, under the generalized Lorentz transformations (2) and (3), the scale transformation D^{-1} of the wave 4-vector is inverse to the scale transformation D of event coordinates, so that the plane wave phase $(k^0x^0 - \boldsymbol{k}\,\boldsymbol{x})$ is an invariant of generalized Lorentz transformations.

The transformation (3) was used in [5] to obtain the relation

$$\omega = \omega'\,\frac{\sqrt{1-\boldsymbol{v}^{\,2}/c^2}}{1-\boldsymbol{v}\boldsymbol{e}/c}\left(\frac{1-\boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1-\boldsymbol{v}^{\,2}/c^2}}\right)^r\,. \tag{4}$$

This is just the exact relativistic formula for the Doppler effect in the locally anisotropic space. With the anisotropy approaching zero $(r \to 0)$, the formula reduces to the classical formula of special relativity theory. In (4), $\boldsymbol{v}$ is the velocity of the moving reference frame; ω' is the frequency of a ray in this frame; ω and $\boldsymbol{e}$ are the frequency and direction of the ray in the initial reference frame; $\boldsymbol{\nu}$ is the unit vector along the preferred direction in the initial reference frame. It is seen that , as may be expected, the Doppler effect in the locally anisotropic space proves to be sensitive to the orientation of the experimental setup (having fixed ω', $|\boldsymbol{v}|$ and $\boldsymbol{v}\boldsymbol{e}$, we obtain the dependence of ω on the angle between $\boldsymbol{v}$ and $\boldsymbol{\nu}$). It is this fact that makes it possible, in principle, to detect the anisotropy of space by measuring the Doppler shift, the real possibility of such detection being limited by the magnitude of the sought anisotropy $(r \ll 1)$, by the degree of source monochromaticity, and by the resolving power of receiver. First, consider a combination of a Mössbauer source and a resonant absorber.

3. The experiments with searching the space anisotropy by measuring the transverse Doppler effect through the Mössbauer effect

As long ago as during the period when the naïve theory of absolute ether was the only alternative to special relativity, Møller [24] suggested the experiments based on precise measurements of the Doppler effect. The respective "null-experiments" were expected either to detect a very weak ether wind or to become the direct and most accurate verification of special relativity.

Considering the possible local anisotropy of real space-time, which is disregarded by special relativity, it would be useful to compare, up to the second order of velocities, among the values of the Doppler shift of frequencies $\Delta\,\omega/\omega = (\omega_a - \omega_s)/\omega_s$, predicted by the pre-relativistic (PR) theory of absolute ether, by special relativity (SR), and by the relativistic theory of locally anisotropic space (AR):

$$(\Delta\,\omega/\omega)^{PR} = \boldsymbol{eu}/c + (\boldsymbol{eu})(\boldsymbol{ev}_s)/c^2 - \boldsymbol{wu}/c^2\,, \tag{5}$$
$$(\Delta\,\omega/\omega)^{SR} = \boldsymbol{eu}/c + (\boldsymbol{eu})(\boldsymbol{ev}_s)/c^2 + (\boldsymbol{v}_a^2 - \boldsymbol{v}_s^2)/(2c^2)\,, \tag{6}$$
$$(\Delta\,\omega/\omega)^{AR} = \boldsymbol{eu}/c + (\boldsymbol{eu})(\boldsymbol{ev}_s)/c^2 + (\boldsymbol{v}_a^2 - \boldsymbol{v}_s^2)/(2c^2) - r\boldsymbol{\nu u}/c\,. \tag{7}$$

Here, $\boldsymbol{v}_s$ and $\boldsymbol{v}_a$ are the velocities of Mössbauer source and absorber with respect to laboratory; $\boldsymbol{u} = \boldsymbol{v}_s - \boldsymbol{v}_a$; $\boldsymbol{e}$ is unit vector along a beam in the laboratory reference frame; $\boldsymbol{w}$ is the ether wind velocity with respect to laboratory; $\boldsymbol{\nu}$ is a unit vector that indicates a locally preferred direction just where the laboratory is located in the space; r is a local magnitude of local space anisotropy; ω_a is the frequency seen by the absorber; ω_s is the source proper frequency. Formulas (5) and (6) have been borrowed from [24], while formula (7) ensues from the exact relation (4) that describes the relativistic Doppler effect allowing for a local space anisotropy. Obviously, formulas (5)–(7) give, generally, different predictions as regards the dependence of $\Delta\,\omega/\omega$ on the source and absorber velocities. As noted in [24], if the source and absorber are both fixed on a rotating rotor, then $(\boldsymbol{eu}) = 0$ with respect to the laboratory reference frame. In this case, the predictions of the theories compared are

$$(\Delta\,\omega/\omega)^{PR} = -\boldsymbol{wu}/c^2\,, \tag{8}$$
$$(\Delta\,\omega/\omega)^{SR} = (\boldsymbol{v}_a^2 - \boldsymbol{v}_s^2)/(2c^2)\,, \tag{9}$$
$$(\Delta\,\omega/\omega)^{AR} = (\boldsymbol{v}_a^2 - \boldsymbol{v}_s^2)/(2c^2) - r\boldsymbol{\nu u}/c\,. \tag{10}$$

Let us find out now what are the results of the Doppler effect measurement experiments. In [25], the Mössbauer source Co^{57} was placed at the rotor center, and the absorber Fe^{57} on the rotor radius. In that experiment, $v_s = 0$. Therefore, formulas (8)–(10) take the form

$$(\Delta\,\omega/\omega)^{PR} = \boldsymbol{wv}_a/c^2\,, \tag{11}$$
$$(\Delta\,\omega/\omega)^{SR} = \boldsymbol{v}_a^2/(2c^2)\,, \tag{12}$$
$$(\Delta\,\omega/\omega)^{AR} = \boldsymbol{v}_a^2/(2c^2) + r\boldsymbol{\nu v}_a/c\,. \tag{13}$$

To increase the statistics strength, work [25] made use of two fixed counters located near the diametrically opposite sides of the rotor. Since the resonance curves were read out by summing up the readouts of both counters, the theoretical values of (11)–(13) averaged over two opposite directions of $\boldsymbol{v}_a$, i.e.

$$\left\langle(\Delta\,\omega/\omega)^{PR}\right\rangle = 0\,, \tag{14}$$

$$\left\langle(\Delta\,\omega/\omega)^{SR}\right\rangle = \left\langle(\Delta\,\omega/\omega)^{AR}\right\rangle = \boldsymbol{v}_a^2/(2c^2) \tag{15}$$

must be compared with the Doppler shift measurement results. Such a comparison shows that the Doppler shifts measured in [25] agree, up to an error of 1.1%, with the predictions (15) of special relativity and relativistic theory of anisotropic space-time, but contradict the pre-relativistic theory of absolute ether, whence, seemingly, it follows unambiguously that other experiments aimed at searching for ether wind (in particular, work [26] that, in terms of special relativity, is of null-experiment type) are senseless. However, this is not the fact, so the experiment [26] deserves particular consideration.

In [26], the Doppler shift of frequency was measured between source and absorber placed at equal and diametrically opposite distances from the center of a rapidly rotating rotor. Since in this case $\boldsymbol{v}_s = -\boldsymbol{v}_a$, then, according to (9) and (8), the special relativity predicts zero frequency shift, while the pre-relativistic theory predicts the effect that differs from zero:

$$(\Delta\,\omega/\omega)^{PR} = 2\boldsymbol{w}\boldsymbol{v}_a/c^2\,. \tag{16}$$

It is remarkable that the effect similar to (16) is also predicted by the relativistic theory of anisotropic space-time. In fact, at $\boldsymbol{v}_s = -\boldsymbol{v}_a$ formula (10) gives

$$(\Delta\,\omega/\omega)^{AR} = 2rc\boldsymbol{\nu}\boldsymbol{v}_a/c^2\,. \tag{17}$$

It should be noted that it is not accidental that the light velocity c is retained in the numerator of the last expression. Comparison between (17) and (16) permits $rc\boldsymbol{\nu}$ to be imparted the sense of vector $\boldsymbol{w}$, i.e. the meaning of ether wind velocity. Thus, whereas the pre-relativistic theory of absolute ether and ether wind fails to withstand an experimental verification, the relativistic theory of anisotropic space-time revives these concepts in a sense, but on the strict relativistic basis. From the very definition of the ether wind velocity as physical quantity $rc\boldsymbol{\nu}$ it follows that it is the same in all physically equivalent Galilean frames. In other words, $rc\boldsymbol{\nu}$ is an invariant of the Lorentz transformations generalized for anisotropic space (1), i.e. is an invariant of transformations (2). It should be noted also that the same invariant enters the definition of the rest momentun $\boldsymbol{P}_{rest} = mrc\boldsymbol{\nu}$ that, together with the rest energy $E_{rest} = mc^2$, characterizes a particle at rest in the anisotropic space. In view of the above, the experiment [26] has a new (active) sense and must be understood to be intended directly for searching a local space anisotropy, so the results presented in [26] must be appraised considering this fact.

First of all, proceeding from [26], we shall find the restriction on the anisotropy magnitude. Let us designate the spherical (rather than geographic) coordinates on the Earth surface as $\Theta\,,\Phi$, where Θ is the polar angle measured from the North Pole; Φ is the azimuthal angle measured eastwards ($0 \le \Theta \le \pi$; $0 \le \Phi < 2\pi$). Let also ϑ designate the angle between a preferred direction in space $\boldsymbol{\nu}$ and the Earth rotation axis. Then,

$$\nu_{e_R}(t) = \boldsymbol{\nu}\boldsymbol{e}_R(t) = \sin\vartheta\sin\Theta\cos(\Phi+\Omega t) + \cos\vartheta\cos\Theta\,, \tag{18}$$

$$\nu_{e_\Theta}(t) = \boldsymbol{\nu}\boldsymbol{e}_\Theta(t) = \sin\vartheta\cos\Theta\cos(\Phi+\Omega t) - \cos\vartheta\sin\Theta\,, \tag{19}$$

$$\nu_{e_\Phi}(t) = \boldsymbol{\nu}\boldsymbol{e}_\Phi(t) = -\sin\vartheta\sin(\Phi+\Omega t)\,, \tag{20}$$

where $\nu_{e_R}(t)$, $\nu_{e_\Theta}(t)$ and $\nu_{e_\Phi}(t)$ are the time- and location on earth-dependent projections of unit vector $\boldsymbol{\nu}$ on a surface normal, meridian, and parallel, respectively; Ω is angular velocity of earth rotation.

The experiment [26] was made using two fixed counters (northern and southern) located near the opposite sides of the rotor. Therefore, the measurements were taken when the absorber velocity $\boldsymbol{v}_a$ was on a geographic parallel and in virtue of (20), the equality

$$\boldsymbol{w}\boldsymbol{v}_a = rc\boldsymbol{\nu}\boldsymbol{v}_a = rc\nu_{e_\Phi} v_a = -rc\sin\vartheta\, v_a \sin(\Phi + \Omega t) = -V v_a \sin(\Phi + \Omega t) \qquad (21)$$

was valid, where

$$V = rc\sin\vartheta \qquad (22)$$

is projection of ether wind velocity on equatorial plane.
Considering (21), the relation (16) shows that a harmonic dependence (with frequency Ω) of the Doppler shift of frequency on time of a day could be expected. At any of the experimental points, the statistics was gathered for six hours, so not more than four experimental points were read out each day, with the entire observation run lasting for a few days with an interval. Considering the doubled standard error, but a few experimental points indicated a positive effect, whereas the standard errors overlapped the expected effect at most of the points. Given this situation, the processing of the entire set of experimental data obtained during a few days has given $V = (1.6 \pm 2.8)$m/s. If $\boldsymbol{\nu}$ is related to the direction to the Galactic center, then, according to (22), this result, when scaled to the space anisotropy magnitude, means that $r = (1.3 \pm 2.4)10^{-8}$. Thus, the experiment [26] has failed to find any ether wind and, therefore, any local space anisotropy. As to the upper limit on the ether wind velocity obtained in 1970 [26], this limit means that, in terms of space anisotropy, $r < 5 \times 10^{-10}$.

4. The planned laboratory experiment to search for space anisotropy using the present-day radio physics techniques for generation of monochromatic oscillations and detection of weak signals

Finally, let us consider another planned laboratory experiment [7] aimed at searching for space anisotropy. The experiment is based on the effect of frequency modulation of harmonic oscillations incoming to a receiver that rotates at a constant velocity about a monochromatic wave emitter. We shall proceed from the exact formula

$$dt' = \left(\frac{1 - \boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1 - \boldsymbol{v}^2/c^2}}\right)^r \sqrt{1 - \boldsymbol{v}^2/c^2}\; dt \qquad (23)$$

which ensues from (1) and, in terms of relativistic theory for locally anisotropic space of events, determines the course of proper time dt' compared with laboratory time dt. Considering the smallness of the anisotropy magnitude r, we obtain in the lowest order of v/c :

$$dt' = \bigl(1 - \boldsymbol{v}^2/(2c^2) - r\boldsymbol{v}\boldsymbol{\nu}/c\bigr)\, dt\,. \qquad (24)$$

Since the transverse Doppler effect arises exclusively from the dependence of the receiver proper time on the receiver velocity, the straightforward way of searching for space anisotropy, i.e. for verifying formula (24), is to analyze the oscillations in the receiver that rotates at a constant velocity around the monochromatic wave source.

More specifically, the experimental design is as follows. Two receivers (1 and 2) are positioned at equal and diametrically opposite distances from an emitter of a monochromatic wave with a frequency ω_0 and rotate with an angular frequency $\Omega = v/R$ around the emitter. Assume for simplicity that vector $\boldsymbol{\nu}$ lies in the rotation plane. Then, we obtain by integrating (24):

$$t' = \left(1 - \frac{v^2}{2c^2}\right) t \mp \frac{rv}{c\,\Omega} \sin \Omega t\,, \tag{25}$$

where t is laboratory time; t' is receiver proper time. Here and henceforth, the upper and lower signs stand for receivers 1 and 2, respectively. To within the same accuracy, we get

$$t = \left(1 + \frac{v^2}{2c^2}\right) t' \pm \frac{rv}{c\,\Omega} \sin \left(1 + \frac{v^2}{2c^2}\right) \Omega t'\,. \tag{26}$$

Let $\Phi(t) = \omega_0 t$ be the phase of oscillations incoming to a receiver as a function of laboratory time. The use of (26) gives the dependence of the phase on the receiver proper time t'. We obtain that the oscillation frequency, $\omega(t') = d\Phi(t')/dt'$, measured in the receiver proper time is

$$\omega\,(t') = \left(1 + \frac{v^2}{2c^2}\right) \omega_0 \pm \frac{rv\,\omega_0}{c} \cos \left(1 + \frac{v^2}{2c^2}\right) \Omega t'\,. \tag{27}$$

Thus the receiver is affected by a frequency-modulated signal, with the modulation depth being proportional to the local space anisotropy r. Detection of low-frequency oscillations (with the frequency $(1 + v^2/(2c^2))\,\Omega$) by the receivers in combination with subsequent subtraction of the oscillations on the rotation axis will give rise to harmonic oscillations with frequency Ω and to doubled amplitude proportional to the anisotropy magnitude r.

The present-day radio physics techniques for recording weak signals permit the above experiment to be realized. An autogenerator stabilized by a superconducting resonator that exhibits a narrow spectral line and low amplitude fluctuations [27], [28] will be used as a monochromatic source. In this case the electromotive force of a signal incoming to receivers 1 and 2 can be presented as

$$\begin{aligned} u\,(t') = u_0 \Bigg\{ \cos \left(1 + \frac{\Omega^2 R^2}{2\,c^2}\right) \omega_0 t' \pm \frac{r\omega_0 R}{2\,c} \cos \left(1 + \frac{\Omega^2 R^2}{2\,c^2}\right) (\omega_0 + \Omega)\, t' \\ \mp \frac{r\omega_0 R}{2\,c} \cos \left(1 + \frac{\Omega^2 R^2}{2c^2}\right) (\omega_0 - \Omega)\, t' \Bigg\}\,. \end{aligned} \tag{28}$$

The experimental design expects a signal, which is the electromotive force component $\Delta\tilde{u}_s$ in the receiver input, whose frequency is tuned off to the left or right from the base frequency:

$$\Delta\tilde{u}_s = \pm u_0 \, \frac{r\omega_0 R}{2\,c} \cos \left(1 + \frac{\Omega^2 R^2}{2\,c^2}\right) (\omega_0 + \Omega)\, t'\,. \tag{29}$$

Equating $\Delta\tilde{u}_s/u_0$ to the relative value $(\Delta u/u)_\alpha$ of the natural amplitude fluctuations of autogenerator at frequency $\omega_0 + \Omega$, we obtain a boundary for the least detectable anisotropy r_{min} determined by the source fluctuations:

$$r_{min} = \frac{c}{R\Omega} \sqrt{\frac{kT_N^A}{WQ^2\tau}}\,, \tag{30}$$

where T_N^A is the noise temperature of the active element of autogenerator, Q is the quality factor of a stabilizing superconducting resonator, W is autogenerator power and τ is measurement time. Substituting $R = 25\,\text{cm}$, $\Omega = 10^2\,\text{rad/s}$ and the real autogenerator parameters $T_N^A = 600\,\text{K}$, $W = 10^{-2}\,\text{wt}$, $Q = 2 \times 10^8$ in (30), we obtain $r_{min} = 5 \times 10^{-11}/\sqrt{\tau}$.

The required receiver sensitivity when recording $\Delta\tilde{u}_s$ is determined from the condition

$$\left(\frac{\Delta\tilde{u}_s}{u_0}\right)^2 = \frac{\Delta W_s}{W} = \frac{r_{min}^2\,\omega_0^2 R^2}{4\,c^2}\,, \tag{31}$$

where $\Delta W_s = kT_N^{Rec}/\tau$ is the least detectable signal power at the receiver input, ω_0 is the frequency of auto-oscillations of source, T_N^{Rec} is the receiver noise temperature.

At $\omega_0 = 2 \times 10^{10}\,\text{rad/s}$ and $W = 10^{-2}\,\text{wt}$, expression (31) gives $T_N^{Rec} = 10^2\,\text{K}$, which is an order higher than the T_N^{Rec} value reached at present in the given frequency range.

5. Conclusion

On examining the experiments aimed at searching for ether wind that were carried out soon after the discovery of the Mössbauer effect, we have concluded that the experiments have given the upper limit $r < 5 \times 10^{-10}$ on the space anisotropy magnitude. The present day use of the radically new rotors ($n \geq 6 \times 10^5$ turns/min) developed by the ITEP team (Moscow), as well as of the Mössbauer sources with a much narrower line width, has made it possible to lower the minimum detectable value of space anisotropy by at least three orders. Accordingly, the experiment [26] is very topical to repeat. At the same time, it should be kept in mind that, in case a new experiment of that type is aimed at searching for space anisotropy originating from the Galaxy, the northern and southern counters are expedient to replace by western and eastern counters because, according to (19), the daily average value of projection of $\boldsymbol{\nu}$ onto the laboratory floor equals $\cos\vartheta \sin\Theta$ and is directed northwards.

As to the experiment described above in section 4, which is expected to base on the effect of harmonic oscillation frequency modulation, the expression (30) shows that the potentiality of such an experiment in detecting a local space anisotropy is restricted solely by the operation period of a respective facility. In any case, the realization of the experiment will make it quite possible to either discover the expected space anisotropy or lower its upper boundary down to $\sim 10^{-14}$.

Finally, it should be noted that, beside the relativistic Finslerian approach, an alternative approach to the space anisotropy problem is being developed actively. The respective string-motivated theory is known to be the Extended Standard Model of strong, weak, and electromagnetic interactions, or the Standard Model Extension (SME) (see [29]–[33] for instance). Since SME is not a relativistically-invariant theory, the anisotropy impact on the fundamental fields dynamics is described by SME in terms of much more numerous parameters as compared with the set of parameters that characterize the space anisotropy proper. As a result, the complete anisotropy measurements involve measuring the very numerous effects predicted by SME. At present, attempts are made to detect and measure the respective effects by such collaborations as, for instance, LSND, KTeV, FOCUS, BaBar, BELLE, OPAL, DELPHI, and BNL g-2.

Contrary to the effects predicted by SME, the Doppler shift is a pure kinematic effect, while the space anisotropy impact on that effect is determined only by the parameters of the anisotropy proper. Therefore, the above described experiments are most expedient as regards searching and measuring the local space anisotropy. Besides, the realization of the experiments does not involve much material cost.

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Vortex-sponge, wave-particle & geometrized space-time

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An important role of analogues is to establish consistency throughout theories developed from them; to disclose links between disconnected interpretations, and to achieve unification. Analogues are disclosing models, not replicas of reality. Analogues may assume several equivalent physical forms whilst having the same describing equations. Examples include gyrostatic media, which derive their properties from infinitely extended arrays of infinitesimally small gyrostats, and the equivalent vortex sponge, which is a frictionless fluid full of infinitesimally small vortex rings moving at random. R. V. L. Hartley showed that the gyrostat analogue, developed by Kelvin and Larmor, and the vortex-sponge which was first devised by Euler, possess the property of internal reflection, arising from the time dependency of the rotational stiffness. This enables spherical standing waves to be set up, to endure, and to represent wave-particles. In 1908 Minkowski geometrized Special Relativity and the Lorentz electron theory, which he interpreted as a geometrized Absolute World. Einstein later presented his General Theory in terms of non-Euclidean geometry. Hartley's wave-particle and vortex-sponge lead to the same equations as the General Theory, and provide the means for constructing a mechanized Absolute World or World Ether.

Hartley's model of a large-scale material particle serves as a combined rod and clock for measuring events. The surveying instruments are disturbed by motion. It is assumed that the particle endures when moving in a gravitational field so that signals sent out to the envelope return simultaneously to centre. This enables the chronotopic interval and relations of General Relativity to be obtained by more than one argument. The measurements made during the surveying operations can be expressed in terms of the non-Euclidean geometry of General Relativity. The geometrized vortex-sponge and Einstein's non-Euclidean space-time are equivalent. The ether interpretation makes provision for a background classical reference frame but this is open to criticism unless the ether is detected. Classical analogues are starting points for deriving consistent relations for describing large-scale matter in gravitational fields.

The small-scale picture of a wave-particle immersed in a vortex-ring atmosphere, acted on by a fluctuating pressure, enables several fundamental relations to be established, including $e = m.c^2$, $e = \hbar\nu$ and the equality of gravitational and inertial mass. Minimum measurable intervals corresponding to the hodon and chronon are intrinsic to the analogue. Vortex sponge interpretations of electromagnetism, and fundamental quantum mechanical phenomena, have been devised by E M Kelly. Particle instability on the small scale gives space-time a quantum foam structure. The modern ether is a model for energy-filled space-time, sometimes described as the Relativistic World Ether. Supercooled helium possesses the properties of the vortex sponge, and has been used by Volovik to interpret General Relativity, Cosmology, and Quantum Mechanics. The paper demonstrates how the formal structure of the General Theory is derived using the mechanical analogue, and briefly indicates how interpretations of small-scale phenomena are derived.

On a Minimum Contradictions Everything

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Many articles have been written on this subject. The earlier articles had as target the unification of the GRT with the QM on the basis of the unification of the physical meaning deriving either from the GRT or the QM. On this basis, it has been shown that the governor equation is Schröndinger's relativistic equation which describes a stochastic space-time regarded as matter itself. The gravitational field coexists with the electromagnetic one the two of them being interconnected. According to this, space-time operators and geometry have been defined. A wide range of phenomena have been explained, the gravitation included, while new phenomena as the ones related to the interaction of the gravitational with the electromagnetic field have been interpreted and technologically approached. In the later papers an effort was made so that the physics proposed to be derived on the basis of mental principles and more specifically from the claim for minimum contradictions lying on the fact that the basic communication system is contradictory. This can be proved through a theorem that is the core where the whole work is based on. Thus, the physics derived has the minimum possible contradictions or equivalently the minimum possible arbitrariness. Note that all principles of the until now dominant physics, even though they condense the experience which has been revealed, are arbitrary. On this basis, the equations of minimum contradictions everything have been stated. The consequences of the later papers are the same with the ones of the earlier papers. There is an ongoing effort for these papers to be written as a unified whole so that any gaps are avoided. According to this point of view, Ether –Everything is the substance within which things exist and from which things are made. The objective of this paper is to present an extended summary of this work regarded as a whole.

1. Introduction to Logic Analysis

Every Physics Theory beyond its particular principles is stated based on the language basic communication system [1,2,3,4]. This system obeys the Aristotle logic (Classical Logic) [5,6], the Leibniz' Sufficient Reason Principle [7] -according to which, for everything we seek the reason of its power- and a hidden axiom which states that "there is anterior-posterior everywhere in communication". In fact, the way in which we communicate is not a simultaneous process but it is characterized by the existence of anterior and posterior; one word is put after another, one phrase after another e.t.c. [1,2,3,4].

The belief that a perfect theory can be found originates from the fact that we believe that the basic communication system is perfect. If this system is contradictory, it is meaningless to seek the statement of a perfect theory through a contradictory system.

According to this work the basic system of communication is contradictory. In fact if we call by Λ a logic consisting of the Classical Logic and the Sufficient Reason Principle, the following can be proved [8,9,10]:

Theorem I: " Any system that includes logic Λ *and a statement that is not theorem of logic* Λ *leads to contradiction."*

On the basis of Theorem I the following lemma can be stated:

Lemma: "Any system that includes logic Λ *and a synthetic sentence leads to contradiction."*

The anterior-posterior axiom constitutes a synthetic sentence; however additionally it can be proved that it is not theorem of Λ. Thus, the following is valid:

Statement I: " Any system that includes logic Λ *and the anterior-posterior axiom leads to contradiction."*

where the anterior – posterior axiom is stated as follows.

Anterior – Posterior Axiom: "There is Anterior-Posterior Everywhere in Communication"

When we try to describe reality through a theory we cannot do it simultaneously but in anterior-posterior terms. Even anterior-posterior, in physical reality, itself is not known but only when it is described. Therefore, this axiom includes every anterior-posterior described; not only the indicating the order of communication elements (words, phrases, e.t.c.). It is noted that in the early papers, which this work is based on, the term earlier posterior was used. However this term applies better to time and not to any countable magnitude. Dr. C.K.Whitney proposed the term anterior-posterior which is more neutral and therefore it can apply beyond time [4].

If we name zero the state before our communication and 1, 2, 3, … the sequent states of this communication we may notice that the anterior-posterior axiom can be related to numbers.

Statement I can also be derived by the aid of Gödel's work and logic Λ [8]. It is noted that Gödel's work, through which his theorems have been stated, cannot be used in the current form of any of his statements; in fact this work is based on an arbitrary hypothesis i.e.:

Gödel's Hypothesis: "There is an algorithm that permits the derivation of only true statements"

The arbitrariness of this hypothesis lies on the fact that the algorithm mentioned is not precisely defined, as it has been noticed by H. Putnam, R. Penrose [11,12] and others [13]. By using logic Λ we can reach Statement I through Gödel's work [14] but without Gödel's hypothesis which could be regarded as a part of Λ. This is a verification of Statement I and Theorem I which is required in order that the basic claim of this work can be applied.

Despite of all these, when we communicate in a way that we consider logical, we could say that we try to understand things through minimum possible contradictions since contradictions are never vanished. On this basis we can state:

The Claim for Minimum Contradictions: " What includes the minimum possible contradictions is accepted as valid".

According to this claim we obtain a logical and an illogical dimension. In fact, through this claim we try to approach logic (minimum possible contradictions) but at the same time we expect something illogical since the contradictions cannot be vanished. However, the question is raised of whether this claim has any sense since one contradiction implies infinite contradictions [14]. The answer to this is that the claim for minimum contradictions creates a modification of the basic communication system since it implies a logic "attractor" through minimum possible contradictions required.

2. Minimum Contradictions Physics

Every theory includes at least the principles of the basic communication system. According to theorem I, further axioms beyond the ones of basic communication must be avoided since they can cause further contradictions. Thus the Claim for Minimum Contradictions operates as a Simplicity Principle. This is compatible with Ockham's razor [15]; however Ockham's razor does not imply any contradiction.

The systems of axioms we use in Physics include the communication system and, therefore, their contradictions are minimized when they are reduced to the communication system itself. Therefore we can state:

We have minimum contradictions in Physics when it is based only on the basic communication system, i.e. on logic Λ and on the "anterior-posterior axiom".

In order that such physics is valid, a unifying principle is required, since everything, *i.e.* matter, field, and space-time, needs to be described in anterior-posterior terms.
At first sight, for a minimum contradictions physics we can make the following statement:

Statement II: Any matter space-time system can be described in anterior–posterior terms.

It is noted that time implies the existence of anterior and posterior; space does, too. If I say 10cm, I mean the existence of anterior-posterior measuring states corresponding to 1,2,3….,10 cm. Therefore, the existence of anterior and posterior is the condition for space and time to exist and *vice-versa*. Thus, because of Statement II, for a least contradictory physics we can state the following statement:

Statement III: Any matter system can be described in space-time terms.

Since everywhere there is space-time and not something else, *Space-Time-Everything* can be regarded as *Matter-Ether*. A matter system, in general, has differences within its various areas. This means that a matter system, in general, is characterized by different rates of anterior - posterior (time) within its various points. Since space is also locally affected by the local rate of anterior-posterior, it can be expected to be deformed due to different rates of anterior-posterior. This means that time can be regarded as a 4^{th} dimension which implies Lorentz' transformations and in extension a relativistic theory [16,17,18].

On this basis space-time can be regarded either *as geometry or as deformable matter- ether*; *this is compatible both with Einstein's and Poincaré's point of view* [19,20].

Basic tool of this work is the Hypothetical Measuring Field (HMF); this is a term initially proposed as "image Field" which has been changed in order to correspond exactly to what it signifies after a proposal by P.F.Parshin [17]. According to M.C.Duffy this term is compatible to an approach taken by Eddington and to recent studies on the physical vacuum based on information science in which material particles, which have a wave particle nature interact with an "image-taking field" [21].

As Hypothetical Measuring Field (HMF) is defined a hypothetical field, which consists of a Euclidean reference space-time, in which at each point A_0 the real characteristics of the corresponding, through the transformations of deformity, point A of the real field exist.

In a space-time description we don't know a priori what energy is; we define energy dE of an infinitesimal space-time element its 'ability to exist'. We may notice that an infinitesimal space-time element with energy dE exists on condition that some corresponding 'anterior-posterior' exist too [16,17]. With respect to the HMF a space-time element is observed during a time dt that is different from the time dt_0 of the corresponding reference space-time element. Various space-time elements in the HMF have different dt for the same dt_0. Thus, dt measures the duration *i.e*. the ability of a space-time element to exist; this ability, by definition is energy; when $dt = dt_0$, this ability is dE_0. Thus, we can write:

$$dE \sim dt \quad \text{and} \quad dE / dE_0 = dt / dt_0 \tag{1}$$

which is a relativistic relation.
Eq. (1) can be viewed in two ways:

a) When dt_0 is a unit of time, Eq. (1) describes the duration dt, with respect to an observer and, as was mentioned, it leads to the relativity theory.

b) When dt is a constant period of time in the HMF, then Eq. (1) can be written in the form:

$$dE / dE_0 = dt / dt_0 = (f / v)/(f / v_0) = v_0 / v \tag{2}$$

where v is the frequency of a periodic phenomenon of comparison and f an arbitrarily constant factor through which we can change the scale of v, v_0. If $v = 1$, v_0 must be different in various points $(\boldsymbol{r}, t)$ of the HMF. If this is the case Eq. (2) can be written in the form:

$$dE / dE_0 = v_0(\boldsymbol{r}, t) \tag{3}$$

Thus, for the same equation we have the following versions:

$$dE / dE_0 = dt / dt_0 \text{ observation} \quad \text{(relativity theory)} \tag{4}$$

$$dE / dE_0 = v_0(\boldsymbol{r}, t) \text{ action} \quad \text{(quantum mechanics)} \tag{5}$$

On this basis, we can reach the basic De Broglie's principle for energy, for $E_0 = h$ (arithmetically) *i.e.* [17,18]:

$$E = h\nu \qquad (6)$$

<u>*At second sight*</u>, because of the claim of the minimum contradictions, we conclude that *Matter-Space-Time-Everything-Aether* can have logical and contradictory behavior at the same time; *this can be valid only if space-time is stochastic.*

According to M.C.Duffy "The modern ether can be treated as a sea of information, and a generator of dynamic algebras, which is revealed as a discretum rather than a continuum on the smallest scales of space-time" [22]. This can be regarded as compatible to stochastic space-time which is not continuum on the smallest scales.

According to A.Pais, Einstein had said: "I consider it quite possible that physics cannot be based on the field concept; i.e., on continuous structures. In that case nothing remains of my entire castle in the air, gravitation theory included, and the rest of modern physics" [23,24].

Despite the fact that space-time may be stochastic, there are basic relativistic relations that continue to be valid; perhaps relativity principle can be stated on the basis of space-time operators as it will be mentioned.

At first sight, QM seems to remain unchangeable. However, what it describes, according to this work, is not a particle wave but the stochastic space-time in the Hypothetical Measuring Field (HMF). As was mentioned a De Broglie's basic principle can be regarded as an other view of a basic relativistic relation of matter space-time; De Broglie's principles can be proved as valid for stochastic space-time. On this basis, we have the frame in which a unified theory can be stated while the operators of relative length in a given direction and relative time can be defined; by the aid of a Ψ wave function the geometry of stochastic space-time can be described.

With starting point R.M. Santilli's paper: " Lie –Admissible Invariant Origin of Irreversibility for Matter and Antimatter at the Classsical and Operator Levels" [25], we may notice the following: An operator can be regarded as the basic acting law which cause all phenomena revealed. On this basis, if invariance is valid *in general* at operator level, it means that the basic laws are invariant. This might be close to a new approach of relativity principle. Space-time operators, according to this work, are invariant to Lorentz' transformations; however the final result i.e. real measurable space-time is non-relativistic, it seems to be fractal.

The stochastic space-time derives from the distribution of the properties of a flat relativistic space-time based on the probability density $P(\boldsymbol{r},t)$ of Schröndinger's relativistic equation which is proved as valid [16,18].

The negative values of $P(\boldsymbol{r},t)$ can correspond to the geometry of the anti-matter. The incomprehensible notion of the negative probability is compatible with the claim for minimum contradictions (since contradictions are always expected). However, the question is raised of whether Schröndinger's relativistic equation or Dirac's equation should be taken into account. As it is known from classical works Dirac's equation is based on the requirement for linear operators correlation. According to the spirit of this work, the linearity that is mentioned constitutes an additional restriction which is not theorem of logic Λ and therefore because of theorem I causes further contradictions beyond the ones imposed by the stochastic space-time consideration.

Schröndinger's relativistic equation, without any potential term, can derive without any further assumption by the aid of Fourier analysis and corresponds to a minimum contradictions description [16,18,20]. It is noted that P. Rowlands has noticed that fermions which derive from Dirac's equation do not describe a whole; "the particle and its "environment" can be considered as two "halves" of a more complete whole" [26]. It is noted that fermions have spin ½ which according to classical point of view corresponds to real particles. According to the spirit of this work we can have spin ½ due to coexisting local equivalent particle fields of gravitational (*g*) and electromagnetic (*em*) space-time even though they are described by Schröndinger's relativistic equation.

The electromagnetic (*em*) space-time is a space-time whose all magnitudes are considered imaginary and behave exactly like the gravitational (*g*). Electromagnetic (*em*) space-time is described by means of space-time wave functions such that:

$$\Psi_{em}(\boldsymbol{r}_{em}, t_{em}) = \Psi_{em}^{g}(\boldsymbol{r}, t) \tag{7}$$

where Eq(g) has meaning due to the coexistence of (*g*) and (*em*) space-time under a scale. The way of coexistence and communication of (*g*) with (*em*) space-time is shown. On this basis space-time as a whole consists of:

1. real (*g*) space-time distributed according to a $P_g(\boldsymbol{r}, t)$ function revealing so (*g*) matter or antimatter for positive or negative values of $P_g(\boldsymbol{r}, t)$.
2. imaginary (*em*) space-time distributed according to a $P_{em}(\boldsymbol{r}, t)$ function revealing so (*em*) matter or antimatter for imaginary positive $(+i)$ or imaginary negative $(-i)$ values of $P_{em}(\boldsymbol{r}, t)$.

At this point we may notice that there are some similarities with P. Rowland's treatment where mass and charge space are independently symbolised and described [27].

The stochastic space-time has the property of self-similarity while, at the same time, it is chaotic (contradictory)- non-deterministic. It is something compatible with fractal geometry, which is a geometry of nature [28,29].

The force of the gravitation is interpreted as a force that is exerted on every infinitesimal element of the stochastic matter space-time in order that it is distributed according to a given probability density. Thus, the following general formula for gravitational acceleration derives:

$$\mathbf{g}(\mathbf{r}, t) = \frac{c^2}{P(\mathbf{r}, t)} \nabla P(\mathbf{r}, t) \tag{8}$$

This formula under certain simplifications, is compatible with Newton law for gravity [17,18]. On this basis, for a symmetric spherical and constant in time field, it can be proved that [30,31]:

$$\mathbf{g}(\mathbf{r}) = -\frac{c^2}{r} \hat{\mathbf{r}} \tag{9}$$

$$\frac{GM}{r} = c^2 \tag{10}$$

It is noteworthy that we can reach the same formula through a differed way related to unification of physics theories [15].

Basic element of this work is that space-time is statistically interpreted. Most of the conclusions derive on the basis of statistical relations related to various space-time magnitudes. These conclusions and consequences are related to new explanations of various phenomena [18]. The reason why we have enough information to draw these conclusions is the clear statistical interpretation which is due to the property of Ψ wave functions to be everywhere self-normalized. In fact according to the claim for minimum contradictions the Ψ wave function of a matter system in general, is equivalent to local Ψ_i wave functions which obey Schröndinger's relativistic equation.

Local Ψ_i wave functions describe coexisting equivalent local (*g*) and (*em*) particle space-time fields which are regarded as extended to the infinity so that Schröndinger's relativistic equation probability density function $P(\boldsymbol{r},t)$ can apply. For this probability density function always is valid that:

$$\int P(\boldsymbol{r},t)dr^3 = 1 \tag{11}$$

Because of the property of Ψ to be self-normalized we have clearly stated statistical relations which permit us to draw conclusions related to forces unification, spin interpretation, matter system quantization, second thermodynamic law derivation, arrow of time and fractal properties interpretation as well as to new explanation of various phenomena. On this basis the possibility to technological applications related to the interaction of the electromagnetic with the gravitational field has been searched; experimental results related to asymmetrical capacitors propulsion and to light water electrolysis excess heat could be regarded as positive. There is not evidence that the statistical relations mentioned are valid in the case of a Dirac treatment; as was mentioned Dirac's equation cannot describe a whole [26]. The property of self-normalization of the theory proposed constitutes a basic difference in relation to existing current theories or new proposed ones which use the requirement for re-normalization [32,33]. This requirement derives from the necessity for various magnitudes to be statistically interpreted but through functions which by themselves do not imply a statistical nature. In these cases problems are raised related to various phenomena and mainly to description and interpretation of gravity. The statistical nature of space-time constitutes the basic consequence of the main principle of this work i.e. of the claim for minimum contradictions.

On this basis the Equations of Minimum Contradictions Everything are stated; geometry and the force per unit of mass at a point $(\boldsymbol{r},t)$ of the HMF is defined. These equations describe a *Space-Time Quantum Mechanics* [34].

Taking into account these equations and using numerical analysis (finite differences) we can conclude that a space-time matter system can not be determined on the basis of initial conditions. Thus, any matter space-time system is self-defined and behaves as if it had an ability to decide for its evolution. This might be the basis for a new approach related to the evolution of biological organizations. This approach is based on the existence of volition, which according to this work characterizes "everything" and therefore biological organizations as well. It is noted that according to minimum contradictions point of view the notion of volition is identified with the notion of “free will”.

This work derives from purely mental conclusions in contrast to theories which condense the experience which has been revealed. According to this work a theorist reaches to conclusions compatible with the ones of an experimentalist. This has similarities with E.J.Post’ point of view where the starting point is the notice that “it is remarkable that Mathematical Theorems can apply so perfectly in Physics” [35]. Thus, in a logically linked way, under certain simplifications, Newton’s gravitation law derives from mental principles and not from empirical observations as the falling apple is.

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РЕЛЯТИВИСТСКИЕ УРАВНЕНИЯ ДЛЯ ВОЛНОВОЙ ФУНКЦИИ В ПРОСТРАНСТВЕ-ВРЕМЕНИ БЕРВАЛЬДА-МООРА

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Приводятся новые свойства системы квадрачисел с коммутативной операцией векторного произведения трехмерных векторов. Определяется характеристическое уравнение для квадрачисла, четыре характеристических числа которого связаны посредством матрицы Адамара с его компонентами. Симметричные формы для набора квадрачисел разных порядков равняются симметричным многочленам для характеристических чисел. Даются релятивистские уравнения четвертого порядка для скалярной волновой функции в случае свободной частицы и находящейся в электромагнитном поле. Изучается представление алгебры квадрачисел недиагональными матрицами порядка четыре. Получены четыре линейных релятивистских уравнения первого порядка для четырех четырех-компонентных волновых функций, описывающие поведение свободных частиц в пространстве-времени Бервальда-Моора в случае чистого ансамбля квантовых систем. Собственные значения энергии частицы не вырождаются для данного значения импульса.

1. Введение

Рассмотрим квадрачисло [1]

$$\mathbf{A} = \left(a_0, \mathbf{a}\right) = a_0 + a_i \mathbf{e}_i \tag{1.1}$$

с вещественными числами a_0 и a_i $\left(i = 1, 2, 3\right)$ и базисными элементами $\mathbf{e}_0 = 1, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$. Закон композиции базисных элементов определяется в виде

$$\mathbf{e}_i \circ \mathbf{e}_j = \delta_{ij}\mathbf{e}_0 + \varepsilon_{kij}\mathbf{e}_k \tag{1.2}$$

и имеет следующие свойства

$$\begin{gathered} \mathbf{e}_1^2 = \mathbf{e}_2^2 = \mathbf{e}_3^2 = 1, \\ \mathbf{e}_1 \circ \mathbf{e}_2 = \mathbf{e}_2 \circ \mathbf{e}_1 = \mathbf{e}_3, \;\; \mathbf{e}_2 \circ \mathbf{e}_3 = \mathbf{e}_3 \circ \mathbf{e}_2 = \mathbf{e}_1, \;\; \mathbf{e}_3 \circ \mathbf{e}_1 = \mathbf{e}_1 \circ \mathbf{e}_3 = \mathbf{e}_2. \end{gathered} \tag{1.3}$$

Здесь δ_{ij} - трехмерный символ Кронекера и ε_{kij} есть трехмерный абсолютно симметричный символ со свойством $\varepsilon_{kij} = 1$ при $i \neq j \neq k$, а остальные значения являются нулевыми и по повторяющимся индексам проводится суммирование. При перестановке любых двух индексов составляющие символа не меняются и справедливы равенства

$$\begin{gathered} \varepsilon_{kij} = \varepsilon_{kji} = \varepsilon_{ikj} = \varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{jik}, \\ \mathbf{e}_{ij} = \varepsilon_{kij}\mathbf{e}_k = \frac{1}{2}\left(\mathbf{e}_i \circ \mathbf{e}_j + \mathbf{e}_j \circ \mathbf{e}_i\right), \;\; \mathbf{e}_i \circ \mathbf{e}_j - \mathbf{e}_j \circ \mathbf{e}_i = 0, \\ \mathbf{e}_m \circ \mathbf{e}_{ij} = \varepsilon_{mij} + \delta_{mi}\mathbf{e}_j + \delta_{mj}\mathbf{e}_i, \\ \mathbf{e}_{ij} \circ \mathbf{e}_m = \varepsilon_{mij} + \delta_{mj}\mathbf{e}_i + \delta_{mi}\mathbf{e}_j. \end{gathered} \tag{1.4}$$

Закон композиции числа (1.1) и $\mathbf{B} = b_0 + b_i \mathbf{e}_i$ дается соотношением

$$\mathbf{C} = \mathbf{A} \circ \mathbf{B} = \left(a_0 b_0 + a_i b_i\right) + \left(a_0 b_k + b_0 a_k + \varepsilon_{kij} a_i b_j\right) \mathbf{e}_k\,, \tag{1.5}$$

где имеем значения компонент

$$\begin{aligned} c_0 &= a_0 b_0 + a_i b_i, \\ c_k &= a_0 b_k + b_0 a_k + \varepsilon_{kij} a_i b_j \end{aligned} \tag{1.6}$$

Представим выражения (1.6) в векторной форме [2]

$$\begin{aligned} c_0 &= a_0 b_0 + (\mathbf{ab}), \\ \mathbf{c} &= a_0 \mathbf{b} + b_0 \mathbf{a} + \{\mathbf{ab}\}. \end{aligned} \tag{1.7}$$

Новое произведение векторов $\{\mathbf{ab}\}_i = \varepsilon_{ikl} a_k b_l$ коммутативно, дистрибутивно относительно сложения, сочетательно относительно умножения на любое число, равно нулю при равенстве нулю одного из векторов.

Выпишем некоторые соотношения для произведений векторов

$$\begin{gathered} \{\mathbf{ab}\} = \{\mathbf{ba}\},\ \{\mathbf{ab}\} + \{\mathbf{ac}\} = \{\mathbf{a}(\mathbf{b}+\mathbf{c})\}, \\ (\{\mathbf{ab}\}\mathbf{c}) = (\{\mathbf{ac}\}\mathbf{b}) = (\{\mathbf{ba}\}\mathbf{c}) = (\{\mathbf{bc}\}\mathbf{a}) = (\{\mathbf{ca}\}\mathbf{b}) = (\{\mathbf{cb}\}\mathbf{a}), \\ \{\mathbf{a}\{\mathbf{bc}\}\} - \{\mathbf{c}\{\mathbf{ba}\}\} = \mathbf{c}(\mathbf{ba}) - \mathbf{a}(\mathbf{bc}), \\ \mathbf{a}^2\mathbf{b}^2 + (\{\mathbf{aa}\}\{\mathbf{bb}\}) = (\mathbf{ab})^2 + \{\mathbf{ab}\}^2, \\ \{\mathbf{aa}\}_i = 2a_k a_l,\ \{\mathbf{a}\{\mathbf{aa}\}\}_i = 2a_i\left(a_k^2 + a_l^2\right),\ (\mathbf{a}\{\mathbf{aa}\}) = 6a_i a_j a_k,\ (i \neq i \neq k), \\ \{\mathbf{a}\{\mathbf{a}\{\mathbf{aa}\}\}\} = \{\mathbf{aa}\}(\mathbf{aa}) - \frac{1}{3}\mathbf{a}(\mathbf{a}\{\mathbf{aa}\}). \end{gathered} \tag{1.8}$$

Целью работы является определение представления алгебры недиагональных матриц порядка четыре изоморфной алгебре квадрачисел для нахождения релятивистских уравнений первого порядка для волновых функций свободной частицы.

2. Матрица Адамара и характеристические значения квадрачисла

Наряду с исходным числом $\mathbf{A}$ определяются еще три числа $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$

$$\begin{aligned} \mathbf{A} &= a_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3, & a_0 &= \frac{1}{4}\left(\mathbf{A} + \mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3\right), \\ \mathbf{A}_1 &= a_0 - a_1\mathbf{e}_1 + a_2\mathbf{e}_2 - a_3\mathbf{e}_3, & a_1\mathbf{e}_1 &= \frac{1}{4}\left(\mathbf{A} - \mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_3\right), \\ \mathbf{A}_2 &= a_0 + a_1\mathbf{e}_1 - a_2\mathbf{e}_2 - a_3\mathbf{e}_3, & a_2\mathbf{e}_2 &= \frac{1}{4}\left(\mathbf{A} + \mathbf{A}_1 - \mathbf{A}_2 - \mathbf{A}_3\right), \\ \mathbf{A}_3 &= a_0 - a_1\mathbf{e}_1 - a_2\mathbf{e}_2 + a_3\mathbf{e}_3, & a_3\mathbf{e}_3 &= \frac{1}{4}\left(\mathbf{A} - \mathbf{A}_1 - \mathbf{A}_2 + \mathbf{A}_3\right). \end{aligned} \tag{2.1}$$

Набор чисел (2.1) только постулируется, а не порождается известным способом, как в случае кватернионов. Действительное число [1]

$$
\begin{aligned}
|\mathbf{A}| &= \sqrt[4]{\mathbf{A}\circ\mathbf{A}_1\circ\mathbf{A}_2\circ\mathbf{A}_3} = \\
&= \left[\left(a_0^2-a_1^2-a_2^2+a_3^2\right)^2-4\left(a_1a_2-a_0a_3\right)^2\right]^{1/4} = \\
&= \left[\left(a_0+a_1+a_2+a_3\right)\left(a_0-a_1+a_2-a_3\right)\left(a_0+a_1-a_2-a_3\right)\left(a_0-a_1-a_2+a_3\right)\right]^{1/4} = \\
&= \left[a_0^4+a_1^4+a_2^4+a_3^4-2\left(a_0^2a_1^2+a_0^2a_2^2+a_0^2a_3^2+a_1^2a_2^2+a_2^2a_3^2+a_3^2a_1^2\right)+8a_0a_1a_2a_3\right]^{1/4}
\end{aligned}
, \qquad (2.2)
$$

называется модулем квадрачисла и, соответственно, имеет место обратное число

$$
\begin{aligned}
\mathbf{A}^{-1} &= \left(\mathbf{A}_1\circ\mathbf{A}_2\circ\mathbf{A}_3\right)\big/|\mathbf{A}|^4 = \\
&= \frac{1}{|\mathbf{A}|^4}\Big\{a_0\left(a_0^2-a_1^2-a_2^2+a_3^2\right)+2a_3\left(a_1a_2-a_0a_3\right)+ \\
&+\left[-a_1\left(a_0^2-a_1^2-a_2^2+a_3^2\right)-2a_2\left(a_1a_2-a_0a_3\right)\right]\mathbf{e}_1+ \\
&+\left[-a_2\left(a_0^2-a_1^2-a_2^2+a_3^2\right)-2a_1\left(a_1a_2-a_0a_3\right)\right]\mathbf{e}_2+ \\
&+\left[a_3\left(a_0^2-a_1^2-a_2^2+a_3^2\right)+2a_0\left(a_1a_2-a_0a_3\right)\right]\mathbf{e}_3\Big\}
\end{aligned}
\qquad (2.3)
$$

с $\left(\mathbf{A}\circ\mathbf{B}\right)^{-1}=\mathbf{B}^{-1}\circ\mathbf{A}^{-1}$, $|\mathbf{A}\circ\mathbf{B}|=|\mathbf{A}||\mathbf{B}|$, $\mathbf{A}_1^{-1}=\left(\mathbf{A}\circ\mathbf{A}_2\circ\mathbf{A}_3\right)\big/|\mathbf{A}_1|^4$, $\mathbf{A}_2^{-1}=\left(\mathbf{A}\circ\mathbf{A}_1\circ\mathbf{A}_3\right)\big/|\mathbf{A}_2|^4$, $\mathbf{A}_3^{-1}=\left(\mathbf{A}\circ\mathbf{A}_1\circ\mathbf{A}_1\right)\big/|\mathbf{A}_3|^4$ и $|\mathbf{A}|=|\mathbf{A}_1|=|\mathbf{A}_2|=|\mathbf{A}_3|$.

Соотношения (2.1) записываются также в матричном виде

$$
\mathbf{A}_m=\sum_r^4 H_{mr}\left(a_r\mathbf{e}_r\right),\ \left(a_m\mathbf{e}_m\right)=\frac{1}{4}\sum_r^4 H_{mr}\mathbf{A}_r, \qquad (2.4)
$$

где числа $\mathbf{A}_m$ и $a_r\mathbf{e}_r$ отождествляются с $\left(\mathbf{A},\mathbf{A}_1,\mathbf{A}_2,\mathbf{A}_3\right)$ и $\left(a_0,a_1\mathbf{e}_1,a_2\mathbf{e}_2,a_3\mathbf{e}_3\right)$.

Симметричная матрица H_{mr} есть матрица Адамара

$$
\mathbf{H}_4=\begin{pmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{pmatrix}=\begin{pmatrix}\mathbf{H}_2&\mathbf{H}_2\\\mathbf{H}_2&-\mathbf{H}_2\end{pmatrix},\ \mathbf{H}_2=\begin{pmatrix}\mathbf{H}_1&\mathbf{H}_1\\\mathbf{H}_1&-\mathbf{H}_1\end{pmatrix},\ \mathbf{H}_1=1 \qquad (2.5)
$$

порядка четыре с элементами равными числам ±1. Матрица Адамара находится методом Сильвестра рекуррентным вычислением из матриц $\mathbf{H}_1$ и $\mathbf{H}_2$ и широко используется в теории информации. Поскольку первая строка и первый столбец состоят из чисел $+1$, то имеем нормализованную матрицу Адамара. Причём элементы строк матрицы являются дискретными значениями ортогональных функций Уолша. Матрица имеет свойство $\mathbf{H}_4\mathbf{H}_4^T=4\mathbf{I}$ (где $\mathbf{H}_4^T$ и $\mathbf{I}$ - транспонированная и единичная четырехмерные матрицы). Матрица определяется так же, как кронекеровское произведение матриц предыдущего порядка. Порядок матрицы Адамара равняется числу $m=2^n$ $\left(n=0,1,2,\ldots\right)$.

Алгебра квадрачисел, впервые введенная в [1], с операциями сложения, умножения на действительное число, с законом композиции и определениями модуля и обратного числа является естественным расширением алгебры двойных чисел.

Приведем некоторые новые свойства квадрачисел.

Определение. Уравнение $\left|\mathbf{A}-\lambda\right|=0$ представляет собой характеристическое уравнение для квадрачисла, а соответствующий набор чисел – характеристические или собственные числа.

С учетом тождества $\lambda=\lambda\mathbf{e}_0$ в значении модуля квадрачисла произведем замену $a_0\to a_0-\lambda$ и, согласно (2.2), получим равенство

$$\left|\mathbf{A}-\lambda\right|^4=\left(\mathbf{A}-\lambda\right)\circ\left(\mathbf{A}_1-\lambda\right)\circ\left(\mathbf{A}_2-\lambda\right)\circ\left(\mathbf{A}_3-\lambda\right), \tag{2.6}$$

из которого вытекает соотношение

$$\left|\mathbf{A}-\lambda\right|^4=\left(-\lambda\right)^4+S_1\left(-\lambda\right)^3+S_2\left(-\lambda\right)^2+S_3\left(-\lambda\right)+S_4. \tag{2.7}$$

Следовательно имеем выражения

$$\begin{gathered}S_1=\varepsilon_m\mathbf{A}_m=\mathbf{A}+\mathbf{A}_1+\mathbf{A}_2+\mathbf{A}_3,\\ S_2=\frac{1}{2!}\varepsilon_{mr}\mathbf{A}_m\circ\mathbf{A}_r=\mathbf{A}\circ\mathbf{A}_1+\mathbf{A}\circ\mathbf{A}_2+\mathbf{A}\circ\mathbf{A}_3+\mathbf{A}_1\circ\mathbf{A}_2+\mathbf{A}_1\circ\mathbf{A}_3+\mathbf{A}_2\circ\mathbf{A}_3,\\ S_3=\frac{1}{3!}\varepsilon_{mrn}\mathbf{A}_m\circ\mathbf{A}_r\circ\mathbf{A}_n=\mathbf{A}_1\circ\mathbf{A}_2\circ\mathbf{A}_3+\mathbf{A}\circ\mathbf{A}_2\circ\mathbf{A}_3+\mathbf{A}\circ\mathbf{A}_1\circ\mathbf{A}_3+\mathbf{A}\circ\mathbf{A}_1\circ\mathbf{A}_2,\\ S_4=\frac{1}{4!}\varepsilon_{mrnl}\mathbf{A}_m\circ\mathbf{A}_r\circ\mathbf{A}_n\circ\mathbf{A}_l=\mathbf{A}\circ\mathbf{A}_1\circ\mathbf{A}_2\circ\mathbf{A}_3,\end{gathered} \tag{2.8}$$

где $\varepsilon_m=1$ и введены четырехмерные абсолютно симметричные символы со свойствами $\varepsilon_{mr}=\varepsilon_{mrn}=\varepsilon_{mrnl}=1$, если $m\neq r\neq n\neq l$, а остальные значения нулевые. Формулы (2.8) есть однородные симметричные формы для набора квадрачисел разных порядков. Из характеристического уравнения для квадрачисла

$$\left|\mathbf{A}-\lambda\right|^4=\left(-\lambda\right)^4+S_1\left(-\lambda\right)^3+S_2\left(-\lambda\right)^2+S_3\left(-\lambda\right)+S_4=0 \tag{2.9}$$

получим соотношения

$$S_1=\varepsilon_m\lambda_m,\ S_2=\frac{1}{2!}\varepsilon_{mr}\lambda_m\lambda_r,\ S_3=\frac{1}{3!}\varepsilon_{mrn}\lambda_m\lambda_r\lambda_n,\ S_4=\frac{1}{4!}\varepsilon_{mrnl}\lambda_m\lambda_r\lambda_n\lambda_l \tag{2.10}$$

с различными четырьмя характеристическими или собственными числами квадрачисла

$$\begin{aligned}\lambda_1&=a_0+a_1+a_2+a_3,\\ \lambda_2&=a_0-a_1+a_2-a_3,\\ \lambda_3&=a_0+a_1-a_2-a_3,\\ \lambda_4&=a_0-a_1-a_2+a_3.\end{aligned} \tag{2.11}$$

Таким образом, симметричные формы для набора квадрачисел разных порядков равняются симметричным многочленам для характеристических чисел.

Запишем собственные числа посредством матрицы Адамара $\lambda_m=H_{mr}a_r$, где a_r отождествляется с $\left(a_0,a_1,a_2,a_3\right)$.

Модуль квадрачисла определяет метрическую функцию финслеровой геометрии Бервальда-Моора

$$\begin{gathered}F=\left|\mathbf{A}\right|=\left(\lambda_1\lambda_2\lambda_3\lambda_4\right)^{1/4}=\left(H_{1i}H_{2j}H_{3k}H_{4t}a_ia_ja_ka_t\right)^{1/4}=\left(\frac{1}{4!}\varepsilon_{mrnl}H_{mi}H_{rj}H_{nk}H_{lt}a_ia_ja_ka_t\right)^{1/4}=\\ =\left[\left(a_0+a_1+a_2+a_3\right)\left(a_0-a_1+a_2-a_3\right)\left(a_0+a_1-a_2-a_3\right)\left(a_0-a_1-a_2+a_3\right)\right]^{1/4},\end{gathered} \tag{2.12}$$

где число $4!$ есть количество различных перестановок индексов в четырехмерном симметричном символе ε_{mrnl}.

3. Релятивистское уравнение четвертого порядка

Уравнение для энергии и импульса свободной частицы в пространстве-времени Бервальда-Моора

$$\prod_{m}^{4}\left[E-c\left(\boldsymbol{\varepsilon}^{m}\mathbf{p}\right)\right]=\left(m_0c^2\right)^4 \tag{3.1}$$

дает релятивистское и инвариантное уравнение четвертого порядка для скалярной волновой функции [3]

$$\prod_{m}^{4}\left(\frac{1}{c}\frac{\partial}{\partial t}+\boldsymbol{\varepsilon}^{m}\frac{\partial}{\partial\mathbf{x}}\right)\varphi\left(\mathbf{x},t\right)=\left(\frac{m_0c}{\hbar}\right)^4\varphi\left(\mathbf{x},t\right). \tag{3.2}$$

Здесь имеем операторы $E=i\hbar\dfrac{\partial}{\partial t}$ и $\mathbf{p}=-i\hbar\dfrac{\partial}{\partial\mathbf{x}}$ и используются известные инвариантные значения компонентов векторов $\boldsymbol{\varepsilon}^1=\left(1,1,1\right)$, $\boldsymbol{\varepsilon}^2=\left(-1,1,-1\right)$, $\boldsymbol{\varepsilon}^3=\left(1,-1,-1\right)$, $\boldsymbol{\varepsilon}^4=\left(-1,-1,1\right)$, которые удовлетворяют равенствам [3]

$$\sum_{m}^{4}\varepsilon_i^m=0,\ \ \frac{1}{4}\sum_{m}^{4}\varepsilon_i^m\varepsilon_j^m=\delta_{ij},\ \ \frac{1}{4}\sum_{m}^{4}\varepsilon_i^m\varepsilon_j^m\varepsilon_k^m=\varepsilon_{ijk},\ \ 1+\left(\boldsymbol{\varepsilon}^m\boldsymbol{\varepsilon}^r\right)=0,\ \ 1+\left(\boldsymbol{\varepsilon}^m\right)^2=4. \tag{3.3}$$

Перепишем уравнение (3.2) в виде

$$\begin{gathered}\left[\frac{\partial^4}{c^4\partial t^4}+\frac{\partial^4}{\partial x^4}+\frac{\partial^4}{\partial y^4}+\frac{\partial^4}{\partial z^4}-2\left(\frac{\partial^4}{c^2\partial t^2\partial x^2}+\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^2\partial z^2}+\frac{\partial^4}{c^2\partial z^2\partial t^2}+\right.\right.\\ \left.\left.+\frac{\partial^4}{c^2\partial t^2\partial y^2}+\frac{\partial^4}{\partial x^2\partial z^2}\right)+8\frac{\partial^4}{c\partial t\partial x\partial y\partial z}\right]\varphi\left(\mathbf{x},t\right)=\left(\frac{m_0c}{\hbar}\right)^4\varphi\left(\mathbf{x},t\right).\end{gathered} \tag{3.4}$$

Форм-инвариантность уравнения (3.4) обеспечивается известными преобразованиями координат и времени [3-5], записанными, например, в векторной форме [3]

$$\begin{gathered}\mathbf{x}'=\frac{1}{N\left(\mathbf{v}'/c\right)}\left\{\mathbf{x}+\mathbf{v}'t+\frac{1}{4c}\sum_{m}^{4}\boldsymbol{\varepsilon}^m\left(\boldsymbol{\varepsilon}^m\mathbf{v}'\right)\left(\boldsymbol{\varepsilon}^m\mathbf{x}\right)\right\},\\ t'=\frac{1}{N\left(\mathbf{v}'/c\right)}\left[t+\frac{1}{4c^2}\sum_{m}^{4}\left(\boldsymbol{\varepsilon}^m\mathbf{v}'\right)\left(\boldsymbol{\varepsilon}^m\mathbf{x}\right)\right]\end{gathered} \tag{3.5}$$

и ее аналоге [2]

$$\begin{gathered}\mathbf{x}'=\frac{1}{N\left(\mathbf{v}'/c\right)}\left[\mathbf{x}+\mathbf{v}'t+\frac{1}{c}\left\{\mathbf{v}'\mathbf{x}\right\}\right],\\ t'=\frac{1}{N\left(\mathbf{v}'/c\right)}\left[t+\frac{1}{c^2}\left(\mathbf{v}'\mathbf{x}\right)\right],\end{gathered} \tag{3.6}$$

а также в координатном виде [5]

$$\begin{gathered}x_i'=\frac{1}{N\left(\mathbf{v}'/c\right)}\left[x_i+v_i't+\frac{1}{c}\varepsilon_{ijk}v_j'x_k\right],\\ t'=\frac{1}{N\left(\mathbf{v}'/c\right)}\left[t+\frac{1}{c^2}\left(v_i'x_i\right)\right]\end{gathered} \tag{3.7}$$

и матричных формах [4, 5]

$$\mathbf{X}' = \frac{\mathbf{A}(\mathbf{v}')}{|\mathbf{A}(\mathbf{v}')|}\mathbf{X} = \frac{\mathbf{B}(\boldsymbol{\alpha})}{|\mathbf{B}(\boldsymbol{\alpha})|}\mathbf{X},$$
$$\mathbf{A}(\mathbf{v}') = \mathbf{H}_4^{-1}\mathbf{D}\mathbf{H}_4 = \frac{1}{4}\mathbf{H}_4^T\mathbf{D}\mathbf{H}_4,\ \mathbf{B}(\boldsymbol{\alpha}) = \mathbf{H}_4^{-1}\mathbf{K}\mathbf{H}_4 = \frac{1}{4}\mathbf{H}_4^T\mathbf{K}\mathbf{H}_4. \tag{3.8}$$

Диагональные матрицы $\mathbf{D} = \{d_1, d_2, d_3, d_4\}$ и $\mathbf{K} = \{k_1, k_2, k_3, k_4\}$ подобны вещественным симметричным матрицам простой структуры $\mathbf{A}(\mathbf{v}')$ и $\mathbf{B}(\boldsymbol{\alpha})$. Матрица Адамара $\mathbf{H}_4$ представляет собой фундаментальную матрицу для рассматриваемых матриц. Вещественные характеристические числа матриц есть $d_m = H_{mr}v_r'/c$ и $k_m = \exp H_{mr}\alpha_r$, где v_r' и α_r отождествляются с $(1, v_x'/c, v_y'/c, v_z'/c)$ и $(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$, соответственно. Величина $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\}$ есть угловая мера [4], $|\mathbf{A}(\mathbf{v}')|^4 = d_1 d_2 d_3 d_4$, $|\mathbf{B}(\boldsymbol{\alpha})|^4 = k_1 k_2 k_3 k_4$ и $\mathbf{X}\{ct, x, y, z\}$.

Здесь для взаимосвязи относительных скоростей имеем следующие формулы для векторов [3]

$$\mathbf{v}' = \left[\frac{1}{4}\sum_m^4 \frac{\boldsymbol{\varepsilon}^m}{1+(\boldsymbol{\varepsilon}^m\mathbf{v})/c}\right]\left[\frac{1}{4}\sum_m^4 \frac{1}{1+(\boldsymbol{\varepsilon}^m\mathbf{v})/c}\right]^{-1} = \left[-\frac{1}{4}\sum_m^4 \frac{\boldsymbol{\varepsilon}^m(\boldsymbol{\varepsilon}^m\mathbf{v})}{1+(\boldsymbol{\varepsilon}^m\mathbf{v})/c}\right]\left[\frac{1}{4}\sum_m^4 \frac{1}{1+(\boldsymbol{\varepsilon}^m\mathbf{v})/c}\right]^{-1} \tag{3.9}$$

и их компонент

$$v_x' = -\frac{\begin{vmatrix} v_x & v_z & v_y \\ v_y & 1 & v_x \\ v_z & v_x & 1 \end{vmatrix}}{\begin{vmatrix} 1 & v_z & v_y \\ v_z & 1 & v_x \\ v_y & v_x & 1 \end{vmatrix}},\ v_y' = \frac{\begin{vmatrix} v_x & 1 & v_y \\ v_y & v_z & v_x \\ v_z & v_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & v_z & v_y \\ v_z & 1 & v_x \\ v_y & v_x & 1 \end{vmatrix}},\ v_z' = -\frac{\begin{vmatrix} v_x & 1 & v_z \\ v_y & v_z & 1 \\ v_z & v_y & v_x \end{vmatrix}}{\begin{vmatrix} 1 & v_z & v_y \\ v_z & 1 & v_x \\ v_y & v_x & 1 \end{vmatrix}}. \tag{3.10}$$

Используя значения векторных произведений (1.7), (1.8), запишем скорость $\mathbf{v}'$ и $N(\mathbf{v})$ в новых векторных формах [2]

$$\mathbf{v}' = -\frac{\mathbf{v}[1-(\mathbf{vv})] - \{\mathbf{vv}\} + \{\mathbf{v}\{\mathbf{vv}\}\}}{1-(\mathbf{vv}) - \frac{1}{3}(\mathbf{v}\{\mathbf{vv}\})},$$
$$N(\mathbf{v}') = 1 + (\mathbf{v}'\mathbf{v}')^2 - 2[(\mathbf{v}'\mathbf{v}') + \{\mathbf{v}'\mathbf{v}'\}\{\mathbf{v}'\mathbf{v}'\}] + \frac{4}{3}(\mathbf{v}'\{\mathbf{v}'\mathbf{v}'\}). \tag{3.11}$$

При преобразованиях остается форм-инвариантным модуль квадрачисла (2.12) (или метрическая функция глобального пространства-времени Бервальда-Моора) в векторной форме

$$\left\{\prod_m^4 \left[ct + (\boldsymbol{\varepsilon}^m\mathbf{x})\right]\right\}^{1/4} = \left\{\prod_m^4 \left[ct' + (\boldsymbol{\varepsilon}^m\mathbf{x}')\right]\right\}^{1/4}. \tag{3.12}$$

Наконец, используя гамильтонов формализм, запишем уравнение (3.2) для частицы в электромагнитном поле в операторном виде

$$\prod_m^4 \left[\left(i\hbar\frac{\partial}{c\partial t} + e\varphi\right) + \boldsymbol{\varepsilon}^m\left(-i\hbar\frac{\partial}{\partial\mathbf{x}} - \frac{e}{c}\mathbf{A}\right)\right]\varphi(\mathbf{x}, t) = (m_0 c)^4 \varphi(\mathbf{x}, t), \tag{3.13}$$

где φ и $\mathbf{A}$ есть скалярный и векторный потенциалы поля.

4. Матричное представление алгебры квадрачисел и релятивистские уравнения первого порядка

Рассмотрим некоторый вариант матричного представления алгебры квадрачисел. Определим двумерные матрицы

$$
\begin{gathered}
T=\begin{pmatrix} a_0-a_3 & a_2-a_1 \\ a_2-a_1 & a_0-a_3 \end{pmatrix},\ \det T=\left(a_0^2+a_3^2\right)-\left(a_1^2+a_2^2\right)+2\left(a_1a_2-a_0a_3\right), \\
T_1=\begin{pmatrix} a_0+a_3 & a_2+a_1 \\ a_2+a_1 & a_0+a_3 \end{pmatrix},\ \det T_1=\left(a_0^2+a_3^2\right)-\left(a_1^2+a_2^2\right)-2\left(a_1a_2-a_0a_3\right), \\
T_2=\begin{pmatrix} a_0-a_3 & -a_2-a_1 \\ -a_2-a_1 & a_0-a_3 \end{pmatrix},\ \det T_2=\left(a_0^2+a_3^2\right)-\left(a_1^2+a_2^2\right)+2\left(a_1a_2-a_0a_3\right), \\
T_3=\begin{pmatrix} a_0+a_3 & -a_2-a_1 \\ -a_2-a_1 & a_0+a_3 \end{pmatrix},\ \det T_3=\left(a_0^2+a_3^2\right)-\left(a_1^2+a_2^2\right)-2\left(a_1a_2-a_0a_3\right).
\end{gathered}
\tag{4.1}
$$

Тогда матричное представление алгебры квадрачисел есть одна из четырех четырехмерных матриц в следующем блочном виде

$$
P_1=\begin{pmatrix} T & 0 \\ 0 & T_1 \end{pmatrix},\ P_2=\begin{pmatrix} T_1 & 0 \\ 0 & T_2 \end{pmatrix},\ P_3=\begin{pmatrix} T_2 & 0 \\ 0 & T_3 \end{pmatrix},\ P_4=\begin{pmatrix} T_3 & 0 \\ 0 & T \end{pmatrix}
\tag{4.2}
$$

с определителем

$$
\begin{gathered}
\det P_1=\det P_2=\det P_3=\det P_4=\left[\left(a_0^2+a_3^2\right)-\left(a_1^2+a_2^2\right)\right]^2-4\left(a_1a_2-a_0a_3\right)^2= \\
=a_0^4+a_1^4+a_2^4+a_3^4-2\left(a_0^2a_1^2+a_0^2a_2^2+a_0^2a_3^2+a_1^2a_2^2+a_2^2a_3^2+a_3^2a_1^2\right)+8a_0a_1a_2a_3,
\end{gathered}
\tag{4.3}
$$

равным четвертой степени модуля квадрачисла $|\mathbf{A}|^4$. Базисными элементами квадрачисла являются матрицы

$$
\begin{gathered}
\mathbf{e}_0=\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{pmatrix},\ \mathbf{e}_1=\begin{pmatrix} 0&-1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&1&0 \end{pmatrix}, \\
\mathbf{e}_2=\begin{pmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0 \end{pmatrix},\ \mathbf{e}_3=\begin{pmatrix} -1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1 \end{pmatrix},
\end{gathered}
\tag{4.4}
$$

вытекающие, например, из представления в виде P_1 и имеющие свойства (1.4) со следами $\operatorname{tr}\mathbf{e}_i=0$, $\operatorname{tr}\left(\mathbf{e}_i\circ\mathbf{e}_j\right)=4\delta_{ij}$.

Рассмотрим набор квадрачисел

$$\begin{cases} E\mathbf{e}_0 + cp_x\mathbf{e}_1 + cp_y\mathbf{e}_2 + cp_z\mathbf{e}_3, \\ E\mathbf{e}_0 - cp_x\mathbf{e}_1 + cp_y\mathbf{e}_2 - cp_z\mathbf{e}_3, \\ E\mathbf{e}_0 + cp_x\mathbf{e}_1 - cp_y\mathbf{e}_2 - cp_z\mathbf{e}_3, \\ E\mathbf{e}_0 - cp_x\mathbf{e}_1 - cp_y\mathbf{e}_2 + cp_z\mathbf{e}_3 \end{cases} \tag{4.5}$$

и, заменяя $\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ на матричные операторы, получим четыре уравнения первого порядка для четырех волновых функций в случае чистого ансамбля квантовых систем

$$\begin{aligned} &i\hbar\mathbf{e}_0 \frac{\partial \psi}{c\partial t} - i\hbar\mathbf{e}_1 \frac{\partial \psi}{\partial x} - i\hbar\mathbf{e}_2 \frac{\partial \psi}{\partial y} - i\hbar\mathbf{e}_3 \frac{\partial \psi}{\partial z} - \boldsymbol{\beta} m_0 c^2 \psi = 0, \\ &i\hbar\mathbf{e}_0 \frac{\partial \psi_1}{c\partial t} + i\hbar\mathbf{e}_1 \frac{\partial \psi_1}{\partial x} - i\hbar\mathbf{e}_2 \frac{\partial \psi_1}{\partial y} + i\hbar\mathbf{e}_3 \frac{\partial \psi_1}{\partial z} - \boldsymbol{\beta} m_0 c^2 \psi_1 = 0, \\ &i\hbar\mathbf{e}_0 \frac{\partial \psi_2}{c\partial t} - i\hbar\mathbf{e}_1 \frac{\partial \psi_2}{\partial x} + i\hbar\mathbf{e}_2 \frac{\partial \psi_2}{\partial y} + i\hbar\mathbf{e}_3 \frac{\partial \psi_2}{\partial z} - \boldsymbol{\beta} m_0 c^2 \psi_2 = 0, \\ &i\hbar\mathbf{e}_0 \frac{\partial \psi_3}{c\partial t} + i\hbar\mathbf{e}_1 \frac{\partial \psi_3}{\partial x} + i\hbar\mathbf{e}_2 \frac{\partial \psi_3}{\partial y} - i\hbar\mathbf{e}_3 \frac{\partial \psi_3}{\partial z} - \boldsymbol{\beta} m_0 c^2 \psi_3 = 0, \end{aligned} \tag{4.6}$$

где введена матрица

$$\boldsymbol{\beta} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \ \boldsymbol{\beta}^4 = 1 . \tag{4.7}$$

Плоская волна для случая первого уравнения в (4.6) для стационарных состояний представляется в виде четырех-компонентной волновой функции

$$\psi = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} e^{\frac{i\mathbf{px}}{\hbar}} \tag{4.8}$$

с постоянными числами u_1, u_2, u_3, u_4. После подстановки волновой функции (4.8) в релятивистское уравнение первого порядка (4.6) получим систему алгебраических уравнений

$$\begin{aligned} -Eu_1 &= -cp_z u_1 + c\left(p_y - p_x\right)u_2 - m_0c^2u_4, \\ -Eu_2 &= c\left(p_y - p_x\right)u_1 - cp_z u_2 + m_0c^2u_3, \\ -Eu_3 &= m_0c^2u_2 - cp_z u_3 + c\left(p_y + p_x\right)u_4, \\ -Eu_4 &= m_0c^2u_1 + c\left(p_y + p_x\right)u_3 + cp_z u_4. \end{aligned} \tag{4.9}$$

Однородная система (4.9) имеет решение при равенстве нулю детерминанта

$$\begin{vmatrix} E-cp_z & cp_y-cp_x & 0 & -m_0c^2 \\ cp_y-cp_z & E-cp_z & m_0c^2 & 0 \\ 0 & m_0c^2 & E+cp_z & cp_y+cp_x \\ m_0c^2 & 0 & cp_y+cp_x & E+cp_z \end{vmatrix}=0. \qquad (4.10)$$

Равенство (4.10) дает соотношение $\prod_m^4\left[E-c\left(\boldsymbol{\varepsilon}^m\mathbf{p}\right)\right]-\left(m_0c^2\right)^4=0$, совпадающее с (3.1). Аналогичные результаты получаются и для других уравнений из (4.6).

5. Заключение

Для случая квадрачисел восполняется пробел в представлении соответствующей алгебры недиагональными четырехмерными матрицами, которые используются при получении четырех уравнений первого порядка для четырех – компонентной волновой функции в квантовом описании движения частиц. Причем каждому значению импульса соответствует три собственных значения энергии принадлежащих неизвестной частице и одно собственное значение неизвестной античастицы, либо наоборот. Возможен также вариант двух частиц и двух античастиц. Так, в случае одного уравнения Дирака в системе бикватернионов соответствующая однородная система, аналогичная (4.10), имеет решение при равенстве нулю соответствующего детерминанта, то есть $\left(E^2-c^2\mathbf{p}^2-m_0^2c^2\right)^2=0$. Здесь для каждого значения импульса имеем дважды вырожденные собственные значения энергии электрона и позитрона. В этом и состоит различие систем бикватернионов и квадрачисел в релятивистских теориях. Нахождение свойств и решений релятивистских уравнений (3.4), (3.13) и (4.6) для различных физических ситуаций представляет отдельную задачу.

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RELATIVITY IN "COSMIC SUBSTRATUM" AND THE UHECR PARADOX

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Abstract

The special theory of relativity predicts an existence of the so called Greisen-Zatsepin-Kuzmin (GZK) phenomenon according to which cosmic ray protons coming from cosmological distances with energies above $5 \times 10^{19} eV$ should not be observed on earth. The cut-off value corresponds to the threshold energy of photo-pion production by protons colliding with soft CMBR photons pervading the universe. Experimentally a number of cosmic ray events have been detected above this GZK limit which is known as the ultra high energy cosmic rays (UHECR) paradox. We suggest a resolution of this paradox through a heuristic modification of the relativistic kinematics keeping in mind that it should not lead to predictions different from those of SR in the well tested domains. It is shown that the absence of GZK limit in UHECR spectrum can be explained in terms of a non-preferred frame effect of the solar system for its motion with respect to the rest frame of the universe (the cosmic substratum). The novel theory can also be called a "doubly special relativistic"(DSR) one but now in a sense different from that of the currently known DSR theories.

1 INTRODUCTION

The possibility that Lorentz invariance can be violated in nature has currently become a subject of interest. People often doubt if the special relativity (SR) is only an approximate symmetry of nature [1, 2]. To give a quantitative measure of Lorentz-invariance violation (LIV), one can build up a test theory where the Lagrangian of electrodynamics can be slightly deformed by adding to it a tiny Lorentz violating term. One such deformation considered by the authors of Ref.[1] (see also [3]) following standard practice causes the speed of light c to differ from the maximum attainable speed c_0 (which hereafter, unless stated otherwise, will be assumed to be equal to 1) by a small velocity parameter ϵ of the theory. The obvious consequence of this consideration is the existence of a preferred inertial frame of reference.

It is a common practice and also reasonable to assume this preferred frame to be "the rest frame of the universe" (Σ_0) with respect to which the cosmic microwave background radiation (CMBR) is isotropic. Let us call it the rest frame of the cosmic substratum (RFCS).

Precision tests for anisotropies in velocity of light due to the motion of the solar system relative to the CMBR frame have set a limit on this ϵ [1, 4],

$$|1 - c| = |\epsilon| < 3 \times 10^{-22}. \qquad (1)$$

However it has been argued [1, 2, 3] that stronger constraints on ϵ can be obtained, not from precision tests, but from observations on ultra high energy cosmic rays (UHECR). For example, if $c < 1$ it has been shown that the mere detection of primary proton energy up to 100 EeV set the bound on ϵ more than one order of magnitude stronger:

$$|\epsilon| < 5 \times 10^{-24}. \qquad (2)$$

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The physical basis for obtaining such a bound is that a particle can be super luminal in vacuum (if $c < 1$) in which case, a proton being a charged particle will in its passage, quickly lose energy through the so called "vacuum Cerenkov radiation" and will therefore fail to be detected with the super luminal speed. The last bound on $|\epsilon|$ is obtained by equating the speed of proton at 100 EeV with the speed of light c, then subtracting it from unity, the latter being the limiting speed of SR. The limit on ϵ thus obtained does not require (unlike the way it is obtained through precision test mentioned before) any assumption regarding the motion of the laboratory frame with respect to Σ_0.

LIV is also much discussed in connection with one of the most puzzling paradoxes in physics concerning UHECRs. One quite robust predictions of special relativity is the existence of the so called Greisen-Zatsepin-Kuzmin (GZK) phenomenon, which tells us that cosmic ray protons coming from cosmological distances with energies above certain limiting value (GZK cutoff), should not be observed on Earth. The predicted value for this catastrophic cutoff is $5 \times 10^{19} eV$. This value corresponds to the threshold energy for photopion production by cosmic ray protons interacting with soft CMBR photons which pervades the universe. However some recent experiments have shown that this relativistically calculated threshold energy seems to be too low. Indeed recently ground based detectors have detected over about a hundred events near and above the GZK cutoff and a double digit number of events with energies at or above $10^{20} eV$. The highest energy cosmic ray so far has been the $3.2 \times 10^{20} eV$ detected by the Fly's Eye air shower detector in Utah [5]. However if the sources of UHECRs are really extragalactic (there are ample reasons to believe so [6]) and since the calculation of GZK limit is so robust that even one event at $10^{20} eV$ "appears surprising"[7]. The arrival of UHECR on Earth with energies above the GZK threshold is known as the UHECR paradox [8, 9, 10] mentioned in the beginning of this paragraph.

There have been exotic proposals in the literature which try to explain the trans-GZK cosmic ray events in the framework of LIV theories which assume the existence of a preferred frame [2, 9, 11]. Let us call them preferred frame theories. As an example, according to one most popular scenarios [12], existence of different maximal speeds for different particle species is assumed and they are also assumed in general to differ from the speed of light in vacuo [see ref. [2] and references therein]. In this way, introduction of small LIV has been shown to have effects that increase rapidly with energy in such a manner that ultimately inelastic collisions with CMBR photons become kinematically forbidden [2].

However there are other class of theories known as the doubly special relativistic (DSR) theories which consider deformation of relativistic dispersion relations for photons and massive particles. Although cosmic ray paradox primarily provides encouragement for such theories, the revision of dispersion relation is often motivated from quantum-gravity considerations, according to which a fundamental length or energy scale (plank length or plank energy) should play a role [8]. DSR theories try to avoid the preferred frame issue prompted by the introduction of such scales in the theory (since length and energy are frame dependent quantities) by introducing the notion of an invariant length or energy scale in addition to the constant c of the usual relativity theory. DSR theories therefore formulates the postulates of SR in ways in order to introduce observer independent length or energy scales. Although one [13] or the other [14] forms of DSR theories are interesting and intellectually satisfying, these are still in a preliminary stage, in so far as their efficacy in solving the threshold anomaly is concerned [10]. In any case, the cosmic ray paradox provides ample reasons for new alternatives to the standard relativity theory. Indeed if the identification of UHECR as protons produced by distant active galaxies eventually turns out to be absolutely correct, one of the varieties of DSR theories or that of the preferred frame ones mentioned earlier can be strong contenders as the candidates describing new physics. The present paper proposes a theory of the latter variety with a very different flavour. It will be shown that the velocity of the solar system with respect to the rest frame of the universe might play a role in explaining the paradox.

In an effort to look for new physics, when one considers theories involving LIV one still believes that behavior of moving rods and clocks is still governed by the Lorentz transformation (LT) however other laws of physics might not strictly remain covariant under LT. For example one may consider the possibility that causal cone need not coincide with the light cone [15], i.e the speed of light may not be the same as the invariant speed "c" of LT.

However if one is prepared to do away with the principle of relativity, or in otherwards if one believes in the existence of a preferred inertial frame, there is no point in holding on to the belief that standard rods and clocks of different inertial frames behave strictly according to LT. Note that after all LT is a consequence of the relativity principle[2].

Hence in search for a new physics one may consider the possibility of a deformed LT (not just a deformed dispersion relation) to relate observations performed by different inertial observers.

Once such a transformation is guessed, other aspects of kinematics such as expressions for momentum $\mathbf{p}$ and energy E of a particle or the dispersion relation can be obtained through a kind of 4-vector formulation (see below).

Clearly the predictions of the deformed LT will be different from those of the relativity theory. However the difference in the predictions must be undetectable in the domain where special relativity has been tested beyond doubt.

In the present paper we shall look for such a transformation that will be capable to explain the UHECR paradox and at the same time will be able to reproduce the standard relativistic results. We know that Einstein obtained his transformations deductively from his relativity and the "constancy of velocity of light" (CVL) postulates. If the relativity postulate is sacrificed what guidelines should one follow in order to guess the transformation equation? The next section will provide an answer to this question.

2 TRANSFORMATION EQUATION

Although the kinematics of relativity theory was obtained by Einstein from a general principle like the relativity of motion and a principle concerning the speed of light, the operative aspects of these postulates used in the derivation can be laid down in more concrete terms. Indeed if one consults a standard text book on relativity, one finds that the derivation of LT starts from the assumption of a linear transformation with unknown coefficients which are determined using essentially the following operative inputs:
(1) The coordinate clocks in any inertial frame are assumed to be synchronized by light signal following the Einstein synchrony or the standard synchrony, according to which the one-way-speed (OWS) of light is assumed to be the same as its two-way-speed (TWS) in any direction[16, 17].
(2) The speed of light[3] is the (i) same and (ii) isotropic with respect to all inertial observers.
(3) Measuring rods placed perpendicular to its direction of motion do not undergo any contraction or elongation with respect to its rest length.
The first of the above is just a synchronization convention but the other two items are the

[2]The relativity principle turns out to be a sufficient condition for LT (if coordinate clocks are synchronized by light signal following Einstein's convention), however it is not a necessary condition. In other words LT may describe the kinematical world even if the principle is seen to be violated in other realms of physics. But here we emphasize that if the relativity principle is sacrificed, LT loses its very foundation.

[3]If standard synchrony is not used, the phrase "speed of light" then means TWS of the same, which is a synchrony independent quantity.

consequences of the relativity principle[4]. A little amplification of this statement in respect of item (2) may be in order. One might think that (2) is equivalent to Einstein's CVL postulate. This is indeed a misconception [19]. The CVL postulate of Einstein refers to constancy with respect to change in the velocity of light source. In effect this postulate emphasises the wave character of light. Once wave is launched it is no longer linked to the source. Indeed Einstein's second postulate concerning the speed of light in conjunction with the principle of relativity only imply the constancy with respect to the change of the inertial observer as well [19].

In a preferred frame theory where the principle of relativity is expected to be violated, the transformation equations cannot be obtained with item (2) as an input which, as explained, depends on the relativity principle although CVL can be used in the stationary frame. As regards input (1) however there is no difficulty but there is no special advantage in synchronizing coordinate clocks using light signal. One may then ask what if the clocks were synchronized by some other signal say an "acoustic signal" for example[5]. One may consider a substratum which can support such a signal and through which different inertial frames are supposed to move. To effect the synchronization, like the standard synchrony we shall stipulate the OWS of the signal along a straight line be equal to its TWS along the line in any frame Σ_k. It has been shown elsewhere [16] that if input (2) is withheld, and the coordinate clocks of any inertial frame is synchronized by "acoustic signal", the transformation equation between a preferred frame Σ_0 and an arbitrary inertial frame Σ_k can be obtained as,

$$x_k = (a_{kx}/a_{ky})(1 - u_{0k}^2/a_0^2)^{-1/2}(x_0 - u_{0k}t_0), \tag{3}$$

$$t_k = (a_0/a_{ky})(1 - u_{0k}^2/a_0^2)^{-1/2}(t_0 - u_{0k}x_0/a_0^2), \tag{4}$$

where x_0, t_0 and x_k, t_k refer to space-time coordinates as measured with respect to the stationary (Σ_0) and moving frame (Σ_k) respectively. The relative velocity of Σ_k with respect to Σ_0 has been denoted by u_{0k}. As regards other terms, a_0 denotes the isotropic "acoustic speed" (two way or one way) in the stationary substratum, whereas a_{kx} and a_{ky} are the TWS' of the synchronizing signal in Σ_k parallel (along the x-direction) and perpendicular (along the y-direction) to its direction of motion respectively. Note that in general a_{kx} and a_{ky} are expected to be functions of u_{0k} and hence the above equations are only formal and not usable unless some phenomenological assumptions are made regarding these functions. For optical signal synchronization we replace the terms a_{kx}, a_{ky} and a_0 in Eqs.(3) and (4) by c_{kx}, c_{ky} and c_0 respectively where the latter three terms represent the respective speeds of the light signal. In the relativistic world, by input (2), one finds in any Σ_k.

$$c_{kx}(u_{0k}) = c_{ky}(u_{0k}) = c_0 \tag{5}$$

and,
the above equations (Eqs.(3) and (4)) turns out to be LT under optical synchronization.

We now ask what if Eq.(5) is approximately valid, so that the speed of light is almost and not quite independent of the speed of the reference frame with respect to a "preferred" one. Note that the transformation equations (3) and (4) are now most appropriate to deal with such questions. We now wish to use input (2) in these equations by modifying the former minimally. We try this by preserving the isotropy component (2 ii) and relaxing the constancy

[4]See any standard text book derivation of LT (for example see[18]) which explains and uses item (3) as a consequence of the relativity principle.

[5]The choice of the phrase "acoustic signal" is just symbolic. We only emphasise here that the transformation equations can be obtained without any reference to the physical nature of the synchronizing agent. However soon we will resort to optical synchronization (see below)

component (2 i) of the said input. Thus TWS of light is assumed to be isotropic in any frame Σ_k and now we conjecture that this isotropic speed depends on u_{0k} in following way,

$$c_{kx} = c_{ky} = c_k = c_0(1 + \alpha u_{0k}^2/c_0^2)^{1/2}, \tag{6}$$

where we have introduced a dimensionless constant α which is assumed to have such a small value that the proposed theory does not differ in its predictions from that so far tested relativistically. Clearly Eq.(5) is now replaced by Eq.(6) which approximately reduces to Eq.(5) for $\alpha u_{0k}^2/c_0^2 << 1$. Note that, depending on the smallness of α, u_{0k} can be very close to c_0 and yet the last condition can still remain valid. We shall show below that if the phenomenological assumption described by Eq.(6) is believed to be true, the UHECR paradox can be explained in terms of the motion of the solar system with respect to the RFCS.

We conclude this section by quoting the relevant transformation equations which are obtained by plugging in Eq.(6) in Eqs.(3) and (4):

$$x_k = (1 - u_{0k}^2/c_0^2)^{-1/2}(x_0 - u_{0k}t_0), \tag{7}$$

$$t_k = (1 + \alpha u_{0k}^2/c_0^2)^{-1/2}(1 - u_{0k}^2/c_0^2)^{-1/2}(t_0 - u_{0k}x_0/c_0^2). \tag{8}$$

3 METRIC AND 4-VECTORS

In SR classical expressions for momentum and energy had to be altered in order for the conservation principles to be Lorentz covariant. These expressions can easily be obtained by writing the energy momentum conservation in terms of a 4-vector relation. The energy momentum 4-vectors are obtained in terms of the invariant interval of SR.

In the present situation, such a thing cannot be obtained easily since one recalls that the notion of invariant interval of SR is an outcome of the existence of an invariant speed c (c_0) of the theory. In the present context in absence of such a speed the invariant interval does not exist in the way it existed in SR. Besides, since there should exist a preferred frame, in order to obtain the correct conservation principle (or to obtain definition of energy and momentum) an appeal to covariance of physical laws cannot be made. In the following we suggest a way out. From the transformations (3) and (4) together with

$$y_k = y_0, \; z_k = z_0, \tag{9}$$

it is evident that

$$(c_{ky}/c_{kx})^2 x_k^2 + y_k^2 + z_k^2 - c_{ky}^2 t_k^2 = x_0^2 + y_0^2 + z_0^2 - c_0^2 t_0^2. \tag{10}$$

Recalling Eq.(6) the above relation reads

$$x_k^2 + y_k^2 + z_k^2 - c_k^2 t_k^2 = x_0^2 + y_0^2 + z_0^2 - c_0^2 t_0^2, \tag{11}$$

and in terms of the differential intervals one obtains the following invariant interval

$$d\tau^2 = dt_k^2 - (1/c_k^2)(dx_k^2 + dy_k^2 + dz_k^2), \tag{12}$$

and by analogy with SR we call $d\tau$ as the proper time interval.

Note that the *expression* for the above invariant interval is frame dependent unlike the case in SR because of the presence of $c_k(u_{0k})$ in the last expression. However one can easily develop a 4-vector formulation like that in SR by defining the 4-momentum of a particle of mass m as

$$\mathcal{P} = (m\gamma_k, m\gamma_k(\mathbf{v_k})_i), \tag{13}$$

with

$$\gamma_k = (1 - (v_k)^2/c_k^2)^{-1/2}, \tag{14}$$

where $(\mathbf{v_k})_i$ represents the i^{th} component of the three velocity $\mathbf{v_k}$ of the particle in Σ_k. Imposing

$$\mathcal{P}.\mathcal{P} = \mathcal{P}^2 = \eta_{\mu\nu} p^\mu p^\nu = invariant \tag{15}$$

where

$$\eta_{\mu\nu} = (1, -1/c_k^2, -1/c_k^2, -1/c_k^2) \tag{16}$$

one obtain the dispersion relation for the particle in any frame Σ_k as

$$E_k^2 = p_k^2 c_k^2 + m^2 c_k^4 \tag{17}$$

where

$$\mathbf{p_k} = m\mathbf{v_k}/(1 - v_k^2/c_k^2)^{1/2} = m\gamma_k \mathbf{v_k} \tag{18}$$

and

$$E_k = m\gamma_k c_k^2 \tag{19}$$

Although Eqs.(17), (18) and (19) look like the corresponding equations in SR, they are different since the relations are dependent on the frame considered, since now $c_k = c_k(u_{0k})$.

Note that expressions for energy, momentum and the dispersion relation reduce to the usual relativistic ones in the preferred frame Σ_0.

4 VELOCITY TRANSFORMATIONS

Our theory therefore does not predict outcomes which are different from those in SR in Σ_0. The question now arises as to whether it is possible to predict a result significantly different from that of SR in a frame of reference (solar system) which is moving with a non-relativistic speed ($u_{0k} \approx 10^{-3}$), with respect to RFCS (Σ_0). The answer seems to be affirmative and we suspect that the resolution of the cosmic ray paradox lies in such a non-preferred frame effect of the theory. To understand this question let us first quote the velocity transformation laws that follow from the transformation relations. We first consider a particle (say a proton) travelling along the x-direction with speed v_0 with respect to Σ_0. The corresponding speed in Σ_k will be obtained from the transformations (7) and (8) as

$$v_k = (1 + \alpha u_k^2/c_0^2)^{1/2}(v_0 \pm u_k)/(1 \pm v_0 u_k/c_0^2), \tag{20}$$

where we have put u_k for u_{0k} for brevity. We shall consider the speeds of the cosmic ray protons in Σ_0 to be very close to unity,

$$v_0 = 1 - \epsilon_0, \tag{21}$$

where ϵ_0 is of the order of 10^{-22} (see below). With this range of values for v_0 and recalling $u_k \approx 10^{-3}$, the velocity transformation formula (Eq.(20)) can be approximated as

$$v_k = (1 + \alpha u_k^2)^{1/2} v_0, \tag{22}$$

where the terms of the order of ${\epsilon_0}^2$ and $\epsilon_0 u_k$ have been neglected in comparison to unity. Although in obtaining Eq.(22) we have assumed the motion of the particles to be along the x-direction, interestingly it can be shown that the above relation holds even for particle travelling along *any* direction under the above mentioned approximation.

5 VELOCITY THRESHOLD AND THE RESOLUTION OF THE PARADOX

Using the usual relativistic energy formula valid in Σ_0

$$E_0 = m/(1 - {v_0}^2)^{1/2}, \tag{23}$$

the velocity threshold for proton in Σ_0 corresponding to the GZK threshold energy $E_{0th} = 5 \times 10^{19}$eV speed can be calculated as

$$v_{0th} = 1 - 1.76 \times 10^{-22}. \tag{24}$$

Now we will provide a possible explanation for the apparent detection of the trans-GZK events in terms of the motion of the solar system with respect to the CMBR frame. A surprisingly small value of the parameter α of the theory will be found to do this job. In order to demonstrate this we first anticipate (see below) this value for α:

$$\alpha = 3.42 \times 10^{-16}. \tag{25}$$

From Eq.(22) the speed of light in the laboratory frame Σ_k (for which $u_k \approx 10^{-3}$) can approximately be written as

$$c_k = 1 + \eta_k \approx (1 + \alpha u_k^2/2), \tag{26}$$

where η_k measures the departure of the light speed value in Σ_k from unity. Clearly

$$\eta_k \approx \alpha u_k^2/2 = 1.71 \times 10^{-22}. \tag{27}$$

However this term is absent in the preferred frame and as we have seen, the special relativistic results (formulas for energy, momentum, dispersion relation etc) hold in Σ_0 and hence GZK cut off value for proton energy obtained from SR is still valid in the CMBR frame. We shall see how this threshold value may appear to be about $3 \times 10^{20} eV$ in Σ_k as detected by Fly's eye air shower detector. Without going into the details of the experimental analysis we now *speculate* that the observed energy of a cosmic ray particle is its *relativistic* energy. We denote it byE_k^{rel} which is given by,

$$E_k^{rel} = mc_0^2/(1 - v_k^2/c_0^2)^{1/2}, \tag{28}$$

where we have explicitly retained c_0 for clarity.

Returning to the energy formula for a particle in our frame Σ_k one notes that its value in the solar system (laboratory) practically does not differ from its relativistic value in Σ_0, as

$$E_k = mc_k^2/(1 - v_k^2/c_k^2)^{1/2} \approx mc_0^2/(1 - v_0^2/c_0^2)^{1/2} = E_0, \tag{29}$$

where we have used

$$v_k^2/c_k^2 = v_0^2/c_0^2, \tag{30}$$

that follows from Eqs.(6) and (22).We have also assumed in Eq.(29), $c_k^2 \approx c_0^2$, since the error involved in such an approximation is only about 1 part in 10^{22}, which can be disregarded since ultimately we will have to explain a discrepancy much bigger than this error (3×10^{20} eV against $5 \times 10^{19} eV$).

The above energy formula (Eq.(29)) can also be expressed as

$$\begin{aligned} E_k &= mc_k^2/[1 - v_k^2/c_0^2(1 + \alpha u_k^2/c_0^2)]^{1/2} \\ &\approx m/[(1 - v_k^2)^{1/2}(1 + \alpha u_k^2/2\epsilon_k)^{1/2}], \end{aligned} \tag{31}$$

where in arriving at the last approximate expression we have put $c_0 = 1$ again and defined,

$$\epsilon_k = 1 - v_k. \tag{32}$$

Using Eq.(28), one obtains from Eq.(31)

$$E_k^{rel} = E_k(1 + \alpha u_k^2/2\epsilon_k)^{1/2}, \tag{33}$$

which by Eq.(29) can be written as

$$E_k^{rel} \approx E_0(1 + \alpha u_k^2/2\epsilon_k)^{1/2}. \tag{34}$$

Note that this is the relativistic energy of a particle moving with speed v_k. We now calculate this relativistic value of energy for a proton having the GZK threshold energy E_{0th}. Using the transformation (22) and assuming $v_0 = v_{0th}$, where the later is given by Eq.(24), one obtains the corresponding v_k as

$$v_k = 1 - \epsilon_k = 1 - 5 \times 10^{-24}, \tag{35}$$

giving

$$\epsilon_k = 5 \times 10^{-24}. \tag{36}$$

Using this value for ϵ_k and assumed value for α (Eq.(25)) and finally putting $E_0 = E_{0th}$, we find from Eq.(34)

$$E_k^{rel} \approx 3 \times 10^{20} eV, \tag{37}$$

which is nothing but the energy of the 300 EeV event detected by the Fly's Eye.

Therefore we conclude that the value of the parameter $\alpha \approx 3.42 \times 10^{-16}$ can explain the apparent detection of trans-GZK events. Note that the above calculation (or the choice of the value for α) depends on the assumption that the 300 EeV event corresponds to the cut off value. However, it may not be so, indeed in future, a bit higher energy event may be detected, in which case the value of α will slightly go up. But this will not pose much problem since the assumed value of α is so small, it has enough flexibility to increase even substantially without contradicting SR in the tested domain.

6 DOUBLY RELATIVISTIC ?

Velocity transformation formulas in the Galilean (classical) world do not contain a constant velocity parameter whereas the same in the relativistic world contains one constant velocity parameter c (c_0 according to the present notation).

Returning to the expression given by Eq.(6),

$c_k = c_0(1+\alpha u_k^2/c_0^2)^{1/2}$ One may note that instead of expressing c_k in terms of a dimensionless constant α, one may also write the same as

$$c_k = c_0(1 + u_k^2/\xi^2)^{1/2} \tag{38}$$

where $\xi = \alpha^{-1/2} c_0$ is a constant velocity parameter of the theory. The consequent velocity transformation laws (Eqs.(22)) therefore are governed by *two* constant velocity parameters (instead of *one* as in SR), c_0 and ξ and hence the present theory can also be called a doubly relativistic one in a sense different from that of currently known doubly special relativistic theories advocated by Amelino-Camelia and others.

7 DISCUSSIONS

In this paper we have shown that the UHECR paradox can be explained in terms of a non-preferred frame effect of the laboratory frame which is moving with velocity ≈ 300 km/sec with respect to the preferred one, assumed to be at rest with CMBR frame. Unlike some earlier efforts (the Coleman Glashow scheme for example) which consider LIV but assume that the physical kinematics is still Lorentzian, we propose to modify the transformation equation itself. Deformed LT are generally discussed in connection with test theories like that of Robertson [20] or Mansouri and Sexl [21] on which improved tests of SR are often based (see for example [22]). But they are not usually considered to represent a new physics that may provide a solution for the UHECR paradox.

Some authors find it troublesome giving up the principle of relativity. In the so called "doubly special relativistic" theories, the particle dispersion relation is modified but the introduction of an invariant length or energy scale in addition to the invariant velocity scale of SR, the "relativity of inertial frames" is still maintained. Such theories, often motivated by quantum-gravity considerations are interesting but are unable to resolve the UHECR paradox quantitatively at the moment.

We here attempt to deform the relativistic kinematics using heuristic means. We do it first by identifying the objective contents of the relativity principle and then go in for modifying these contents minimally to obtain a new transformation that relates space-time of an arbitrary frame of reference with that of the universal rest frame of the cosmic substratum. The only phenomenological assumption regarding the speed of light in Σ_k, $c_k = c_0(1 + \alpha u_k^2/c_0^2)^{1/2}$ (in contrast to the assumption, $c_k = c_0$ in SR) for which $c_k - c_0 = \eta_k \approx 1.71 \times 10^{-22}$ in the laboratory frame ($u_k \approx 300$ km/sec), is the only speculative aspect that has been used to derive the new kinematics. Since the isotropy ingredient of the second relativity postulates has not been disturbed, Michelson-Morley type experiments cannot distinguish the proposed kinematics with that of the relativistic one. Also the limit on ϵ given in Eq.(1) as a result of precision test becomes inconsequential, since the expected result in the present case would be zero. The recent improved test of time dilation in SR using laser spectroscopy sets a new limit of 2.2×10^{-7} for deviation of time dilation factor [22]. This even does not match with the smallness of η_k which is also the measure of this deviation according to the new kinematics. Hence the precision tests possibly will be unable to discern any deviation from SR in the near future, yet one may find an explanation of the cosmic ray paradox in the proposed deformed relativistic kinematics.

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THE INCORRECTNESS OF THE CLASSICAL PRINCIPLE OF EQUIVALENCE, AND THE CORRECT PRINCIPLE OF EQUIVALENCE, THOUGH NOT NEEDED FOR A THEORY OF GRAVITATION

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In this article, we show that the analogy between the *effect of acceleration* and the *effect of gravitation,* making up the classical Principle of Equivalence (PE), which is the basis of the General Theory of Relativity (GTR), constitutes a *non-conform analogy,* i.e. it does not embody a *one to one correspondence between the two worlds* coming into play. This will constitute a starting point to show the inadequacy of the classical PE. On the basis of a *quantum mechanical theorem* previously established, we prove that, the classical PE is further *inaccurate.* For one thing, is happens to constitute a *clear violation* of the *law of energy conservation.* More specifically, owing to the *law of energy conservation,* broadened to embody the *mass & energy equivalence* of the Special Theory of Relativity (STR), next to the mass dilation due to the motion in question, the force field too, is to alter the rest mass subject to an accelerational motion *(which happens something totally overlooked by the GTR).* Then, we establish a *correct* PE. The approach we present, leaves unnecessary the classical PE, thus the GTR, and yields a *whole new theory about gravitation,* along with all end results of this theory, up to a third order Taylor expansion, yet with no singularity *(thus, no black holes),* and with an incomparable ease, with a *different metric* too. Our approach in fact is *(not restricted to gravitation, and)* extendable to all fields.

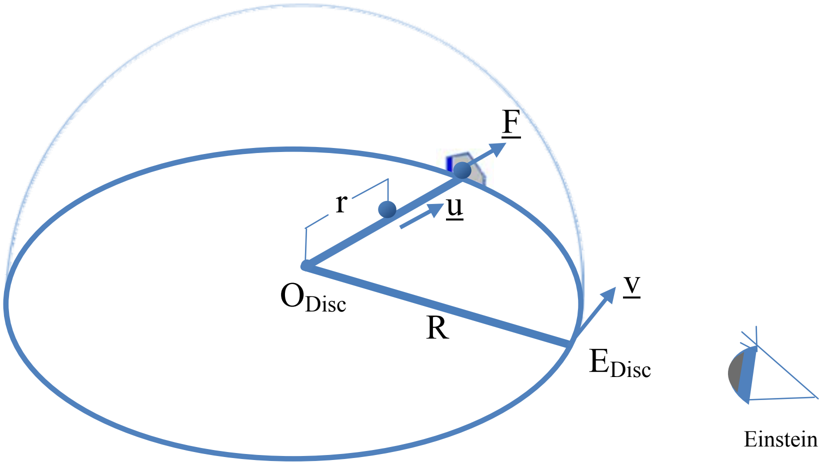
BASIC PITFALL OF THE CLASSICAL PRINCIPLE OF EQUIVALENCE

The classical Principle of Equivalence (PE),[*] draws a parallel between the *effect of acceleration* and the *effect of gravitation (and, much work is based on it, starting with the General Theory of Relativity).*[1] Herein we will first show that the *analogy* in question, the way it is established *(though capable to furnish satisfactory results),* is non-conform *(i.e. it does not embody, a one to one correspondence, between gravitation and acceleration).* This will constitute a starting point to show the inadequacy of the classical PE. To prove our claim, we envisage a *rotating disc,* with an *angular velocity* ϖ. Thus, an object of mass "m", situated at rest, a distance r, from the origin O_{Disc}, is *(due to the centrifugical force),* subject to an *outward radial force.* Accordingly, if set free, it can go, as far as R, up to the *edge of the disc,* E_{Disc} (see Figure 1). We propose to gear such a *"universe"* to be *equivalent* to a *universe* made of the *same mass,* this time though, attracted by a gravitational source of mass $\mathcal{M}$, thus extending from the center of $\mathcal{M}$, up to infinity.[†] So, we have, the *two worlds* to be compared with each other; we call the first one, the *"Disc World" ($\mathcal{DW}$),* and the second one, the *"Gravitation World" ($\mathcal{GW}$).*[‡]

[*] Some may think of the PE, as the *sameness of paths (both in time and in space),* of different masses left to a free fall, in a gravitational field; here, we do not mean this occurrence.

[†] Visibly, the *rotating disc* is a two dimensional world, and the gravitation world, can be envisaged as a three dimensional world. This is not important, since the comparison we aim to, is based on *the magnitude of the acceleration* depicted by the *two worlds,* and not the *equality* of the number of dimensions of these worlds. Moreover, both worlds, just along the *acceleration vector* coming into play, can well be considered to be simply one-dimensional-worlds. Or, we could very well consider a *rotating cylinder* or *sphere,* instead. But as mentioned, this is not the issue.

[‡] We could of course as well, consider an *accelerated elevato.* The basic idea we want to paint herein, would not anyway differ. Yet, the reason we consider a *rotating disc* instead, is merely that, it provides us, as we will see, with a simple, *linearly* varying



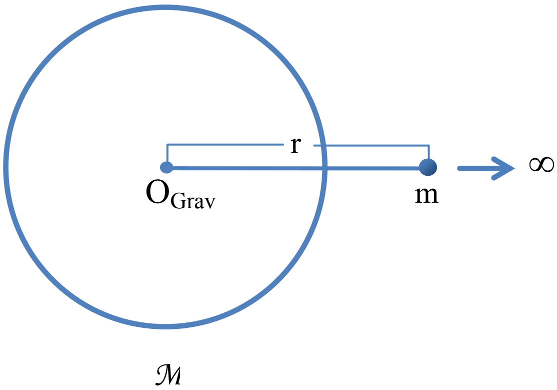


Figure 1: The "Disc World" Versus The "Gravitation World"

ROTATING SYSTEM VERSUS GRAVITATION: THE ANALOGY MAKING UP THE CLASSICAL PE, IS NON-CONFORM

In order to establish a *one to one correspondence* between the effect caused by the *rotating disc* on the mass m, and that caused by *a gravitational field,* we have to assume that, E_{Disc}, i.e. the edge of the disc *(the radial direction of concern, points to),* is to be associated with the center O_{Grav} of $\mathcal{M}$; this is, indeed *(were it capable, to penetrate),* how far m can go, if set free. On the other hand, the *Disc World,* to be made similar to the *Gravitation World,* must be a *closed* and *isolated* universe, delineated by the *interior* of the rotating disc, solely, extending from O_{Disc} up to E_{Disc}, the edge of the disc, thus excluding any *exterior*. Thence, an *"outsider"* belonging to the *"Outside World"* (which we will call $\mathcal{OW}$, in short), does not belong to the $\mathcal{DW}$. We will call the *outsider* we revealed, the *"alien – outsider"*. (He belongs to the $\mathcal{OW}$.) For an *exact comparison* of $\mathcal{DW}$ and $\mathcal{GW}$,

acceleration *(along with the radial direction),* and not a *constant* one *(which does not land itself to a convenient description we aim to).*

in the $\mathcal{DW}$, there should strictly be *nothing* beyond E_{Disc}, since a "region beyond E_{Disc}" would correspond to a "region, beyond O_{Grav}", and there is nothing beyond O_{Grav}.

One other *vital point,* is the following: The location, free of gravitation, in the field created by $\mathcal{M}$, is situated at infinity. Whereas, the location free of acceleration on the Disc World *(is not, "outside of the disc", since, as conveyed, there cannot be "any outside", thus),* is the center of the disc, O_{Disc}. This observation will prevent us from committing any related mistake. Thence, we sketch in Table 1, the *conform correspondences* in between the Gravitation World and the Disc World.

Table 1 Conform Correspondences in Between the Gravitation World ($\mathcal{GW}$)
and the Disc World ($\mathcal{DW}$)

	Gravitation World ($\mathcal{GW}$)	Disc World ($\mathcal{DW}$)
"Edge" of the world of concern	O_{Grav} *(center of the gravitational source)*	E_{Disc} *(edge of the disc)*
"Effect free" location	infinity	O_{Disc} *(center of the disc)*

Within this frame, as a conclusion, we should not have any room, for an alien – outsider, who may have set the disc into motion. Otherwise, his correspondent would be the *"instant creator of the gravitational source"*, nearby the mass m, and there is no such creation, we would envisage. Any approach tempting to compare the *effect of acceleration* with the *effect of gravitation,* based on the outsider's measurements *(whatever would be, its usefulness),* is bound to be *non-conform.* Unfortunately, this is exactly what the classical PE does. The observer scrutinizes the *rotating disc* from the outside! As we will soon elaborate on, he furthermore commits an essential mistake, assuming that the *rotating mass* does not undergo any other change than *that due to the motion.* We will how ever see that the force field, all by itself, as well, influences the *overall relativistic energy* of the rotating object. Therefore, the classical PE is unfortunately, ill born, and leads to irremediable inconsistencies *(what ever may be, so far, the accuracy of experimental proofs, on its side).*

STUDY OF THE EFFECT OF THE ACCELERATION ON THE ROTATING OBJECT, AS ASSESSED FROM THE CENTER OF THE DISC WORLD

For a *one to one correspondence* between the $\mathcal{DW}$ and the $\mathcal{GW}$, we have to study what happens to the object of mass "m", when it is moved on the disc, as referred to O_{Disc} *(and not according to an alien, situated outside of the disc).* We propose to move m, quasistatically, on a radial direction. Note that, in the $\mathcal{DW}$ *(during the rotation),* the direction O_{Disc}-E_{Disc} stays at rest, as referred to O_{Disc}, since the observer at O_{Disc} will not be aware of the rotation *(given that he can not receive any information from the outside, he will be limited to feel the outward acceleration, without knowing the source of it),* and that all radii evidently pass by the center. In the $\mathcal{DW}$, it is easy to measure the strength of the acceleration $\gamma_{Disc}(r)$, at the location r; it is proportional to r, more specifically,

$$\gamma_{Disc}(r) = k\,r\,. \qquad (1)$$

(acceleration as measured by the observer in the $\mathcal{DW}$ at the given location)

The *distance* from the center of the disc to r, is measured as referred to O_{Disc}. We will soon see that, it is not the same distance, if for instance, assessed by an observer at r. Then, Eq.(1), should be altered; the dependence of $\gamma_{Disc}(r)$ to r, becomes more complicated. But here we do not really have to get into such complexity. The alien *(who does not belong to the Disc World),* knows that

$$k = \varpi^2 \quad , \tag{2}$$

ϖ being the *angular velocity* of the rotating disc, as viewed from the $\mathcal{OW}$.

In order to delineate a one to one correspondence with the *Gravitation World,* as mentioned, the *Disc World* must be a *closed system,* i.e. somehow *isolated* from the *outsider's world,* so that, one can insure the *energy conservation.* For the sake of completeness, this should be stated as a law drawn by the Disc World's Observer.

Law 1: In the closed, isolated Disc World, energy must be conserved.

Suppose then, the object m is situated at E_{Disc}. If now, one wants to carry it *quasistatically,* from the attraction field, up to the location O_{Disc}, he has to furnish to it, a given amount of energy, while working against the outward centrifugal force of strength

$$F(r) = m_{0\mathcal{D}isc}(r)\gamma_{Disc}(r) = m_{0\mathcal{D}isc}(r)kr \quad . \tag{3}$$

(outward force's strength at the given location)

The mass $m_{0\mathcal{D}isc}(r)$, is the *rest mass* of the object at the location r, and it should be noted that, owing to the law of energy conservation, as will be detailed, it must be subject to a change. On the other hand, it is true that here we have *tacitly* borrowed the law of force from Newton. Yet the observer in the Disc World, can well make experiments, the way Newton did, and can establish, as we anticipate, the following law,[2] to be precise, for a *static mass,* exclusively:

Law 2: In the Disc World, a static mass is submitted to a static force given by
Force = [Rest Mass] x [Centrifugal Acceleration]. (4)

This law, corresponds to Newton's law of gravitation (i.e. gravitational force = rest mass x gravitational acceleration), with respect to the Gravitation World, but the way we consider, strictly for *static masses.* On the other hand, the *law of conservation of energy,* broadened to embody *the mass & energy equivalence* of the Special Theory of Relativity, requires that the rest mass of the object should be increased as much, when carried, toward O_{Disc} . Thus, we can further stress the following law, in fact nothing else, but once again, the *law of energy conservation,* where though, *energy* and *mass* are essentially not different from each other.[3]

Law 3: The rest mass of an object bound to a location in the Disc World's accelerational field, amounts less than its rest mass measured at the center of the disc, and this, as much as its binding energy, vis-à-vis the location of concern.

Thus, as assessed by the observer situated at O_{Disc},[§] one can write

$$-\,dm_{0\mathcal{D}isc}(r)c_0^2 = F(r)dr = m_{0\mathcal{D}isc}(r)(kr)dr\ , \tag{5}$$

where c_0 is the speed of light.

Via integration, we can write

$$m_{0\mathcal{D}isc}(r) = m_0 \exp\left(-\frac{\int_0^r \gamma_{Disc}(r')dr'}{c_0^2}\right) = m_0 \exp\left(-\frac{kr^2}{2c_0^2}\right)\ . \tag{6}$$

(rest mass of the object at r, *in the* $\mathcal{DW}$*, as referred to the center of the disc)*

Similarly, for the *Gravitation World,* as assessed by the observer situated at infinity, one would write[4]

$$dm_{0\mathcal{G}rav}(r)c_0^2 = \frac{G\mathcal{M}m_{0\mathcal{G}rav}(r)}{r^2}dr = \gamma_{Grav}(r)m_{0\mathcal{G}rav}(r)dr\ , \tag{7}$$

or via integration

$$m_{0\mathcal{G}rav}(r) = m_0 \exp\left(-\frac{\int_r^\infty \gamma_{Grav}(r')dr'}{c_0^2}\right) = m_0 \exp\left(-\frac{G\mathcal{M}}{rc_0^2}\right)\ ; \tag{8}$$

(rest mass of the object at r, *in the* $\mathcal{GW}$ *as referred to infinity)*

here, $\gamma_{Grav}(r)$ visibly, is used for $G\mathcal{M}/r^2$, and $m_{0\mathcal{G}rav}(r)$ is the *rest mass* of the object in hand, at the altitude r, in the gravitational field, measured from the center of the celestial body in consideration, as referred though to the observer located in at infinity; $\mathcal{M}$ is the mass of the celestial body of concern, and G the gravitation constant, still as assessed by the observer locaged at infinity).

OUR QUANTUM MECHANICAL THEOREM

The first author has previously established the following *general quantum mechanical theorem.*[5]

Theorem 1: Consider a relativistic or non-relativistic quantum mechanical description of a given object, depending on whichever, may be appropriate. This description points to an *internal dynamics* which consists in a *"clock motion",* achieved in a *"clock space",* along with a *"unit period of time".* The description excludes *"synthetic potential energies", which may otherwise lead to incompatibilities with the Special Theory of Relativity.* The object is supposed to embody $\mathcal{K}$ particles, altogether. If then, different masses m_{k0}, k = 1, …, $\mathcal{K}$, involved by this description of the object at rest, are over all multiplied by the arbitrary number γ, the following two general results are conjointly obtained: a) The *total energy* E_0 associated with the given clock's motion of the object is increased as much, or the same, the unit period of time T_0, of the motion associated with

[§] We tacitly assume that the object m is very small as compared to the disc's mass, so that moving it on the disc, does not alter the angular momentum, thus the rotational speed, of the disc.

this energy, is decreased as much. b) The *characteristic length,* or the *size* $\mathcal{R}_0$ to be associated with the given clock's motion of concern, contracts as much.
In mathematical words this is:

$$[(m_{k0}, k = 1, \ldots, \mathcal{K}) \rightarrow (\gamma\, m_{k0}, k = 1, \ldots, \mathcal{K})] \Rightarrow [(E_0 \rightarrow \gamma\, E_0) \text{ or } (T_0 \rightarrow \frac{T_0}{\gamma}), \text{ and } (\mathcal{R}_0 \rightarrow \frac{\mathcal{R}_0}{\gamma})].$$

This, together with Law 3, yields at once the next theorem.

Theorem 2: A clock interacting with any accelerating field, electric, nuclear, gravitational, or else (without loosing its identity), retards as imposed by its quantum mechanical description, due to the mass deficiency, which amounts to the equivalent of the binding energy it displays in the field in consideration; at the same time, and for the same reason, the space size in which it is installed, stretches just as much.

This can further be grasped rather easily, as follows. The mass deficiency, the object displays in the accelerating field, weakens its internal dynamics as much, which makes it slow down. Thence, one arrives at the principal results, stated above. This leads, via Eq.(8), for the total energy $E_{0\mathcal{D}isc}(r)$ to be associated with the internal dynamics of the object in hand (assumed at rest, at the location r of the accelerational field of the $\mathcal{DW}$),

$$E_{0\mathcal{D}isc}(r) = E_0 \exp\left(-\frac{\gamma r}{2c_0^2}\right) \quad , \tag{9}$$

(total energy delineated by the internal dynamics of the object at r, *on the disc, as referred to the center of the disc)*

where, to simplify our notation, we wrote straight γ, instead of $\gamma_{\mathrm{Disc}}(r)$. Similarly, we have for the period of time $T_{0\mathcal{D}isc}(r)$, and the unit length $\mathcal{R}_{0\mathcal{D}isc}(r)$, to be associated with the internal dynamics of it, at r, respectively,

$$T_{0\mathcal{D}isc}(r) = T_0 \exp\left(\frac{\gamma r}{2c_0^2}\right) \quad , \tag{10}$$

(period of time delineated by the internal dynamics of the object at r, *on the disc, as referred to the center of the disc)*

$$\mathcal{R}_{0\mathcal{D}isc}(r) = \mathcal{R}_0 \exp\left(\frac{\gamma r}{2c_0^2}\right) \quad . \tag{11}$$

(unit length delineated by the internal dynamics of the object at r, *on the disc, as referred to the center of the disc)*

Thus, in the $\mathcal{DW}$, the unit length stretches, as much as the period of time, and this uniformly. Note that the application of the foregoing calculation to the Gravitation World is immediate, and does not require any analogy. All we have to do *(as we will soon detail),* is to replace the centrifugal force by the Newton attraction force, and that is all.

AND, HOW DOES THE ALIEN - OUTSIDER, CLASSICALLY SPEAKING, ASSESS THE CHANGES TAKING PLACE IN THE $\mathcal{DW}$?

The object in the $\mathcal{DW}$, according to the outside observer, is not at rest, but in motion. One thus, classically, proposes to apply the Special Theory of Relativity (STR), to the object, supposing that the rest mass is, as in effect, adopted by the General Theory of Relativity (GTR), not altered due to the acceleration *(and this is not correct at all).* Anyway, the relativistic mass $m_{\mathcal{R}}(r)$, as *classically* assessed, by the outside observer, becomes

$$m_{\mathcal{R}}(r) = \frac{m_0}{\sqrt{1 - v^2 / c_0^2}} = \frac{m_0}{\sqrt{1 - \varpi^2 r^2 / c_0^2}} = \frac{m_0}{\sqrt{1 - \gamma r / c_0^2}} \quad . \tag{12}$$

(relativistic mass of m *at* r, *on the disc,*
as classically assessed by the outside observer)

Relatedly, $T_{\mathcal{R}}(r)$ and $R_{\mathcal{R}}(r)$ to be associated with the internal dynamics of it, thus, become

$$T_{\mathcal{R}}(r) = \frac{T_0}{\sqrt{1 - \gamma r / c_0^2}} \quad , \tag{13}$$

(the classical relativistically dilated period of time, delineated by the internal dynamics of the object at r, *on the disc, as observed by the outside observer)*

$$\mathcal{R}_{\mathcal{R}}(r) = \mathcal{R}_0 \sqrt{1 - \gamma r / c_0^2} \quad . \tag{14}$$

(the classical relativistically contracted unit length, delineated by the internal dynamics of the object at r, *on the disc, as observed by the outside observer)*

Note that, accordingly, as considered by the GTR, the unit length contracts, and this, only along the direction of motion.

INEXACTNESS OF THE PRINCIPLE OF EQUIVALENCE

Let us summarize our results in Table 2. The classical PE aims to equate, the quantities presented in, respectively, the second raw and third raw of this table, and as we see, none of the corresponding quantities are equal to each other. Mainly, $m_{0\mathcal{D}isc}(r) = m_0 \exp(-\gamma r / (2c_0^2))$ [cf. Eq.(6)], as assessed by the center of the *Disc World,* and $m_{\mathcal{R}}(r) = m_0 / \sqrt{1 - \gamma r / c_0^2}$ [cf. Eq.(12)], as classically assessed by the *Outside World,* are quite different. With respect to m_0, the first one is a *decreased mass,* whereas the second one is an *increased mass.* This result alone, shows that based on our approach, the classical PE, is inaccurate.

Table 2 Disc World ($\mathcal{DW}$), Versus Outsider's World ($\mathcal{OW}$)

	Mass	*Unit length*	*Period of time*
As seen from the center of the disc *($\mathcal{DW}$)*	$m_{\mathcal{D}isc}(r) = m_0 \exp\left(-\frac{\gamma r}{2c_0^2}\right)$ *(rest mass of the object)*	$\mathcal{R}_{\mathcal{D}isc}(r) = \mathcal{R}_0 \exp\left(\frac{\gamma r}{2c_0^2}\right)$ *(uniform stretching)*	$T_{\mathcal{D}isc}(r) = T_0 \exp\left(\frac{\gamma r}{2c_0^2}\right) \cong T_0\left(1 + \frac{\gamma r}{2c_0^2} + \frac{\gamma^2 r^2}{8c_0^4}\right)$

As classically seen from the outside world *(OW)*	$m_{\mathcal{R}}(r) = \dfrac{m_0}{\sqrt{1-\gamma r/c_0^2}}$ *(relativistic mass of the object)*	$\mathcal{R}_{\mathcal{R}}(r) = \mathcal{R}_0\sqrt{1-\gamma r/c_0^2}$ *(contraction along the direction of motion)*	$T_{\mathcal{R}}(r) = \dfrac{T_0}{\sqrt{1-\gamma r/c_0^2}}$ $\cong T_0\left(1+\dfrac{\gamma r}{2c_0^2}+\dfrac{\gamma^2 r^2}{8c_0^4}\right)$

Furthermore, the unit length *stretches* in the $\mathcal{DW}$, in all directions, whereas it *contracts,* in the $\mathcal{OW}$, and this, only along the direction of the acceleration. On the other hand, both results, referenced to respectively the Disc World and the Outside World, yield a *red shift* (cf. the last column of Table 2). The related mechanisms, yet, differ. The expressions $T_{\mathcal{D}isc}(r) = T_0 \exp(\gamma r/(2c_0^2))$, and $T_{\mathcal{R}}(r) = T_0/\sqrt{1-\gamma r/c_0^2}$ are, though very close to each other, not anyhow *identical.* The *red shift* remains the only measurable quantity, amongst mass, length and period of time. Thus, strictly speaking, the *non-equality* of the two expressions in question, along with our approach, shows that the classical PE, in the most optimistic case, is still *inexact.* Let us see, how the mechanisms regarding to the *red shift,* differ from each other, as seen form the two worlds. As referenced to the *center of the disc,* the *relativistic energy* is decreased, and this *at rest,* yielding a *weakening* of the internal dynamics, or the same, the weakening of the frequency of light emitted, thus a red shift. Rigorously speaking, this latter result is implied by *quantum mechanics.* As referred to the outside observer *(classically speaking),* due to motion *(alone),* the relativistic energy is increased, but time is dilated, thus the frequency of the light emitted by the object is decreased, i.e. still a *red shift,* and this is implied by the bear *relativity of motion,* which *(classically)* assumes, that, the acceleration besides the motion, does not affect in any way the frequency of light, in consideration *(which, according to our approach, is utterly erroneous).* On the whole the classical PE, turns out to be a *non-conform analogy,* leading to inconsistencies. The way it is set up, it could only lead to acceptable results. But in any case, it is *inexact,* as far as the *red shift* prediction is concerned, and *erroneous* as far as transformations that *masses, and lengths* would undergo in an accelerated motion. As we will see the remedy of it, leaves it needless.

DISCUSSION ABOUT THE VALIDITY OF THE CLASSICAL PE, AND THE REMEDY OF IT

It is already *astounding* that [cf. Eq.(10)],

$$T_{\mathcal{D}}(r) = T_0 \exp\left(\frac{\gamma r}{2c_0^2}\right), \tag{10}$$

(the period of time associated with the internal dynamics of the object at rest, as referred to the center of the disc)

and [cf. Eq.(13)],

$$T_{\mathcal{R}}(r) = \frac{T_0}{\sqrt{1-\gamma r/c_0^2}}, \tag{13}$$

(the relativistic period of time associated with the internal dynamics of the object, as referred to the outside alien

are *equal* to each other, up to a third order Taylor expansion (cf. the last column of Table 2), *and* the following question arises:

Is it a coincidence that the two expressions are equal? If not, why are they not identical? Unfortunately, it looks that this is, a *diabolic coincidence,* which in fact insures that the end results of the GTR coincide out of the blue satisfactorily, with the experimental results. And surely, the fact that they are not *identical,* is crucial. As explained, the analogy on which the classical PE lies, first of all, is *non-conform.* One, for an *appropriate correspondence,* should view the phenomena taking place in the *Disc World,* from O_{Disc}, and not from the *outside.* Having failed to do so, seems to be the cause of major error. Henceforth, the classical PE assumes that the acceleration does not alter in any way the *object's rest mass (besides the Lorentz mass increase, due to the motion, the $\mathcal{OW}$'s observer visualizes).* But if so, the observer at O_{Disc} , in the *closed* Disc World, would not observe any change *(since, with respect to* O_{Disc} , as we have elaborated, *the change is solely due to the rest mass decrease, resulting from the law of energy conservation, embodying the mass & energy equivalence of the STR).* For the *outside observer,* how ever, the location O_{Disc} is situated in the $\mathcal{OW}$, since O_{Disc} is well at rest, as referred to the $\mathcal{OW}$. Then, whatever is observed in the $\mathcal{OW}$, should be observed as referred to O_{Disc}. So two different observers, one at O_{Disc} and the other outside, both at rest in the $\mathcal{OW}$, come to different conclusions! This is a *clear contradiction.* Thence, something must be *wrong,* somewhere. The remedy is that the $\mathcal{OW}$'s observer, must take into account the *rest mass change,* due to the *acceleration,* next to the *instantaneous change of the rest mass* induced by the motion. If the rest mass of "m" is decreased as referred to O_{Disc}, it should also be decreased for the $\mathcal{OW}$. Then, the *overall relativistic mass* $m_{OverAll}(R)$ due to the rotation of the disc becomes

$$m_{OverAll}(R) = m_0 \frac{\exp\left(-\varpi^2 R^2/(2c_0^2)\right)}{\sqrt{1-\varpi^2 R^2/c_0^2}} \cong m_0\left(1+\frac{\varpi^4 R^4}{4c_0^4}\right) \cong m_0 \quad . \tag{15}$$

(corrected overall relativistic mass, due to the rotational motion)

.

On the other hand, the *overall rotational relativistic period of time* $T_{\mathcal{R}}(R)$ to be associated with the rotating object m, situated at a distance R, on the disc, can be written, via taking into account both the *rest mass decrease (arising from the mass & energy equivalence)* we have unveiled [cf. Eq.(10)], and the *usual relativistic time dilation due to the motion* [cf. Eq.(13)], as

$$T_{OverAll}(R) = T_0 \frac{\exp\left(\varpi^2 R^2/(2c_0^2)\right)}{\sqrt{1-\varpi^2 R^2/c_0^2}} \cong T_0\left(1+\frac{\varpi^2 R^2}{c_0^2}+\frac{\varpi^4 R^4}{4c_0^4}\right). \tag{16}$$

(corrected period of time, displayed by the object in rotation)

In a similar way, finally, the outside observer, is expected to measure the outcome $\mathcal{R}_{OverAll}(r)$ of the superposition of the two occurrences in question, as regards to a unit length too, i.e. [via Eqs. (11) and)14)]

$$\mathcal{R}_{OverAll}(r) = \mathcal{R}_0 \exp\left(\frac{\varpi^2 R^2}{2c_0^2}\right)\sqrt{1-\frac{\varpi^2 R^2}{c_0^2}} \cong \mathcal{R}_0\left(1-\frac{\varpi^4 R^4}{4c_0^4}\right) \cong \mathcal{R}_0 \quad . \tag{17}$$

(corrected unit length, displayed by the object in rotation)

Note that, our prediction displayed by Eq.(16), is in full agreement with the bound muon decay rate retardation. That is, the period of the bound muon is retarded not only due to its motion around the nucleus, but also, due already to its binding to the nucleus.[6] On the other hand, our on the whole, non - Lorentz - contracted - unit length, displayed by Eq.(37), is in full compatibility with previous predictions made about the rotating disc.[7] Recall that, Kündig, almost half a century ago, measured the time dilation of an object in rotation, and published results that were in agreement with the classical prediction;[8] however, he seems to have misprocessed the data.[9]

RESTATEMENT OF THE PRICIPLE OF EQUIVALENCE (PE), ALONG WITH A NEW THEORY OF GRAVITATION – WHICH IS NOT BASED ON IT

Here, we state the *correct principle of equivalence,* which we would not really need as a basis of any new theory. It is a simple derivation based on the *law of energy conservation* broadened to embody the *mass & energy equivalence of the special theory of relativity.*

The Correct PE: Both *gravitation* and *accelerational motion,* in fact any force field, alter a given *"rest mass",* held at rest, in the same manner. It is that the rest mass of the given object, decreases as much as the energy necessary to furnish to this object, in order to remove it, from the force field.

This finding, though, leaves unnecessary the use of the *analogy* between acceleration and gravitation, for a subsequent theory of gravitation. Our approach is general, and can be directly applied to any force field, the object interacts with. Let us work out, our statement mathematically, for the *case of gravitation.*[3,4] To simplify things, suppose that, the object starts falling from very far. All along, its rest mass, must be decreased as much as its *static binding energy* with respect the field. It can be shown that the rest mass $m_0(r)$ of the object bound to a celestial body of mass $\mathcal{M}$, at the altitude r, can be (supposing that $\mathcal{M}$ is infinitely more massive than the falling object) written (with the usual definitions), as[2]

$$m_0(r) = m_0 e^{-\alpha} \ , \ \ \alpha = G\mathcal{M}/(rc_0^2) \quad . \tag{18}$$

Recall that, here, the universal gravitational constant G, and the distance r of the object to the center of the celestial body, are assessed by the distant observer. Through the fall, the rest mass *(while getting decreased due to binding),* is at the same time, increased by the corresponding Lorentz dilation factor, due to the motion. Thus, the *overall relativistic energy* $m_{OverAll}(r)c_0^2$ of the object freely falling with the velocity u(r) will be given by

$$m_{OverAll}(r)c_0^2 = m_0 c_0^2 \frac{\exp\left(-G\mathcal{M}/(rc_0^2)\right)}{\sqrt{1-u^2(r)/c_0^2}} = \text{Constant} \quad . \tag{19}$$

(overall relativistic mass of the object moving with the velocity u, *at the given location)*

The overall relativistic energy must indeed remain constant, due to the law of energy conservation, and this requirement frames the equation of motion [whose integral form, is Eq.(19)]. The *overall relativistic energy at infinity* is $m_0c_0^2$. Thus, the *constant* coming into play throughout the free fall, is nothing else, but *this latter quantity.* In other words,

$$m_{OverAll}(r)c_0^2 = m_0 c_0^2 \frac{\exp\left(-G\mathcal{M}/(rc_0^2)\right)}{\sqrt{1-u^2(r)/c_0^2}} = m_0c_0^2 \quad . \tag{20}$$

(overall relativistic energy through the motion)

Eq.(5) immediately yields

$$\frac{\exp\left(-G\mathcal{M}/(rc_0^2)\right)}{\sqrt{1-u^2(r)/c_0^2}} = 1 \quad . \tag{21}$$

(the new principle of equivalence,
stated for a free fall)

This is the expression of our *new* PE. It provides us with the capability of predicting u, at the given location, once $\mathcal{M}$ is known. Again this is nothing else, but a plain result implied by the law of energy conservation.

CONCLUSION

In this article, we have shown that the *analogy* between the *effect of acceleration* and the *effect of gravitation,* making up the *(classical)* Principle of Equivalence (PE), which is the basis of the GTR constitutes a *non-conform analogy,* i.e. it does not embody a *one to one correspondence between the two worlds,* coming into play. This appears to have led historically, to many inadequacies. On the basis of a *quantum mechanical theorem* previously established, we have proven that, the classical PE is further, inaccurate. It is that, owing to the *law of energy conservation,* broadened to embody the *mass & energy equivalence* of the STR next to the *mass dilation* due to the motion in question, the force field too, is to alter the rest mass of the object of concern *(which happens something totally overlooked by the GTR).* In short, the classical PE constitutes a clear violation of the law of energy conservation, though diabolically leading to satisfactory results *(and claimed to be proven via high precision measurements, which we left aside in this article).* We established a *correct* PE. Yet the remedial of the the analogy making up the *"principle of equivalence",* leaves it, needless. The same holds for the GTR *(which seems to be left obsolete);* thus, our approach yields a whole new theory about gravitation, based on just *energy conservation (along with the mass & energy equivalence of the STR)* and *quantum mechanics,* delineating a *new metric,* yielding all end results of the GTR, up to a third order Taylor expansion, yet with no singularity *(thus no black holes),* and with an incomparable ease. Our approach in fact is *(not restricted to gravitation, but)* extendable to all fields, allowing the unification of micro and macro worlds.

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REFORMULATION OF THE DEFLECTION OF THE ELECTRON IN A CAPACITOR, TAKING INTO ACCOUNT THE VARIATION OF ELECTRON'S REST MASS, AS IMPOSED BY THE LAW OF CONSERVATION OF ENERGY

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Herein, we derive the deflection, an electron would undergo while flying through a parallel-plate capacitor, for first, the *usual relativistic case,* then for the *controversial case* we introduce, i.e. the case where the rest mass of the electron, due to the law of conservation of energy, broadened to embody the mass & energy equivalence based on the special theory of relativity, is altered by the presence of the field, and this as much as the binding energy coming into play. Thus, the rest mass of the electron is increased, if an electric field of negative charges is suddenly created around it; it would decrease in the opposite case of the creation of an electric field of positive charge. Furthermore, if the electron in consideration is *quasistatically* moved to regions, where its binding energy with respect to the field, increases, *its rest mass decreases, just as much.* Our approach makes that the effective Coulomb Force, acting on the moving electron, is different from that acting on the electron at rest. Although the difference between the two is very small, it still remains measurable. Our approach leads to a quantization on the deflection, versus the speed of the electron.

INTRODUCTION

J. J. Thompson, in 1897, via measuring the deflection of an electron flying through the plates of a capacitor had determined the ratio charge/mass, for an electron.[1]

The first author has previously anticipated that the *rest mass of the electron,* in the presence of an electric field, should be modified, not only due to its motion, thus by the usual Lorentz factor $(1-v^2/c^2)^{-1/2}$, but also already at rest, directly due to the presence of the field. The latter is required by the law of conservation of energy, broadened to embody the mass & energy equivalence *(of the special theory of relativity).* The variation of the rest mass in consideration, thereby, ought to be, as much as the binding energy coming into play.[2] Thus, the rest mass of the electron is increased, if an electric field of negative charge, is suddenly created around it. It would decrease in the opposite case of the creation of an electric field of positive charge. Furthermore, if the electron in consideration is *quasistatically* moved to regions, where its binding energy with respect to the field, increases, its rest mass decreases, just as much.

This paper proposes that, an accurate Thompson experiment can determine the difference between the classical relativistic approach and the present approach.

Let us define the following quantities:

e : electric charge intensity of the electron
E : field intensity reigning in between the plates of the capacitor
$m_{0\infty}$: electron rest mass at infinity
L : capacitor plate length
v_0 : electron's velocity, before it enters the capacitor

We assume that the electron, initially, moves along the center line of the median plane of a parallel-plate capacitor. We call this direction x, and the direction perpendicular to that, y.

Below, we recall the derivation of the standard deflection for the non-relativistic case, then we derive the deflection for the relativistic case, and finally for the case where the rest mass of the statically bound electron is altered by the presence of the field. Our approach induces that, the effective Coulomb Force acting on the moving electron, is different from that acting on the electron at rest. Although, the difference between the two is very small, it still remains measurable. In what follows, for a first approximation, we will overlook the electromagnetic radiation generated by the accelerated electron.

THE CLASSICAL NON-RELATIVISTIC CASE

Originally J. J. Thompson applied in an elegant manner Newton's Second Law of Motion to the deflection of the electron through the plates. Since the electric force applies only along the y direction, the initial velocity v_0 is altered along this direction, while remaining the same along the x direction.[1]

The electric force strength F, can be expressed, as customarily, as

$$F = eE \ . \tag{1}$$

Classically speaking, along Newton's Second Law of Motion, for a non-relativistic case, one can write

$$F = m_{0\infty} \frac{dv_y}{dt} \ , \tag{2}$$

where v_y is the y component of the electron's velocity, i.e.

$$v_y = \frac{dy(t)}{dt} \ ; \tag{3}$$

$y(t)$ is the deflection acquired by the electron at time t, t=0 being the time the electron enters the capacitor. Thus one, as usual, can write

$$\frac{eE}{m_{0\infty}} = a = \frac{dv_y}{dt} \ ; \tag{4-1}$$

$$v_y(t) = \frac{dy(t)}{dt} = \int_{t=0}^{t} a\,dt = at \ ; \tag{4-2}$$

$$y(t) = \int_{t=0}^{t} at\,dt = \frac{1}{2}at^2 \ ; \tag{4-3}$$

hence

$$Y = \frac{1}{2}aT^2 \ , \tag{4-4}$$

T being the period of time the electron spends in between the plates of the capacitor, and Y being the overall deflection at the exit.

One can, on the other hand, write

$$v_0 = \frac{L}{T} \ . \tag{5}$$

Thus

$$Y = \frac{1}{2}\frac{Ee}{m_{0\infty}}\frac{L^2}{v_0^2} \ . \tag{6}$$

(overall deflection, in the non-relativistic case)

THE CLASSICAL RELATIVISTIC CASE

If v_0 is not far from the velocity of light, and the change of it through the flight of the electron in between the plates of the capacitor should not be neglected, then, one should elaborate on Eq.(2), without though *(classically speaking)*, having to alter Eq.(1):

$$eE = \frac{d}{dt}\left(\frac{m_{0\infty} v_y}{\sqrt{1 - v^2/c^2}}\right) , \tag{7}$$

where v is the velocity of the electron at time t, so that

$$v^2 = v_x^2 + v_y^2 ; \tag{8}$$

since there is no force along the x direction, then

$$v_x = v_0 . \tag{9}$$

Let us elaborate on Eq.(7):

$$\frac{eE}{m_{0\infty}} = \frac{1}{\sqrt{1 - v^2/c^2}}\frac{dv_y}{dt} + \frac{(v/c^2)(dv/dt)}{\sqrt{1 - v^2/c^2}}\frac{v_y}{\left(1 - v^2/c^2\right)} . \tag{10}$$

According to Eqs. (8) and (9), one can write,

$$vdv = v_y dv_y . \tag{11}$$

Thus, Eq.(10) becomes

$$\frac{eE}{m_{0\infty}} = \frac{\left(1 - v^2/c^2\right)}{\left(1 - v^2/c^2\right)^{1/2}}\frac{dv_y}{dt} + \frac{(v_y^2/c^2)(dv_y/dt)}{\left(1 - v^2/c^2\right)^{1/2}} = \frac{\left[1 - (v^2 + v_y^2)/c^2\right]}{\left(1 - v^2/c^2\right)^{1/2}}\frac{dv_y}{dt} . \tag{12}$$

Via using Eqs. (8) and (9), one can write

$$\frac{eE}{m_{0\infty}} = \frac{dv_y}{dt}\frac{1 - v_0^2/c^2}{\left[1 - (v_0^2 + v_y^2)/c^2\right]^{3/2}} . \tag{13}$$

A rigorous solution can be obtained via the integration of the above equation:

$$\int_{t=0}^{t}\frac{eE}{m_{0\infty}}dt = \left(1 - \frac{v_0^2}{c^2}\right)\int_{v_y=0}^{v_y}\frac{dv_y}{\left[1 - (v_0^2 + v_y^2)/c^2\right]^{3/2}} . \tag{14}$$

This leads to

$$\frac{eE}{m_{0\infty}}t = \frac{v_y}{\left[1 - (v_0^2 + v_y^2)/c^2\right]^{1/2}} . \tag{15}$$

This yields

$$\left(1 - \frac{v_0^2}{c^2}\right)\left(\frac{eE}{m_{0\infty}}t\right)^2 = v_y^2\left[1 + \left(\frac{eE}{m_{0\infty}c}t\right)^2\right] , \tag{16}$$

or

$$v_y = \frac{dy}{dt} = \sqrt{1 - \frac{v_0^2}{c^2}}\left(\frac{eE}{m_{0\infty}}\right)\frac{t}{\sqrt{1 + \left[eEt/(m_{0\infty}c)\right]^2}} . \tag{17}$$

Thus

$$Y = \sqrt{1 - \frac{v_0^2}{c^2}}\left(\frac{eE}{m_{0\infty}}\right)\int_{t=0}^{T}\frac{tdt}{\sqrt{1 + \left[eEt/(m_{0\infty}c)\right]^2}} = \frac{\sqrt{1 - v_0^2/c^2}}{eE/(m_{0\infty}c^2)}\left[\sqrt{1 + \left[eET/(m_{0\infty}c)\right]^2} - 1\right]. \tag{18}$$

or

$$Y = \frac{\sqrt{1 - v_0^2/c^2}}{\left(eE/(m_{0\infty}c^2)\right)}\left[\sqrt{1 + \left[eEL/(m_{0\infty}cv_0)\right]^2} - 1\right]. \tag{19}$$

(rigorous, overall deflection, in the relativistic case)

THE CASE WHERE THE REST MASS OF THE BOUND ELECTRON IS ALTERED BY THE PRESENCE OF THE FIELD

Our approach is slightly different than the classical one. We propose to alter the rest mass of the bound electron as much as its binding energy vis-a-vis the electric field, as imposed by the law of conservation of energy, broadened to embody the mass & energy equivalence of the special theory of relativity.[2,3] Thus, the *rest mass* of the electron is increased, if an electric field of negative charge is suddenly created around it. It would decrease in the opposite case of the creation of an electric field of positive charge. Furthermore, if the electron in consideration is *quasistatically* moved into regions, where its binding energy with respect to the field, increases, its rest mass still decreases, just as much. In effect, consider an electron bound to an electric field. In order to carry it to infinity, free of field, one has to deliver to it an amount of energy, equal to its binding energy *(at the initial location in consideration).*

The law of conservation of energy, requires that, the electron's rest mass at infinity, is increased as much. Thus when set free, in the field, in order to accelerate, the electron will undergo the transformation of a minimal part of its rest mass into the extra kinetic energy it will acquire on the way. We can suppose that, the electron is infinitely less massive than the source creating the field.

Now, let ask the following question: What is the *relativistic energy of an electron* engaged with the electric field of a capacitor? If the electron is initially sitting on the negatively marked plate of a capacitor of normally 1 eV of electric potential difference between the plates, and the capacitor is charged all of a sudden, then the rest mass of the electron would experience an increase of 1eV.

If on the other hand, the electron is at first sitting on the positively marked plate, but initially *neutral,* and the capacitor is charged all of a sudden, then the rest mass of the electron practically remains the same, assuming that the electron's binding energy coming into play is negligible; this indeed seems a fair assumption given that, such an electron would anyway belong to the *conduction band* of the material of the capacitor, in consideration. With regards to a creation of charges in the vicinity of the electron, the situation is not any different when a freely flying electron enters in between the plates of a capacitor. As soon as it crosses the border, it witnesses the creation of a source of energy, and at that moment, there must occur a change in its relativistic energy; thus, its rest mass must increase accordingly. If it enters the plates of the given capacitor of 1 Volt, while flying right in between the plates of it, thereafter, its relativistic energy must increase as much as 1eV/2. The same must hold with regards to situations displayed by accelerators, and so forth. Our claim is well supported by experimental and theoretical results, though involving totally different setups than the one we presented above.[4,5]

The law of conservation of energy, in regards to the isolated system composed of the capacitor and the electron, necessitates that, the overall energy remains the same. The capacitor's plates being fixed, thence the initial relativistic energy of the electron must remain the same throughout its flight in between the plates of the capacitor. Let us then make the following definitions:

$m_0(y)$: rest mass of the bound electron deflected as much as the latitude y, at time t

eEy : binding energy of the static electron, at y

Thus, based on the foregoing discussion, $m_0(y)$ becomes

$$m_0(y) = \left(m_{0\infty} + \frac{Eed}{2c^2} \right) \left(1 - \frac{eEy}{m_{0\infty} c^2 + \dfrac{Eed}{2}} \right) ; \qquad (20)$$

the first term on the RHS of this equation, tells us how much the rest mass of the electron, is increased, due to the field, right after the electron experiences the creation of the field on the way, at the entrance of the capacitor; d, here is the width of the capacitor, and we assume that, as conveyed, the electron enters the capacitor, on the median plane of this; the term in between the second parentheses, on the RHS of this equation, describes how the initial overall rest mass, decreases through the field. The overall energy $m_\gamma(y)$ of the electron at the given latitude, then becomes

$$m_\gamma(y)c^2 = M_{0\infty}c^2 \frac{1 - eEy/(M_{0\infty}c^2)}{\sqrt{1 - v^2/c^2}} \ , \tag{21-a}$$

where we define $M_{0\infty}$ as

$$M_{0\infty} = m_{0\infty}\left(1 + \frac{Eed}{2m_{0\infty}c^2}\right). \tag{21-b}$$

This ought to be constant throughout, and equal to the initial relativistic energy of the electron:

$$m_\gamma(y)c^2 = M_{0\infty}c^2 \frac{1 - eEy/(M_{0\infty}c^2)}{\sqrt{1 - v^2/c^2}} = \mathit{Constant} = \frac{M_{0\infty}c^2}{\sqrt{1 - v_0^2/c^2}} \ . \tag{22}$$

(overall energy of the flying electron, predicted by the authors at the latitude y, *as referred to the median plane to the parallel-plate capacitor)*

This equation can be written as

$$\left(1 - \frac{eEy}{M_{0\infty}c^2}\right)^2 = \frac{1 - v^2/c^2}{1 - v_0^2/c^2} = \frac{1 - (v_x^2 + v_y^2)/c^2}{1 - v_0^2/c^2} \ , \tag{23}$$

or, following Eq.(14),

$$\left(1 - \frac{eEy}{M_{0\infty}c^2}\right)^2 = 1 - \frac{v_y^2/c^2}{1 - v_0^2/c^2} \ , \tag{24}$$

leading to

$$v_y = \frac{dy}{dt} = c\sqrt{1 - \left(1 - \frac{eEy}{M_{0\infty}c^2}\right)^2}\sqrt{1 - \frac{v_0^2}{c^2}} \ . \tag{25}$$

This equation, via integration, leads to

$$\int_{y=0}^{y} \frac{dy}{\sqrt{1 - \left[1 - eE/(M_{0\infty}c^2)\right]^2}} = c\sqrt{1 - \frac{v_0^2}{c^2}} \int_{t=0}^{t} dt \ . \tag{26}$$

The integral taking part at the LHS of this equation is still easy to achieve:

$\int \frac{dx}{\sqrt{X}} = -\frac{1}{\sqrt{-c}} \sin^{-1} \frac{2cx + b}{\sqrt{-q}}$, $X = a + bx + cx^2$, $q = 4ac - b^2$, $a = 0$, $b = 2\frac{eE}{m_0c^2}$, $c = -\left(\frac{eE}{m_0c^2}\right)^2$.

$$\int_{y=0}^{y} \frac{dy}{\sqrt{2eEy/(M_{0\infty}c^2) - \left(eEy/(M_{0\infty}c^2)\right)^2}}$$

$$= -\frac{1}{eE/(M_{0\infty}c^2)}\left[\sin^{-1}\frac{-2\left(eE/(M_{0\infty}c^2)\right)^2 Y + 2eE/(M_{0\infty}c^2)}{2\left(eE/(M_{0\infty}c^2)\right)} - \frac{\pi}{2}\right] \quad . \tag{27}$$

$$= \frac{1}{eE/(M_{0\infty}c^2)}\left[\frac{\pi}{2} - \sin^{-1}\left(1 - \frac{eEY}{M_{0\infty}c^2}\right)\right]$$

$$= c\sqrt{1-\frac{v_0^2}{c^2}}\int_{t=0}^{T} dt = c\sqrt{1-\frac{v_0^2}{c^2}}\frac{L}{v_0},\ 0 \le \left(1 - \frac{eEY}{M_{0\infty}c^2}\right) \le 1$$

This yields

$$\sin^{-1}\left(1 - \frac{eEY}{M_{0\infty}c^2}\right) = \frac{\pi}{2} - \frac{eEL}{M_{0\infty}c^2}\frac{c}{v_0}\sqrt{1-\frac{v_0^2}{c^2}} \ , \tag{28}$$

or

$$\frac{eEY}{M_{0\infty}c^2} = 1 - \cos\left(\frac{eEL}{M_{0\infty}c^2}\frac{c}{v_0}\sqrt{1-\frac{v_0^2}{c^2}}\right) \ , \tag{29}$$

A summary of our results are presented in Table 1, for each of the cases taken into consideration, along with the related Second Order Taylor expansions.

Table 1 Overall Deflection for Different Cases

Cases	**Overall Deflection of the Electron at the Exit, Per Unit Length of Course** $\left(X = \frac{EeL}{m_{0\infty}c^2},\ k = \frac{v_0}{c}\right)$
Non-Relativistic Approach	$\frac{Y}{L} = \frac{1}{2}\frac{EeL}{m_{0\infty}c_0^2}\frac{c_0^2}{v_0^2} = \frac{1}{2}\frac{X}{k^2}$
Classic-Relativistic Approach	$\frac{Y}{L} = \frac{\sqrt{1-\frac{v_0^2}{c^2}}}{\left(\frac{eEL}{m_{0\infty}c^2}\right)}\left[\sqrt{1+\left(\frac{eEL}{m_{0\infty}c^2}\frac{c}{v_0}\right)^2} - 1\right] = \frac{\sqrt{1-k^2}}{X}\left(\sqrt{1+\frac{X^2}{k^2}} - 1\right)$ *(rigorous expression)* $\frac{Y}{L} \cong \frac{1}{2}\frac{eEL}{m_{0\infty}c^2}\sqrt{1-\frac{v_0^2}{c^2}}\frac{c^2}{v_0^2}\left[1 - \frac{1}{4}\left(\frac{eEL}{m_{0\infty}c^2}\right)^2\frac{c^2}{v_0^2}\right]$ *(second order Taylor expansion of the rigorous expression)*

Present Approach	$\frac{Y}{L}=\frac{m_{0\infty}c^2\left(1+\frac{eEd}{2m_{0\infty}c^2}\right)}{eEL}\left\{1-\cos\left[\frac{eEL}{m_{0\infty}c^2\left(1+\frac{eEd}{2m_{0\infty}c^2}\right)}\frac{c}{v_0}\sqrt{1-\frac{v_0^2}{c^2}}\right]\right\}$ $=\frac{\left(1+\frac{1}{2}\frac{d}{L}X\right)}{X}\left\{1-\cos\left[\frac{X}{k\left(1+\frac{1}{2}\frac{d}{L}X\right)}\sqrt{1-k^2}\right]\right\}$ *(rigorous expression yeld by the present approach)* $\frac{Y}{L}\cong\frac{1}{2}\frac{eEL}{m_{0\infty}c^2}\left(1-\frac{v_0^2}{c^2}\right)\left(1-\frac{eEd}{2m_{0\infty}c^2}\right)\frac{c^2}{v_0^2}\left[1-\frac{1}{12}\left(\frac{eEL}{m_{0\infty}c^2}\right)^2\left(1-\frac{v_0^2}{c^2}\right)\left(1-\frac{eEd}{2m_{0\infty}c^2}\right)^2\frac{c^2}{v_0^2}\right]$ *(second order Taylor expansion)*

ANALYSIS OF THE RESULTS

The analysis of Table 1 shows that, all of the three results (i.e. classical non-relativistic result, classical relativistic result, and the present result) bear, *quite different mathematical forms,* but they ultimately lead to the same result, for a weak field, and ordinary velocities. Still, the characteristics of our result [i.e. Eq.(29)], is very different than of those of the two others, for it yields to a *quantization* of the deflection versus to the velocity of the electron.

This is something to dig in separately. Nonetheless, in order to grasp it shortly, let us first make the following definitions:

$$X=\frac{EeL}{m_{0\infty}c^2}\quad , \tag{30}$$

$$k=\frac{v_0}{c}\ , \tag{31}$$

$$A=\frac{X}{1+dX/(2L)}\quad . \tag{32}$$

We can then write X, in terms of A:

$$X=\frac{1}{A-(d/2L)}\ . \tag{33}$$

Thus, the deflection per unit length Y/L of the capacitor, for a given length L and width d, and for a given electric field strength E, oscillates between 0, and $2/A$, as the cosine varies between 1 and -1, as a function of k. For a small X, A becomes X; in this case, the deflection Y/L per unit length of the capacitor, oscillates in the range of [0, 2/X].

The cosine function takes the value of unity, when its argument takes the value of $n2\pi, n=0,1,2,...$:

$$\frac{A}{k}\sqrt{1-k^2}=n(2\pi), n=0,1,2,...\ . \tag{34}$$

This yields

$$k^2=\frac{v_{n0\,min}^2}{c^2}=\frac{1}{1+n^2 4\pi^2/A^2}\ ,\ n=0,1,2,...\ , \tag{35}$$

(nth velocity making the deflection vanish)

where, $v_{0n\,min}$ represents the nth velocity making the deflection vanish.

The cosine function takes the value of -1, when its argument takes the value of $(2n+1)\pi, n=0,1,2,...$:

$$k^2 = \frac{v_{n0\,max}^2}{c^2} = \frac{1}{1+\frac{(2n+1)^2\pi^2}{A^2}}, \ n=0,1,2,... \ , \tag{36}$$

(nth velocity making, for a given field strength, the deflection maximum)

where, $v_{0n\,max}$ represents the nth velocity making the deflection vanish.

Note that $v_{0n\,max}$ is smaller than $v_{0n\,min}$. Note further that for n=0, Eq.(35), anyway leads to $v_{00\,min} = c$.

The splitting between the two velocities, can be formulated as

$$\frac{c^2}{v_{n0\,max}^2} - \frac{c^2}{v_{n0\,min}^2} = \frac{(3n^2+4n+1)\pi^2}{A^2}, \ n=0,1,2,... \ . \tag{37}$$

In short, for every n (for given E, L and d), one can determine, a pair of velocities $v_{0n\,max}$ and $v_{0n\,min}$. *(Recall that, the velocity $v_{0n\,max}$ of the electron, making the deflection maximum, is smaller than the velocity $v_{0n\,min}$ of the electron, making the deflection minimum.)* Thereby, as the velocity gets increased from $v_{0n\,max}$, toward $v_{0n\,min}$, the deflection within the range in question, decreases from 2/A to 0. As n tends toward infinity, the velocity of the electron, furnished, either by Eq.(35), or by Eq.(36), tends toward zero; furthermore, $v_{0n\,max}$ and $v_{0n\,max}$ get very close to each other, so that under the given circumstances, there can be no distinction between these two velocities. Note that, both the classical non-relativistic deflection, and the classical relativistic deflection, are increasing functions with respect to X, i.e. $eEL/(m_{0\infty}c^2)$. Our deflection is not. Although, all of the three results are virtually indistinguishable, up to very high X's, our result, interestingly delineate maxima. To see this, let us take the derivative of our result $Y/L = (1/A)\{1-\cos(A\sqrt{1-k^2}/k)\}$, and set it to zero. Then we have

$$Z\sin Z + \cos Z = 1 \ , \tag{38}$$

where Z is defined as

$$Z = \frac{A}{k}\sqrt{1-k^2} \ . \tag{39}$$

There is a trivial solution to this equation; it is that

$$Z = \frac{A}{k}\sqrt{1-k^2} = n2\pi, n=0,1,2,... \ . \tag{40}$$

It is interesting to note that, this latter relationship furnishes the velocities making the deflection vanish, for a given A [cf. Eq.(34)], or the value of A making the deflection exhibit an extremum, for a given velocity. For n=0, Eq.(40) leads Z=0, i.e. A=0, thus X=0, or E=0. In other words, n=0 points to a minimum; thus one can conjecture that, Eq.(40) furnishes the minima, the deflection should exhibit.

Note however that, since A cannot increase indefinitely, Z can assume only few $n2\pi$, if not just one, i.e. that for n=1. In any case, we should expect Eq.(38) to exhibit maxima, next to the minima delineated by Eq.(40).

We show in Figure 1, a typical behavior of the deflection, in the classical relativistic case, and in our case, versus the electric field intensity (namely, X), for a given velocity, and "capacitor's size properties", namely, "distance between the plates" / "length", i.e. d/L. The quantization we conjectured is clearly observed. It is interesting to note that, all three predictions we studied (cf.

Table 1) are practically indistinguishable), up to quite high values of X, and very law values of electron velocity (which appears to be the reason for which our findings are left in the dark, up to now. Further in Figure 2, we show the deflection, for a given field intensity, versus the speed of the electron. We still observe the expected quantization.

Figure 1 Deflection versus strength of the field, for d/L=1, k=0.1

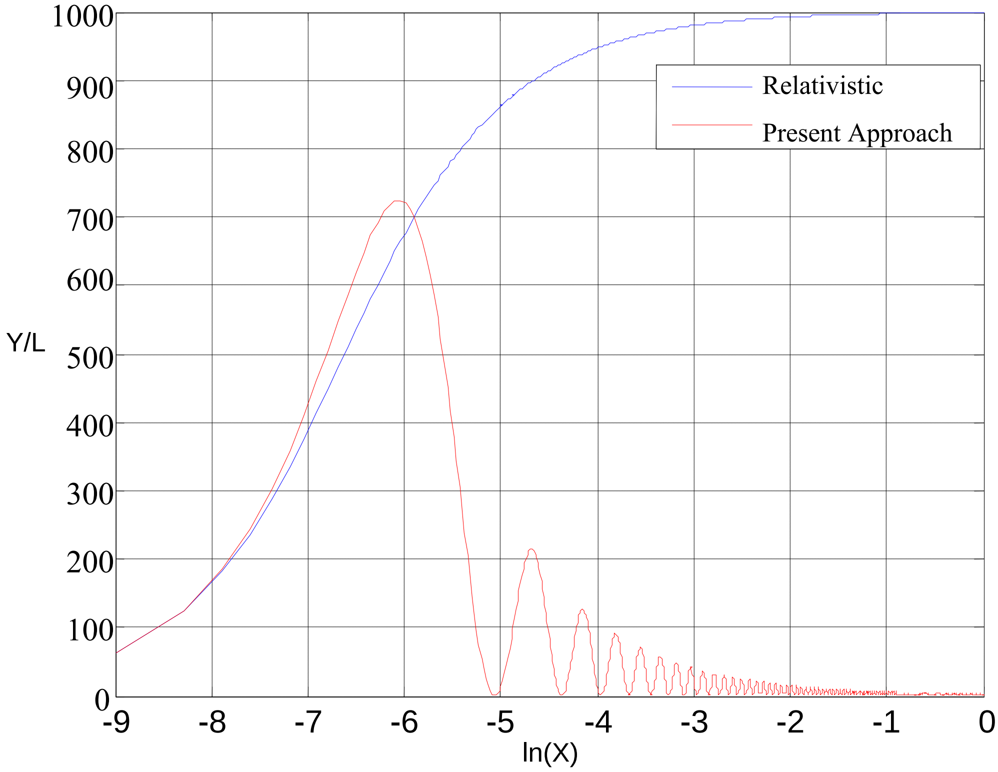
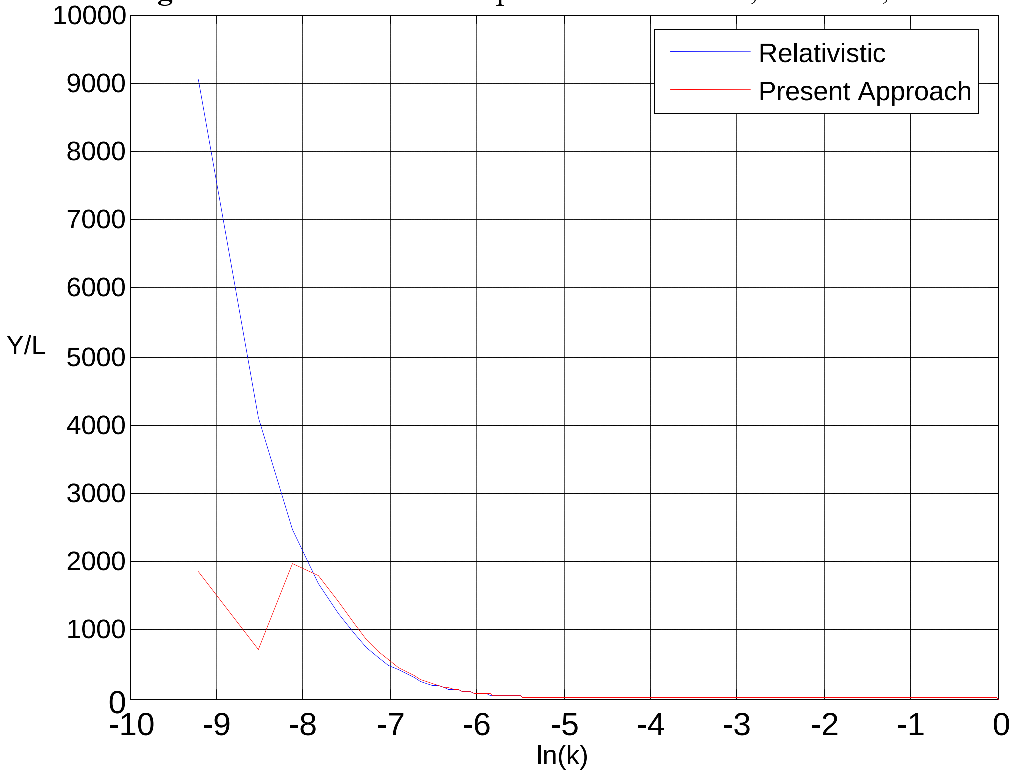


Figure 2 Deflection versus speed of the electron, for d/L=1, X =0.001



CONCLUSION

The most interesting feature we have discovered appears to be the quantization of the deflection of the electron. This is due to the emergence of the cosine function in the expression of the deflection. Thus, for a given set of length L, width d, and electric field strength E, the capacitor assumes, the deflection oscillates between 0, and $2/A$ [cf. Eq.(32)], as a function of the initial speed of the electron. For specific values of this speed, it vanishes or draws maxima [cf. Eqs. (35) and (36)]. Note that the appearance of the cosine function in the expression of the deflection does not depend on our anticipation regarding the practically "immediate" increase of the rest mass of the electron, as soon as it is engaged in its endeavor in between the plates f the capacitor. Besides, a comparative plot of the three results sketched in Table 1, versus the variable X [cf. Eq.(30)], shows that, they all run very close to each other, up to very strong field strengths. Only then a slight divergence appears. The two relativistic results (i.e. the classical relativistic result and our own result), still run very close to each other, and this may be the reason why there has been no suspicion at all, about the validity of the classical relativistic results, up to now. It is also striking that our deflection draws a maximum, with respect to the field strength, and then repeatedly decreases, vanishes, and increases, as the field gets stronger. This is also a basic feature, which related to the quantization property of the deflection, and which evidently, characterizes our approach.

Note that the classical relativistic deflection, increases monotonously with respect to the field strength, climbing up to the upper limit of $L\sqrt{1-k^2}/k$, for a hypothetic infinite field strength [cf. Eq.(19)].

One other characteristic our approach displays, is that the deflection, next to the length of the capacitor, also depends on the width of it.

Note further that, both the classical prediction and our own prediction, lead to a zero deflection, as the velocity tends to the velocity of light. *(This may constitute another reason for which, the difference we predict, between the experimental result and the classical relativistic result, was not so far captured.)* Thus, it seems important not to work with very high velocities, to be able to intercept the difference we unveiled.

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Geometry of Observation: Space-time perspective

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1. The issue

Special Relativity is vexed by preternatural effects: time-dilation and Fitzgerald contraction. They are problematic in the first instance because they are counter intuitive. In addition, they are believed to be the reason that no relativistic formulation of Hamiltonian Mechanics for interacting particles (not a single particle in an exterior field) exists. In other words: because there is believed to be no coherent proper time for a system of particles, or in other words the proper times of individual particles cannot be coordinated, it is thought that there is no variable conjugate to the system Hamiltonian.[1] The simplest example of this problem is captured by the legendary "twin paradox" in which asymmetric ageing implies that the integral of arc lengths along the two different world lines that cross twice, are not equal.

2. The resolution

This writer holds that these opinions are in error and caused by confusion resulting from not distinguishing between emission times and reception times of signals. I argue that the geometry of particles as emitters is Euclidean 3+1, with no metric relation between the 3-space and the 1-space (time). But, I will also argue, that the geometry of the very process of observation using light rays induces the Minkowski-hyperbolic structure of Special Relativity (SR). That is, the physical circumstances of observation with light are a realization of the mathematical structure of projective geometry from the 3+1 Euclidean ontological space onto the celestial sphere of an observer (i.e, the retina [of *one* eye]).[2]

3. Technicalities

The mathematics of projective geometry is complex enough to challenge one's patience. For present purposes it is not necessary to go into details; I shall only suggest some thoughts intended to motivate further detailed study. Projective geometry, which was static and intended for for realistic drawing, was originally developed implicitly with the idea that projective rays have infinite velocity. This assumption must be extended by considering a physical projective ray to be a light ray as described by geometric optics. Such an extention brings with it two well verified empirical features from geometrical optics:

1. Light rays traverse straight lines in empty 3-space.
2. The ratio of variations in 3-space to the index of variation (time), is a constant.

What can be said from projective mathematics applied to this situation is:

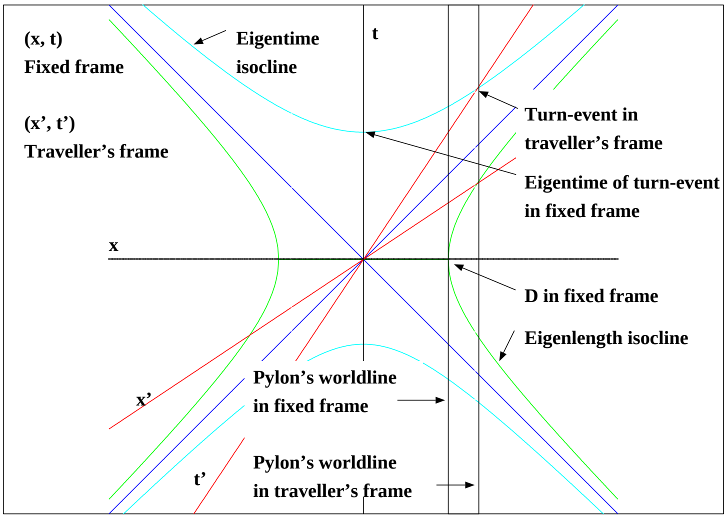
1. Signals arriving at an eye from the same direction at any given instant along a specific direction can have originated at various distances. Their "resolution distance" can be null whatever their physical (3-space) distance if their time-separation interval is appropriate. This is a physical effect or fact unaltered by any change of the observer's eye's position or state of motion.
2. Transformations at or of the eye that leave the null-distance of incoming signals invariant are know to be the Lorentz transformations.[3, 4]

These facts can lead only to the conclusion, that effects derived from Lorentz transforms are artifacts of observation, they are not caused by, nor effect, nor pertain to the source of the signals by themselves.

4. An application: the "twin" paradox

The application of the principles outlined above to the "clock" or "twin" paradox can be greatly simplified by considering the trip to be compose of two components, an outgoing and an incoming. For each component, all accelerations are considered to have occurred in preparation and that the traveler's outbound clock is started by instantaneous contact as the preaccelerated traveler passes the stationary twin. Then the outbound traveler starts the clock of a preaccelerated inbound traveler by instantaneous contact. Finally, the inbound traveler stops his clock and that of the stationary twin also by contact so that the trip is comprised of two segments, none with acceleration.

Minkowski Charts for Relative Motion



This figure is comprised of two Minkowski charts superimposed on each other. The world line of the reversal point in the fixed frame passes through the point 'D' on the -axis. The corresponding point on the x'-axis is found by sliding up the *eigen*length isocline to the intersection with the x'-axis. The world line of the reversal point passes through this point on the prime chart. The intersection of the reversal point's world line with the x'-axis is the point on the traveler's chart representing the 'turn-around' event. The *eigen*time of the turn-around event in the fixed frame is found by sliding down that *eigen*time isocline which passes through the turn-around event to its intersection with the t-axis. It is clear that this value is identical with the time assigned by the fixed twin to the turn-around event as it may be projected horizontally over to the intersection of the reversal point's world line in the fixed frame with the time axis of the traveler. The paradox arises by using, incorrectly, that *eigen*time isocline which passes through the intersection of the traveler's and the reversal point's fixed frame worldlines.[5]

For analysis of the trip now, the crucial issue is the location of the contact point of the inbound and outbound travelers (or the reversal point). According to the analysis above, the changes brought about the the changes due to the boost given the traveler do not affect object in the frame of the stationary twin, including the distance of the reversal point; thus, it is invariant. This means that its location on the Minkowski chart of the traveling twin is located along the hyperbolic isocline defied by the Lorentz transformation, and is further out along the world line of the traveler in his frame than the usual analysis of such a trip would have it. In fact the difference exactly compensates the time-dilation or FitzGerald contraction factor.

5. Experiments

Textbooks on Special Relativity all cite certain experiments considered to verify either time dilation or FitzGerald contraction. Upon examination, it turns out, there are really only two such experiments; and, both can be criticized.

One is the clocks-around-the-world in which atomic clocks were transported around the world in commercial airliners and compared afterwards with similar clocks stationary on the ground. The published conclusions of this experiments seemed to confirm the reality of time dilation. In fact, however, subsequent analysis by A. Kelly showed that the stability of the clocks involved was at least two orders of magnitude too weak to show the effect at appropriate scale.

The other common experiment is that based on the observation of decay mesons at ground level when the common analysis seems to show that all such mesons should have decayed within 200-300 meters based on the observed life time and velocity. However, the standard analysis of this phenomenon overlooks some technical points. In the first place, the governing equation for decay:

$$N(x) = N_0 e^{\frac{-t}{\lambda}}.$$

According to the standard analysis, λ, as the half-life, is subject to time dilation under Lorentz transformation.

In this writer's view, this analysis is too simple. First, note that this function is finite out to infinity, which means that some mesons will always be detected at the surface of the earth, the issue is: how many? To draw the usual conclusion the answer must be many more than would be seen for an identical ensemble that is stationary. It is just this requirement, however, that is affected by space-time perspective; the coefficient N_0 has the units of [No./L^3], and one of the L's is transformed because of the change of inertial frame between a stationary and moving ensemble. When this is taken into account, the whole decay curve for the moving ensemble from the point of view of a stationary earthbound observer is magnified and therfore greater at longer times in comparison to the imagined equivalent stationary ensemble. Again, this is just a space-time perspective effect, and like all such effects is an artifact of the geometry of observation and does not represent an ontological modification of the observed objects.

6. Conclusion

This analysis leads to the conclusion that the individual proper times of two or more particles comprising a system do not conflict, the integrated arc length along all world lines crossing twice are identical between the crossing points; i.e., the integrand of arc-length is singled valued and analytic and therefore path independent.

The importance of this analysis is most significant with regard to the existence of a Hamiltonian formulation of relativistic mechanics as it removes the presumed exclusion of a system proper time as the variable conjugate to the system Hamiltonian.[6, 7]

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Алгебраическая динамика и концепция комплексного случайного времени

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В последнее время увеличилось количество работ, в которых делается попытка понять происхождение физических закономерностей, наблюдаемых в эксперименте, на основе некоторого первичного принципа, "Кода Природы", на элементарном уровне предшествующего известному *генетическому коду* -- коду живого. В развиваемой автором концепции *алгебродинамики* законы движения частиц, их свойства, как и геометрия физического пространства-времени, являются следствиями внутренних "числовых" свойств исключительной алгебры *комплексных кватернионов,* обобщающих "школьную" алгебру комплексных чисел.

Показано, в частности, что эта алгебра порождает наблюдаемую геометрию 4-мерного физического пространства-времени -- геометрию Минковского, однако в дополнение к этому приводит к объяснению свойства необратимости физического времени. Первичная алгебра обуславливает также существование внутренней "фазовой" степени свободы, позволяющей непринужденно объяснить загадочные *волновые* свойства материи, включая явления квантовой интерференции.

Само время в рамках такой расширенной геометрии Минковского оказывается комплексным (двумерным), поэтому хотя сами физические события жестко предопределены (детерминированы), *порядок их следования* не определен и, более того, - *случаен.* С этим связана наблюдаемая "квантовая" неопределенность движения микрочастиц и, с другой стороны, - глобальные "сверхпричинные" корреляции подобные, возможно, наблюдавшимся в серии экспериментов С.Э. Шноля.

В рамках расширенной геометрии Минковского реализуется также давняя идея Фейнмана-Уилера обо "всех электронах как одном единственном электроне", т.е. о частице, "наблюдающей" саму себя в различных (но строго определенных!) точках своей единственной Мировой линии (т.н. концепция "дубликонов", см. Кассандров, PIRT_2005). Заметим в заключение, что алгебродинамика тесно связана с концепцией *твисторов* Р.Пенроуза и с *теорией катастроф* Тома-Арнольда.

Kerr geometry, Spinning particle and Quantum theory

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Abstract

Dirac electron theory and QED do not take into account gravitational field, while the corresponding Kerr-Newman solution with parameters of electron has very strong stringy, topological and non-local action on the Compton distances, polarizing space-time and deforming the Coulomb field.

We argue that the Kerr geometry may be hidden beyond the Quantum Theory. In particular, we show that the the Dirac electron theory may be combined with the Kerr-Schild formalizm and describe an electron as an extended Kerr particle. We consider complex representation of the Kerr geometry, and corresponding complex string-like source. We show that the Kerr theorem provides a vortex twistorial structure of the exctended electron and describe its wave excitations, which shows the physical origin of the wave function.

1 Introduction. Quantum gravity at the Compton level.

Superstring theory is based on the extended stringy elementary states:

Points $\longrightarrow$ *Extended Strings*,

and on the unification of the Quantum Theory with Gravity on Planckian level of masses M_{pl}, which correspond to the distances of order 10^{-33} cm. Such a penetrating into the deep structure of space-time has been based on the convincing evidences:

a/ The brilliant confirmation of the predictions of QED, which ignored gravitational field and has been tested up to distances of order 10^{-16} cm, convinces that boundary of Quantum Gravity may be shifted at least beyond the distances 10^{-16} cm.

b/ The dimensional analysis, showing that M_{pl} corresponds to the energies $E_{pl} = \sqrt{\hbar c^5/G}$ which are formed from the fundamental constants relating quantum theory, $\hbar$, special relativity, c, and gravity, G.

c/ Estimation of the masses M_q and distances, where the action of gravity may be comparable with the action of quantum effects, which is done by the comparison of the corresponding gravitational radius of the Schwarzschild black hole $r_g = 2M_q$ with the Compton radius of the corresponding quantum particle $r_c = 1/M_q$ (we use here the Planck units $\hbar = c = G = 1$). One sees that the equality $r_g \sim r_c$ is achieved by the Planckian masses $M_q \sim 1$, i.e. by

$M_q \sim 10^{-33} cm$. It leads to the conclusion that quantum gravity has to act on the Planckian scale $r_g \approx r_c = \frac{1}{M_q} \sim 1$.

All that is convincing, except for the argument c/ . The Schwarzschild geometry does not take into account spin of quantum particles which is indeed very high with respect to the masses. In particular, for electron $S = 1/2$, while $m \approx 10^{-22}$. So, to estimate gravitational field of spinning particle, one has to use the Kerr, or Kerr-Newman solutions [1]. Of course, there may be objections that quantum processes are strongly non-stationary because of the vacuum fluctuations, and they cannot be described by the stationary Kerr and Kerr-Newman solutions. However, QED tells us that electromagnetic radiative corrections are not too large. On the other hand, we do not know another solution which could better describe the gravitational field of a spinning particle. In any case, estimations on the base Kerr solution have to be much more correct then on the base of Schwarzschild solution.

Performing such estimation, we obtain a striking contradiction with the above scale of Quantum Gravity !

Indeed, for the Kerr and Kerr-Newman solutions we have the basic relation between angular momentum J, mass m and radius of the Kerr singular ring a :

$$J = ma. \tag{1}$$

Therefore, Kerr's gravitational field of a spinning particle is extended together with the Kerr singular ring to the distances $a = J/m = \hbar/2m \sim 10^{22}$ which is of the order of the Compton length of electron 10^{-11} cm. [1] Therefore, in analogy with string theory *the 'point-like' Schwarzschild singularity turns in the Kerr geometry into an extended closed string of the Compton size.*

Notice, that the Kerr string is not only analogy. It was shown that the Kerr singular ring is indeed the string [6] and in the analog of the Kerr solution to low energy string theory the field around the Kerr string is similar to the field around a heterotic string [7].

The use of Kerr geometry for estimation of scale for Quantum Gravity gives the striking discrepancy with the respect to the estimation done with the Schwarzschild solution. We arrive at the conclusion that the Kerr geometry has to play an important role in Quantum processes on the Compton distances of electron, of order $\sim 10^{-11}$ cm

Note, that basically the local gravitational field at these distances is extremely small, and is really negligible. The curvature is concentrated near an extremely narrow vicinity of the Kerr singular ring which forms a closed string extending to the Coppton distances.

[1] See also [2, 3, 4, 6].

2 The real structure of the Kerr-Newman solution.

The Kerr-Newman metric may be represented in the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} - 2Hk_\mu k_\nu, \tag{2}$$

where $\eta_{\mu\nu}$ is auxiliary Minkowski metric and
$H = \frac{mr-q^2/2}{r^2+a^2\cos^2\theta}$,
one sees that metric is Minkowskian almost everywhere, for exclusion of the negligibly small subset of the space-time. However, this stringy subset has very strong dragging effect which polarizes space-time leading to a very specific polarization of the electromagnetic fields. As a result, the electromagnetic field of the corresponding Kerr-Newman solution $F_{\mu\nu}$, which cannot be consider as a weak one for parameters of charged particles, turns out to be aligned with the Kerr principal null congruence. Electromagnetic and gravitational fields are formed by the twisting vector field $k_\mu(x)$, principal null congruence (PNC), and acquire the Kerr stringy circular singularity as a caustic of PNC.

The explicit form of the field k_μ is determined by the one-form

$$k_\mu dx^\mu = dt + \frac{z}{r}dz + \frac{r}{r^2+a^2}(xdx+ydy) - \frac{a}{r^2+a^2}(xdy-ydx). \tag{3}$$

It is a twisting family of null rays, fig.1, forming a vortex which is described by the Kerr theorem in twistor terms[8, 9].[2]

PNC plays very important role, since the field k_μ determines not only the form of Kerr-Newman metric with mass m and charge q, but also the Kerr-Newman electromagnetic vector potential $A_\mu = \frac{qr}{r^2+a^2\cos^2\theta}\, k_\mu$, , and the flow of radiation in the radiative rotating solutions.

The congruence covers spacetime twice, and the Kerr ring is a branch line of the space on two sheets: positive sheet of the 'outgoing' fields ($r > 0$) and negative sheet of the 'ingoing' fields. ($r < 0$). Notice, that for $a^2 >> m^2$ the black hole horizons are absent, and space-time acquires a twofold topology [2, 4].

There appears the Question: "Why Quantum Theory does not feel such drastic changes in the structure of space time on the Compton distances?"

How can such drastic changes in the structure of space-time and electromagnetic field be experimentally unobservable and theoretically ignorable in QED?

The negative sheet of Kerr geometry may be truncated along the disk $r = 0$. In this case, inserting the truncated space-time into the Einstein-Maxwell equation, one obtains on the 'right' side of the equations the source with a disk-like support. This source has a specific matter with superconducting properties

[2]Complicate form of the field $k_\mu(x)$ determines the complicate form of the Kerr metric, contrary to the extremely simple Kerr-Schild representation (2).

[2, 4]. The 'negative' sheet of space appears now as a mirror image of the positive one, so the Kerr singular ring is an 'Alice' string related to the mirror world. Such a modification changes interpretation, but does not simplify problem, since it means that Quantum Theory does not feel this 'giant' mirror of the Compton size, while the virtual charges have to be very sensitive to it.[3]

The assumption, that QED has to be corrected taking into account the peculiarities of the space-time caused by the Kerr geometry, may not be considered as reasonable because of the extraordinary exactness of the QED.

There is apparently the unique way to resolve this contradiction: to conjecture that the Kerr geometry is hidden beyond the Quantum Theory, i.e. is already taken into account and play there essential role.

From this point of view there is no need to quantize gravity, since the Kerr geometry may be the source of some quantum properties, i.e. may be primary with respect to the Quantum Theory.

3 Microgeon with spin.

Let us consider the Wheeler's model of **Mass Without Mass – 'Geon'.** The photons are moving along the ring-like orbits, being bound by the own gravitational field. Such field configuration may generate the particle-like object having the mass and angular momentum. Could such construction be realized with an unique photon? In general, of course - not, because of the weakness of gravitational field. However, similar idea on 'mass without mass' is realized in the theory of massless relativistic strings and may be realized due to the stringy properties of the Kerr solution with $a >> m$. In the Kerr geometry, one can excite the Kerr circular string by an electromagnetic field propagating along this singular string as along of a waveguide. Electromagnetic excitations of the Kerr source with $a >> m$ has the stringy structure, and leads to a contribution to the mass and spin. In particular, the model of microgeon with spin turns out to be self-consistent [3, 6, 11].

Analysis of the exact *aligned* electromagnetic excitations on the Kerr background shows an unexpected peculiarity [11, 10]: the inevitable appearance of two axial singular semi-infinite half-strings of opposite chiralities. There appears the following stringy skeleton of a spinning particle, fig. 2.

The spin of this microgeon may be interpreted as excitation of the Kerr string by a photon moving along a circular orbit, which is reminiscent of the electron self-energy diagram in QED.

In the Kerr's gravity, the virtual photon line of this diagram does not leave the Compton region of the particle due to the Kerr stringy waveguide. As it was shown in [11], the axial half-strings are the null-strings (the Schild, or the pp-

[3]Note, that this disk is relativistically rotating and has a thickness of the order of classical size of electron, $r_e = e^2/2m$,[4, 5].

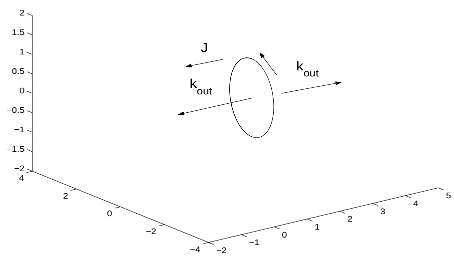

Figure 1: Skeleton of the Kerr Spinning Particle.

wave strings) and may be described by the bilinear spinor combinations formed from the solutions of the Dirac equation.[4]

Moreover, there is a wonderful fact, that the basic quantum relation $E = \hbar\omega$ is already contained in the basic relation of the Kerr geometry $J = ma$, (1). Indeed, setting $J = \frac{\hbar}{2}$, one writes (1) as $a = \frac{\hbar}{2m}$.

So far, we considered the constant $\hbar$ not as a quantum constant, but as an experimentally constant characterizing the spin of electron. Let us consider now the classical fields propagating along the Kerr ring with speed of the light and with the winding number of phase $n = 1/2$. The corresponding length of wave will be $\lambda = 2\pi a/n = 2\pi\hbar/m$ and the corresponding frequency $\omega = 2\pi c/\lambda = cm/\hbar$. It yields

$$\frac{E}{c} \equiv mc = \hbar\omega. \tag{4}$$

Up to now, we have not used the quantum operators at all. We have used only the topological properties providing the two-valued representations by rotations, or the classical quantization of phase (winding number). As a result, we have obtained the quantum relation (4) from the classical Kerr relation $J = ma$:

$$J = ma \quad \Rightarrow \quad E = \hbar\omega. \tag{5}$$

It suggests that *'Kerr's geometry' may cause the origin of Quantum properties,*

4 Dirac equation and the complex Kerr geometry.

Dirac equation in the Weyl basis splits:

$$\sigma^{\mu}_{\alpha\dot{\alpha}}(i\partial_\mu + eA_\mu)\chi^{\dot{\alpha}} = m\phi_\alpha, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha}(i\partial_\mu + eA_\mu)\phi_\alpha = m\chi^{\dot{\alpha}},$$

[4] The axial and circular singularities form a specific multischeeted topology of space-time, which admits the spinor two-valuedness.

and the Dirac spinors form a null tetrad. The null vectors $k_L^\mu = \bar{\chi}\sigma^\mu\chi \qquad k_R^\mu = \bar{\phi}\bar{\sigma}^\mu\phi$, characterize polarization of a free electron in the state with a definite projection of angular momentum on axis z, [11]. Two others null vectors $m^\mu = \phi\sigma^\mu\chi$, and $\bar{m}^\mu = (\phi\sigma^\mu\chi)^+$ are controlled by the phase of wave function and set a synchronization of the null tetrad in the surrounding space-time, playing the role of an order parameter which carries the de Broglie wave.

It is well known [**?**] that the Kerr-Newman solution has the same gyromagnetic ratio ($g = 2$), as that of the Dirac electron. There appears a natural question: is it an incident or there is a deep relationship between the Dirac equation and the Kerr-Newman geometry? This problem is related to the problem of description of electron in coordinate representation, and to the problem of localized states in the Dirac theory [15, **?**].

4.1 Problem of the coordinate description

It is known that operator of coordinate $\hat{\vec{x}} = \nabla_{\vec{p}}$ is not Hermitean in any relativistic theory, $(\Psi, \hat{x}\Phi) \neq (\hat{x}\Psi, \Phi)$.

It suggests that coordinate of electron may be be complex. In the terms of the null vectors $k_L = (1, \vec{k}_L) = (1, 0, 0, 1)$ and $k_R = (1, \vec{k}_R) = (1, 0, 0, -1)$, it takes the form

$$(\bar{\Psi}\hat{X}\Psi) = x + ia(k_L + k_R), \tag{6}$$

where x is a center of mass and $a = \frac{\hbar c}{2m}$ is the Compton length. In the Weil representation, the vectors k_L and k_R transform independently by Lorentz transformations and transfer to each other by the space reflection (inversion) $P = \eta_P\gamma_4, \ |\eta_P| = 1$. It gives a hint that the Dirac particle may be formed by two complex point-like particles $X = \frac{1}{2}(X_L + X_R)$ propagating along the complex world-lines

$$X_L^\mu(t) = x^\mu(t) + ia(1, 0, 0, 1) \quad X_R^\mu(t) = x^\mu(t) + ia(1, 0, 0, -1). \tag{7}$$

Such a representation turns out to be close related to the complex representation of the Kerr geometry [18, 3, 12, 8, 11].

4.2 Complex representation of the Kerr geometry

In 1887 (!) Appel [17] consider a simple complex transformation of the Coulomb solution $\phi = q/r$, a complex shift $(x, y, z) \to (x, y, z+ia)$ of the origin $(x_0, y_0, z_0) = (0, 0, 0)$ to the point $(0, 0, ia)$. On the real section (real(x, y, z)), the resulting solution

$$\phi(x, y, z) = \Re e \ q/\tilde{r} \tag{8}$$

acquires a complex radial coordinate $\tilde{r} = \sqrt{x^2 + y^2 + (z - ia)^2}$. Representing $\tilde{r}$ in the form

$$\tilde{r} = r - ia\cos\theta \tag{9}$$

one obtains for $\tilde{r}^2$

$$r^2 - a^2 \cos^2\theta - 2iar\cos\theta = x^2 + y^2 + z^2 - a^2 - 2iaz. \tag{10}$$

Imaginary part of this equation gives $z = r\cos\theta$, which may be substituted back in the real part of (10). It leads to the equation $x^2 + y^2 = (r^2 + a^2)\sin^2\theta$, which may be split into two conjugate equations $x \pm iy = (r \pm ia)e^{\pm i\phi}\sin\theta$. Therefore, we obtain the transfer from the complex coordinate $\tilde{r}$ to the Kerr-Schild coordinate system

$$\begin{aligned} x + iy &= (r + ia)e^{i\phi}\sin\theta, \qquad (11)\\ z &= r\cos\theta,\\ t &= r + \rho. \end{aligned}$$

Here r and θ are the oblate spheroidal coordinates, and the last relation is a definition of the real retarded-time coordinate ρ. The Kerr-Schild coordinates θ, ϕ, ρ fixe a null ray in M^4 (twistor) which is parametrized by coordinate r.

One sees, that after complex shift, the singular point-like source of the Coulomb solution turns into a singular ring corresponding to $\tilde{r} = 0$, or $r = \cos\theta = 0$. This ring has radius a and lies in the plane $z = 0$. The space-time is foliated on the null congruence of twistor lines, shown on fig. 1. It is twofolded having the ring-like singularity as the branch line. Therefore, for the each real point $(t, x, y, z) \in \mathbf{M^4}$ we have two points, one of them is lying on the positive sheet of space, corresponding to $r > 0$, and another one lies on the negative sheet, where $r < 0$.

It was obtained that the Appel potential corresponds exactly to electromagnetic field of the Kerr-Newman solution written on the auxiliary Minkowski space of the Kerr-Schild metric (2), [3]. The vector of complex shift $\vec{a} = (a_x, a_y, a_z)$ corresponds to direction of the angular momentum J of the Kerr solution, and $|a| = J/m$.

Newman and Lind [18] suggested a description of the Kerr-Newman geometry in the form of a retarded-time construction, in which it is generated by a complex source, propagating along a *complex world line* $\overset{\circ}{X}{}^{\mu}(\tau)$ in a complexified Minkowski space-time $\mathbf{CM^4}$. The rigorous description of this representation was given in the Kerr-Schild approach [1] based on the Kerr theorem and the Kerr-Schild form of metric (2)[5] The complex retarded time is determined in analogy with the real one, but is to be based on the complex null cones [18, 8, 11].

Let's consider the complex radial distance from a real point x to a complex point X_L of the 'left' complex world-line

$$\tilde{r}_L = \sqrt{(\vec{x} - \vec{X}_L)^2} = r_L - ia\cos\theta_L. \tag{12}$$

[5]It is related to the existence of auxiliary Minkowski metric $\eta^{\mu\nu}$, [8, 9].

To determine a retarded-time parameter τ_L one has to write down the light-cone equation $ds^2 = 0$, or

$$\tilde{r}_L^2 - (t - \tau_L)^2 = 0 \tag{13}$$

It may be split into two retarded-advanced-time equations $t - \tau_L = \pm \tilde{r}_L$. The retarded-time equation corresponds to the sign + and, due to (9), leads to relation

$$\tau_L = t - r_L + ia\cos\theta_L. \tag{14}$$

One sees that τ_L turns out to be complex

$$\tau_L = \rho_L + i\sigma_L, \quad \sigma_L = a\cos\theta_L. \tag{15}$$

4.3 The complex worldline as a string

In the complex retarded-time construction, the left complex world line $X_L(\tau_L)$ has to be parametrized by complex parameter $\tau_L = \rho_L + i\sigma_L$. It has a few important consequences.

i/ Being parametrized by two parameters ρ and σ, the complex world-line is really a world-sheet and corresponds to a *complex string*. This string is very specific, since it is extended in the complex time direction σ.

ii/ A fixed value of σ_L corresponds to the fixed value of $\cos\theta_L$, and, in accordance with (12), together with the fixed parameter ϕ, it selects a null ray of the Kerr congruence (twistor).

iii/Since $|\cos\theta| \leq 1$, parameter σ is restricted by interval $\sigma \in [-a, a]$, i.e. complex string is open and the points $\rho \pm ia$ are positioned at its ends. The world-sheet represents an infinite strip: $(t, \sigma) : -\infty < t < \infty, \sigma \in [-a, a]$.

iv/ From (7) and (14) one sees that the left complex point of the Dirac x-coordinate $X_L = ia(1, 0, 0, 1)$ has $\Im m\ \tau_L = ia\cos\theta_L)$, which yields $\cos\theta_L = 1$.

Therefore, this is the boundary point of the complex world line and coordinate relations (12 show that the family of complex light cones positioned at this boundary have the real tracks along the axial null line $z = r, \quad x = y = 0$.

Similar treatment for the right complex point of the Dirac x-coordinate $X_R = ia(1, 0, 0, -1)$ show that it is also placed on the same boundary of the stringy strip (the same timelike component ia), however, $\Im m\ \tau_R = -ia\cos\theta$, which yields $\cos\theta_R = -1$ and corresponds to the axial null line propagating in opposite direction $z = -r, \quad x = y = 0$.

Therefore, two complex sources of the Dirac operator of coordinate have the real image in the real space-time in the form of the considered above two axial semi-infinite half-strings: left and right.[6]

Note, that there is an asymmetry in the complex left and right coordinates $X_L = ia(1, 0, 0, 1)$ and $X_R = ia(1, 0, 0, -1)$. The time-like components of the

[6]For more details see [12, 6, 11].

both sources are adjoined to the same right end of the complex string interval $[-ia, ia]$. This asymmetry is removed by a remarkable stringy construction - orientifold [12, 6, 11].

Orientifold.

The models of relativistic strings contains usually two stringy modes: left and right. So, the modes $X_L(\tau_L)$ and $X_R(\tau_R)$ represent only the half-strings on the interval $\sigma \in [-a, a]$. Orientifold is formed from two open half-strings which are joined forming one closed string, and this closed string is to be folded one. The interval $[-a, a]$ is covered by parameter σ twice: the first time from left to right, and, say the left half-string has the usual parametrization. While the interval $[-a, a]$ is reversed and covers the original one in opposite orientation for the right lalf-string. Therefore, the parameter σ covers interval twice and string turns out to be closed, but folded. The right and left string modes are flipping between the the initiate and the reversed intervals. One sees that for the complex interval the revers is equivalent to complex conjugation of the parameter τ. So, one has to put $\tau_R = \bar{\tau}_L$.[7] After orientifolding, the complex timelike coordinates of the points X_L and $\bar{X}_R$ turns out to be sitting on the opposite ends of the interval $[-a, a]$, while their imaginary space-like coordinates will be coinciding, which corresponds to one of the necessary orientifold condition $X_L(\tau_L) = \bar{X}_R(\bar{\tau}_R)$.

5 Conclusion

The above treatment shows that the electron may possess the nontrivial real and complex structures which are related to the real and complex structures of the Kerr geometry. The Dirac equation works apparently in the complex Minkowski space-time, and the space-time source of the naked electron is not elementary, but represents a specific complex string with two quark-like sources sitting on the ends of this string. While, after orientifolding this string, the space coordinates of these sources are merging, turning into a complex point shifted in the imaginary direction on the Compton distance a. This complex position of the source is, apparently, the origin of the problems with localized states and with the operator of coordinate in the Dirac theory.

The obtained recently multiparticle Kerr-Schild solutions [9] shed some light on the multiparticle structure of the dressed electron considered in QED. This treatment is based on the remarkable properties of the Kerr theorem. There is also remarkable renormalization of the Kerr singularity by gravitational field [5]. However, these questions go out of the frame of this paper.

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Signals in Reverse Time from Heliogeophysical Random Processes and their Employment for the Long-term Forecast

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Quantum mechanical principle of weak causality admits signaling in reverse time for the genuine random processes. It reflects in the heuristic equation of macroscopic nonlocality. Series of the long-term experiment had been revealed availability of the advanced response of random dissipative probe processes in the lab detectors to the large-scale dissipative heliogeophysical processes with big random component. The level of advanced correlation and time shift allowed to put forward the forecasting problem. This problem has been solved and successfully tested on all available experimental data of enough volume for the of long-term forecast series of the solar and geomagnetic activity.

1. Introduction

Since 1930-th phenomenon of quantum nonlocality has been attracting attention, above of all in connection with apparent violation of relativity. Indeed quantum correlations occur through a spacelike interval, that is possible namely due to absence of any local carriers of interaction. But it remains to be strange, because such correlations imply possible reversal of time ordering. The mainstream of quantum information research avoids this question, as from the outset it had been realized that quantum nonlocal channel could transmit through spacelike interval only *unknown* information, and therefore for the communication purposes one should use an ancillary classical channel. Therefore that question becomes irrelevant.

Cramer [1,2] suggested an elegant transactional interpretation of quantum nonlocality leaned upon Wheeler-Feynman action-at-a-distance theory. By Cramer transaction through spacelike interval need not superluminal speed, it carries out by a couple of the signals traveling in direct and reverse time. It is very natural idea, because Wheeler-Feynman theory is nonlocal itself. Cramer was also the first who explicitly distinguished the principles of strong (local) and weak (nonlocal) causality [1]. The latter implies a possibility of signal transmission in reverse time, but only related with unknown states, or in other terms with genuine random processes. The weak causality admits extraction information from the future without any classical paradoxes. Although Cramer's works had some internal contradiction – explanation of quantum phenomena on the base of classical Wheeler-Feynman theory, now the successively quantum versions of action-at-a-distance theory have been developed [3,4]. On the other hand, as it was generally believed that quantum nonlocality existed only at the micro-level, Cramer supposed that that strong causality might be violated only at this level. However the idea about persistence of nonlocality in the macroscopic limit was put forward from different standpoints [5-9]. In addition the important experimental results were obtained by Kozyrev before the emergence of these ideas, in the framework of causal mechanics concept (and interpreted in another terms), which demonstrated phenomena very similar to macroscopic nonlocality [10], in particular advanced correlations for the (random) dissipative processes [11-13].

The progress in quantum mechanics shed a new light on Kozyrev's works inspired the authors on performance of own experiments [14-25]. As a result, the availability of advanced correlations, that is in literal sense the signals in reverse time, has been reliable revealed for some large-scale random dissipative astrophysical and geophysical source-processes and the probe-processes in the lab detectors highly protected against the local impacts. The correlation magnitude and advancement value proved to be large. It allowed to suggest employment of this phenomenon for the forecast of such source-processes.

In this paper we present the approach to and result of solving the forecast problem for the random component of solar and geomagnetic activity on the base of measurement of nonlocal correlation detector signals in reverse time.

2. Model of macroscopic nonlocal transaction

In spite of the recent progress [5-9], development of the successive theory of macroscopic entanglement (which has to resemble classical thermodynamics, i.e. to operate with the macroscopic parameters) is difficult task and such theory is absent at present. For some cases the macroscopic consequences of entanglement were theoretically predicted, and corresponding experiments were performed [26,27], but they included only deterministic processes, irrelevant to the time reversal problem. At microscopic level the idea of experimental detection of time reversed events was suggested [28], but it have not been realized yet. On the other hand, an important feature of Kozyrev's experiments was dissipativity of the processes. Although it is known that dissipativity leads to decoherence, recently the constructive role of dissipativity in entanglement generation was discovered [29,30].

On the base of those ideas the following heuristic equation of macroscopic nonlocality, relating the entropy production per particle in the probe-process (detector) $\dot{S}_d$ and the density of total entropy production in the sources $\dot{s}$ with symmetrical retardation and advancement has been suggested [14,16,18,19]:

$$\dot{S}_d = \sigma \int \frac{\dot{s}}{x^2} \delta\left(t^2 - \frac{x^2}{v^2}\right) dV , \tag{1}$$

where a cross-section $\sigma \sim \hbar^4 / m_e^2 e^4$, m_e is electron mass, e is elementary charge, $\dot{s}$ is density of the entropy production in the sources, x is distance, t is time, propagation velocity v for diffusion entanglement swapping can be very small, the integral is taken over the source volume.

Let us demonstrate correspondence of heuristic (1) with the strict quantum mechanical result developed for a dilute spin gas [9]. In Ref. [9], for partition of the system A-B, the following equation is obtained:

$$S_A \approx \frac{N_A N_B}{N-1} rt(2 - \log_2 e) , \tag{2}$$

where numbers of particles $N_A + N_B = N$, r is collision rate.

For adaptation (1) to conditions of model (2), forget about time shift and integrate over time, neglecting the irrelevant integration constant. Then (1) in the steady-state regime reduces to:

$$S_d = \sigma \int \frac{s}{x^2} dV . \tag{3}$$

Consider the detector as a small part A of the large homogeneous system. Correspondingly our "sources" proves to be the part B. Then:

$$\frac{S_A}{N_A} = \sigma \frac{S_B}{L^2} , \tag{4}$$

where L is the space size of the system.

Now slightly transform (2), taking into account assumption that mean free path compatible to the size of the enclosing volume [9]. That is $t = L / \langle v_r \rangle$, therefore $rt = \sigma Ln$, where $n = N / V$. On the other hand, $Ln \approx N / L^2$, $rt \approx \sigma N / L^2$. Assume $N >> 1$. At last use ln (not $\log_2$) in the entropy definition (because it was always adopted in our entropy calculation [14,16,18,19]). As a result we can rewrite (2):

$$\frac{S_A}{N_A} \approx \sigma \frac{0.3863 N_B}{L^2} . \tag{5}$$

We have obvious correspondence (4) and (5) with $S_B \approx 0.3863N_B$.

This correspondence encourage to consider the equation of macroscopic nonlocality (1) as at least a not too bad approximation of reality.

Eq.(1) in it simples form its completely time symmetric. It is a consequence of time symmetry of original Wheeler-Feynman approach. Known agreement with observed time asymmetry was achieved by *ad hoc* emitter-absorber phase relation leading to destructive and constructive interference for the advanced and retarded fields respectively. Hoyle and Narlikar [4] have proved that observed time asymmetry emerges from absorption asymmetry: efficiency of absorption of the advanced field is less than (perfect) of the retarded one (although their theory does not predict how much less). They have explained it by the cosmological reasons: the fact is only Steady-state and Quasi-steady-state cosmological models provide such asymmetry. But their proof itself [4] did not refer to any cosmological conditions and could be applied, e.g. for a radiating charge in a cavity. Therefore absorption asymmetry reflects time asymmetry at more deep level in spirit of Kozyrev [10]. Observational consequence of the absorption asymmetry, if there is an intermediate medium, has to be prevailing advanced nonlocal correlation over retarded one.

Nonlocal nature of macroscopic correlations can be tested by two ways. They both are based on the causal analysis [25, 31-35].

The first way is verification of violation of strong causality. The causality function of arbitrary classical variables X and Y defined as $\gamma = i_{Y|X} / i_{X|Y}$ that is as the ratio of independence functions: $i_{Y|X} = S(Y|X) / S(Y)$, $i_{X|Y} = S(X|Y) / S(X)$, where S are corresponding Shannon conditional and marginal entropies By definition $\gamma > 1$ means that Y is cause and X is effect. Principle of strong causality is:

$$\gamma > 1 \Rightarrow \tau < 0, \tag{6}$$

where τ is time shift of the correlation maximum of Y relative to X. Violating of (6) means signaling in reverse time, that is sufficient condition of nonlocality. Note, for quantum variables we have to use von Neumann entropies and consequently, instead of γ, more complicated function of course of time [25]. But as below we use only classical output of measuring device, we may employ γ without limitations.

The second way is verification of the following Bell-like inequality:

$$i_{X|Z} \geq \max\,(i_{X|Y}, i_{Y|Z}), \tag{7}$$

where local connection of the processes X, Y, Z is possible only along the causal chain $Z \rightarrow Y \rightarrow X$. Violation of (7) is sufficient condition of nonlocal nature of correlation X and Z. Note, that similar to usual Bell inequalities, violation of (7) does not forbid existence of *nonlocal* hidden variables [25].

3. Experiments

As it is not possible to measure $\dot{S}_d$ and $\dot{s}$ in (1) directly, we have to evaluate for the concrete source and probe processes the theoretical expressions relating the entropies with the observables: $\dot{S}_d = F(P_d, \{p_d\})$, $\dot{s} = f(P_s, \{p_s\})$, where P_s is measured parameter of the source-process, P_d is the same of the probe-process (detector signal), $\{p\}$ is set of other parameters of the processes, influencing on the entropy, which must be known unless they are stable. This problem has been solved for three types of the probe-processes: spontaneous variations of weakly polarized electrodes in an electrolyte [14, 16-18, 22], spontaneous variations of dark current of the photomultiplier [22] and fluctuations of ion mobility in a small electrolyte volume [36]. The problem is quite solvable also for any source-process, though we used for quantitative verification of (1) only a rather simple example of Ohmic dissipation [14,16-18, 22].

The experiments were performed with mentioned three types of detectors. In their construction the main attention was paid to exclusion of all possible local impacts (temperature and the like). The design of the experimental setups and their parameters are described in detail in [14-18].

The experiments with controlled (deterministic) lab source-processes (phase transition, etc) demonstrated, of course, only retarded correlations [15, 36].

The main effort was directed to detection of correlations with the spontaneous (random) source-processes in the environment: the meteorological, ionospheric, geomagnetic and solar activity in the long-term experiments in 1993-2003. The full description of the data, their processing and interpretation is presented in [14, 16-25]. The main results are:

1. Signals of different detectors spaced up to 40 *km* turned out correlated and this correlation can not be explained by a local impact of any common factors.

2. Magnitudes of the detector signals are satisfactory corresponded to predictions of Eq.(1).

3. The most prominent fact is reliable detection of the advanced response of the probe-processes to the all above source ones. Both inequalities (6) and (7) are violated. Maxima of the correlation functions of the detector signals and the indices of source-activity are observed at advancement of order 10 hours – 100 days and its magnitude is as much as 0.50 – 0.95. Both the advancement and correlation magnitudes increase with the source spatial scale. Advanced correlation always more than retarded, their ratio is 1.1 – 2.6.

Of course, existence of the different sources called for data prefiltration for signal separation. But the prefiltration also was called for suppression of deterministic periodic components for increase the signal/noise ratio in the advanced domain.

It turned out that contributions of the solar and geomagnetic activity in the detector signal could be more readily separated. The best index of the solar activity proved to be the radio wave flux *R* at frequency range 610 – 2800 *MHz*, that is emitted from the level of upper chromosphere – low corona (that is level of maximum dissipation in the solar atmosphere). The optimal frequency with in this range changed from year to year. The best index of the geomagnetic activity proved to be *Dst*-index reflecting the most large-scale dissipative processes in the magnetosphere.

In Fig 1 an example of correlation function of the solar activity and detector signal is shown, in Fig. 2 – the same for geomagnetic activity (by the same realization). In both cases maximal correlation corresponds to advancement 42 days (known retardation of geomagnetic activity relative to solar one is insignificant in this scale). The value 42 days was rather typical, although the processes turned out strongly non-stationary and for different realizations position of the maximum varied from 33 to 130 days.

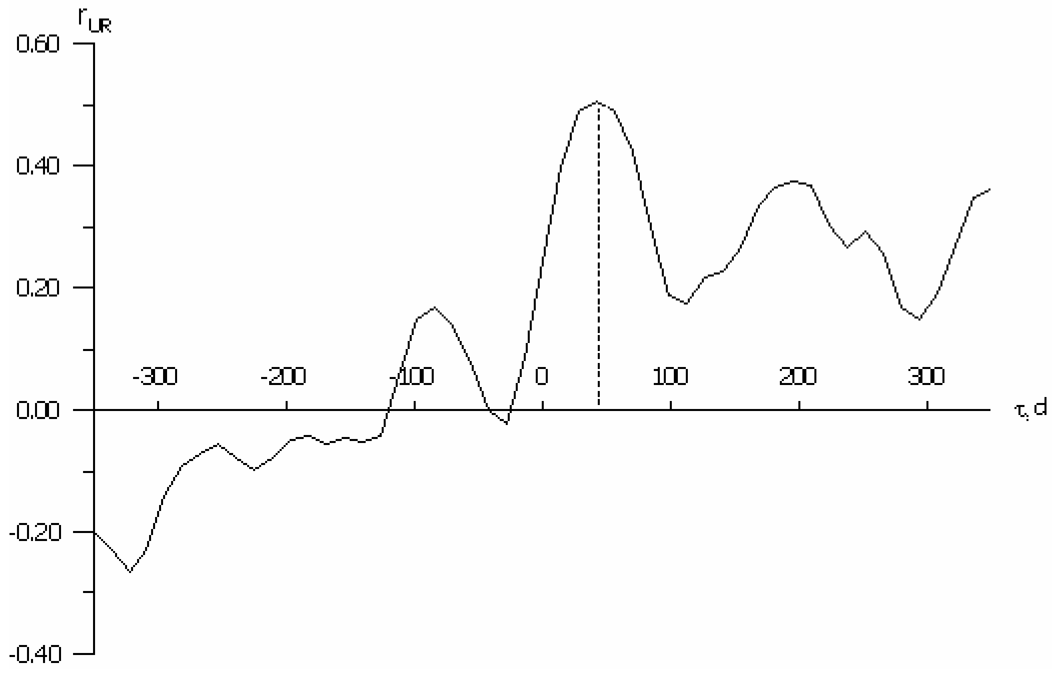

Fig.1. *Correlation function r_{UR} of the detector signal U and solar activity R by low-pass filtered data T > 28 dqys. Negative time shift τ, days, corresponds to retardation U relative to R, positive one - to advancement.*

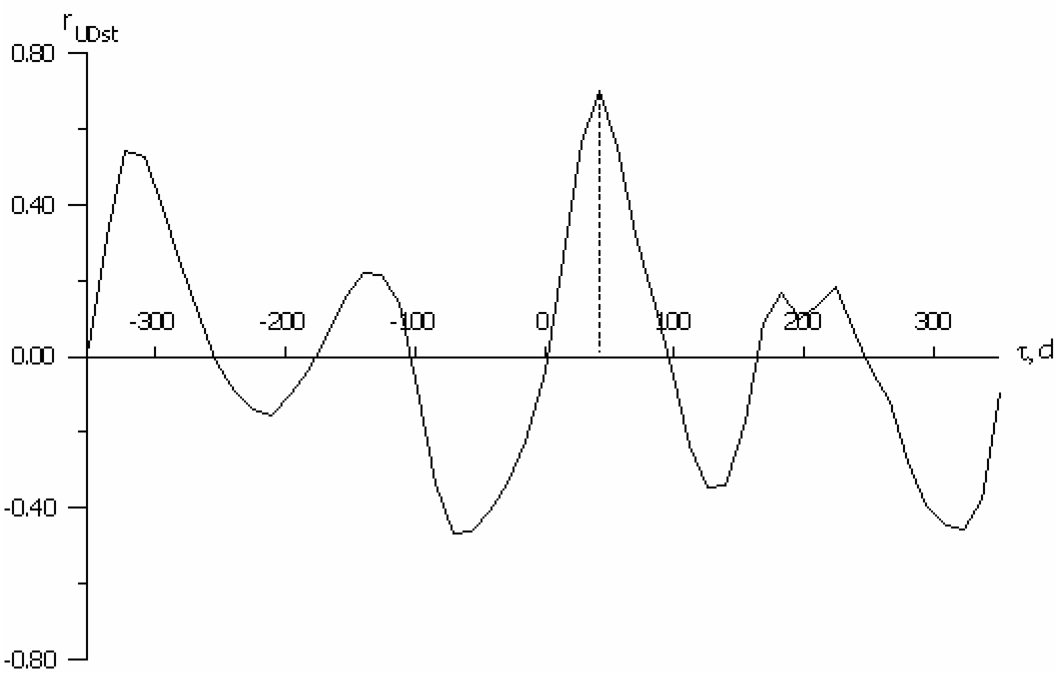

Fig. 2. *Correlation function r_{UDst} of the detector signal U and geomagnetic activity Dst by data filtered in period range 364 > T > 28 days. Negative time shift τ, days, corresponds to retardation U relative to Dst, positive one – to advancement.*

4. Forecasting algorithm

Availability of the advanced correlation allowed to demonstrate the possibility of the forecast of random component of the solar and geomagnetic activity by the detector signal by means of shift of the realizations [17-24].

But for the real forecast such simplest approach fails, since, first, the processes are far from δ-correlated ones, therefore big errors are unavoidable and, second, position of the main correlation maximum is instable because of non-stationarity of the processes and one can use it only for *a posteriori* demonstration.

For the solution of the real problem the algorithm has been elaborated, based on the convolution of impulse transfer characteristic with multitude of the preceding detector signal values. On the "training" interval $[t_1, t_n]$ the impulse transfer characteristic $g(\tau)$ is computed, which relates the detector signal X and forecasted parameter (activity index) Y with advancement $\Delta t = t - t_n$, by solving the convolution equation:

$$Y(t) = \int_{t_1}^{t_n} g(\tau) X(t-\tau) d\tau \, . \tag{8}$$

Solving of (8) in the discrete form is reduced to the system of linear equations $\{\boldsymbol{Y}=\boldsymbol{XK}\}$. The components of $\boldsymbol{K}$ vector are equivalent to coefficients of plural cross-regression (for the case of Gaussian distribution). The number of equations n equals to the advancement of the forecast. $\boldsymbol{X}$ is the square matrix $n \times n$, the strings are formed from values of the detector signal on the training interval. The first string consists of the values with time index from 1 to n, the second – from 2 to n+1, etc. The sequential values of the Y are corresponding to the each string of matrix. The system is solved by Gauss method. The stability of the results are achieved by an optimal regularization. Practically the advancement is chosen equal to expected average position of correlation maximum. The total training interval for Y ends by the last observed value, while for X – preceding on Δt.

The computed by such way transfer characteristic then is used for the calculation of the only value of the forecasted parameter Y with the advancement Δt. For this purpose the direct problem (8) is solved by X interval ended by the last observed value. On the next day the training interval moved forward and the next value Y is forecasted. Such procedure allow to minimize influence of non-stationarity. To suppress the residual instability the received sequence goes through an optimal low-pass postfiltration.

This method is more preferential over often employed in the akin context (of uncertainty of the cross-correlation function maximum) the plural regression method on correlation matrix calculation, since the suggested one does not require any additional hypothesis about the probability distribution. It is essential, for the reason that distribution very seldom is the eigendistribution, what is needed for singleness of the regression problem traditional solution, and it is not nearly always Gaussian,. what is needed for correspondence of this solution to the maximal likelihood criterion.

5. Experimental forecasting

For test of the method in the regime of real forecast simulation, all obtained detector signal hourly time series of sufficient length – not less than one year for R and two years for Dst (because of shortcoming of the series length, especially valuable with wide-band prefiltration necessary for Dst). Only data of the electrode detector U (which was the most reliable) satisfied this requirement. Results of day by day forecasting series (with duration less than observed ones at the expense of corresponding prefiltration and employment of initial segments as training ones) were compared with factual evolution of Dst or R. Quality of the forecast was assessed by standard deviation of the curves ε in corresponding absolute units, that is nT for Dst and 10^{-22} $Wm^{-2}Hz^{-1}$ for R. The optimal postfiltration in the almost all cases had pass period $T > 14$ days.

In the algorithm described above, the every point of forecasted curves, presented below. is result of computation by selected observed data, minimal volume of which is determined by the forecast advancement (determining duration of the training interval) and by the filter parameters. It should be stressed that only the long-period random component is forecasted, that is the forecast is background, although the nonlocality effect in itself admits the forecast of individual powerful events [20].

In Fig. 3 the solar forecast by the same data (and with the same prefiltration $T > 28^d$) as for Fig. 1 is shown. Advancement of the forecast $\Delta t = 35^d$, error $\varepsilon = 0.88$. Without postfiltration $\Delta t = 42^d$, $\varepsilon = 1.16$.

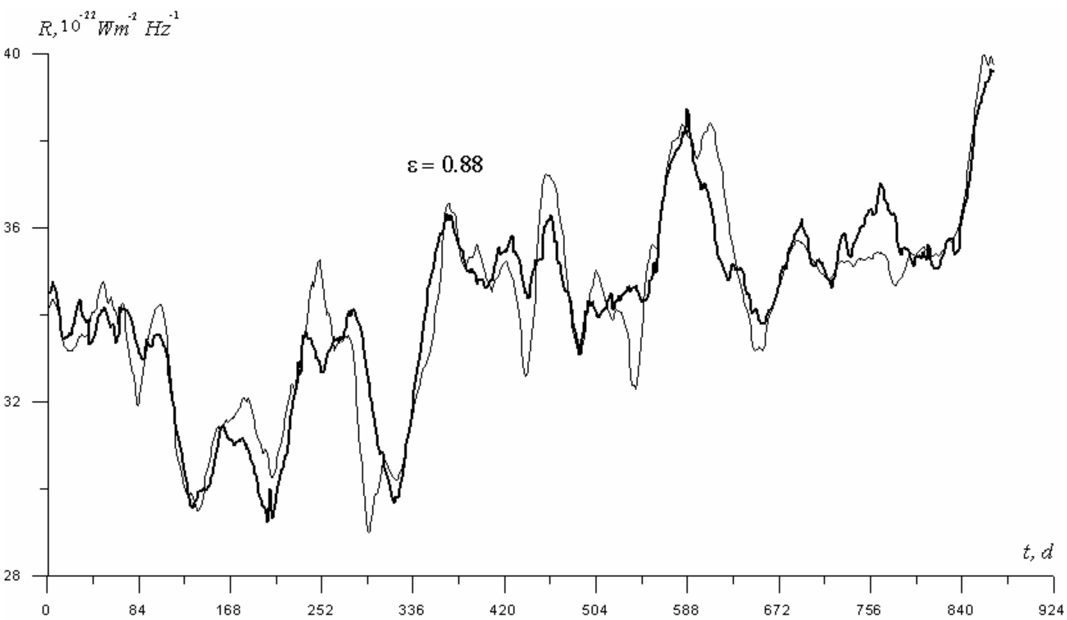

Fig. 3. *The forecast of solar activity R (at 610 MHz) with advancement 35 days(fine line) compared to the factual curve(thick line). The origin of time count corresponds to 3/20/1995.*

In Fig. 4 the geomagnetic forecast by the same data (and with the same prefiltration $364^d > T > 28^d$) as for Fig. 2 is shown. Advancement of the forecast $\Delta t = 35^d$, error $\varepsilon = 1.7$. Without postfiltration $\Delta t = 42^d$, but $\varepsilon = 2.4$.

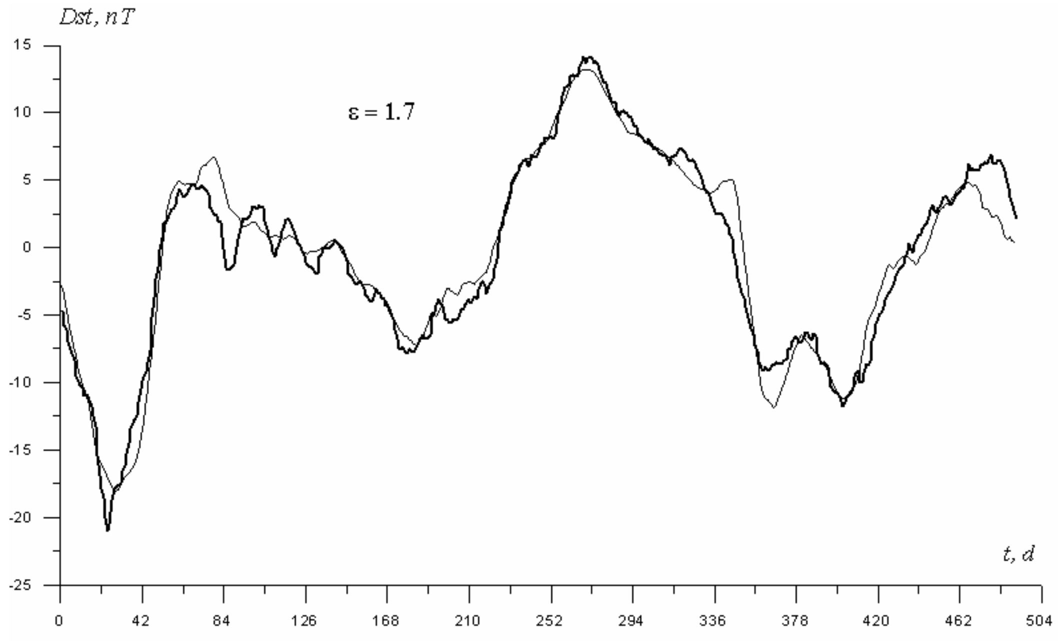

Fig. 4. *The forecast of geomagnetic activity Dst with advancement 35 days (fine line) compared to the factual curve (thick line). The origin of time count (days) corresponds to 9/19/1995.*

In Fig. 5 the solar forecast for the time of beginning of the next in turn solar cycle is shown. As the moment of beginning is a random event, it is interesting to test capability of the method. For this reason prefiltration for this case is $T > 7^d$ 1 The forecasting curve was postfiltered also with $T > 7^d$. Resulting advancement $\Delta t = 39^d$ and error $\varepsilon = 5.2$ are only slightly less than without postfiltration: $\Delta t = 42^d$, $\varepsilon = 5.4$. It is seen that cycle beginning (sharp increase of R at 125 d) is well predicted.

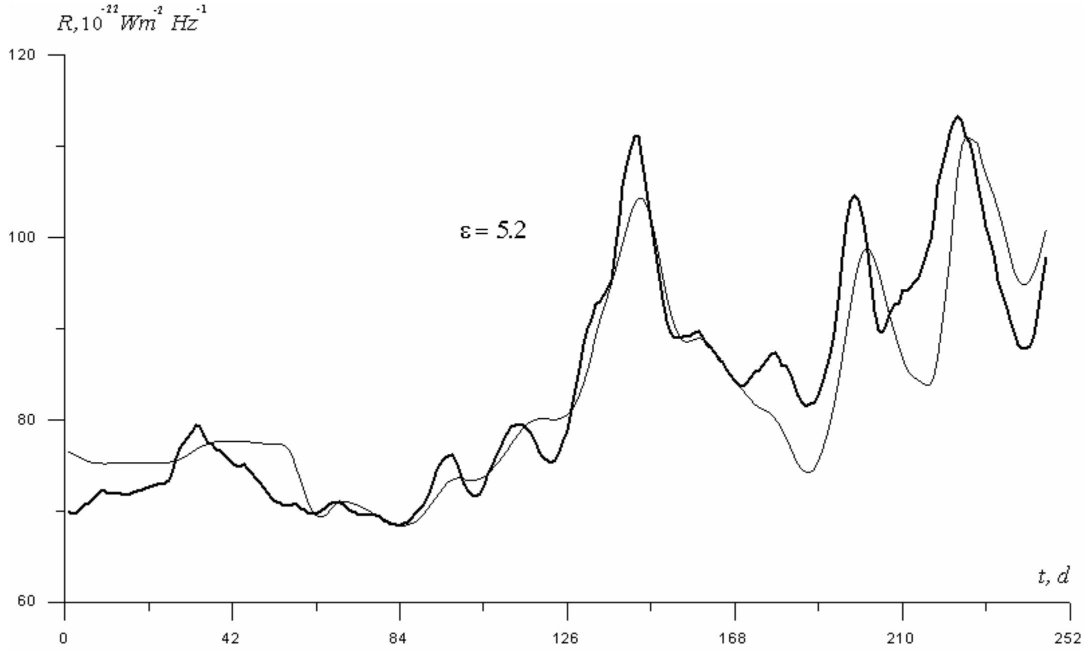

Fig. 5. *The forecast of solar activity R (at 2800 MHz) with advancement 39 days (fine line) compared to the factual curve (thick line). The origin of time count (days) corresponds to 3/21/1997.*

In Fig. 6 the solar forecast by data of the most recent experiment [22, 23] provided the most advancement is shown. Prefiltration was $28^d < T < 183^d$, postfiltration – $T > 14^d$. Resulting $\Delta t = 123^d$, $\varepsilon = 2.0$, while without postfiltration $\Delta t = 130^d$, $\varepsilon = 2.4$.

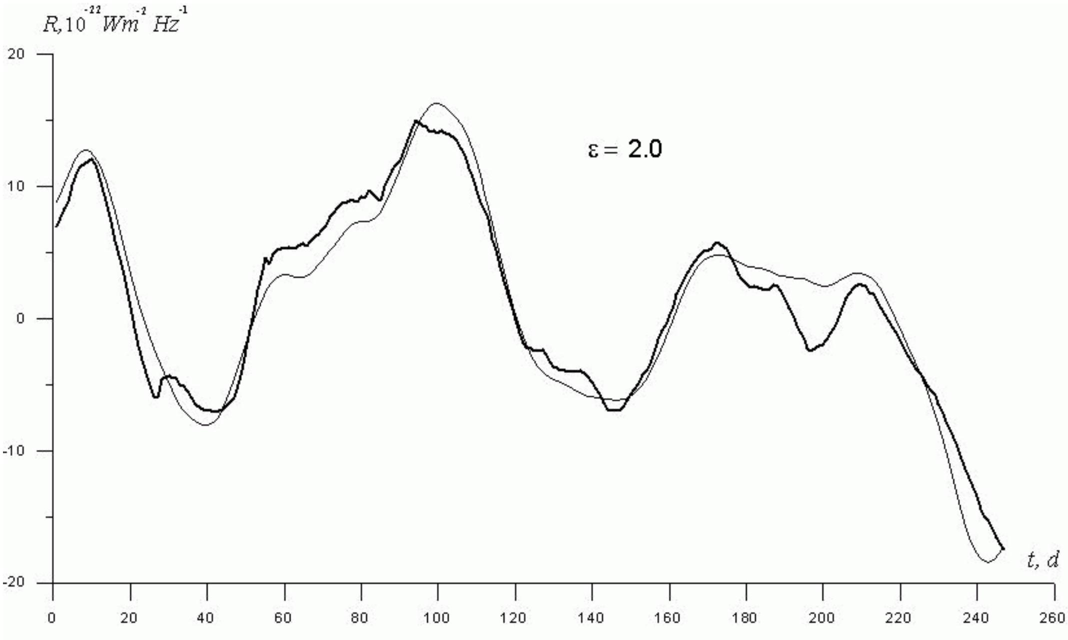

Fig. 6. *The forecast of solar activity R (at 1415 MHz) with advancement 123 days (fine line) compared to the factual curve (thick line). The origin of time count (days) corresponds to 2/20/2003.*

In Fig. 7 the geomagnetic forecast by the same data and with the same pre- and postfiltration as for Fig. 6 is shown. Resulting $\Delta t = 123^d$, $\varepsilon = 2.9$, while without postfiltration $\Delta t = 130^d$, $\varepsilon = 3.5$.

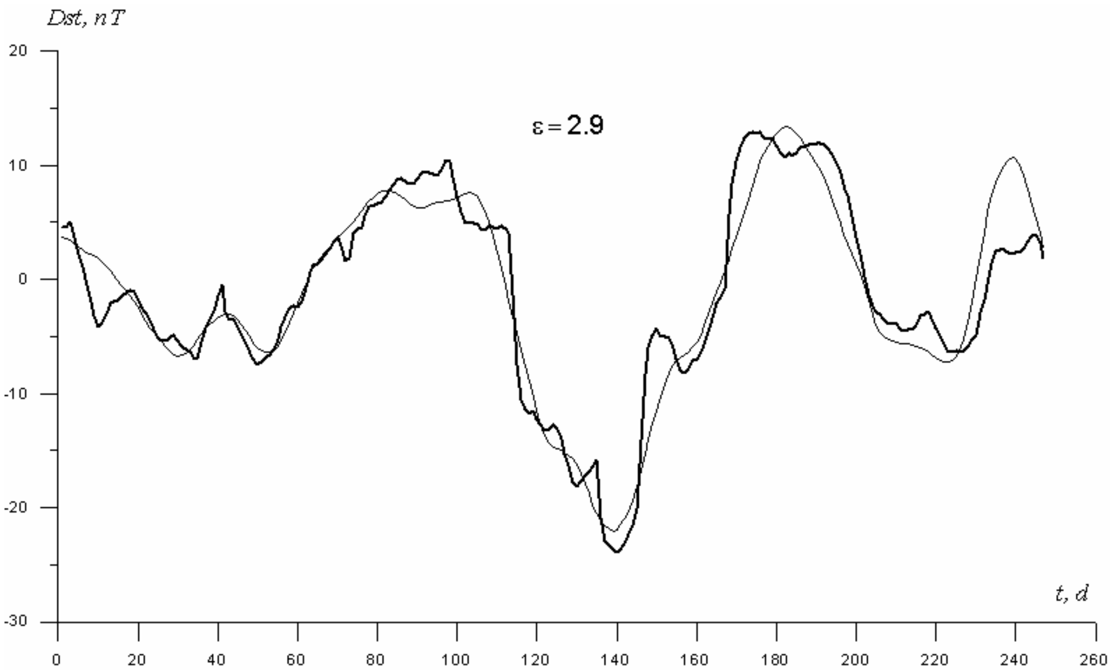

Fig. 7. *The forecast of geomagnetic activity Dst with advancement 123 days (fine line) compared to the factual curve (thick line). The origin of time count (days) corresponds to 2/20/2003.*

6. Conclusion

We have considered the model of macroscopic nonlocality describing the unusual advanced correlation of the dissipative processes. The experimental data have confirmed observability of such correlation for large-scale natural dissipative processes. Among them the most easy for detection proved to be the random component of solar and geomagnetic activity. The pragmatic forecasting algorithm on the nonlocal correlations has been elaborated.

Employment of nonlocal correlation allowed to realize the background long-term forecast of solar and geomagnetic activity with acceptable for all the practical purposes accuracy. Probably, this idea may be also implemented for the forecasts of the dissipative processes with big random component. It should be stressed that suggested method is unique one namely by the possibility of forecasting of the *random* component. All existing approaches to the forecasting problem are deterministic (in spite of employment of statistical cross- or auto-regression algorithms), the random component represents for them unavoidable error. Therefore the described method is essentially complimentary to the customary ones.

Acknowledgement

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Space-time structure revealed during investigation of local-time effect

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The results of many years of investigation of macroscopic fluctuation phenomena can be considered as evidence of an essential heterogeneity and anisotropy of space-time. This statement is based upon the results of studies of α-decay rate fluctuations of ^{239}Pu sources measured by plane semiconductor detectors and detectors with collimators cutting α-particle beams, carried out in the years 1985–2005 [1–6]. For reasons of methodology, the time resolution reached in those years was about one minute, and the studied spatial scale about a hundred kilometers. This work presents results of further investigations of macroscopic fluctuations phenomena with time resolution to 0.5 milliseconds. Such resolution allows studies of local time effects for distances down to one metre between sources of fluctuations [7, 8]. On the one hand, this result has an independent importance as a lower scale end for the existence of macroscopic fluctuations phenomena, but on the other hand, it has great methodological importance due to the possibility of systematic laboratory investigations, which were previously unavailable because of very large spatial distances between places of measurement. Several such investigations are the subject of this paper.

1. Introduction

Our previous works [1 – 4] give a detailed description of macroscopic fluctuations phenomena, which consists of regular changes in the fine structure of histogram shapes built on the basis of short samples of time series of fluctuations in different process of any nature – from biochemical reactions and noises in gravitational antennae to fluctuations in α-decay rate. From the fact that fine structure of histograms, which is the main object of macroscopic fluctuation phenomena investigations, doesn't depend on the qualitative nature of the fluctuating process, so it follows that the fine structure can be caused only by the common factor of space-time heterogeneity. Consequently, macroscopic fluctuation phenomena can be determined by gravitational interaction, or as shown in [5, 6], by gravitational wave influence.

The present work was carried out as further investigations into macroscopic fluctuation phenomena. The local time effect, which is the main subject of this paper, is synchronous in the local time appearance of pairs of histograms with similar fine structure constructed on the basis of measurements of fluctuations in processes of different nature at different geographical locations. The effect illustrates the dependence of the fine structure of the histograms on the Earth's rotation around its axis and around the Sun.

The local time effect is closely connected with space-time heterogeneity. In other words, this effect is possible only if the experimental setup consists of a pair of separated sources of fluctuations moving through heterogeneous space. It is obvious that for the case of homogeneous space the effect doesn't exist. Existence of a local-time effect for some space-time scale can be considered as evidence of space-time heterogeneity, which corresponds to this scale.

The existence of a local time effect was studied for different distances between places of measurement, from a hundred kilometres up to the largest distance possible on the Earth (~ 15000 km). The goal of the present work is an investigation of the existence of the effect for distances between places of measurements ranging from one metre up to tens of metres. Such distances we call 'laboratory scale'.

2. Experimental investigations of the existence of a local-time effect for longitudinal distances between places of measurements from 500 m to 15 km.

The main problem of experimental investigations of a local-time effect at small distances is resolution enhancement of the macroscopic fluctuations method, which is defined by histogram duration. All investigations of a local-time effect were carried out by using α-decay rate fluctuations of ^{239}Pu sources. Histogram durations in this case are one minute. But such sources of fluctuations become uselessness for distances in tens of kilometres or less when histogram durations must be about one second or less. For this reason in work [7-8] we rejected α-decay sources of fluctuations and instead used as a source, noise generated by a semiconductor diode. Used diodes give a noise signal with a frequency band of up to tens of megahertz and because of this satisfy the requirements of the present investigations.

To check the suitability of the selected diode noise source for local-time effect investigations, comparative tests were made at distances for which existence of the effect was proved by using α-decay sources of fluctuations [7]. This work confirmed the suitability of diode semiconductor noise for studies of the local-time effect.

Below we present a short description of our experiments for investigation of a local-time effect for longitudinal distances of 500 m up to 15 km between locations of measurements. The first experiment studied the local-time effect for a longitudinal distance of 15 km between locations of measurements, the second one for a set of longitudinal distances from 500 m to 6 km. A more detailed description of these experiments is given in [8].

In the first experiment a series of synchronous measurements were carried out in Pushchino (Lat. 54°50.037′ North, Lon. 37°37.589′ East) and Bolshevik (Lat. 54°54.165′ North, Lon. 37°21.910′ East). The longitudinal difference α between places of measurements was α = 15.679′. This value of α corresponds to a difference of local time Δt = 62.7 sec and longitudinal distance Δl=15 km.

To study the local-time effect in Pushchino and Bolshevik, we obtained 10-minute time series by digitizing fluctuations from noise generators with a sampling frequency of 44100 Hz. From this initial time series with three different steps of 735, 147 and 14 points, we extracted single measurements and obtained three time series with equivalent frequency equal 60 Hz, 300 Hz and 3150 Hz. On the basis of this time series, in a standard way [1-3] using a 60-point sample length for the first and second time series and a 63-point sample length for third time series, we constructed three sets consisting of histograms with duration 1 sec, 0.2 sec and 0.02 sec.

Fig. 1 depicts the intervals distribution obtained after comparisons of the 1-sec histogram sets. The distribution has a peak, which corresponds to a time interval of 63±1 sec, and which accurately corresponds to a local time difference Δt = 62.7 sec between places of measurements.

Local time peaks ordinarily obtained on the interval distributions are very sharp and consist of 1-2 histograms [1-3] i.e. are practically structureless. The peak in Fig. 1 *a*) can also be considered as structureless. This leads us to the further investigation of its structure.

The fact that all sets of histograms were obtained on the basis of the same initial time series on the one hand, enables enhancement of time resolution of the method of investigation, and on the other hand, eliminates necessity of very precise and expensive synchronization of spaced measurements. The intervals distribution obtained for the 1-sec histograms set allows the use of information about the location of a local-time peak alignment of time series. The alignment makes possible the use of the set of histograms of the next order of smallness.

Using the 0.2-sec histograms set increased resolution five times and allowed more detailed investigations of local-time peak structure and its position on the time axis. Since the positions of the peak on the 1-sec intervals distribution (Fig. 1) are known, it is possible to select their neighbourhood by means of 60 sec relative shift of initial time series and prepare after this a 0.2-sec histograms set for further comparison.

The intervals distribution obtained from comparisons for the 0.2-sec histograms set is presented in Fig. 1*b*). One can see that maximum similarity of histogram shape occurs for pairs of histograms separated by an interval of 63±0.2 sec. This value is the same as for the 1-sec histogram intervals distribution, but in the latter case it is defined with an accuracy of 0.2 sec. It's easy to see from the intervals distribution, Fig. 1*b*), that after fivefold enhancement of resolution, the distribution has a single sharp peak again. So a change of time scale in this case doesn't lead to a change of intervals distribution. This means that we must enhance the time resolution yet again to study the local time peak structure. We can do this by using the 0.02-sec histograms set.

The intervals distribution for the case of 0.02-sec histograms is presented in Fig. 1*c*). Unlike the intervals distributions in Fig. 1*a*) and in Fig. 1*b*), distribution in Fig. 1*c*) consists of two distinct peaks. The first peak corresponds to a local time difference of 62.98±0.02 sec, the second one to 63.16±0.02 sec. The difference between the peaks is $\Delta t' = 0.18 \pm 0.02$ sec.

Splitting of the local-time peak in Fig 1 *c*) is similar to splitting of the daily period in two peaks with periods equal to solar and sidereal days [9-11]. This result will be considered in the next section.

The experiment described above demonstrates the existence of a local-time effect for longitudinal distance between locations of measurements at 15 km, and splitting of the local-time peak corresponding to that distance. It is natural to inquire as to what is the minimum distance for the existence of a local time effect. The next step in this direction is the second experiment presented below.

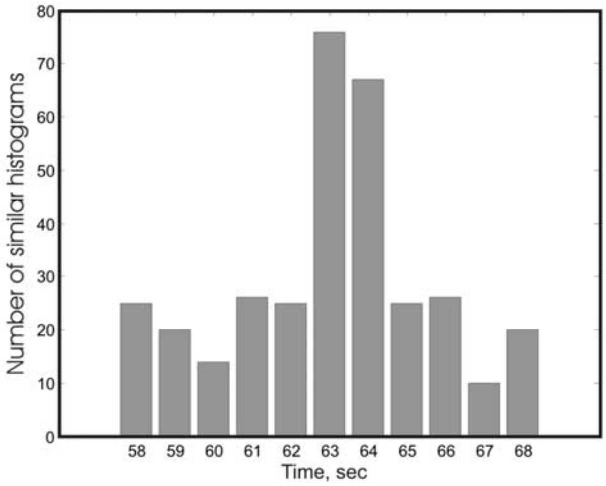
In this experiment two measurement systems were used: stationary and mobile. Four series of measurements were carried out. The longitudinal differences of locations of stationary and mobile measurement systems was 6 km, 3.9 km, 1.6 km and 500 m. The method of experimental data processing used was the same as for first experiment. It was found that for each of foregoing distances, a local-time effect exists and the local-time peak splitting can be observed.

3. Second-order splitting of the local-time peak. Preliminary results.

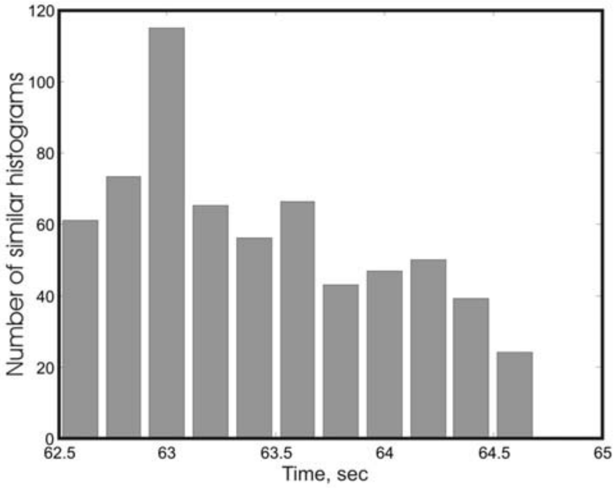
Four-minute splitting of the daily period of repetition of histogram shape on solar and stellar sub-periods was reported in [3]. In that paper the phenomenon is considered as evidence of existence of two preferential directions: towards the Sun and towards the coelosphere. After a time interval of 1436 min the Earth makes one complete revolution and the measurement system plane has the same direction in space as one stellar day before. After four minutes from this moment, the measurement system plane will be directed towards the Sun. This is the cause of a solar-day period $T = 1440$ min.

Let us suppose that the splitting described in the present paper has the same nature as splitting of the daily period. Then from the daily period splitting $\Delta T = 4$ min it is possible to obtain a constant of proportionality k :

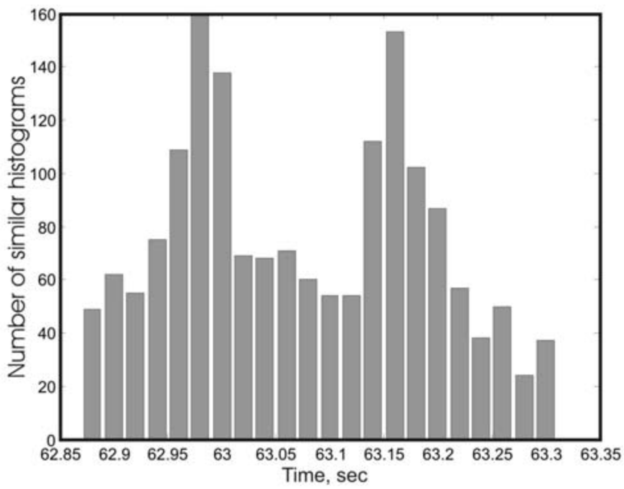
$$k = \frac{\Delta T}{T} = \frac{240\,\text{sec}}{86400\,\text{sec}} \approx 2.78 \cdot 10^{-3}. \tag{1}$$



a)



b)



c)

Fig.1. Intervals distributions obtained after comparisons of 1-sec (*a*), 0.2-sec (*b*), and 0.02-sec (*c*) histogram sets. The Y-axis depicts the number of histograms, which were found to be similar; the X-axis – time interval between pairs of histograms, sec.

The longitudinal difference between places of measurements presented in the second section is Δt = 62.7 sec and we can calculate splitting of the local-time peak for this value of Δt:

$$\Delta t' = k\Delta t = 62.7 \times 2.78 \cdot 10^{-3} \approx 0.17\,\text{sec}\,. \qquad (2)$$

It is easy to see from Fig. 1*c*) that splitting of the local-time peak is amounts to 0.18±0.02 sec. This value agrees with estimation (2). Values of splitting of the local-time peak, which were obtained for the mobile experiment, are also in good agreement with values obtained by the help of formula (2).

This result allows us to consider sub-peaks of local-time peak as stellar and solar and suppose that in this case the cause of splitting is the same as for daily-period splitting. But the question about local-time peak structure remains open.

In order to further investigations of the local-time peak structure an experiment was carried out using synchronous measurements in Rostov-on-Don (Lat. 47°13.85′ North, Lon. 39°44.05′ East) and Bolshevik (Lat. 54°54.16′ North, Lon. 37°21.91′ East). The local-time difference for these locations of measurements is Δt = 568.56 sec. The value of the local-time peak splitting, according to (2), is $\Delta t'$ = 1.58 sec. The method of experimental data processing was the same as described in section 2.

In Fig. 2, a summation of all comparative results is presented. For the considered case we omit presentation of our results in the form of interval distributions, like those in Fig 1, because it involves ten graphs.

Fig. 2 consists of four lines. At the leftmost side of each line is the duration of a single histogram in the four sets of histograms, which were prepared for comparison. So we have four sets consisting of 1-sec, 0.2-sec, 0.0286-sec and 1.36 ms histograms. The rectangle in the first line schematically shows a local-time peak, obtained as a result of comparisons for the 1-sec histograms set. Taking into account synchronization error (about one second), the result is 567±2 sec. This value is in agreement with the calculated longitudinal difference of local time Δt = 568.56 sec (throughout Fig. 2, calculated values are given in parentheses).

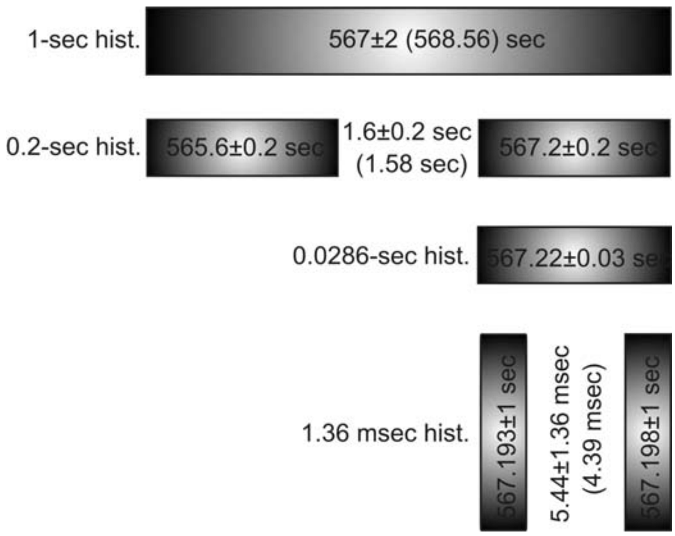


Fig. 2. Local-time peak splitting obtained in the experiment with synchronous measurements of fluctuations of a pair of semiconductor noise generators, carried out in Rostov-on-Don and Bolshevik.

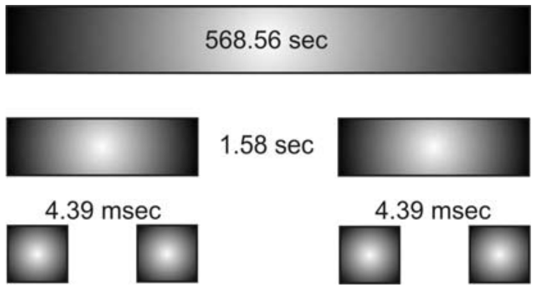


Fig. 3. Expected structure of local-time peak splitting for experiment with synchronous measurements in Rostov-on-Don and Bolshevik, calculated on the base of formula (4).

The second line in Fig. 2 presents results for the 0.2-sec histograms set. The values in the rectangles show sidereal and solar sub-peaks of the local-time peak. The value between the rectangles gives the splitting of the local-time peak. The experimentally obtained splitting value is 1.6±0.2 sec, which is in good agreement with the value calculated on the basis of formula (2).

The third and fourth lines of Fig.2 present the results of additional investigations of local-time peak structure. In the third line is the result of comparisons of the 0.0286-sec histograms set for intervals, which constitute the closest neighbourhood of 567.2±0.2-sec peak. Using the 0.0286-sec histograms set increased resolution almost ten times and defines peak position on the intervals distribution at 567.22±0.03 sec. The obtained peak is structureless. Further increase of resolution moves to the 1.36-ms histograms set, presented in fourth line. In this case resolution enhancement revealed splitting of 567.22±0.03 sec peak.

The splitting presented in last line of the diagram, can be regarded as second-order splitting. It can be calculated using first-order splitting $\Delta t' = 1.58$ sec by the analogue of formula (2):

$$\Delta t'' = k\Delta t'. \tag{3}$$

It easy to see from (3) and from Fig. 2, for second-order splitting $\Delta t''$ the value of first-order splitting $\Delta t'$ plays the same role as the local-time value Δt for $\Delta t'$. Numerical calculations using (3) gives $\Delta t'' = 4.39$ ms, which is in good agreement with the experimentally obtained splitting value 5.44±1.36 ms.

Experimental evidence for the existence of second-order splitting leads us to conjecture the possibility of *n*-order splitting. It easy to see from (2) and (3) that the *n*-order splitting value Δt^n can be obtained in the following way:

$$\Delta t^n = k^n \Delta t. \tag{4}$$

Fig. 3 presents an idealized structure of local-time peak splitting for the considered experiment, which was calculated on the base of formula (4). Unlike Fig. 2, the structure of local-time peak splitting in Fig. 3 is symmetrical. Studies of a possible splitting of 565.6± 0.2 sec peak is our immediate task. At this time the results presented in the Fig. 2 can be considered as preliminary.

4. Experimental investigations of the existence of a local-time effect for longitudinal distances between places of measurements from 1 m to 12 m

The experiments described in two previous sections demonstrate the existence of a local-time effect for a longitudinal distance of 500 m between locations of measurements, and the existence of second-order splitting of the local-time peak. The next step in our investigations is a study of the local-time effect on the laboratory scale.

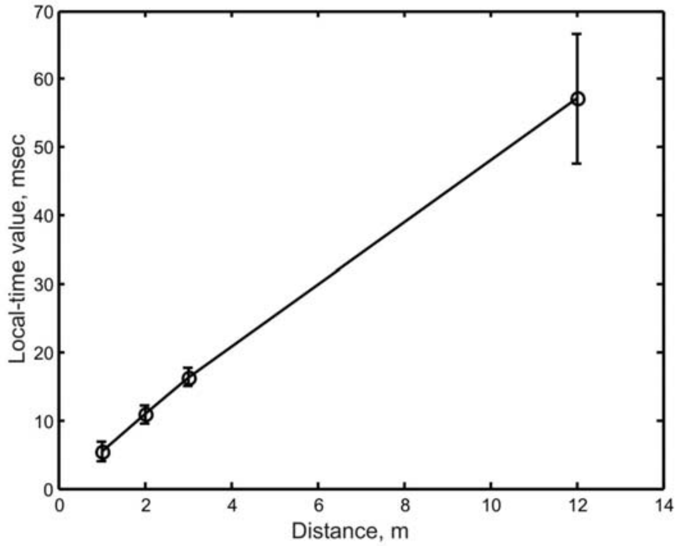


Fig. 4. Values of local-time shift as a function of distance between two sources of fluctuations. The graph presents results of investigations of the local-time effect for distances 1 m, 2m, 3m, and 12 m.

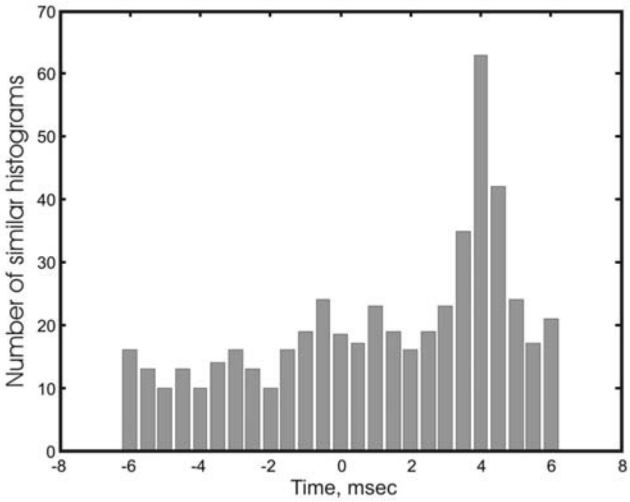


Fig. 5. Example of intervals distribution for longitudinal distance between two sources of fluctuations at one metre separation. Single histogram duration – 0.5 ms.

The main difference between local-time effect investigations on the laboratory scale and the experiments described above is an absence of a special synchronization system. In the laboratory case the experimental setup consists of two synchronous data acquisition channels and two spaced noise generators, which are symmetrically connected to it. A LeCroy WJ322 digital storage oscilloscope was used for data acquisition. Standard record length of the oscilloscope consists of 500 kpts per channel. This allowed capture of two synchronous sets of 50-point histograms. The maximum length of every set is 10000 histograms.

Fig. 4 presents values of local time shift as a function of distance between two noise generators. The graph presents the results of investigations of a local-time effect for distances of 1 m, 2 m, 3 m, and 12 m. Local-time values were found with an accuracy of ±9.52 ms for the 12 m experiments and with an accuracy of ±1.36 ms for the 1 m, 2 m, and 3 m experiments.

An example of an intervals distribution for 1 m longitudinal distance between two noise generators is presented in Fig. 5. The intervals distribution was obtained on the basis of the 0.5-ms histograms set. Using the Earth's equatorial radius value (6378245 m) and the latitude of the place of measurements (54°50.0.37′), it is possible to estimate (for details see Section 6) the local-time difference for a 1 m longitudinal distance. The estimated value is 3.7 ms. It is easy to see from Fig. 5 that the experimentally obtained value of local-time peak is 4±0.5 ms, which is in good agreement with the theoretical value.

The results of our investigations for the laboratory scale, which are presented in this section, confirm a local-time effect for distances up to one metre. So we can state that a local-time effect exists for distances from one metre up to thousands of kilometers. This is equivalent to the statement that space heterogeneity can be observed down to the 1m scale.

5. The dependence of a local-time effect on spatial direction

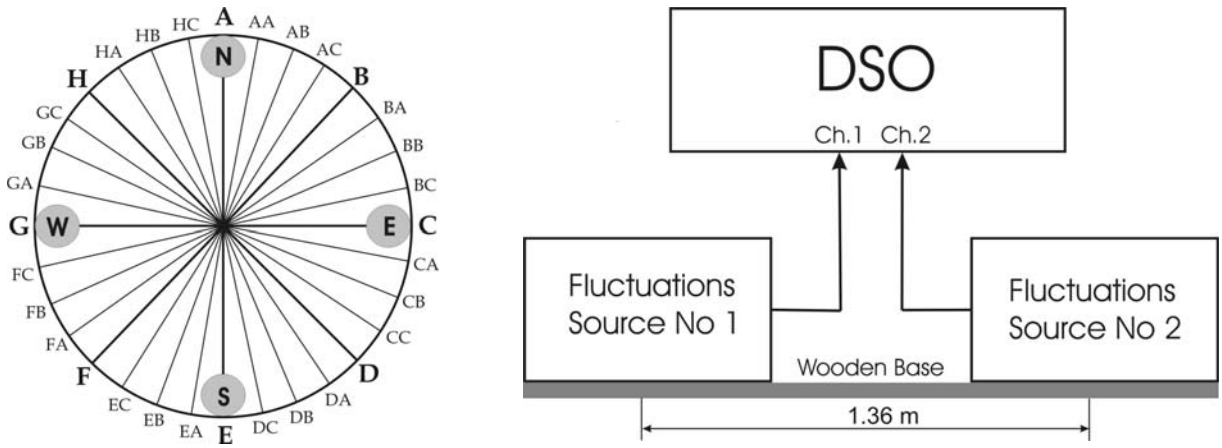
A functional diagram of the experimental setup is presented in Fig. 6 *b*). It consists of two sources of fluctuations, which are fixed to a wooden base. The distance between the sources was 1.36 m. The base, with the sources of fluctuations, can revolve on its axis and can be positioned in any desired direction. A two-channel LeCroy WJ322 digital storage oscilloscope (DSO in Fig. 6 *b*) was used for data acquisition.

The digitizing frequency used for all series of measurements was 100 kHz. Consequently, the duration of 50-point histograms, which were used in the experiment, is 0.5 milliseconds. This means that all local-time values in the experiment are defined with an accuracy of ±0.5 milliseconds. Fig. 6 *a*) depicts the spatial directions which were examined in the experiment. In Fig. 6 *a*) every one of these directions is denoted by letters outside the circle. For example, direction AA means that the base with the sources of fluctuations is aligned in the EA-AA direction in such a way that source No 1 is placed on the AA end of the base and source No 2 is placed on the EA end. Correspondingly, direction EA means that source No 1 is on the EA end, and source No 2 is on the opposite end. Letters N, S, E, and W denote directions to the North, South, East, and West respectively. Directions A and E lie on an Earth meridian, and directions G and C lie on an Earth parallel.

The angular difference between two neighboring directions is 11.25°, so we have 32 spatial directions. To examine all the directions one series of measurements must include 32 pairs of synchronous records. Every record consists of 500,000 points. This allowed acquisition of two synchronous sets of 50-point histograms for every direction. Every set contains 10,000 histograms. The experimental results, which are presented below, are based on 8 series of measurements.

It is important to note that pairs of directions presented in Fig. 6 *a*), for example, A-E and E-A, are actually the same because the pair of fluctuations sources used in the experiment are non-directional. For this reason the total number of directions examined is half that denoted by letters in Fig. 6 *a*). The second measurement in an opposite pair of directions can be considered as a control.

Fig. 7 shows the interval distributions obtained for each of the 32 spatial directions. Every one of these distributions is averaged through the interval distributions from all of the series of measurements for every one of the spatial directions.



a) *b*)

Fig. 6. Diagram of spatial directions, which was examined in experiments with fixed spatial base 1.36m (a) and functional diagram of the experimental setup (b).

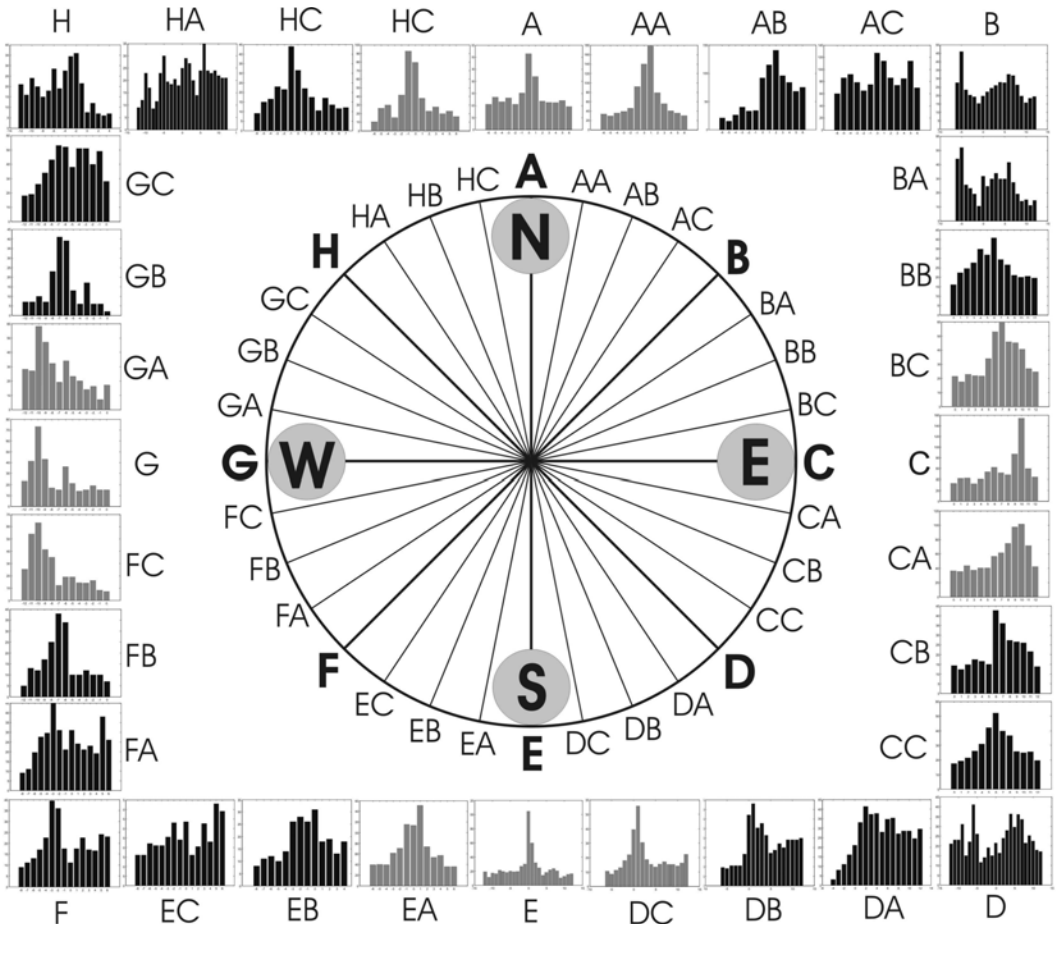


Fig. 7. Averaged interval distributions obtained for every spatial direction.

The circle inside Fig. 7 is the same as in Fig. 6 *a*) and shows spatial directions in relation to the presented interval distributions. All the distributions presented in Fig. 7 can be divided into two distinct groups. The first group consists of distributions in the neighbourhoods (approximately ±11.25◦ of the directions A-E and C-G; labeled as A, E, C, and G. To the first group also can be related distributions that are closest to A, E, C, and G: HC, AA, BC, CA, DC, EA, FC, GA. To the second group can be related all remaining distributions. The distribution from the first group we call ‘non-diagonal’, and from the second, ‘diagonal’. The first group in Fig. 7 is highlighted by the gray color.

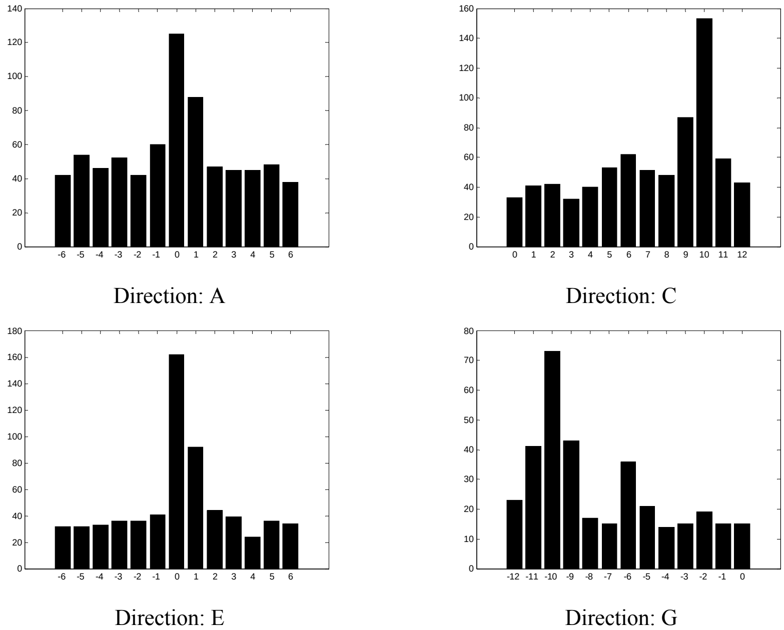


Fig. 8. Non-diagonal interval distributions for meridian (North-South) directions A and E, and for parallel (East-West) directions C and G.

The main difference between the two groups lies in the following: non-diagonal distributions always have a single peak, which corresponds to the same interval value in all series of measurements. In the case of the non-diagonal distributions, every spatial direction can be characterized by a stable, reproducible pattern of interval distribution. Contrary to non-diagonal distributions, a diagonal distribution is multipeaked and cannot ordinarily be characterized by a stable, reproducible pattern. Non-diagonal interval distributions are presented in Fig. 8. For Earth meridian directions (A and E), patterns of interval distributions always have a stable peak at zero intervals. In the case of Earth parallel directions (C and G), interval distributions have a peak at the interval that is equal to the local-time-difference for the spatial base of 1.36 m.

This difference has the same magnitude but different sign for opposite directions. It is easy to see from Fig. 8 that interval distributions for directions C and G have peaks at the intervals 10 and −10.

6. Value of local-time-difference

As follows from previous investigations [1 – 4] the value of the local-time effect depends only on the longitudinal difference between places of measurements, not on latitudinal distance. From this it follows that the factor, which determines the shape of fine structure of histograms must be axial-symmetric.

Longitudinal dependence of local-time effect phenomenology can be considered as dependence of shape of the fine structure of histograms on spatial directions defined by the centre of the Earth and the two points where measurements are taken [14]. In this case the results of measurements depend on the solid angle between two planes defined by the axis of the Earth and the two points of measurement; such angle depends on the longitudinal difference, not on the latitudinal difference.

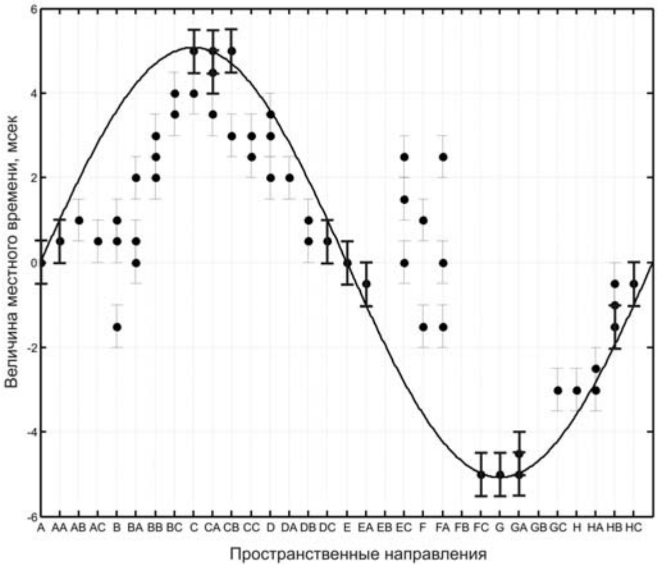


Fig. 9: Theoretical estimation (solid line) and experimentally obtained local-time values. Points with bold error bars show local-time values for non-diagonal directions.

But for the case of separated measurements with fixed spatial base $\Delta L_0 = Const$, the results of the experiment become dependent on latitude, θ. Really, the time Δt , after which fluctuation source No 2 will define the same direction as source No 1 before, depends on the velocity of the measurement system $v(\theta, h)$:

$$\Delta t = \frac{\Delta L_0}{v(\theta, h)} \sin\alpha \, , \tag{5}$$

where $\alpha \in [0, 2\pi]$ is an angle, counter-clockwise from the direction to the North (direction A). It is important to note that the theoretical estimation of the longitudinal difference is given by (5) obtained on the assumption that the factor determining the fine structure of histograms is axial-symmetric.

The value $v(\theta, h)$ is determined by:

$$v(\theta, h) = \frac{2\pi}{T} \left(\sqrt{\frac{R_p^2}{\frac{R_p^2}{R_e^2} + tg^2(\theta)}} + h \right), \tag{6}$$

where $R_p = 6356863m$ and $R_e = 6378245m$ are the values of the polar and equatorial radii of the Earth [15] respectively, $T = 86160\sec$ is the period of the Earth's revolution. For the place of measurements (Pushchino, Moscow region) we have latitude $\theta_P = 54^\circ 50.037'$ and height above sea level $h_P = 170m$. So the velocity of the measurement system is $v(\theta_P, h_P) = 268\, m/\sec$. For near-equatorial regions $v(\theta, h)$ can exceed $v(\theta_P, h_P)$ by almost twice the latter. Consequently, for measurements with a fixed spatial base we have sufficiently strong dependence of local-time-difference (6) on latitude θ.

The value of the velocity $v(\theta_P, h_P)$ allows, on the basis of (6), calculation of the local-time-difference $\Delta t(\alpha)$ as function of spatial directions examined in the experiment.

The solid line in Fig. 9 shows the results of this calculation. Points with error bars in Fig. 9 show local-time values obtained for all series of measurements.

It is easy to see from Fig. 9 that the experimental results are in excellent agreement with the theoretically predicted local-time values only for a narrow neighbourhood around the directions North-South (directions A and E) and East-West (directions C and G) i.e. for non-diagonal directions. At the same time, for diagonal directions, the experimental results in most cases don't follow the theoretical predictions. Results presented in Fig. 9 are in agreement with results summarized in Fig. 7, and linked to the dependence of local-time effect on spatial directions.

The results reveal the character of near-Earth space anisotropy. As pointed out above, the theoretical estimation of local-time effect values in Fig. 4 were obtained under the hypothesis that the effect is caused by some axial-symmetric structure, which has permanent properties along an Earth meridian. According to this hypothesis, the dependence of local-time effect must be the same for all spatial directions, and local-time values obtained in the experiment must follow the theoretically predicted values. But the fact that the diagonal directions experimental results don't confirm this hypothesis leads to the conclusion that at the laboratory scale local-time effects cannot be caused by some axial-symmetric structure.

Evidently, dependence of local-time effects in East-West directions is linked to the rotational motion of the Earth. In this case, after the time interval Δt, which is equal to local-time difference for the spatial base used, the position of the 'West' source of fluctuations will be exactly the same as the position of 'East' previously. In the case of diagonal spatial directions such a coincidence is absent. However, for North-South direction such an explanation is inapplicable.

Dependence of the local-time effect in the direction of a meridian is probably linked to the velocity component along the path of the Solar System in the Galaxy. This hypothesis is preliminary and may possibly change in consequence of future investigations.

7. Discussion

Local-time effect, as pointed out in [1], is linked to rotational motion of the Earth. The simplest explanation of this fact is that, due to the rotational motion of the Earth, after time Δt, measurement system No. 2 appears in the same place where system No. 1 was before. The same places cause the same shape of fine structure of histograms. Actually such an explanation is not sufficient because of the orbital motion of the Earth, which noticeably exceeds axial rotational motion. Therefore measurement system No. 2 cannot appear in the same places where system No. 1 was. But if we consider two directions defined by the centre of the Earth and two points were we conduct spaced measurement, then after time Δt measurement system No. 2 takes the same directions in space as system No. 1 before. From this it follows that similarity of histogram shapes is in some way connected with the same space directions. This conclusion also agrees with experimental results presented in [12-13].

In speaking of preferential directions we implicitly supposed that the measurement system is directional and because of this can resolve these directions. Such a supposition is quite reasonable for the case of daily period splitting, but for splitting of the local-time peak observed on the 1 m scale it becomes very problematic because an angle, which must be resolved by the measurement system, is negligible. It is most likely that in this case we are dealing with space-time structure, which are in some way connected with preferential directions towards the Sun and the coelosphere. Second-order splitting of local-time peaks can also be considered as an argument confirming this supposition. Apparently we can speak of a sharp anisotropy of near-earth space-time. Existence of a local-time effect leads us to conclude that this anisotropy is axially symmetric.

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Light propagation in the field of non-minimal Dirac monopole[1]

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In the paper [1] new exact solutions of the three-parameter non-minimal Einstein-Yang-Mills model with the Lagrangian

$$\mathcal{L} = \frac{R}{8\pi} + \frac{1}{2} F^{(a)}_{ik} F^{ik(a)} + \frac{1}{2} \mathcal{R}^{ikmn} F^{(a)}_{ik} F^{(a)}_{mn} \qquad (1)$$

and the non-minimal susceptibility tensor [2, 3]

$$\mathcal{R}^{ikmn} = \frac{q_1}{2} R \left(g_{im} g_{kn} - g_{in} g_{km} \right) + \frac{q_2}{2} \left(R_{im} g_{kn} + R_{kn} g_{im} - R_{in} g_{km} - R_{km} g_{in} \right) + q_3 R_{ikmn} \quad (2)$$

are obtained. These solutions are associated with a point-like Abelian magnetic non-minimal Wu-Yang monopole. In the model with $q_1 = -q < 0$, $q_2 = 4q$, $q_3 = -6q$ the gravitational field of such a monopole is characterized by the regular metric

$$ds^2 = N(r) dt^2 - \frac{dr^2}{N(r)} - r^2 d\Omega^2 \,, \qquad N(r) = 1 + \frac{r^2(\kappa - 4Mr)}{2(r^4 + \kappa q)} \,. \qquad (3)$$

The exact solution of the non-minimal Einstein-Maxwell model attributed to the *non-minimal Dirac monopole*, the properties of which we discuss here, is obtained from the mentioned solution by the reduction of the gauge group to $U(1)$. We have considered the problem of propagation of the electromagnetic waves in the monopole field background, by using two approaches: first, we have analyzed the null geodesics in the regular metric (3), second, we have reconstructed two optical metrics taking into account the non-minimal susceptibility (2). It was shown that the effect of birefringence takes place and a light beam coming from infinity splits into two flows with different polarization and phase velocities. In contrast to the regular metric (3) both optical metrics are irregular. This fact was interpreted as an appearance of a spherical surface near the Dirac monopole, separating the region, which is inaccessible for the incoming light, and reflecting photons.

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The symmetry of quantum systems and the differential geometry

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We shell follow Lobachevsky which considered that the geometrical structures are imaginary among these it is necessary to select ones corresponding to the experimental data adequately. As a result it is suggested to use the methods of the differential geometry for the description of stochastic processes which are inherent in physical systems, considering that the realizations of the local Lie loop which characterize the possible symmetry of non-homogeneous space are the maximum plausible only.

1. Introduction

In November 2003 it was published the paper [1] of Kadomtsev on the matter of his lectures of 1997 year which showed that the discussion of the 30th years of the twentieth century between Bohr and Einstein on basic principles of quantum mechanics did not lose its topicality in present. As is known Einstein supposed that the probabilistic laws of the quantum mechanics are the consequence of the incompleteness of the physical systems description. By this the incompleteness can be eliminated through the introduction of the additional hidden parameters. Thereby it supposed the availability of the classical laws on the more intimate subquantized level of a matter. On the contrary Bohr supposed the principled impossibility to reach of this as many characteristics of a microcosm are showing exclusively thanks to the availability of macroscopic instruments and which's were not be accredited to elementary particles in the absence of measurement.

In the twentieth century it was made endeavours to break down the determinism illusion which firmly established in the science to the nineteenth century close. Certainly, the quantum mechanics inserted the principal contribution in this, the formation of which was initiated by results of experiments in atomic and nuclear physics. But and in the base of bases in which the determinism was founded - in the classical mechanics - it was marked “disadvantages” causing to the loss of illusions [2]. Despite the fact that with illusions were finished it is difficult to give up the determinism idea, as the planning of the physical experiments was based on the accounts relying on the methods which appeared in the science during the determinism domination.

In the first place to these methods it is necessary to attribute the infinitesimal calculus. It is difficult to overestimate the successes in the field. We can indicate only at a field of mathematics - the Lie group theory - which exercised the huge influence on all theoretical physics. Of course, here the useful results can be received owing to the “good” properties of the used spaces (it is used the Hausdorff spaces in spite of the quantum nature of laws acting in the microcosm). What is more the availability of the smooth congruence's which are the solutions of differential equations plays the important role in the Lie group theory. At the same time in the quantum mechanics the existence of the elementary particles paths are negated.

In consequence of this instead of the Lie derivatives it is becoming necessary to use the more general operators which can induce the more general algebraic structures in comparison with the Lie groups. Specifically it can be the Lie local loops [3]. These structures allow taking into account the absence of the determinism in the real physical processes and to use the Feynman formalism of the continual integration in the quantum theory. So we shall consider solutions of standard differential equations only as maximum plausible functions used for a condensed (short) description of physical systems. Of course by this we take account of laws acting in the microcosm and regard them more the fundamental ones than those which are used for the description of macroscopic bodies motions. As a result of such approach the Prigogine's thesis on the time irreversibility receive the further development on the level of the elementary particles description [4].

2. The Lie local loop of realizations

Let M is the topological space and G is the local loop. Following for the work of Barut and Rączka [5] we shall say that $G(x_\circ)$ is the local (left) loop of realizations in M, if for an arbitrary point $x_\circ \in M$ there exists such a neighbourhood $U(x_\circ) \subset M$, what

1) with each $a \in G(x_\circ)$ and $x \in U(x_\circ)$ is juxtaposed the homeomorphism $x \to ax$ from $U(x_\circ)$ in $U(x_\circ)$,
2) a mapping $(a, x) \to ax$ is a continuous one from $G(x_\circ) \times U(x_\circ)$ in $U(x_\circ)$,
3) $(a_1, a_2)x = a_1(a_2 x)$ for $a_1, a_2 \in G(x_\circ)$ and $x \in U(x_\circ)$.

We shall say what $G(x_\circ)$ acts transitively in M, if for each a pair of points $x_1, x_2 \in U(x_\circ)$ there exists such an element $a \in G(x_\circ)$, what $x_2 = ax_1$.

We shall name the topological space M as the local quasi-homogeneous one, if the local loop $G(x_\circ)$ acts transitively in $U(x_\circ) \subset M$. If the loop G acts transitively on M, then we shall name M as the quasi-homogeneous space.

Let E_{n+N} is the vector fiber space with the base M_n and the projection π_N, $\Psi(x)$ is the arbitrary section of fibre bundle E_{n+N}, ∂_i is the partial derivative symbol. Let us to consider the infinitesimal substitutions defining the vector space mapping of the neighbour points x and $x + \delta x$ ($x \in U,\ x + \delta x \in U,\ U \subset M_n$) and holding the possible linear dependence between vectors. We write given substitutions as:

$$\Psi'(x + \delta x) = \Psi(x) + \delta\Psi(x) = \Psi(x) + \delta T(x)\Psi(x) \tag{2.1}$$

where $\delta T(x)$ is the infinitesimal affinor fields. By this the vector field change in consequence of the transition in the neighbour point has the form: $\Psi(x + \delta x) - \Psi(x) \approx \delta x^i \partial_i \Psi(x)$ and the change of the field Ψ in the point $x + \delta x$ will equal

$$\Psi'(x + \delta x) - \Psi(x + \delta x) = \Psi'(x + \delta x) - \Psi(x)$$
$$-[\Psi(x + \delta x) - \Psi(x)] \approx \delta T(x)\Psi(x) - \delta x^i \partial_i \Psi(x).$$

Further we shall denote

$$\delta_\circ \Psi(x) = \delta T(x)\Psi(x) - \delta x^i \partial_i \Psi(x). \tag{2.2}$$

Let the formula (2.1) defines the infinitesimal substitution of the Lie local loop $G_r(x)$ moreover the unit e of the Lie local loop, the co-ordinates of which equal to zero, corresponds to the identity substitution. Then the infinitesimal substitutions of the Lie loop in co-ordinates are written as

$$x^i \to x^i + \delta x^i = x^i + \delta\omega^a(x)\xi_a^i(x), \tag{2.3}$$

$$\Psi^A(x) \to \Psi^A(x) + \delta\omega^a(x) T_{aB}^{\ A}(x) \Psi^B(x), \tag{2.4}$$

where x^i are the co-ordinates of the point x, $x^i + \delta x^i$ are the co-ordinates of the point $x + \delta x$, $\Psi^A(x)$ are the components of the vector field $\Psi(x)$ and $\delta\omega^a(x)$ are the components of the infinitesimal vector field $\delta\omega(x)$ being the section of the vector fibre bundle E_{n+r} with the base M_n and with the projection π_r (here and further Latin indices *i, j, k, ...* will run the values of integers from 1 to n and Latin capital indices *A, B, C, D, E* will run the values of integers from 1 to N).

As a result the formula (2.2) is rewritten in the following form:

$$\delta_\circ \Psi = \delta\omega^a X_a(\Psi), \tag{2.5}$$

where

$$X_a(\Psi) = T_a\Psi - \xi_a^i \partial_i \Psi \tag{2.6}$$

or in the co-ordinates

$$X_a^A(\Psi) = T_{aB}^{\ A}\Psi^B - \xi_a^i \partial_i \Psi^A.$$

In the general case a type of geometrical objects can do not conserving with the similar substitutions. Therefore below we shall consider only such substitutions which conserve a type of geometrical objects.

In first in the formula (2.5) it ought to become to the covariant derivative. Let

$$\delta_\circ \Psi^A = \delta\omega^a X_a^A(\Psi) = \delta\omega^a (L_{aB}^{\ A}\Psi^B - \xi_a^i \nabla_i \Psi^A), \tag{2.7}$$

where

$$L_{aB}^{\ A} = T_{aB}^{\ A} + \xi_a^i \Gamma_{iB}^{\ A}, \qquad \nabla_i \Psi^A = \partial_i \Psi^A + \Gamma_{iB}^{\ A}\Psi^B,$$

and we demand that $L_{aB}^{\ A}(x)$ and $\xi_a^i(x)$ should be the components of intermediate [6] tensor fields. Hence if $\Psi(x)$ are the components of the vector field then $\Psi(x) + \delta_\circ\Psi(x)$ also are the components of the vector field.

We shall name the fields $X_a(\Psi)$ as the generators of the Lie local loop $G_r(x)$, if the multiplication $[X_a X_b]$ satisfies the following two axioms:

$$1)\quad [X_a X_b] + [X_b X_a] = 0,$$

$$2)\quad [[X_a X_b]X_c] + [[X_b X_c]X_a] + [[X_c X_a]X_b] = 0. \tag{2.8}$$

So, let

$$[X_a X_b] = X_a X_b - X_b X_a = C_{ab}^c X_c. \tag{2.9}$$

As a result the intermediate tensor fields $L_{aB}^{\ A}(x)$ and $\xi_a^i(x)$ must satisfy to the following correlations:

$$L_{aC}^{\ B} L_{bB}^{\ A} - L_{bC}^{\ B} L_{aB}^{\ A} + \xi_a^i \nabla_i L_{bC}^{\ A} - \xi_b^i \nabla_i L_{aC}^{\ A} - \xi_a^i \xi_b^j R_{ijC}{}^A = -C_{ab}^c L_{cC}^{\ A},$$

$$\xi_a^i \nabla_i \xi_b^k - \xi_b^i \nabla_i \xi_a^k - 2\xi_a^i \xi_b^j S_{ij}^k = -C_{ab}^c \xi_c^k,$$

where $S_{ij}{}^k(x)$ are the components of the torsion tensor and $R_{ijC}{}^A(x)$ are the curvature tensor components of the connection $\Gamma_{iC}^{\ A}(x)$. The components $C_{ab}{}^c(x)$, alternating on down indices owing to (2.9) of the structural tensor, must satisfy in consequence of (2.8) to the generalized Jacobi identities

$$C_{[ab}^d C_{c]d}^e - \xi_{[a}^i \nabla_{|i|} C_{bc]}^e + \xi_{[a}^i \xi_b^j R_{|ij|c}{}^e = 0 \tag{2.10}$$

($R_{ijc}{}^e(x)$ are the curvature tensor components of the connection $\Gamma_{ia}^{\ b}(x)$).

Note that if the Lie local loop $G_r(x)$ operates in the space of the affine connection as transitively so and effectively ($n = r$), then choosing the components ξ_a^k of the intermediate tensor field equaled to the Kronecker symbols δ_a^k it can show that the correlations (2.10) become in the Ricci identity when $C_{ab}^c = 2S_{ab}^c$ [7].

Now if we want to generalize the Klein adjunction principle introduced for the classification of homogeneous spaces using Lie local loops of substitutions instead groups of transformations then the non-homogeneous (we shall name their quasi-homogeneous ones) spaces of the affine connection will come within the scope of the classification. In particular as the Lie local loop substitutions contain the transformations of the projective group, then the Klein homogeneous spaces will introduce in the new extended classification. On the other hand all particular cases of affine connection spaces quoted in the book of Norden [6] must be contained in the extended classification as the parallel translation (the translation on the manifold M_n) introduces in the substitutions of the Lie local loop $G_r(x)$. Thus the use of Lie local loops instead Lie groups does the Klein program (although and in the general sense) the practicable one [7].

3. The maximum plausible realizations

Let the functions Φ, Ψ, Θ, ... belong to the functional complex linear space L with the semi-scalar product [8]. The complex-valued function $\langle\Psi,\Phi\rangle$, being the semi-scalar product, must satisfy the following conditions:

1) $\langle\Psi,\Phi\rangle = \langle\Phi,\Psi\rangle^*$, 2) $\langle\lambda\Psi+\nu\Theta,\Phi\rangle = \lambda\,\langle\Psi,\Phi\rangle + \nu\langle\Theta,\Phi\rangle$, 3) $\langle\Psi,\Psi\rangle \geq 0$,

where * is the symbol of the complex conjugation, λ and ν are complex numbers. Usually for the description of quantum systems the Hilbert space is used. But we do not want to be bounded even the pre-Hilbert space, in which the completeness condition is absent in a sense of a metric induced by a scalar product and we exclude the axiom according to which only the null vector satisfies to the condition

$$4)\ \ \langle\Psi,\Psi\rangle = 0\ .$$

By this one in main conditions of a functions set - the possibility of their orthogonalization will absent, but we go on this deliberately, in order to have the possibility to describe the generalized coherent states [9].

Let us to consider the packet $\{\Psi(\omega)\}$ of functions and let the substitutions

$$\Psi \to \Psi + \delta\Psi = \Psi + \delta T(\Psi)$$

are the most general infinitesimal ones (in contradistinction to linear substitutions (2.1)), where δT are infinitesimal operators of a transition (we do not concretize at first which type of symmetries by them are given). We note that in the elementary particles theory the operators δT, defined by a scattering matrix, generate symmetries, characterizing studied interactions.

We draw smooth curves through the common point $\omega \in M_r$ with the assistance of which we define the corresponding set of vector fields $\{\delta\xi(\omega)\}$. Further we define the deviations of fields $\Psi(\omega)$ in the point $\omega \in M_r$ as $\delta_\circ\Psi = \delta X(\Psi) = \delta T(\Psi) - \delta\xi(\Psi)$ and we shall require that these deviations were minimal ones even if in “the mean”. If we state the task – to find the smooth fields $\Psi(\omega)$ in the studied domain Ω_r of the parameters space M_r then it can turn out to be unrealistic one (possibly $r \gg 1$ and possibly $r \to \infty$). That’s precisely therefore the task of the finding of the restrictions $\Psi(x)$ on the manifold M_n ($x \in M_n \subset M_r$, $n \leq r$) will present an interest.

Let the square of the semi-norm $|X(\Upsilon)|$ has the form as the following integral

$$\mathrm{A} = \int\limits_{\Omega_n} \Lambda\, d_n V = \int\limits_{\Omega_n} \kappa \overline{X}(\Psi) \rho X(\Psi) d_n V\ . \tag{3.1}$$

(we shall name A as an action and Λ as a Lagrangian also as in the field theory) [10]. Here and further κ is a constant; $\rho = \rho(x)$ is the density matrix (tr $\rho = 1$, $\rho^+ = \rho$, the top index "+" is the symbol of the Hermitian conjugation) and the bar means the generalized Dirac conjugation which must coincide with the standard one in particular case that is to be the superposition of Hermitian conjugation and the spatial inversion of the space-time M_4. We shall name solutions $\Psi(x)$ of differential equations, which are being produced by the requirement of the minimality of the integral (3.1), as the maximum plausible realizations of Lie local loops and shall use for the construction of the all set of functions $\{\Psi(x)\}$ (generated by the transition operators).

Of course for this purpose we can use the analog of the maximum likelihood method employing for the probability amplitude, but not for the probability as in the mathematical statistics. As is known, according to the Feynman's hypothesis the probability amplitude of the system transition from the state $\Psi(x)$ in the state $\Psi'(x')$ equal to the following integral

$$K(\Psi,\Psi') = \int\limits_{\Omega(\Psi,\Psi')} \exp(i\mathrm{A}) D\Psi = \lim_{N\to\infty} I_N \int d\Psi_1 \ldots \int d\Psi_k \ldots \int d\Psi_{N-1} \exp\left(i \sum_{k=1}^{N-1} \Lambda(\Psi(x_k)) \Delta V_k \right) \tag{3.2}$$

(it is used the system of units $h/(2\pi) = c = 1$, where h is the Planck's constant and c is the light speed; $i^2 = -1$; the constant I_N is chosen so that the limit is existing). So, the formula (3.2) allows describe the most adequately the physical process in the quantum theory. At the same time the functions $\Psi(x)$, being the solutions of differential equations, received from the requirement of the minimum of the action A, may be the maximum likelihood ones only, but then they allow describing in condensed (short) form the same physical system. In this approach the Lagrangian Λ plays the more fundamental role than differential equations which are generated by it. As the transition operators are constructed on the base of experimental data, then the differential equations, obtained in a result of the Lagrangian special choice in the action (3.1), can name as the differential equations of the root-mean-square regression Ψ on x.

So let us go into the construction of the condensed (approximate) description of the physical processes in which the base role play the fundamental differential equations of the field theory interpreting theirs as the regression differential equations of the field functions on the point coordinates of the space-time manifold.

4. Generalization of Green theorem and its applications

We must receive the generalization of Green theorem in a non-homogeneous space for the implementation of the suggested program. Let $\eta_{i_1 \ldots i_n}(x)$ are the basic n-vector components of the affine connection space M_n [6], $\Gamma_{ij}^k(x)$ are the components of the internal connection, ∇_i is the symbol of the covariant derivative in regard to the connection $\Gamma_{ij}^k(x)$. We shall consider the domain $\Omega_n \subset M_n$ bounded by the hypersurface Ω_{n-1} covered by one map. Let us to introduce the symbol of the covariant derivative ∇_i° in regard to the connection $\Gamma_{ij}^{\circ k}(x)$ given on Ω_n such what

$$\nabla_i^\circ \eta_{j_1 \ldots j_n} = 0, \quad \Gamma_{ij}^{\circ k} = \Gamma_{ji}^{\circ k}.$$

We write the generalization Stokes formula [11]:

$$\oint\limits_{\Omega_{n-1}} W_{i_1 \ldots i_{n-1}} d\tau^{i_1 \ldots i_{n-1}} = \int\limits_{\Omega_n} \partial_{i_n} W_{i_1 \ldots i_{n-1}} d\tau^{i_1 \ldots i_{n-1} i_n}, \tag{4.1}$$

where $W_{i_1...i_{n-1}}(x)$ are the tensor field components, $d\tau^{i_1...i_{n-1}}$ is the form of the hypersurface Ω_{n-1} volume and $d\tau^{i_1...i_{n-1}i_n}$ is the form of the domain Ω_n volume. Let $T_i = T^j_{ji}$, where the components T^k_{ij} of the affine strain tensor [6] are defined in the form

$$T^k_{ij} = \Gamma^k_{ij} - \Gamma^{\circ k}_{ij} \tag{4.2}$$

and besides let

$$W^{i_n} = W_{i_1...i_{n-1}} \eta^{i_1...i_{n-1}i_n}.$$

As [11]

$$d\tau^{i_1...i_n} = \eta^{i_1...i_n} d_n V, \qquad d\tau^{i_1...i_{n-1}} = \eta^{i_1...i_{n-1}i_n} N_{i_n} \varepsilon(N) d_{n-1}V,$$

where $N_{i_n}(x)$ is the normal to the hypersurface Ω_{n-1}, $\varepsilon(N) = \pm 1$ is the indicator of the vector N_{i_n}, $\eta^{i_1...i_n}$ is the n-vector being the relative one to the basic n-vector, and

$$\eta^{i_1...i_{n-1}i_n} \partial_{i_n} W_{i_1...i_{n-1}} = \eta^{i_1...i_{n-1}i_n} \nabla^{\circ}_{i_n} W_{i_1...i_{n-1}} = \nabla^{\circ}_{i_n} W^{i_n} = \partial_{i_n} W^{i_n} + \Gamma^{\circ}_{i_n} W^{i_n},$$

then the formula (4.1) can be written in the form

$$\oint\limits_{\Omega_{n-1}} W^{i_n} N_{i_n} \varepsilon(N) d_{n-1}V = \int\limits_{\Omega_n} (\nabla_i W^i - T_i W^i) d_n V. \tag{4.3}$$

We received the generalization of Green formula in a non-homogeneous space, which will be used for the derivation of field equations and conservation laws.

Let E_{n+N} is the vector fiber space with the base M_n and with the projection π_N, $\Psi(x)$ is the section of the vector bundle E_{n+N}, ∇_i is the symbol of the covariant derivative in regard to the connections $\Gamma^k_{ij}(x)$ and $\Gamma_i(x)$. Let us to consider a variation of an action A in regard to following arbitrary infinitesimal substitutions:

$$x \to x, \quad \Psi \to \Psi + \delta_\circ \Psi, \quad \nabla_i \Psi \to \nabla_i \Psi + \delta_\circ \nabla_i \Psi,$$

where

$$\delta_\circ \nabla_i \Psi = \nabla_i \delta_\circ \Psi - T_i \delta_\circ \Psi, \tag{4.4}$$

by this the changes $\delta_\circ \Psi(x)$ of functions must be reduced in zero on a bound Ω_{n-1} of a domain Ω_n. Let a Lagrangian Λ depend on covariant derivatives of fields $\Psi(x)$ not over the first order. As a result a variation of an action is written in the form:

$$\begin{aligned} \delta_\circ \mathrm{A} &= \int\limits_{\Omega_n} \left(\frac{\partial \Lambda}{\partial \Psi} \delta_\circ \Psi + \frac{\partial \Lambda}{\partial \nabla_i \Psi} \delta_\circ \nabla_i \Psi \right) d_n V = \\ &= \int\limits_{\Omega_n} \left(\frac{\partial \Lambda}{\partial \Psi} - \nabla_i \left(\frac{\partial \Lambda}{\partial \nabla_i \Psi} \right) \right) \delta_\circ \Psi d_n V + \\ &+ \int\limits_{\Omega_n} \left(\nabla_i \left(\frac{\partial \Lambda}{\partial \nabla_i \Psi} \delta_\circ \Psi \right) - T_i \frac{\partial \Lambda}{\partial \nabla_i \Psi} \delta_\circ \Psi \right) d_n V. \end{aligned} \tag{4.5}$$

Using the formula (4.3) we transform the last integral of the expression (4.5) in the integral on a surface which will equal to zero in consequence of a conversion in zero changes $\delta_\circ \Psi(x)$ on this surface. Thus for arbitrary changes $\delta_\circ \Psi(x)$ of functions in a domain Ω_n a variation $\delta_\circ \mathrm{A}$ of the action A can be converted in zero only by the following condition:

$$\frac{\partial \Lambda}{\partial \Psi}-\nabla_i\left(\frac{\partial \Lambda}{\partial \nabla_i \Psi}\right)=0. \tag{4.6}$$

Solving given equations it can receive the maximum plausible realizations $\Psi(x)$ of the local Lie loop $G_r(x)$ induced by its generators.

We demand that the action A was the invariant one with respect to the infinitesimal substitutions (2.3) and (2.4) of the local Lie loop $G_r(x)$ conserving the type of geometrical objects. We write the n-dimensional volume element d_nV in the form

$$d_nV=\eta(x)d_{\circ n}V=\eta(x)dx^1dx^2...dx^n.$$

By infinitesimal substitutions (2.3) the volume element $d_{\circ n}V$ is changed as

$$d_{\circ n}V \rightarrow d_{\circ n}V+\frac{\partial\left(\delta\omega^a \xi_a^i\right)}{\partial x^i}d_{\circ n}V.$$

Let us to consider the corresponding variation of the action (3.1):

$$\delta \mathrm{A}=\int\limits_{\Omega_n}\left(\left(\frac{\partial \Lambda}{\partial \Psi}\delta_\circ \Psi+\frac{\partial \Lambda}{\partial(\nabla_i\Psi)}\delta_\circ(\nabla_i\Psi)\right)\eta+\Lambda\eta\partial_i\left(\delta\omega^a\xi_a^i\right)+\partial_i(\Lambda\eta)\delta\omega^a\xi_a^i\right)d_{\circ n}V \tag{4.7}$$

Taking account of the condition (4.4) we write the expression (4.7) as:

$$\delta \mathrm{A}=\int\limits_{\Omega_n}\left(\left(\frac{\partial \Lambda}{\partial \Psi}-\nabla_i\left(\frac{\partial \Lambda}{\partial(\nabla_i\Psi)}\right)\right)\delta\omega^a X^a(\Psi)-\nabla_i\left(I_a^i\delta\omega^a\right)+T_iI_a^i\delta\omega^a\right)d_nV,$$

where

$$I_a^i=-\Lambda\xi_a^i-\frac{\partial \Lambda}{\partial(\nabla_i\Psi)}\left(L_a\Psi-\xi_a^j\nabla_j\Psi\right).$$

In that case when arbitrary parameters $\delta\omega^a$ are constants we receive the following correlations from the requirement of the action (3.1) invariance in regard to substitutions (2.3) and (2.4) (Noether theorem):

$$\left(\frac{\partial \Lambda}{\partial \Psi}-\nabla_i\left(\frac{\partial \Lambda}{\partial(\nabla_i\Psi)}\right)\right)X_a(\Psi)=\nabla_iI_a^i-T_iI_a^i. \tag{4.8}$$

As a result on the extremals (4.6) we have the differential conservation laws

$$\nabla_iI_a^i-T_iI_a^i=0 \qquad \text{or} \qquad \partial_i\left(\eta I_a^i\right)=0.$$

5. Conclusion

The last astronomical data do not let to doubt what the Universe is classified among the physical systems, the information's on which's it is impossible to consider the full one. It is precisely therefore the use of standard cosmological model, received within the scope of General Relativity adopting the deterministic conception, is the hindrance in the future researches. Specifically, it is necessary to return the original sense to the mathematical concepts such as the space curvature, which's must give the estimation of the physical reality rather than must replace it. We shall suppose that the greater part of weakly interacting particles constituting the great background of Universe exist in the degenerate (basic) state inserting the minor contribution in the vacuum polarization for the estimation of which the space curvature is used.

In consequence of the low temperature $T_\circ$ of the Universe matter ground state (the density of the Universe matter ground state $n_\circ\sim 10^{-3}GeV^3$ [12]) the excited states of color fermions – the quarks in the form of baryons are distributed inhomogeneously and with a marginal density $n_b \sim T_\circ^3$ (n_b is the density of the Universe baryon matter, the temperature $T_\circ \sim 10^{-13}\, GeV$, an estimation of which may be the temperature of the cosmic microwave background detected of A.A. Penzias and R.W. Wilson in 1964 [13]) Therefore the geometrical structure of the space is distinguished from the flat space no too distinct. What is more it can assume that the symmetry of the Minkowski space-time is induced by physical properties of Universe fermions in a degenerate state when $T_\circ$=0. It allows to solve not only the problem of the Universe planeness [14] but also to solve the problem of the observer horizon (the isotropy problem of the cosmic microwave background from the observer horizon of the Universe [14]).

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Quasiparticles of cristalline lattices and physical vacuum

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1. Introduction.
Modern field theory is based on continuous model of physical vacuum. However, if physical vacuum is some media, we should consider, that such media should have distinct microscopic structure. We know that limit equilibrium state of every media at low temperature is crystalline lattice state. So, from this point of view, we may propose, that physical vacuum is the space ordered structure, resemble to crystalline lattice.

In this paper we would like to discuss some properties of physical vacuum as structure, resemble to crystalline lattice.

2. Quasiparticles of crystalline chains and globular photonic crystal.
At the first step we remind the properties of quasiparticles in one dimensional crystalline structure – crystalline chain. The simplest example of crystalline chain is so called monoparticle one with additional bonds (see Fig.1). Taking into account only nearest neighbor for such type chain we have following law of motion [1]:

$$m\ddot{u}(l) = -\gamma_0 u(l) - \gamma\left[2u(l) - u(l-1) - u(l+1)\right], \tag{1}$$

where $u(l)$ - is the deviation of particle with l number $(l = 0, 1, ...)$.

Dispersion law for such type chain is

$$\omega^2(k) = \frac{\gamma_0}{m} + 4\frac{\gamma}{m}\sin^2\frac{ka_0}{2}, \tag{2}$$

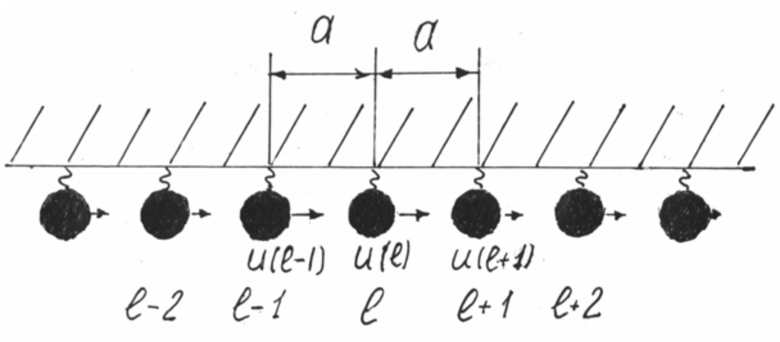

Fig.1. Monoparticle chain with additional bonds

where a_0 - is the lattice constant, γ_0 and γ - corresponding force constants.

If $\gamma_0 = 0$ and $\gamma > 0$ from (2) we found acoustical branch dispersion law of crystalline chain:

$$\omega = 2\frac{S}{a_0}\sin\frac{ka_0}{2}, \tag{3}$$

where $S^2 = \frac{\gamma}{m}a_0{}^2$, and k - wave vector of the corresponding flat wave. Dispersion law (3) corresponds to known quasiparticles of solids – acoustical phonons. Dependence of the energy E from impulse p for such quasiparticles is;

$$E = 2\frac{\hbar S}{a_0}\sin\frac{pa_0}{2\hbar},\tag{3a}$$

So the simplest one-dimensional crystalline model of the physical vacuum is the monoparticle chain with the dispersion law (3). In this case value S has a sense of velocity of light ($S = c_0$) in vacuum and fundamental constant a_0 is the elemental translation of crystalline vacuum.

Dimension considerations (see later) and fundamental constants value (ħ=1,05x10^{-34} Js; C$_0$=2,998x10^{8}m/s; G=6,67x10^{-11}m^3/kgs^2) result in following value of physical vacuum lattice constant: a_0 ~10^{-35} m. At small wave vector k according to (3) we can use linear relations:

$$\omega = c_0 k \;;\quad E = c_0 p\tag{3b}$$

Such equations is the first approximation for photonic dispersion law and energy from impulse dependence for physical vacuum. However at finite wave vector k we should use more complicate relations:

$$\omega = 2\frac{c_0}{a_0}\sin\frac{ka_0}{2};\qquad E = 2\frac{\hbar c_0}{a_0}\sin\frac{pa_0}{2\hbar}\tag{3c}$$

If $\gamma_0 > 0$ and $\gamma > 0$ we have dispersion law of crystalline chain with additional bonds, corresponding to positive effective mass of quasiparticles – so called optical phonons:

$$\omega^2 = \omega_0^2 + 4\frac{S^2}{a_0^2}\sin^2\frac{ka_0}{2},\tag{4}$$

where $\omega_0^2 = \frac{\gamma_0}{m}$ and $S^2 = \frac{\gamma}{m}a^2$.

Accordingly, when $\gamma_0 > 0$ and $\gamma < 0$ we have optical branch with negative mass of corresponding quasiparticles:

$$\omega^2 = \omega_0^2 - 4\frac{S^2}{a_0^2}\sin^2\frac{ka_0}{2}.\tag{5}$$

At last when $\gamma_0 < 0$ and $\gamma > 0$ we have:

$$\omega^2 = -\omega_0^2 + 4\frac{S^2}{a_0^2}\sin^2\frac{ka_0}{2}.\tag{6}$$

where $\omega_0^2 = \frac{|\gamma_0|}{m}$ and $S^2 = \frac{\gamma}{m}a_0^2$.

Last relation corresponds to nonequilibrium lattice, existing only for open system.

Real crystals may be unstable under temperature, pressure and other factors acting. As a result of such kind reasons structural phase transition in crystal may take place. One of the known phase transition in real crystalline structures is ferroelastic one. As a result of initial lattice instability in this case superlattice with a new lattice constant d>a emerges. Such instability is connected with some acoustical mode – so called soft acoustical mode. In some cases new lattice constant may be essentially larger with comparing to initial one, i.e. d>>a .

In three-dimensional case such type of phase transition corresponds to forming of so called globular photonic crystal. Globular photonic crystal is constructed from equal diameter globules packed in cubic face centered crystalline lattice (see Fig.2). There are the resemble structures in nature, known as natural opals. Artificial opals consist from amorphous silica globules. Size of such globules is equal to D=200-400 nm. Amorphous silica globules form cubic face centered lattice. Corresponding lattice constant $a = \sqrt{2/3}d$ (d -is the diameter amorphous silica globules) is comparable to wavelength of visible or ultraviolet electromagnetic waves. Bragg-diffraction of visible light in opals is illustrated by see Fig.3.

Relations (3-5) may be used for description of photonic band laws in photonic crystal, if constants a_0 and S are replaced by a and c. In this case the relation $\omega = 2\frac{c}{a}\sin\frac{ka}{2}$ corresponds to the lowest photonic band; (5) - to the second and (4) - to the third one.

Thus we have the following dispersion laws for first three photonic bands:

$$\omega = 2\frac{c}{a}\sin\frac{ka}{2} \tag{7a}$$

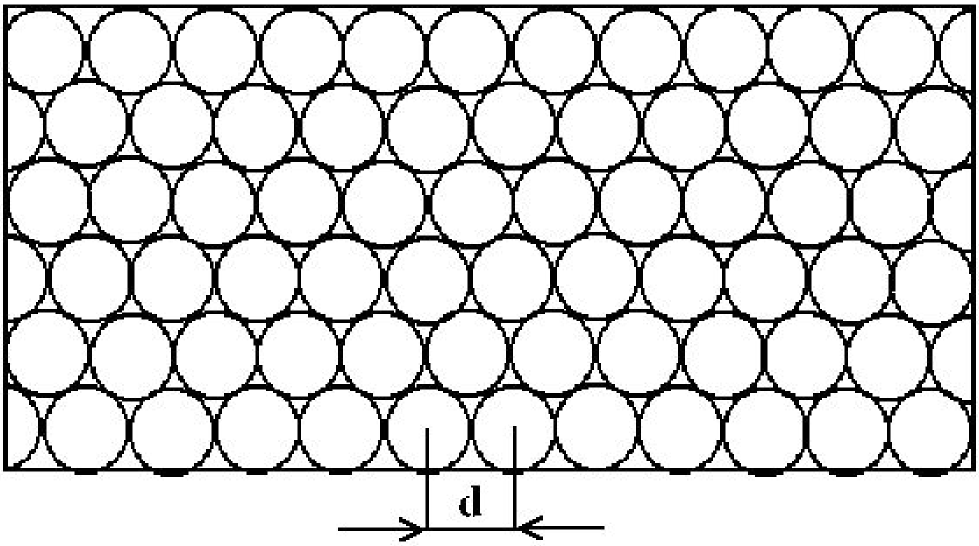

Fig.2. Schematic shape of globular photonic crystals.

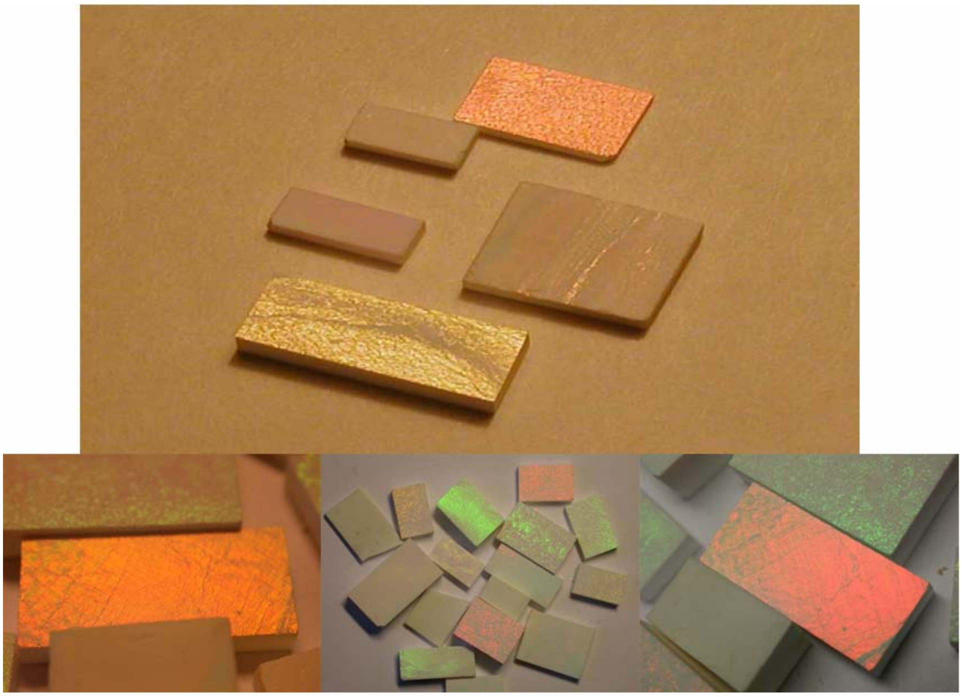

Fig.3 The shape of artificial opals illuminated by continuous visible source of light .

$$\omega^2 = \omega_{01}^2 - 4\frac{c^2}{a^2}\sin^2\frac{ka}{2} \tag{7b}$$

$$\omega^2 = \omega_{02}^2 + 4\frac{c^2}{a^2}\sin^2\frac{ka}{2} \tag{7c}$$

For the simplicity we consider, that $\omega_{01} = \omega_{02} = \omega_0$. Accordingly we have:

$$\omega = 2\frac{c}{a}\sin\frac{ka}{2} \tag{8a}$$

$$\omega^2 = \omega_{01}^2 - 4\frac{c^2}{a^2}\sin^2\frac{ka}{2} \tag{8b}$$

$$\omega^2 = \omega_{02}^2 + 4\frac{c^2}{a^2}\sin^2\frac{ka}{2} \tag{8c}$$

The shape of three first dispersion photonic bands is illustrated by Fig.4 for the next value of parameters: $\omega_0 = 5{,}39\text{x}10^{15}$ 1/s, $a = 1{,}68\text{x}10^{-7}$ m.

At small wave-vector value we may use quasirelativistic approximations as :

$$\omega_1 = ck \tag{9a}$$

$$\omega_2^2 = \omega_0^2 - c^2k^2 \tag{9b}$$

$$\omega_3^2 = \omega_0^2 + c^2k^2 \tag{9c}$$

$$\omega^2 = -\omega_0^2 + c^2k^2 \tag{9d}$$

In these relations c -constant closed to velocity of light in vacuum. So we can conclude that photon-like dispersion law (9a) corresponds to photons, (9c) - to relativistic particles with positive rest mass, (9b) - to relativistic particles with negative rest mass, (9d) - to relativistic particles with imagine rest mass. Thus in photonic crystals we have the unusual situation, when photons become heavy particles with negative or positive rest mass.

3. Group velocity of electromagnetic waves , effective mass of photons and refraction dispersion law in globular photonic crystals

Group velocity of electromagnetic waves $v = \frac{d\omega}{dk}$ may be determined if dispersion law for photonic branch is known. Accordingly for the first photonic branch we have [2]:

$$v = c\sin(ka/2)\,. \tag{10}$$

For the second photonic branch it takes place:

$$v = -\frac{2c^2\sin(ka/2)\cos(ka/2)}{\sqrt{\omega_0^2a^2 - 4c^2 + 4c^2(\cos(ka/2))^2}}\,. \tag{11}$$

For the third photonic branch accordingly:

$$v = \frac{2c^2\sin(ka/2)\cos(ka/2)}{\sqrt{\omega_0^2a^2 + 4c^2 - 4c^2(\cos(ka/2))^2}} \tag{12}$$

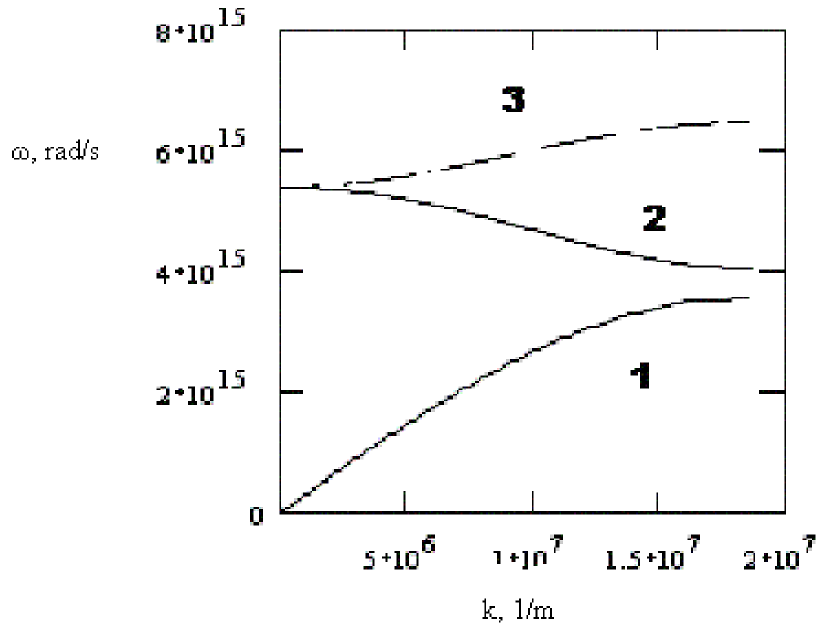

Fig.4. Dispersion laws for three photonic branches

Fig.5 illustrates the results of calculations of dependencies (10-12) for the following value of parameters: $\omega_0 = 5{,}39\text{x}10^{15}$ rad/s, $a = 1{,}68\text{x}10^{-7}$ m.

Upper (solid) curve corresponds to the first photonic branch and the lowest curve - to the second photonic branch. We can see, that for the first and third photonic branches the directions of the group and phase velocity vectors are always the same. Dotted curve, corresponding to the second photonic curve, corresponds to the waves with opposite directions of group and phase velocity vectors.

Calculations of effective mass dependency from wave vector k value have been fulfilled by using known formula :

$$m = \frac{h}{2\pi \dfrac{d^2\omega}{d\text{k}^2}} \quad , \tag{13}$$

where h – Plank constant. Accordingly, for the first photonic branch we obtain:

$$m = -\frac{h}{\pi c a \sin(ka/2)} . \tag{14}$$

Corresponding dependence (for h=1) is illustrated by Fig. 6a.
Accordingly, for the second branch we have:

$$m = -\frac{ha[\omega_0^2 a^2 - 4c^2 \sin^2 \dfrac{ka}{2}]^{1/2}}{2\pi[4c^4\omega^{-2} \cos^2 \dfrac{ka}{2} \sin^2 \dfrac{ka}{2} + c^2 a^2 \cos^2 \dfrac{ka}{2} - c^2 a^2 \sin^2 \dfrac{ka}{2}]} \tag{15}$$

Obtained dependence is shown at Fig. 6b. For the third branch we have:

$$m = -\frac{ha[\omega_0^2 a^2 + 4c^2 \sin^2 \dfrac{ka}{2}]^{1/2}}{2\pi[4c^4\omega^{-2} \cos^2 \dfrac{ka}{2} \sin^2 \dfrac{ka}{2} - c^2 a^2 \cos^2 \dfrac{ka}{2} + c^2 a^2 \sin^2 \dfrac{ka}{2}]} \tag{16}$$

Obtained dependence is shown at Fig. 6c.

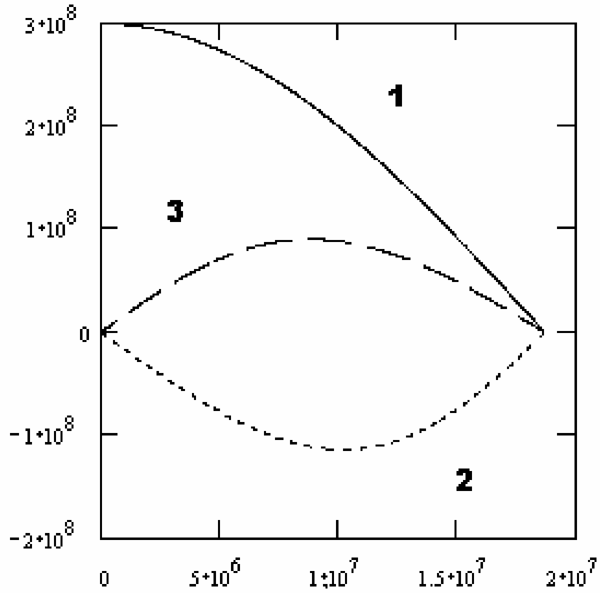

Fig.5. Group velocity dependence from wave vector value in globular photonic crystal.

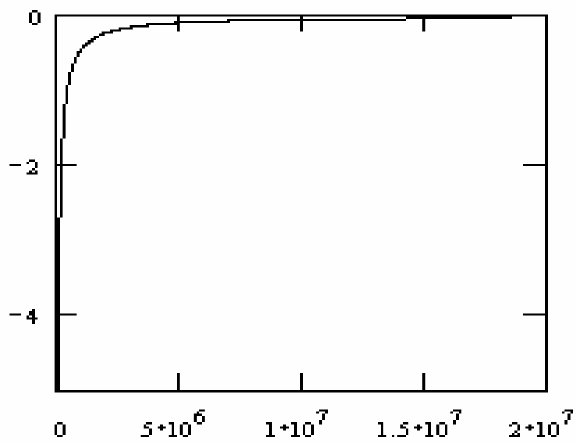

Fig.6a. Effective mass dependence from wave vector value for the lowest photonic branch

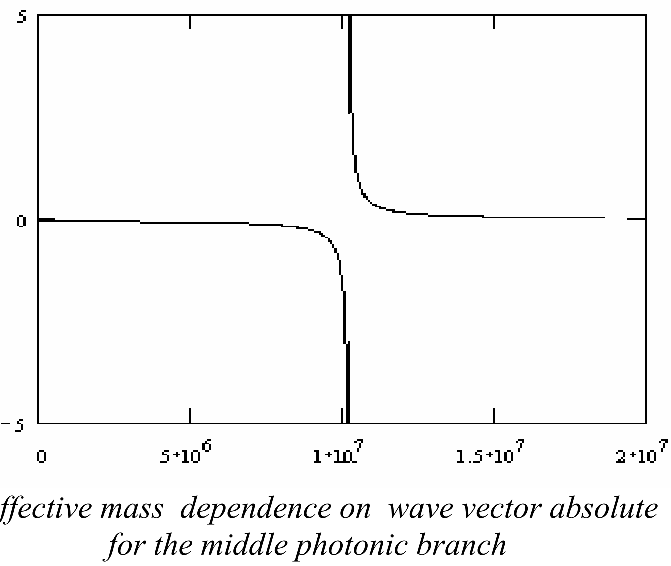

Fig. 6b. Effective mass dependence on wave vector absolute value for the middle photonic branch

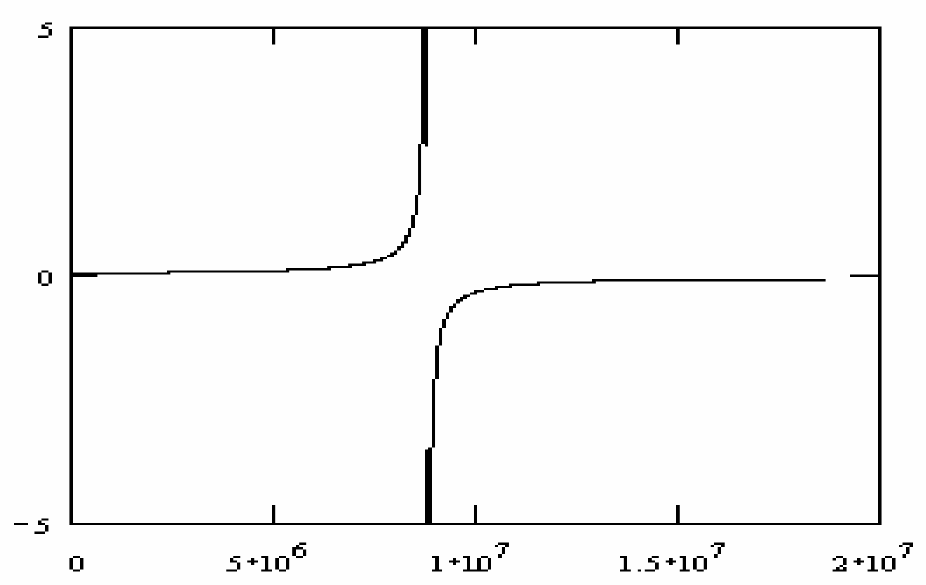

Fig. 6c. Effective mass dependence from wave vector value for the upper photonic branch

Write the following relation for index of refraction for normal incidence of light onto the surface of photonic crystal:

$$\omega(\vec{k})\frac{\vec{k}}{k}=\frac{\vec{c}_0}{n} \tag{17}$$

In this relation the direction of wave vector $\vec{k}$ is coinciding with vector of phase velocity direction in crystal, and vector $\vec{c}_0$ direction **(** vector light velocity in vacuum**)** is coinciding with vector of group velocity direction in crystal.

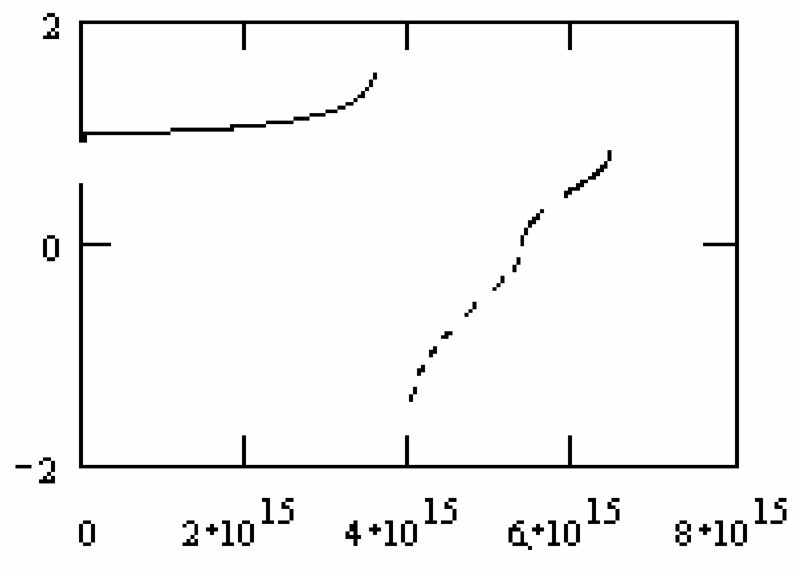

Fig.7. Index of refraction dependence from frequency value

Thus, if the directions of group and phase velocity vectors are coinciding, refraction index is positive (first and third branch). If the directions of group and phase velocity vectors are not coinciding, refraction index is negative (second branch). .

Fig. 7 illustrates the obtained dependencies of refractin index for all discussed photonic branches. At points, corresponding to the edges of forbidden photonic zone, we can see the singularities of obtained index of refraction dependencies.

4. Quasiparticles in physical vacuum

Globular photonic crystal is the crystalline structure, formed as a result of globules packing; the diameter "d" of which is essentially larger than the size "a_0" of microparticles, from which globules are constructed.

We have proposed , that initial physical vacuum was constructed as a result of very small particles (praparticles) packing, attracting according to Newton gravitational law at large distance and repulsing at small distance. The vacuum lattice constant "a_0" and mass value "M" of praparticles can be evaluated from dimension considerations, using fundamental constants: velocity of light ($c=3x10^8$m/s), gravitational constant ($G=6.67x10^{=11}m^3/kgs^2$) and Max Plank constant ($\hbar=1.05x10^{-34}$Js). As a result we have the order of mentioned value: vacuum lattice constant (so called Plank length): $a_0=(\hbar G/c^3)^{1/2}=1,6x10^{-35}$ m (closed to the dimension of praparticles) and mass of praparticles: $M=(\hbar\ c/G\ )^{1/2}=2x10^{-8}$kg. Thus, we consider the initial physical vacuum structure as face centered cubic lattice with lattice constant $a\sim10^{-35}$ m, packed from particles with mass equal to $M\sim10^{-8}$kg.

If we take into account attracting between praparticles only, we have gravitational crystalline lattice, which is unstable and should be compressed into point state. State of strongly compressed vacuum maybe really have existed in past, but now we have large enough extent of physical vacuum. According to proposition of compression and next expansion of physical vacuum, the crystalline lattice of physical vacuum should be inhomogeneous and may be resemble to mentioned photonic crystal structure.

So we come to conclusion, that many years ago in physical vacuum ferroelastic phase transition has taken place due to instability of some longitudinal acoustic mode – soft mode, inducing phase transition. As a result of such transitions physical vacuum became globular photonic crystal with new lattice contant L, value of which is essential larger than initial lattice constant a_0: L» a_0 .

So we have proposed that physical vacuum is some type of globular photonic crystal with lattice constant L=a » a_0, corresponding to weak interaction length. According to the known theory heavy boson energy is close to $E_w = 90\Gamma eV$. So we can evaluate weak interaction lattice constant L from the relation:

$$L = \frac{2\hbar c\pi}{E_w} = \frac{2\cdot 1,05\cdot 10^{-34}\cdot 3\cdot 10^{8}\cdot 3,14}{90\cdot 10^{9}\cdot 1,6\cdot 10^{-19}} = 2,75\cdot 10^{-17}\, m\, . \qquad (18)$$

Such value is in correspondence with figure, known in weak interaction field theory .

So if we accept globular photonic crystals model for physical vacuum, theory predicts the existence of heavy photon in accordance with relations (14-16). From Fig. 6(a-c) we can conclude that photonic mass sign may be negative or positive, and group velocity of corresponding waves may be very small (see Fig.5). So far such type photons in physical vacuum have not been observed. Note, that according to dispersion laws (6) and (7d), existence of longitudinal tachyon-like photons –"longtons"- in physical vacuum should take place. Such particles as elemental excitations of physical vacuum correspond to longitudinal electromagnetic waves, which are unstable at small wave vector value due to gravitational attraction between globules of physical vacuum.

Each elemental globule of real photonic crystal is a spherical resonator. Total symmetric mode of such resonator corresponds to scalar waves, propagating along the photonic crystal. Elemental excitations, corresponding to such mode (breathing mode), are some type of scalar Higgs boson, emerging in weak interaction field theory. Main resonance frequency of globular resonator is close to value: $\omega_r = S\frac{\pi}{L}$, where L - is the size of the globule, S – sound velocity value. In common case the globule resonator modes with frequency ω_r have deformational nature and behave as tensor-type waves. Dispersion law of the corresponding tensor-type waves may be written as:

$$\omega^2 = \omega_r^2 + S^2 k^2 \ . \tag{19}$$

Elemental excitations of such kind waves are even type ones and accordingly one particle absorption or emission of such type excitations is forbidden by selection rules for vector type electromagnetic processes and also for vector type acoustical processes. Accordingly such type excitations we shall call "darktons". If S-constant is the electromagnetic wave velocity($S=c_0$), we have a new type of electromagnetic waves –even type electromagnetic waves. Note, that gravitational waves have also tensor type symmetry properties. So we think that gravitational waves are some kind of deformational perturbations of physical vacuum as real media and are one type of even electromagnetic waves. Thus elemental excitations of gravitational waves - gravitons- are some type of darktons.

On the other hand, the presence in physical vacuum of photons with very small velocity - "slowtons'- and with negative or positive mass sign explains us the nature of "dark substance" and "dark energy" origin.

5. Opportunity of generation and observation of new types photons in media and in physical vacuum

Now there is the task of experimental observation of slowtons, darktons, longtons and other types of physical vacuum quasiparticles. We consider, that this task might be solved in laboratory, with using of modern laser technique. The example of such kind experiment is the pumping with the help of intensive laser radiation quasiparticles in real crystal, having the same symmetry properties as slowtons, darktons and longtons. For instance longitudinal Frenkel excitons in dielectrics ($NaNO_2$ and others) may be excited already at room temperature. Even-type elemental excitations should emerge in media as a result of two-photonic processes. One type of such processes is Stimulated Raman effect in real condensed media (crystal or liquids) and also in globular photonic crystal. Even-type waves in this case should emerge as a result of coherent breathing type molecular vibrations excitations due to stimulated Raman scattering in molecular media. As a rule such type scattering takes place for total symmetrical molecular modes in liquids (C_6H_6, CS_2 and others) and also some crystals. In these case corresponding wave numbers are 500-1000 cm^{-1} , i.e. frequencies are close to 10^{13} Hz – range. The energy of even-type waves, emerged under stimulated Raman scattering, is comparable with the initial energy of pulsed laser emission. Infrared region waves, corresponding to two-photon emission from excited even-type level as a result of stimulated Raman process , also may be generated and then detected.

Note, that recently [3,4] stimulated globular scattering of light in globular photonic crystal have been observed. Such scattering was excited by using of giant (Q-switched) ruby (694.3 nm) laser pulses with intensity close to 10^8-10^{10}W/cm^2. The experiments have been fulfilled fore pure opal and also for opals, filled by acetone or ethanol. One or two Stokes satellites with frequency shifts 10^{10} Hz have been observed. The intensity of S-satellites for stimulated globular light scattering was comparable with intensity of exciting line (694.3 nm). As a result of stimulated globular light scattering even-type waves, corresponding to Higgs bosons, were generated.

Thus we consider that now there is the opportunity to generate in laboratory new types of photons, corresponding to elemental excitations of physical vacuum, with the help of powerful laser emission and another equipments, permitting to excite longitudinal or forbidden for dipole emission waves.

4. Conclusion

Thus we have come to the following results.

1. Dispersion law of acoustical branch of real crystalline lattices is resemble to photon dispersion law of physical vacuum. Accordingly we should wait for dispersion of light velocity at large enough wave vector value.
2. Physical vacuum may be described as globular photonic crystal with super lattice constant equal to $\sim 10^{-17}$m (distance of weak interactions).
3. Relativistic law of dispersion of elemental particles may be obtained from more common relations by using of lattice models of physical vacuum. The photons with negative or imaginary masses (heavy photons), very "slow" photons (slowtons) and even type photons (darktons) are predicted.
4. Longtons, darktons and slowtons may be generated in laboratory and then detected in real crystals or molecular media at some distance from emitters.

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Gravitationally bound quantum systems with leptons and mesons

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Primordial black holes (mini-holes) can capture elementary particles due to gravitational interaction. Bound quantum systems maintaining a particle in orbit around a mini-hole are called graviatoms. The graviatoms prove to exist only under some conditions, which are satisfied for systems containing leptons and mesons. Electromagnetic and gravitational radiations of charged particles in the graviatoms are comparable with Hawking's mini-hole radiation. The graviatoms containing neutrinos have macroscopic dimensions. Baryon interaction with mini-holes is describable in the framework of quantum accretion.

1. Introduction

As known, there exist bound quantum systems[1] due to electromagnetic interaction, for example atoms and molecules. Atomic nuclei and hadrons are examples of the quantum systems bound via strong interaction.

Planets, stars and galaxies are classical systems bound by gravity. Newton's gravitational force is about forty orders weaker than Coulomb's force for micro-particles. That is why gravity is usually neglected in elementary particle physics.

However, if one component of the system is assumed to be considerably massive and has a small dimension and the other is an elementary particle, then a quantum system can be formed, for example, as a result of capturing particles by mini-holes in the early Universe. Such systems are called graviatoms [1].

Another example of the quantum systems bound by gravity is macro-bodies capturing neutrinos having de Broglie's wave length of macroscopic value.

2. Approach

We consider Schrödinger's equation for hydrogen-like graviatoms, which describes a radial motion of a particle in the mini-hole potential, and find their energy levels. The angular wave function is the well-known eigen function of the orbital momentum operator for a hydrogen-like atom.

Schrödinger's equation for the graviatom

[1] Quantum system definition: the de Broglie wave length for a particle is about a system size

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{pl}}{dr}\right)-\frac{l(l+1)}{r^2}R_{pl}+\frac{2m}{\hbar^2}\left(E+\frac{mc^2 r_g}{2r}\right)R_{pl}=0 \tag{1}$$

describes a radial motion of a particle with the mass m in the mini-hole potential, where $r_g = 2GM/c^2$ and M are the mini-hole gravitational radius and mass respectively.

The energy spectrum is of hydrogen-like form

$$E=-\frac{G^2M^2m^3}{2\hbar^2n^2}. \tag{2}$$

3. Graviatom existence conditions

A graviatom can exist if the following conditions are fulfilled [2, 3]:

- the geometrical condition $L > r_g + R$, where L is the characteristic size of the graviatom and R is the characteristic size of a particle;
- the stability condition given by
  (a) $\tau_{gr} < \tau_H$, where τ_{gr} is the graviatom lifetime and τ_H is the mini-hole lifetime, and
  (b) $\tau_{gr} < \tau_p$, where τ_p is the particle lifetime (for unstable particles);
- the indestructibility condition (due to tidal forces and the Hawking effect) $E_d < E_b$, where E_d is the destructive energy and E_b is the binding energy.

The charged particles satisfying these conditions are the **electron, muon, tau lepton, wino, pion** and **kaon**.

The conditions of existence of the graviatoms reduce to the relation between the masses of the mini-hole and particle, with their product being approximately constant equal to the Planck mass (figure 1).

4. Graviatom radiation

The intensity of the electric dipole radiation of a particle with mass the m and charge e in the gravitational field of a mini-hole is

$$I^d_{fi}=\frac{2\hbar e^2\omega^3_{if}f_{if}}{mc^3}, \tag{3}$$

where $\omega_{if} = (E_i - E_f)/\hbar$ is the frequency of the transition $i \to f$ and f_{if} is the oscillator strength.

The electric quadrupole radiation intensity for the transition $3d \to 1s$ is

$$I^q_{13}=\frac{6\hbar e^2\omega^3_{31}}{mc^3}f_{3d\to 1s}. \tag{4}$$

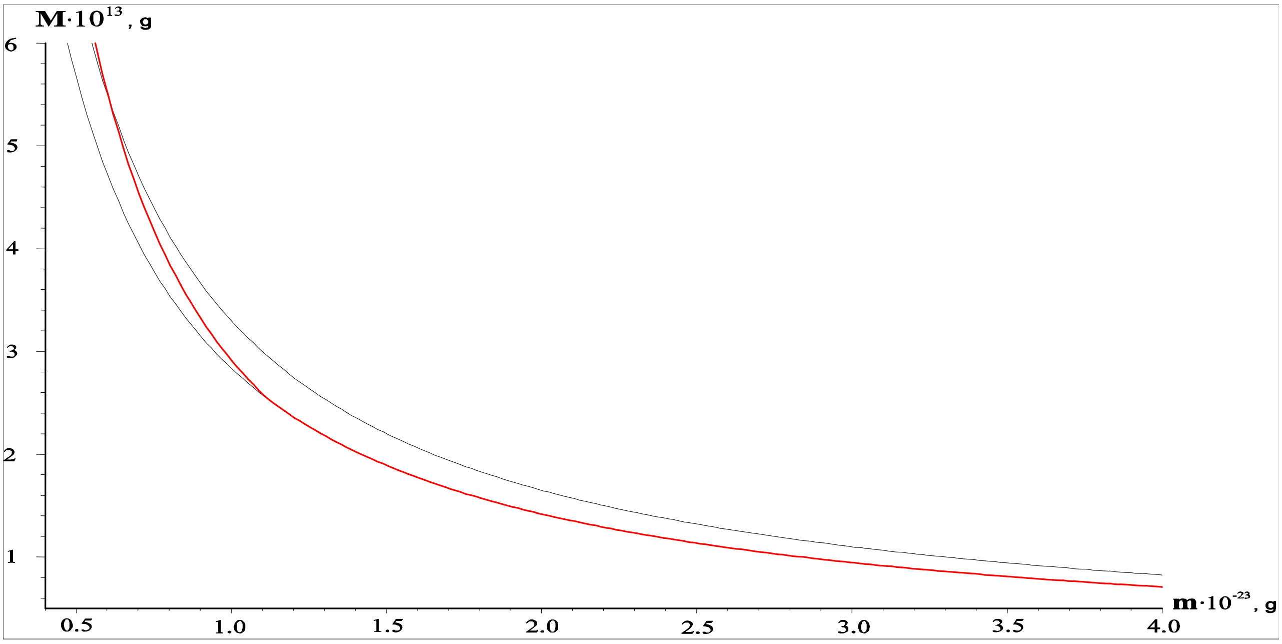

Fig. 1. The upper and lower curves indicate the range of values related to the geometrical condition (the upper one) and to Hawking effect (the lower one). The middle curve is related to the particle stability condition ($\tau_p = 10^{-22}$ s).

The gravitational radiation intensity for the transition $3d \rightarrow 1s$ is

$$I_{13}^{g} = \frac{24\hbar G M \omega_{31}^{3}}{c^{3}} f_{3d\rightarrow 1s}. \tag{5}$$

The mini-hole creates particles near its horizon due to Hawking's effect, its power

$$P_H = \frac{\hbar c^6}{15360\pi G^2 M^2}. \tag{6}$$

The Hawking energy

$$E_H = \frac{b\hbar c^3}{8\pi G M}, \tag{7}$$

where b=2,822, following Wien's displacement law.

We evaluate the intensity of electric dipole and quadrupole radiation of a charged particle in the gravitational field of the mini-hole. The oscillator strengths are given by the well-known formulae for the hydrogen-like atoms. The gravitational radiation for transition 3d to 1s is also calculated. The graviatom radiations are compared with Hawking's effect. The formulae for Hawking's radiation power and energy are also presented.

The electric dipole, gravitational and Hawking's radiation prove to be comparable. It is possible for graviatoms involving electrons, muons and pions to have existed up to now. The gravitational fine-structure constant appears to be greater than the electromagnetic one but does not exceed the unity, so the perturbation theory remains valid.

The mini-hole masses for the graviatoms, involving electrons, muons and pions, exceed the value of 4.38×10^{14} g, which means that it is possible for such graviatoms to have existed up to now. The gravitational equivalent of the fine-structure constant

$$\frac{GMm}{\hbar c} = 0.608 \div 0.707. \tag{8}$$

Thus, the perturbation theory remains valid.

5. Systems with neutrinos

The de Broglie wave length for a neutrino [4]
$$\lambda_{dB} = \frac{\hbar^2}{GMm_\nu^2}, \tag{9}$$

where m_ν is the neutrino mass, is of macroscopic value ($10^1 \div 10^6$ cm).

Macro-bodies capture neutrinos onto both Bohr's hydrogen-like levels (outside the body) and Thomson's oscillatory ones (inside the body). The oscillation frequency $\omega = \left(\frac{4}{3}\pi\rho G \right)^{1/2}$, the gravitational radiation intensity

$$I_{mb} = \frac{\hbar^{7/3} \left(\frac{4}{3}\pi\rho G \right)^{9/4}}{c^5 m_\nu^{5/2}}, \tag{10}$$

where ρ is the macro-body density.

Bound quantum systems with the neutrino has a macroscopic size, nevertheless, the neutrino energy levels are quantized. For macro-bodies two cases are possible: Bohr's levels outside the body and Thomson's – inside it. The gravitational radiation proves to be negligibly small.

6. Conclusion

The graviatom can contain only leptons and mesons. The observable stellar magnitude for graviatom electromagnetic radiation exceeds 23^m.

Stable graviatoms with baryon constituents are impossible. The internal structure of the baryons, consisting of quarks and gluons, should be taken into account. There occurs a so-called quantum accretion of baryons onto a mini-hole. The whole problem is solvable within the framework of quantum chromodynamics and quantum electrohydrodynamics.

Neutrinos can form quantum macro-systems.

Of interest is the fact that from the galaxy rotation curves it follows that the dark matter mass is proportional to the distance from the galaxy centre. Similarly, the total mass of neutrinos on the nth level is proportional to their orbit radius.

The description of gravitationally bound macro-systems with neutrinos may be helpful for solving the dark matter problem in the Universe.

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Principle of Mach in the five-measurement cosmological solution of Ross

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In the five-measurement cosmological solution of Ross on the basis of the standard cosmological closed model and the induced energy-momentum tensor the function influencing the inertial properties of a trial particle and permitting to point to the connection, between it and a mass distribution, including a dark substance in the Universe and in its nearest environment is postulated. This can be an argument for the proof of a principle of Mach.

1. Introductoin

Ernst Mach realized the historical and critical analysis of mechanics [1], he paid attention to its feeble aspects; in particular, to the Newtonian concept of absolute Space and absolute Motions. In 1921 in the Royal college of London A. Einstein [13,p.111] said on this point " in the theory of Newton if one considers motion not from the causative from the only descriptive point of view, it exists only as a relative movement of subjects in relation to each other. However from this point of view the acceleration appearing in the equations of Newton is not clear. Newton has to invent physical space, in relation to which acceleration must exist. The principle of Mach is reduced to a motion not in relation to terrain clearance space, but in respect to plurality of all the rest of the ponderables bodies, which one determine the properties of space, in particular Universe"… The speculations of A.Einstein brought him to more concrete formulations of this principle [14, p.90] :

1. The inertia of a body should increase in accordance with an aggregation of the ponderables masses near to it.
2. The body should test accelerating force, when the neighboring masses are accelerated, this force in a direction should coincide with the a direction of acceleration.
3. Rotating hollow body should create "a Coriolis field of forces" inside itself which seeks to decline the propellented bodies in a current of traffic, and also radial centrifugal field of forces.

A.Pais [2] wrote, that he tried to follow the this principle in his theory as the quantities $g_{\mu\nu}$- the gravitational and metric potentials cause the activity of inertia, they must be, in turn, completely determined by a mass distribution in the Universe. A. Einstein assumed Space as the closed three-dimensional spherical Universe, where there is no Newtonian perpetuity; further, for the stationary solution he entered the term Λ, but then with the appearing of the theory of the nonstationary Universe of Fridman he refused from it ; the realization of a principle of Mach remained a problem. As A.Pais wrote,"in my opinion, the problem of origin of inertia was and remains the most dark problem in the theory of particles and fields".R. Tolman [3] in his Book wrote, that " in addition to principles of a covariance and equivalence we should, apparently, enter some the complementary element into the theory of gravitation".. and further " now we should enter the third principle of the relativistic theory of gravitation, namely: the precise formulation of the law, reflecting the dependence of metric and gravitational fields from condition of space- time, the knowledg of which allow one to compute the gravitate effects at given allocations of a substance and energy".. The Editor of translation of the Work of R.Tolman J.A.Smorodinski noted, that "at the present time (70 s) "no.. evidence for any connection between a mass distribution in the Universe and mass of any body or particle" [p.192, there]. The writers of the collection of problems on a Theory of relativity and gravitation [4] wrote" that no body for the present did not manage to achieve more or less noticeable successes in deduction of a principle of Mach from the equation of half general Theory of relativity " [p.348].

J.Weber [5], studying this problem, wrote, that Einstein and Wheeler studied a problem on opportunity of explanation of a principle of Mach not as a consequence of a field equation, but as a requirement imposed on boundary conditions. If system is "isolated", it is necessary to enter the requirement, that over large distances the metric should properly transfer in the metric of the remaining Universe. It would mean the interaction of a substance of the remaining part of the Universe with a mass of "isolated" systems. The realization of such program could give a relation linking the inert properties with a configuration of a substance in the remote areas of the Universe" [5,p.197-198]. At last, it is necessary to note the work of F.Hoyle [6] (middle 70 s), where he suggested the modification, based on a principle of Mach (partitioning of the Universe into "compartments" than longer one looks into the depth of the Universe, the more decrease the mass; in this case the size of particles decreases also, resulting in the extremity of "compartment" [6,p.158].

The present work deals with the attempt of the partial solution of this problem on the basis of the five- measurement theory of Kaluza-Klein, as in the parameters and functional capability of constructing the induced energy-momentum tensor operating these parameters are built in it. Such parameters are a builder of a metric tensor at a footstep to coordinate γ_{55} and coupling coefficient of the five-measurement and four-measurement metric ſ, equal [11]:

$$ſ=1-\varepsilon n^2/\gamma_{55}=dS^2/ds^2, \quad \varepsilon=\pm1 \tag{1}$$

where the scalar field n can be presented, for example, from two components

$$n_1=(e/m)/(1-(e/m)^2)^{1/2}, \tag{2}$$

where e/m –specific charge, if a particle charged [11], and let's assume

$$n_2=A+\gamma, \tag{3}$$

where A- constant, γ- gravitational perturbation of a total energy neighboring to a trial of bodies; by character, the function of γ_{55} is bound to density of a substance (customary and dark) of the Universe (see below). With allowance made for the achievements of a modern satellite astrometry it is already possible to set a task of measuring estimation of gravitational perturbation of our Galaxy [15].

2. Interpretation of the cosmological solution of Ross on the basis of the standart close cosmological model and the induced energy-momentum tensor

So,we shall consider the precise cosmological solution of Ross, Basu and Rey [8], [9], [10] :

$$dS^2=(dx^0)^2+\varepsilon\gamma_{55}(x^0,x^5)(dx^5)^2-f^2(x^0,x^5)[dr^2/(1-kr^2)+r^2d\theta^2+r^2\sin^2\theta d\varphi^2], \tag{4}$$

where

$$f^2=[(1/þ(x^5)-k)(x^0)^2+2\mu(x^5)x^0+F], \; F=(\mu^2+Q)/(1/þ-k), \;\; \gamma_{55}=(\partial f/\partial x^5)^2þ, \tag{5}$$

k, Q –constants, þ, μ –function from a footstep of dimensionless coordinate, below will be shown; at $x^5=0$ the solution (4) formally becomes the solutions of Robertson-Walker. Let's consider, according to A.Einstein [13], the close Universe (Riemann sphere), where there is no Newtonian perpetuity. From (5) it is visible, that f – the quadratic function on x^0 and exists a point, where the derivative $\partial f/\partial x^0$ is peer to zero point, that contradicts definion of parameter Habbl'e H. Therefore, it is necessary to accept diverse model for reseaech, where the indicated derivative is not peer to zero point, namely

$$f^2=(1/p-k)(x^0)^2+Q/(1/p-k), \; \mu=0. \tag{6}$$

From the not static solution of the homogeneous Universe it is possible to estimate parameters p,Q

$$3f''/f=8\pi G\Omega/2, \; \Omega=\rho/\rho_{cr}, \tag{7}$$

Among the "miracles" of the theory Kaluza it is necessary to mark an opportunity of obteinung of an induced energy-momentum tensor [11],[12] used for obtaining builders of a tensor γ_{55} and, as a consequent, definition of the pressure p_t and the density ρ_t of dark energy:

$$T_1^{\ 1} = T_2^{\ 2} = -p_t,\ T_0^{\ 0} = \rho_t\ . \qquad (8)$$

Allowing an the equation of condition for dark energy [7] $p_t = -w\rho_t$,w>0, and the components of a metric tensor (4) we shall receive from [11] the differential equations of a view

$$w(\partial^2(\partial f/\partial x^5)/\partial^2 x^0)/2-(\partial f/\partial x^0)(\partial(\partial f/\partial x^5)/\partial x^0)/f+(1-3w/2)(\partial f/\partial x^5)/pf^2 = 0. \qquad (9)$$

From (9) with allowance for definitions of function Habbl'e

$$H = ((\partial f/\partial x^0)/f) = (1/p-k)x^0/f\ , \qquad (10)$$

the expression follows

$$((\partial p/\partial x^5)/p^2)L(w,p,Q,H) = 0\ . \qquad (11)$$

From (11) the relevant consequent follows: a case of equaling to zero point of the first multiplicand parameter p for obtained from the solution Ross of value of (5) becomes independent from a footstep of coordinate and we automatically transfer from the fiftdimensional Space in the four-dimensional Space-Time, i.e. to the standard cosmological model! All problems, bound with the proof of a principle of Mach are in this case maintained. In case of equaling to zero point of the second multiplicand the parameter Q about expresses through the quantities H,p the above mentioned arguments of the proof of a principle of Mach also are maintained.

Parameter p can be presented for a simplicity by a symmetric function from x^5 : $p=\lambda x^5$, whence we have $\gamma_{55} = \gamma_{55}(x^5,\lambda)$; the constant of λ is determined from the equation of Fridman's, where

$$(\partial(\partial f/\partial x^0)/\partial x^0)/f = [1/p-k-H^2]. \qquad (12)$$

As parameter p enters definition of a the stationary value of Habble, the fifth measuring will be bo-und to red bias, i.e. with and measurand; the fifth builder of a metric tensor is the indicator of the performances of allocation of a substance of the Universe.

3. Discussion of results

From (4) it is visible, that in theory the builders of ametric tensor depend on a fifth of coordinate, the physical sense by which one demands discussion.While there is a hypothesis about her smallness and difficulty of her detection [17]. It is possible to present this measuring as product of a dimensionless quantity x^5 and $\hat{h}$, where $\hat{h}$ –the constant size of a compactification, small as contrasted to sizes the macrocosm [17], and x^5 is possible to refer to identify as "redshift", $x^5 = \Delta v/v$ (for the solar–earth system $x^5 \sim 10^{-6}$). While it is possible to refer to Judgement E.Mach , which one wrote< that ..”before promulgation of publish of Riemann I viewed already multidimensional spaces as a the mathematical-physical auxiliary resort".

From the covariant equation of an one–dimensional motion of a concentrated point of a variable rest mass [21] it is possible to receive on the basis of the metric of Gross-Perry [20] solutions (dependence of mass on velocity) already with allowance for of parameter ſ and gravitational interaction of the "near" bodies. Allowing a Lagrangian, neglecting a smallness dx^5/ds,

$$L=1 = ſ(1-\alpha/r)^a(x'^0)^2 - ſr'^2/(1-\alpha/r)^{a+b}\ ,\ a^2+ab+b^2 = 1, \qquad (13)$$

It is possible to record the differential equation, in a right member which one the Jet acceleration incipient at the anisotropic effluxion of mass and gravitational force F^1 is submitted:

$$d((q/(1-q^2/c^2)^{1/2})/ds = -(v_{eff}/c)(d(m/M)/ds)/(m/M)(1-q^2/c^2)^{1/2} + F^1/(1-q^2/c^2)^{1/2}, \qquad (14)$$

where\`$q=v/(1-\alpha/r)^a$, m-variable mass, the for v_{eff} – velocity of effluxion, $F^1 = -c^2\alpha a(1-\alpha/r)^{a+b-1}/2r$.

Integration (14) gives

$$v/c = \{[1-2(v_{eff}/c)(\ln(m/M))/(\eta_0 ſ) - \int F^1 ds]/[1+2(v_{eff}/c)(\ln(m/M))/(\eta_0 ſ) + \int F^1 ds]\}(1-\alpha/r)^a, \qquad (15)$$

here $\eta_0 = (1+q/c)/(1-q/c)$.

From (15) follows, that at decrease of mass the velocity grows , at growth of a the gravitation slow down. If to apply this formula the gravitational formation of variable mass to all Universe, that, agre- es Hoyle, we at expansion "observation" a decrease of mass of the bodies and particles in our part of "compartment" of the Universe with transition of the particles in photons at partial gravitational "slowing down" of this process.

From the same Lagrangian it is possible to record expression for acceleration of a trial particle

$$r''\approx-(a+b)\alpha r'^2/r^2(1-\alpha/r)-r'(d\ln f)/ds + \ldots \quad (16)$$

As $d\ln f/ds < 0$, we have padding force conterminous on a direction to acceleration, that confirms [14]. The point third in the requirement of Einstein about a body, rotaried by a floor, in the present operation was not explored, though, is interquartile, he too can have a positive take in the given multivariate approach.

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A BLACK HOLE INTERPRETED AS A POINT MASS IN GR

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Abstract

A description of a point mass in general relativity (GR) is given in the framework of the field formulation of GR where all the dynamical fields, including the gravitational field, are considered in a fixed background spacetime. With the use of stationary (not static) coordinates non-singular at the horizon, the Schwarzschild solution is presented as a point-like field configuration in a whole background Minkowski space. We directly and simply continue the Newtonian derivation with the use of the Dirac δ-function, at the same time, the new derivation does not contradict GR corresponding its principles.

A one of important notions in physics is a test particle, which usually is treated as a point mass without volume and with a restricted set of characteristics. Of course, in the framework of the theory under consideration, a representation of this object has to be non-contradictive. In spite of a visible simplicity one has to provide this representation very carefully. Thus, a solution for the point mass in general relativity (GR), as a central symmetrical one in vacuum, is the well known Schwarzschild solution [1]. However, an analysis shows that the notion of a point particle has disappeared, but one has to consider a complicated geometry with a horizon and a true singularity. In this paper we interpret the Schwarzschild black hole, conserving all its properties, as a point particle in GR.

The derivation of the point particle in Newtonian gravity is given in a natural and simple way. One has to assume that the mass distribution has the form

$$\rho(\mathbf{r}) = m\delta(\mathbf{r}) \tag{1}$$

where the Dirac δ-function satisfies the ordinary Poisson equation, which in spherical coordinates is

$$\nabla^2 \left(\frac{1}{r}\right) \equiv \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right)\frac{1}{r} = -4\pi\delta(\mathbf{r})\,. \tag{2}$$

Then, the Newtonian potential $\phi = Gm/r$ satisfies to the Newtonian equation in the whole space including the point $r = 0$ if the rules for using generalized functions [2] are taken into account. Besides, the whole mass of the system is defined by the *unique* expression

$$m = \int_\Sigma dx^3 \rho(\mathbf{r}) \tag{3}$$

where both an usual regular distribution and the point particle density (1) can be integrated providing the standard mass m.

Concerning GR, as a rule, one concludes that at spacetime singularities GR does not work without a possibility to apply mathematical operations, unlike Newtonian gravity. Sometimes, to avoid singularities, various modified physical models are suggested [3, 4]. On the other hand, in spite of that a black hole singularity is a real physical singularity, other authors suggest describe it in mathematical terms. Thus, in [5] using various transformations of the radial coordinate a description of a static spherically symmetric spacetime in GR with a point singularity at the center and vacuum outside is given. In [6], a substitution of metrical coefficients of various black hole solutions into the lhs of the Einstein equations

$$G_{\mu\nu} = \kappa T_{\mu\nu} \tag{4}$$

leads to appearance of generalized functions. Then the authors treat that these solutions satisfy Einstein equations everywhere if the energy-momentum at the rhs of (4) has a relevant singularities of generalized functions.

However, if one considers GR in the usual geometrical description, then this interpretation meets conceptual difficulties (for details see the paper by Narlikar [7] and a discussion in the paper [8]). If we assume that the metric coefficients of the Schwarzschild solution in his coordinates are fulfilled in the whole spacetime, including $r = 0$, then due to (4) the matter distribution will have the form:

$$T_0{}^0 = T_1{}^1 = 0, \qquad T_2{}^2 = T_3{}^3 = \frac{mc^2}{2}\,\delta(\mathbf{r}). \tag{5}$$

It will not be possible to obtain the correct total mass for this distribution if the ordinary volume integration, like (3), is used. Indeed, for (5) the mass density is equal to zero even if one remembers that the time coordinate and the radial coordinate change their sense inside the horizon. Dirac's δ-functions of a timelike coordinate are non explainable also. Thus, by this way one cannot follow the description (1) - (3) in Newtonian gravity.

In [8], it was shown that, analogously to the Newtonian prescription, the point mass in GR can be described in a non-contradictory manner in the framework of a so-called field-theoretical formulation (or simply "field formulation") of GR, where all the dynamical fields, including the gravitational field (metric perturbations), are considered in a background (fixed, auxiliary) spacetime (curved or flat). The field formulation has been developed in [9] - [11], it is four-covariant and very similar to a gauge invariant field theory in a fixed spacetime. At the same time, the field description can be constructed with the help of a *simple* decomposition of the variables of the geometrical formulation into a sum of background and dynamical variables of the field formulation [10]. Anyway, both the formulations of GR are equivalent. Therefore, any solutions to GR can be treated in the framework of the field formulation.

Here, to repeat the main notions of the field formulation of GR [9] it is enough to present the equations for the gravitational field $h^{\mu\nu}$ on Ricci-flat backgrounds:

$$G^L_{\mu\nu}(h^{\alpha\beta}) = \kappa t^{tot}_{\mu\nu}\,. \tag{6}$$

The lhs is linear in the symmetric tensor $h^{\mu\nu}$:

$$G^L_{\mu\nu}(h^{\alpha\beta}) \equiv \tfrac{1}{2}\left(h_{\mu\nu}{}^{;\alpha}{}_{;\alpha} + \gamma_{\mu\nu}h^{\alpha\beta}{}_{;\alpha\beta} - h^{\alpha}{}_{\mu;\nu\alpha} - h^{\alpha}{}_{\nu;\mu\alpha}\right) \tag{7}$$

where $\gamma_{\mu\nu}$ is the background metric; $\gamma \equiv \det \gamma_{\mu\nu}$; $(;\alpha)$ means the covariant derivative with respect to $\gamma_{\mu\nu}$. The total energy-momentum tensor

$$t^{tot}_{\mu\nu} \equiv t^{g}_{\mu\nu} + t^{m}_{\mu\nu} \tag{8}$$

is obtained after varying the action of GR in the field form with respect to $\gamma^{\mu\nu}$. The explicit expression for the pure gravitational part of (8) can be find in [9]. The equivalence between the field and geometrical formulations of GR can be stated by

$$\sqrt{-\gamma}\,(\gamma^{\mu\nu} + h^{\mu\nu}) \equiv \sqrt{-g}g^{\mu\nu} \tag{9}$$

where $g \equiv \det g_{\mu\nu}$. Then, the equations (6) change over to the usual form of the Einstein equations (4) with the dynamic metric $g_{\mu\nu}$. The matter energy-momentum tensor in (8) is connected with $T_{\mu\nu}$ in (4) by

$$t^m_{\mu\nu} = T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T_{\alpha\beta}g^{\alpha\beta} - \tfrac{1}{2}\gamma_{\mu\nu}\gamma^{\alpha\beta}\left(T_{\alpha\beta} - \tfrac{1}{2}g_{\alpha\beta}T_{\pi\rho}g^{\pi\rho}\right)\,. \tag{10}$$

The gauge transformations in the field-theoretical derivation are directly connected with mapping a spacetime onto itself. For example, for the different maps one has $g^{\mu\nu}(x)$ and $g'^{\mu\nu}(x)$. Then, choosing the same background metric in (9) one finds that $h^{\mu\nu}(x)$ and $h'^{\mu\nu}(x)$ are connected by the gauge transformations and each of them satisfies the Einstein equations in the form (6). In the general case, a manifold which supports a physical metric has not to coincide with a manifold which supports a background auxiliary metric. As a result, non-physical "singularities", "membranes", "absolute voids", *etc.*, can appear in a field configuration propagating on the background. This can lead to cumbersome explanations, confused interpretations, *etc.* Considering the field formulation as a convenient tool for a resolution of several theoretical problems in GR it is reasonable to avoid such difficulties. Here, we exploit the model when a spacetime of the standard Schwarzschild solution and a background Minkowski space are in one-to-one correspondence.

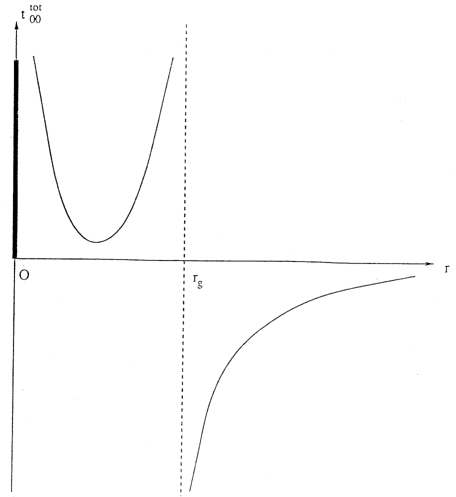

Figure 1: The energy distribution for the field configuration associated with the Schwarzschild solution. Schwarzschild gauge fixing

The usual Schwarzschild metric in his coordinates is

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) . \tag{11}$$

The background Minkowski space is presented by the metric in the polar coordinates:

$$d\bar{s}^2 = c^2 dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{12}$$

where we will numerate the coordinates as $x^0 = ct$, $x^1 = r$, $x^2 = \theta$ and $x^3 = \phi$. Then with the use of (9) we present the solution (11) in the form of a field configuration

$$h^{00} = \frac{r_g}{r}\left(1 - \frac{r_g}{r}\right)^{-1}, \qquad h^{11} = \frac{r_g}{r} . \tag{13}$$

The energy-momentum tensor for the configuration (13) can be constructed directly calculating (8) with the use of (10) and (5), or with the use of (7) — the lhs of (6). Its components are presented by expressions proportional to $\delta(\mathbf{r})$ and by free gravitational field outside $r = 0$. Thus, the 00-component presenting the energy density for a point in the Minkowski space is

$$t^{tot}_{00} = \frac{mc^2}{2}\delta(r)\left[1 - \left(1 - \frac{r_g}{r}\right)^{-2}\right] - \frac{{r_g}^2}{\kappa r^4}\left(1 - \frac{r_g}{r}\right)^{-3} . \tag{14}$$

It is natural that the total energy obtained with the use of (14) over the volume integral analogous (3) is equal to mc^2. If one calculates the energy outside the horizon only one will obtain $-\infty$; the energy inside the horizon is equal to $+\infty$ (see Fig. 1). However the infinite contributions near horizon are compensated. The contribution to the total energy from the δ-function is equal to $mc^2/2$, while the contribution from the free gravitational field outside $r = 0$ is also equal to $mc^2/2$.

As is seen, the situation is more comprehensive than for the point mass in Newtonian gravity. Nevertheless, the problem of the point mass is resolved. Namely, one has accepted result by the volume integration, and the configuration (13) satisfies the Einstein equations (6) at all the points of the Minkowski space, including $r = 0$, if the operations with the generalized functions are valid. Thus the concept of Minkowski space is extended from spatial infinity (frame of reference of a distant observer) up to the horizon $r = r_g$, and even under the horizon including the worldline $r = 0$ of the true singularity.

Let us discuss the Fig. 1. In reality the distant observer cannot see the space within the horizon. So, if we approach the horizon from outside, then we have the infinite negative density for the gravitational energy. Naively this picture can be explained as follows. From the point of view of the distant observer (and absolutely) if the test particle moves closer from outside to the horizon then one finds it more difficult to escape from the black hole. Indeed, the negative density of the gravitational energy (and, consequently, the attraction) is stronger near the horizon. At the horizon (the density $t^{tot}_{00} = -\infty$) it is impossible to escape the black hole.

As is seen, at $r = r_g$ the energy density and other characteristics have discontinuities. Although it is not a real singularity, a "visible" boundary between the regions outside and inside the horizon exists and does not allow to consider an evolution of events continuously. In the field formulation this situation is interpreted as a "bad" fixing of gauge freedoms, which can be improved. That is the break at $r = r_g$ can be countered with the use of an appropriate choice of a flat background, which is determined by related coordinates for the Schwarzschild solution, i.e., by w related mapping. At least, the use of the coordinates without singularities at the horizon could resolve the problem locally at neighborhood of $r = r_g$. Next, it would be natural to describe the true singularity by the world line $r = 0$ of the chosen polar coordinates. Besides, it is desirable to have appropriate coordinates, in which the Schwarzschild solution conserves the form of an asymptotically flat spacetime.

Recently Pitts and Schieve [12] defined and studied properties of a so-called "η-causality". Its fulfilment means that the physical light cone is inside the flat light cone at all the points of the Minkowski space. It is necessary to avoid interpretation difficulties under the field-theoretical presentation of GR. By this requirement all the causally connected events in the physical spacetime are described by the right causal structure of the Minkowski space. A related position of the light cones is not gauge invariant. We consider this requirement *only* to construct a more convenient in applications and interpretation field configuration for the Schwarzschild solution. The requirement of the η-causality can be strengthened by the requirement of a "stable η-causality" [12]. The last means that the physical light cone has to be *strictly* inside the flat light cone, and this is important when quantization problems are under consideration. Indeed, in the case of tangency a field is on the verge of η-causality violation. Returning to the presentation in the Schwarzschild coordinates (11) - (14) we note that it does not satisfy the η-causality requirement.

In the paper [13], taking into account the above requirements we have found a desirable description. A more appropriate gauge fixing corresponds to the *stationary* (not static) coordinates presented in [14, 15] and recently improved in [12]. We consider also the contracting Eddingtom-Finkenstein coordinates in stationary form [16]. These two coordinate systems belong to a parameterized family where all of systems satisfies all the above requirements. The transformation

$$ct' = ct + r_g \ln \left| \left(\frac{r}{r_g} - 1 \right) \left(\frac{r_g}{r} \right)^{\alpha} \right| \tag{15}$$

gives just the parameterized by $\alpha \in [0, 2]$ family of such metrics. The cases $\alpha = 1$ and $\alpha = 0$ correspond to the first and to the second examples above. The *stable* η-causality *is not* satisfied with $\alpha = 0$ at $0 \le r \le \infty$. Properties of field configurations corresponding to $\alpha \in (0, 2]$ qualitatively are the same as for $\alpha = 1$.

To construct a field configuration corresponding to any metric of the family obtained by the transformation (15) one has to change $t' \to t$ and again to use (12) and (9). Such field configurations are connected by the gauge transformations between themselves and with (13) and are physically equivalent. Both the gravitational potentials and the components of the total energy-momentum tensor for the field configurations with $\alpha \in [0, 2]$ have no a break at $r = r_g$.

Concretely for the case $\alpha = 1$ one has for the Schwarzschild solution after changing $t' \to t$ the metric in the stationary form:

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - 2 \frac{r_g^2}{r^2} c\, dt\, dr - \left(1 + \frac{r_g}{r}\right) \left(1 + \frac{r_g^2}{r^2}\right) dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right) . \tag{16}$$

The correspondent field configuration is

$$h^{00} = \frac{r_g}{r} + \frac{r_g^2}{r^2} + \frac{r_g^3}{r^3}, \qquad h^{01} = -\frac{r_g^2}{r^2}, \qquad h^{11} = \frac{r_g}{r} . \tag{17}$$

The non-zero components of $t^{tot}_{\mu\nu}$ are significantly simpler than for the configuration (13):

$$\begin{aligned} t^{tot}_{00} &= mc^2 \delta(\mathbf{r}) + mc^2 \frac{r_g}{r} \left(1 + \frac{3}{2} \frac{r_g}{r}\right) \delta(\mathbf{r}) - \frac{mc^2}{4\pi} \frac{r_g}{r^4} \left(1 + 3 \frac{r_g}{r}\right) , \\ t^{tot}_{11} &= -mc^2 \delta(\mathbf{r}) , \\ t^{tot}_{AB} &= -\tfrac{1}{2} \gamma_{AB}\, mc^2 \delta(\mathbf{r}) ; \qquad A,\, B = 2,\, 3. \end{aligned} \tag{18}$$

The energy distribution is described by the 00-component of the energy-momentum tensor, and for $\alpha \in (0, 2]$ qualitatively is presented on Fig. 2. Then the total energy of the system

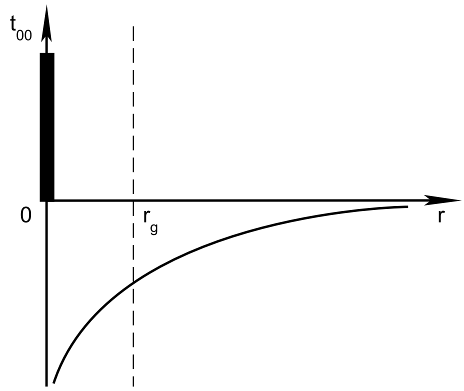

Figure 2: The energy distribution for the field configuration associated with the Schwarzschild solution. The case: $\alpha \in (0,\, 2]$

can be calculated both by the volume integration

$$mc^2 = \lim_{r\to\infty} \int_V t^{tot}_{00} r^2 \sin\theta dr d\theta d\phi\,, \tag{19}$$

or by the surface integration over the 2-sphere:

$$mc^2 = \lim_{r\to\infty} \frac{1}{2\kappa} \oint_{\partial V} \left(h_{00}{}^{;1} + \gamma_{00} h^{1\alpha}{}_{;\alpha} - 2h^1{}_{0;0} \right) r^2 \sin\theta d\theta d\phi\,. \tag{20}$$

For the case $\alpha = 0$ really we have to examine the contracting Eddington-Finkelstein metric for the Schwarzschild geometry [16]:

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - 2\,\frac{r_g}{r}\, c\, dt\, dr - \left(1 + \frac{r_g}{r}\right) dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)\,. \tag{21}$$

The gravitational field configuration corresponding to (21) is

$$h^{00} = \frac{r_g}{r}\,, \qquad h^{01} = -\frac{r_g}{r}\,, \qquad h^{11} = \frac{r_g}{r}\,. \tag{22}$$

All the non-zero components of the total energy-momentum tensor for this field configuration have the simplest form:

$$\begin{aligned} t^{tot}_{00} &= mc^2 \delta(\mathbf{r})\,, \\ t^{tot}_{11} &= -mc^2 \delta(\mathbf{r})\,, \\ t^{tot}_{AB} &= -\tfrac{1}{2} \gamma_{AB}\, mc^2 \delta(\mathbf{r})\,. \end{aligned} \tag{23}$$

This energy-momentum is concentrated *only* at $r = 0$, see the energy distribution on Fig. 3. The other component t^{tot}_{11} and the angular ones t^{tot}_{AB} (both in (23) and (18)) formally could be interpreted as related to the "inner" properties of the point. Indeed, they are proportional only to $\delta(\mathbf{r})$ and, thus, describe the point "inner radial" and "inner tangent" pressure. At last, again the total energy for (23) is mc^2.

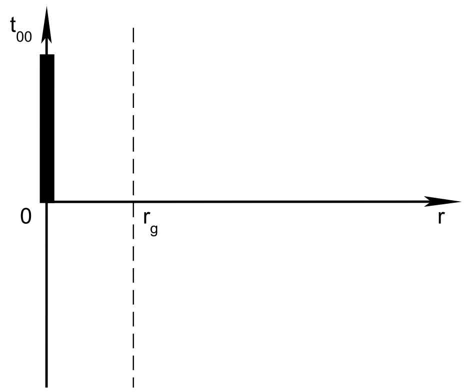

Figure 3: The energy distribution for the field configuration associated with the Schwarzschild solution. The case: $\alpha = 0$

The field configurations for $\alpha \in [0, 2]$ satisfy the Einstein equations (6) at all the points of the Minkowski space including $r = 0$. Then, keeping in mind the presentations on the figures 2 and 3 one can conclude that an appropriate description of a point particle in GR is approached. It is directly and simply continues the Newtonian derivation and, at the same time, does not contradict GR corresponding its principles.

Except a pure theoretical interest the description given in the present paper could be interesting and useful for experimental gravity problems. Gravitational wave detectors such as LIGO and VIRGO will definitely discover gravitational waves from coalescing binary systems comprising of compact relativistic objects. Therefore it is necessary to derive equations of motion of such components, e.g., two black holes. As a rule, at an *initial* step the black holes are modeled by point-like particles presented by Dirac's δ-function, like in (1). Then consequent post-Newtonian approximations are used (see the works with excellent mathematics [17] - [19] and references therein). However this approach meets difficulties related to the non-linear nature of the Einstein equations. Different regularization methods have been suggested to bypass them. However, in spite of a significant progress, so far the problem of motion of the black holes in GR has many of open questions. Our way of definition of a point-like source of gravity is different. Not making *initial* assumptions on its structure we use the *exact* Schwarzschild solution itself to define it. A resulting field configuration includes a description of the true singularity in the the form of a point-like particle. It is easy for applications and allows to reproduce the Schwarzschild solution as is, i.e., without approximations, with correctly-defined position of the horizon, *etc.* It is interesting to note also that in our exact derivation the "structure" of the point itself is more complicated than under the simplest assumption, like in (1). Thus, possibly our results (18) and (23) could play a role of examples for checking cumbersome series in the approach of the work [17] - [19].

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Темная материя – интерпретации и проверка

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Приведены и проанализированы известные экспериментальные данные, из которых следует представление о существовании темной материи. Обсуждаются различные альтернативные интерпретации этих данных, возможные при изменении существующих теоретических представлений. Показано, что для решения вопроса о выборе той или иной альтернативы необходим специальный эксперимент. Такой эксперимент – наблюдение сигнала оптико-метрического резонанса в специально подобранных астрофизических системах, включающих космические мазеры и источники периодических гравитационных волн (пульсары или тесные двойные) – предложен, и приведена его краткая теория. Приведен план соответствующего исследования нашей галактики. Обсуждается, чему будет соответствовать каждый из всех возможных исходов такого исследования, что позволит выбрать наиболее пригодную интерпретацию данных в том числе о темной материи.

1. Эксперименты, интерпретации и проблемы

С точки зрения проблемы существования темной материи в настоящее время наибольшее внимание привлекают два типа экспериментальных результатов, полученных для двух масштабов, – для одной галактики и для галактических кластеров.

Первый тип представляет собой кривые вращения, построенные для спиральных галактик, т.е. зависимости орбитальных скоростей вращения звезд от расстояния до центра галактики [1-4]. Для иллюстрации приведем рисунки из работы [3] (см. Рис.1).

Экспериментальные точки, полученные при измерении орбитальных скоростей v звезд спиральных галактик в зависимости от расстояния R до центра галактик, описываются эмпирической зависимостью следующего вида [5]

$$v^2 = \frac{\beta^* c^2 N^*}{R} + \frac{\gamma^* c^2 N^* R}{2} + \frac{\gamma_0 c^2 R}{2} \tag{1}$$

где c – скорость света, N^* - число звезд в галактике (обычно порядка 10^{11}), β^* для Солнца имеет значение $\beta^* = \frac{M_S G}{c^2} = 1.48 \cdot 10^5$ см (M_S – масса Солнца, G – гравитационная постоянная), γ^* и γ_0 – универсальные параметры со значениями $\gamma^* = 5.42 \cdot 10^{-41}$ см$^{-1}$, $\gamma_0 = 3.06 \cdot 10^{-30}$ см$^{-1}$. На расстояниях порядка размера галактики все три параметра становятся сопоставимыми по величине, в то время, как результат теории Ньютона, а также и решения Шварцшильда уравнений ОТО, предсказывает только спад с расстоянием, соответствующий первому слагаемому в формуле (1). При расчетах учитывался экспоненциальный характер распределение звезд по расстоянию до центра галактики. Следуя историческому примеру, связанному с предсказанием существования, а затем открытием планеты Нептун в Солнечной системе, для того, чтобы обеспечить наблюдаемое движение светящихся звезд, предполагают существование дополнительной материи, взаимодействующей со звездами за счет гравитации. При этом масса этой материи должна быть примерно втрое больше массы видимых звезд, она должна быть сосредоточена периферии галактики, а не в ее центре, и электромагнитных волн она не излучает и не поглощает. Такую материю, частицы которой до сих пор не наблюдались, называют темной.

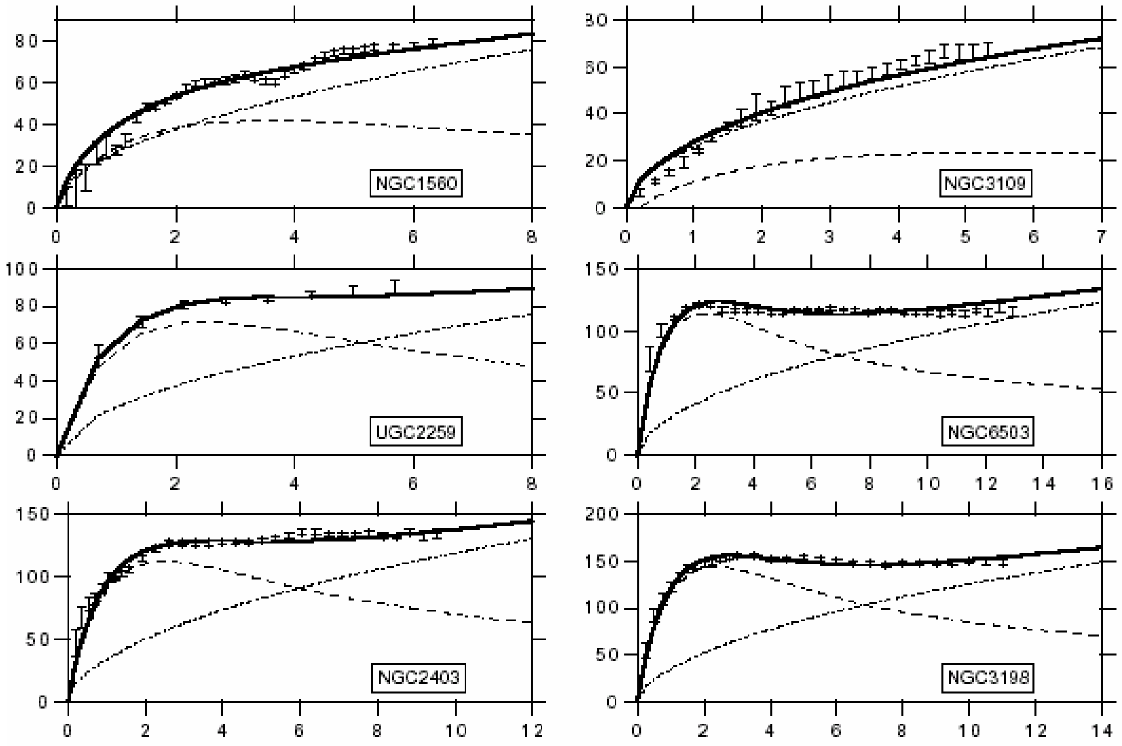

Рис.1. [3] Скорости орбитального движения (в км/с) в зависимости от R/R_0, где R_0 – характерный масштаб для каждой конкретной галактики. Штрихованная линия – Ньтоновский потенциал, (совпадающий с решением Шварцшильда), создаваемый наблюдаемой светящейся материей при учете экспоненциального распределения звезд в пределах галактики.

Обзор теорий, позволяющих сформулировать целый ряд интерпретаций данных о кривых вращения, имеется в [6]. Известны следующие два принципиальных подхода, которые можно использовать для решения возникших проблем в интерпретации экспериментальных данных. Первый из них предполагает добавление чего-либо в существующую теорию. Например,

1. Учет членов четвертого порядка в выражении Эйнштейна-Гильберта для действия [7]
2. Введение нового дополнительного (скалярного) поля [8]
3. Увеличение числа мировых констант, приводящее к модификации закона динамики Ньютона [9] (возможна переформулировка в терминах дополнительного скалярного поля в лагранжиане [10])
4. Увеличение числа измерений [11]

Второй подход предполагает не добавление чего-либо к старой теории, а изменение ее внутренней структуры. Например,

5. Отказ от симметрии метрики по индексам [12, 13]
6. Переход к другой геометрии (например, работа Г.Вейля [14], где была предпринята попытка объединения гравитации и электричества)
7. Использование другого вариационного принципа [15]

Первые три варианта, так или иначе, приводят к представлению о темной материи, и можно предположить, что будет построен соответствующий раздел теории элементарных частиц, а сами они будут обнаружены в эксперименте. В частности, возлагают надежды на мощный ускоритель, который скоро будет использоваться в ЦЕРНе.

Однако, в идеях, связанных с темной материей, имеется и логическое затруднение. С одной стороны, одна лишь светящаяся материя не обеспечивает того гравитационного потенциала, который приводит к наблюдаемым кривым вращения. И для того, чтобы объяснить эти результаты наблюдений, т.е. получить необходимый гравитационный потенциал, необходимо ввести темную материю. А она, как следует из наблюдений, не участвует ни в каком электромагнитном взаимодействии испускания или поглощения излучения. С другой стороны, имеется известный эмпирический закон Тулли-Фишера, устанавливающий соотношение между светимостью (спиральной) галактики и скоростью движения звезд на ее периферии: $v^4 \sim L$. Т.е. скорость движения, обусловленная гравитацией галактики, оказывается все-таки связанной именно со светящейся материей.

Четвертый вариант в настоящее время превратился в развитую теорию струн и бран, пригодную для описания некоторых фундаментальных свойств нашего мира. Неясно, однако, как использовать его для решения рассматриваемой конкретной задачи.

Пятый вариант, по-видимому, представляет собой своеобразную многопараметрическую подгонку.

Геометрия Вейля, упомянутая в шестом варианте, имела вполне конкретное предназначение, но идея той инвариантности, которая в современной теории гравитации называется конформной, оказалась плодотворной. Так, в седьмом варианте в основу вариационного принципа в выражении для действия был положен тензор Вейля. В результате в стационарном сферически симметричном случае (т.е. том, который соответствует теории Ньютона и решению Шварцшильда) был получен гравитационный потенциал, удовлетворяющий не уравнению Пуассона второго порядка, как обычно, а уравнению Пуассона четвертого порядка. А в решение последнего входит не только слагаемое, убывающее, как $1/r$, но и слагаемое, растущее, как r, а также постоянное слагаемое. Это в точности соответствует формуле (1). Здесь, однако, тоже имеются проблемы. Самая естественная из них состоит в том, что полученное из нового вариационного принципа уравнение Эйнштейна в применении к пустому пространству не имеет структуры волнового уравнения. Поэтому в рамках такой новой теории гравитационные волны не могут существовать. Однако результаты наблюдений Р. Халса и Дж. Тэйлора [16] совпадают с теоретическими расчетами, основанными на гипотезе о существовании гравитационных волн, следующей из обычной ОТО, с точностью до 2 %. Кроме того, уже первый из перечисленных вариантов предполагал изменение вариационного принципа, и понятно, что сделать это можно многими различными способами. В принципе, можно мыслить своеобразное «разложение действия в ряд» по различным скалярам с последующим вычислением коэффициентов. Пример аналога такого подхода имеется в термодинамике реального газа при выписывании членов вириального разложения.

Важной и пока еще не учтенной при интерпретации особенностью исследуемой ситуации является существенная (пространственная) анизотропия объекта наблюдения на галактическом масштабе. Спиральные галактики имеют выделенное направление вдоль оси вращения. Развернутое анизотропное Финслерово обобщение теории относительности предпринималось Р. Пименовым [17], а также другими авторами (напр., [18]) более узко. Было, в частности, показано, что три известных эффекта: смещение перигелия Меркурия, отклонение луча света и красное смещение, имеются и в теории на основе Римановой геометрии, и в теории на основе Финслеровой геометрии, а их величины неразличимы в пределах одной и той же точности наблюдений. Эксперимента, способного выявить различия между геометриями, т.е. установить геометрическую структуру физического пространства-времени, до сих пор предложено не было.

Второй тип экспериментальных результатов, по мнению авторов [19], непосредственно подтверждает существование темной материи на метагалактическом масштабе. С помощью оборудования телескопа «Чандра» выполнялось наблюдение уникальной пары кластеров галактик, проходящих друг сквозь друга.

Согласно [20-22], для кластеров галактик, проявляющих свойства гравитационных линз, лишь 1-2% массы приходится на видимые звезды и примерно 5-15% массы приходится на межгалактическую (барионную) среду – плазму, способную испускать рентгеновское излучение. При этом в обычном (стационарном) случае центры масс галактик и плазмы в кластере совпадают. Отметим, что имеется пятикратное расхождение в значениях масс галактических кластеров, определяемых, с одной стороны, традиционными астрофизическими методами, а с другой, – на основании расчетов, следующих из ОТО и связанных с гравитационным линзированием. Это обстоятельство также может учитываться при выдвижении гипотезы темной материи, на которую в межгалактическом случае приходится уже 4/5 всей массы.

Но наблюдения [19], дают еще более интересный результат. На рис.2 представлены результаты измерений, связанных с указанным кластерами галактик. Видно, что распределение масс, полученное на основе измерений гравитационного линзирования (зеленые линии), связано со светящимися галактиками, массы которых считаются меньшими. В то время, как центры масс соответствующих – предположительно более массивных, чем все звезды галактик, – облаков плазмы, полученные на основании измерений интенсивности рентгеновского излучения, смещены и соответствуют областям сжатия газа, т.е. ударным волнам, возникшим при прохождении галактик друг сквозь друга. По мнению авторов отсюда следует, что светящимся галактикам сопутствовала связанная с ними бесстолкновительная темная материя, обеспечившая наблюдаемый эффект линзирования.

Рассматривая эту точку зрения, мы могли бы указать на уже упоминавшееся выше логическое затруднение: темная материя никак не проявляет себя в электромагнитном диапазоне, но взаимодействует со световым лучом, отклоняя его. Но это соответствует метрической природе гравитации, лежащей в основе современных представлений. Сопоставляя данные для кластеров галактик с данными для отдельных спиральных галактик, где выполняется закон Тулли-Фишера, мы видим, что масштабы несопоставимы, и одной и той же причиной в обоих случаях не обойтись.

Объединяя обе группы экспериментальных данных, можно заключить, что, во-первых, существующая проблема интерпретации связана с галактиками, а не со вселенной в целом. Именно к светящимся галактикам привязана – бесстолкновительная – темная материя в соответствии с результатами [19] измерений «Чандры». А во вторых, внимание следует обратить на периферийные области конкретных галактик, где и сосредоточена эта гипотетическая темная материя, согласно экспериментальным кривым вращения. При этом рассматриваемые – спиральные – галактики имеют выраженную анизотропию, а их внутренние области удовлетворительно описываются в рамках существующей теории.

Оба эти обстоятельства указывают на то, что необходим эксперимент, на результаты которого можно было бы опереться, выбирая направление дальнейших фундаментальных исследований, как в теоретическом, так и в экспериментальном плане. Перечислим некоторые вопросы, на которые хотелось бы получить ответ с помощью такого эксперимента:

1. Имеются ли ограничения на выбор скаляра в вариационном принципе?

2. Имеются ли какие-либо преимущества у скаляров определенного типа?
3. Нельзя ли обойтись без привлечения понятия темной материи?
4. Какую роль играет анизотропия для случая спиральных галактик?
5. Проявляет ли себя неоднородность распределения массы в окрестности галактики, следующая из кривых вращения, в каких-либо других наблюдениях?

Заметим, что, поскольку наша галактика – Млечный Путь – также является спиральной, для необходимых наблюдений имеется сравнительно доступный регион.

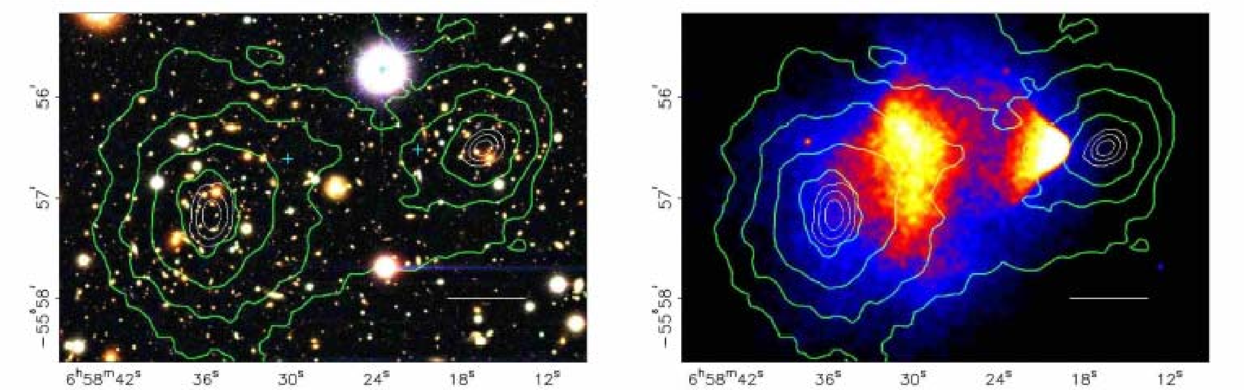

Рис 2. [19]. Зеленые линии на обеих частях характеризуют гравитационные линзы. Слева приведено распределение галактик в кластерах. Справа – рентгеновское излучение межгалактической плазмы, видны области сжатия, соответствующие ударным волнам.

2. Оптико-метрический параметрический резонанс

Основой предполагаемого эксперимента являются наблюдения за сигналами определенных космических мазеров, расположенных в разных местах нашей галактики. Эти сигналы при определенных условиях, приводят к оптико-метрическому параметрическому резонансу (ОМПР). Физический смысл явления ОМПР подробно обсуждался в [23-26], а возможность его приложения к астрофизическим наблюдениям в [27] и [28]. Вкратце его суть в рассматриваемом случае состоит в следующем. Космический мазер представляет собой источник электромагнитной волны (ЭМВ), обладающей высокой степенью монохроматичности. Атомы космического мазера можно считать двухуровневыми. Если мазер насыщенный, то его поле является спектроскопически сильным. Если на такой мазер падает гравитационная волна (ГВ) от периодического источника, расположенного, как показано на рис.3, то расстояние между атомами мазера и радиотелескопом на Земле, а значит, и их скорость будут изменяться с частотой, равной частоте ГВ.

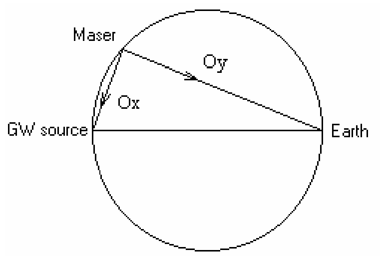

Рис.3

Источником ГВ может служить пульсар или тесная двойная система. Интенсивность излучения мазера характеризуется параметром, имеющим размерность частоты, – так называемой частотой Раби $\alpha_1 = \frac{\mu E}{\hbar}$, где μ – дипольный момент, E – электрическая напряженность, $\hbar$ = $1.05 \cdot 10^{-27}$ эрг·с – постоянная Планка. Воздействие ГВ будет проявляться еще и в перестройке уровней атома [29], и в фазовой модуляции ЭМВ. Можно показать [27], что поправки, возникающие в связи с перестройкой уровней значительно меньше, чем поправки, связанные с воздействием на ЭМВ и на координату атома. Поскольку безразмерная амплитуда

ГВ является малой, фазовую модуляцию можно представить в виде амплитудной, что эквивалентно появлению двух дополнительных слабых ЭМВ, мало отстоящих от основной – собственно ЭМВ мазера. В результате получается система уравнений, состоящая из уравнений Блоха для матрицы плотности, описывающих динамику двухуровневого атома в сильном поле,

$$\begin{aligned}
&\frac{d}{dt}\rho_{22} = -\gamma\rho_{22} + 2i\alpha_1 \cos(\Omega t - k_1 y)(\rho_{21} - \rho_{12}) \\
&[\frac{\partial}{\partial t} + v\frac{\partial}{\partial y}]\rho_{12} = -(\gamma_{12} + i\omega)\rho_{12} - 2i\alpha_1 \cos(\Omega t - k_1 y)(\rho_{22} - \rho_{11}) \\
&\rho_{22} + \rho_{11} = 1
\end{aligned} \tag{2}$$

уравнения эйконала для фазы, описывающего воздействие ГВ на ЭМВ,

$$g^{ik}\frac{\partial\psi}{\partial y^i}\frac{\partial\psi}{\partial y^k} = 0 \tag{3}$$

и уравнения геодезической, позволяющей описать движение атома в поле ГВ

$$\frac{d^2x^i}{ds^2} + \Gamma^i{}_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0 \tag{4}$$

Здесь ρ_{22} , ρ_{11} - населенности уровней; ρ_{12}, ρ_{21} – поляризационные члены; γ, γ_{12} - продольная и поперечная постоянные распада (считая нижний уровень основным, получаем $\gamma_{12} = \gamma/2$); Ω – частота ЭМВ мазера, ω – частота атомного перехода, k_1 – волновой вектор ЭМВ; v – скорость атома вдоль оси Oy; $\gamma << \alpha_1$ - условие сильного поля. Рассматривая сначала обычную ситуацию Римановой геометрии, получаем для метрического тензора в пустом пространстве выражение

$$g^{ik} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 + h\cos\frac{D}{c}(x^0 - x^1) & 0 \\ 0 & 0 & 0 & -1 - h\cos\frac{D}{c}(x^0 - x^1) \end{pmatrix} \tag{5}$$

где h – безразмерная амплитуда ГВ, D – частота ГВ. В этом случае система уравнений может быть решена в частном случае с помощью метода асимптотического разложения по малому параметру ε, определяемому соотношением $\frac{\gamma}{\alpha_1} = \Gamma\varepsilon; \Gamma = O(1); \varepsilon << 1$. Этот случай характеризуется выполнением набора условий, сформулированных в [27-28], соответствующих параметрическому резонансу. Они имеют вид:

- ЭМВ является спектроскопически сильной

$$\frac{\gamma}{\alpha_1} = \Gamma\varepsilon; \Gamma = O(1); \varepsilon << 1 \tag{6}$$

- Амплитудное условие ОМПР, связанное с трихроматическим полем

$$\frac{\alpha_2}{\alpha_1} = \frac{\omega h}{4D} = a\varepsilon; a = O(1); \varepsilon << 1 \tag{7}$$

- Амплитудное условие ОМПР, связанное с периодическим изменением скорости атома

$$\frac{kv_1}{\alpha_1} = \frac{\omega h}{\alpha_1} = \kappa\varepsilon; \kappa = O(1); \varepsilon << 1 \tag{8}$$

- Частотное условие ОМПР

$$(\omega - \Omega + kv_0)^2 + 4\alpha_1{}^2 = D^2 + O(\varepsilon) \quad \Rightarrow D \sim 2\alpha_1 \tag{9}$$

Если условия (6-9) выполнены, то можно получить главный член асимптотического разложения для Im(ρ_{21}) – величины, характеризующей поток энергии рассеянного излучения. На частоте, смещенной на D от центрального пика, он не пропорционален ε^0, т.е. не зависит от амплитуды ГВ, и имеет вид

$$\mathrm{Im}(\rho_{21}) \sim \frac{\alpha_1}{D}\cos 2Dt + O(\varepsilon) \tag{10}$$

Этот результат означает, что сигнал мазера приобретает так называемую нестационарную компоненту на определенной частоте, близкой к частоте атомного перехода. Иными словами, на этой частоте будет происходить периодическое усиление и ослабление сигнала с частотой колебаний атома. Это явление связано с перераспределением энергии по частотам в результате параметрического резонанса. При обычных наблюдениях такой сигнал наблюдаться не будет из-за усреднения по времени. Однако его можно обнаружить либо инструментальным путем при использовании устройства, известного в спектроскопии как детектор ворот, либо путем специальной обработки обычного сигнала. Можно показать, что сигнал такого типа можно уверенно идентифицировать как сигнал ОМПР. Оценки показывают, что его величина сопоставима с величиной пика обычного сигнала, характеризующего взаимодействие атома и резонансного поля, т.е. для его наблюдения не нужно специальное сверхчувствительное оборудование, а значит, существенно уменьшается проблема сигнал/шум.

Если удастся выполнить успешные наблюдения такого сигнала, то будет решена задача детектирования ГВ, а обсуждаемая методика может стать основой гравитационно-волновой астрономии. Однако с точки зрения целей данной работы следует обратиться к анализу всех возможных исходов такого эксперимента по регистрации сигнала ОМПР.

3. Исследование альтернатив, связанных с гипотезой темной материи

Прежде всего отметим, что оценки параметров астрофизических систем, сделанные в [27-28], показывают, что необходимые расстояния между источниками ГВ (пульсарами или двойными) и мазерами имеют межзвездный масштаб. Это означает, что один и тот же источник ГВ – своеобразный маяк – может воздействовать на несколько мазеров. Это позволит исследовать влияние анизотропии пространства-времени, если она имеет место. Действительно, если условия ОМПР для одного и того же источника формально выполнены для мазеров, находящихся в разных направлениях, но сигнал ОМПР наблюдаются не во всех из них, то есть основания думать, что направление играет роль, и можно получить характеристику анизотропии экспериментальным путем. Такой результат наблюдений имел бы и другие далеко идущие последствия [30].

Предлагается выполнить поиск сигнала ОМПР для центральной (ЦОГ) и периферийных (ПОГ) областей нашей галактики. С учетом сказанного в предыдущем абзаце всего будет возможно девять возможных исходов такого исследования, каждый из которых позволит сделать тот или иной вывод в отношении вопросов, сформулированных в конце первого раздела. Перечислим все из них, укажем смысл соответствующих результатов и укажем проблемы, которые и станут направлением фундаментальных исследований.

1. Нет сигнала ОМПР для всех мазеров, соответствующих источникам ГВ в ЦОГ, нет сигнала ОМПР для всех мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: гравитационные волны не существуют (и возможность ГВ-астрономии отсутствует). Это означает, что уравнение Эйнштейна в пустом пространстве не имеет структуры волнового уравнения. Тогда пригодна, например, конформная теория [15], и нет необходимости вводить понятие темной материи и искать соответствующие ей частицы. Для описания физического пространства-времени пригодна Риманова геометрия. **Проблемы**: Узнав, каких скаляров быть в вариационном принципе не должно, мы по-прежнему не знаем, какие следовало бы предпочесть. Становится непонятными, как сама интерпретация результатов [16], так и ее успех на основе представления о ГВ.

2. Есть сигнал ОМПР для всех мазеров, соответствующих источникам ГВ в ЦОГ, нет сигнала ОМПР для всех мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: зависимость от масштаба (возможно, конформная гравитация вне галактики). Риманова геометрия подходит для описания пространства-времени. В ЦОГ пригодна ОТО и возможна ГВ-астрономия.

3. Нет сигнала ОМПР для всех мазеров, соответствующих источникам ГВ в ЦОГ, есть сигнал ОМПР для всех мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: зависимость от масштаба (возможно, конформная гравитация внутри галактики). Риманова геометрия подходит для описания пространства-времени. В ПОГ пригодна ОТО и возможна ГВ-астрономия.

4. Есть сигнал ОМПР для всех мазеров, соответствующих источникам ГВ в ЦОГ, есть сигнал ОМПР для всех мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: Риманова геометрия подходит для описания пространства-времени. ОТО пригодна на масштабах галактики, возможна ГВ-астрономия. **Проблемы**: Проблема темной материи.

Остальные результаты связаны с возможным проявлением анизотропии пространства-времени, когда наблюдаются сигналы ОМПР лишь от некоторых мазеров из тех, что удовлетворяют условиям ОМПР.

5. Есть сигнал ОМПР для *некоторых* мазеров, соответствующих источникам ГВ в ЦОГ, есть сигнал ОМПР для *некоторых* мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: для описания пространства-времени подходит скорее Финслерова геометрия. На масштабах галактики необходимо расширение ОТО, возможна ГВ-астрономия.

Проблемы: Проблема темной материи.

6. Есть сигнал ОМПР для *некоторых* мазеров, соответствующих источникам ГВ в ЦОГ, нет сигнала ОМПР для всех мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: зависимость от масштаба (возможно, конформная гравитация вне галактики). Финслерова геометрия подходит для описания пространства-времени в ЦОГ, где возможна ГВ-астрономия.

7. Нет сигнала ОМПР для всех мазеров, соответствующих источникам ГВ в ЦОГ, есть сигнал ОМПР для *некоторых* мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: зависимость от масштаба (возможно, конформная гравитация внутри галактики). Финслерова геометрия подходит для описания пространства-времени в ПОГ, где возможна ГВ-астрономия.

8. Есть сигнал ОМПР для всех мазеров, соответствующих источникам ГВ в ЦОГ, есть сигнал ОМПР для *некоторых* мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: Риманова геометрия подходит для описания пространства-времени в ЦОГ. Финслерова геометрия подходит для описания пространства-времени в ПОГ. Возможна ГВ-астрономия. **Проблемы**: Проблема темной материи.

9. Есть сигнал ОМПР для *некоторых* мазеров, соответствующих источникам ГВ в ЦОГ, есть сигнал ОМПР для всех мазеров, соответствующих источникам ГВ в ПОГ.

Вывод: Финслерова геометрия подходит для описания пространства-времени в ЦОГ. Риманова геометрия подходит для описания пространства-времени в ПОГ. Возможна ГВ-астрономия. **Проблемы**: Проблема темной материи.

4. Обсуждение

Экспериментальные данные последних лет привели к необходимости введения и обсуждения понятия «темная материя». Одновременно стало ясно, что обычная ОТО не всегда пригодна для описания эффектов в масштабе галактики и выше, на что указывал еще В.А.Фок [31]. Поэтому возможен ряд альтернативных интерпретаций полученных данных наблюдений. Для выбора между ними и последующей разработки теории в соответствующем направлении необходим специальный эксперимент.

Изложенные выше теория эффекта нулевого порядка, пригодного для детектирования ГВ, план наблюдения сигнала ОМПР от космических мазеров и интерпретация возможных результатов позволяют рассчитывать на возможность проверки имеющихся в настоящее время альтернативных гипотез, связанных с представлениями о темной материи. Хотелось бы обратить внимание на информативность результатов предлагаемого плана эксперимента: каждый из возможных исходов позволяет получить ответы на вопросы, сформулированные в конце первого раздела и существенно продвинуться в понимании фундаментальных проблем. В Приложениях приведены примеры астрофизических систем, пригодных для наблюдений.

Приложение 1. Координаты и параметры астрофизических систем, пригодных для детектирования сигнала ОМПР [32-33]

	Название	RaJ	DecJ	d(pc)	D(Hz)
1.Пульсар	J1022+1001	10:22:58.006	+10^{0}01’52.8”	300	60.7794489280
Мазер	AF Leo	11:25:16.4	+15^{0}25’22”	270	
2.Пульсар	B0656+14	06:59:48.134	+14^{0}14’21.5”	290	2.59813685751
Мазер	U ORI	05:52:51.0	+20^{0}10’06.0”	280	
3.Пульсар	J0538+2817	05:38:25.0632	+28^{0}17’9.07”	1770	6.9852763480
Мазер	HH 4	05:37:21.8	+23^{0}49’24.0”	1700	
4.Пульсар	B0031-07	00:34:08.86	-07^{0}21’53.4”	720	1.0605004987
Мазер	U CET	02:31:19.6	-13^{0}22’02.0”	660	
5.Дв. зв.	RXJ0806.3+1527	08:06.3	+15^{0}27’	100	0.00311526
Мазер	RT Vir	13:00:06.1	+05^{0}27’14”	120	

Приложение 2. Координаты и параметры астрофизических систем, пригодных для детектирования ОМПР (проверка наличия анизотропии пространства-времени) [32-33]

	Name	RaJ	DecJ	*d* (pc)	*D* (Hz)
6.Пульсар	J1908+0734	19:08:17.01	+07^{0}34’14.36”	580	4.70914721426
Мазер-1	IRC+10365	18:34:59.0	+10^{0}23’00.0”	500	
Мазер-2	RT AQL	19:35:36.0	+11^{0}36’18.0”	530	
7.Пульсар	J0205+6449	02:05:37.92	+64^{0}49’42.8”	3200	15.223855772
Мазер-1	IRAS00117+6412	00:11:44.6	+64^{0}12’04.0”	3170	
Мазер-2	W3 (1)	02:21:40.8	+61^{0}53’26.0”	3180	

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Ternary relative velocity

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Abstract

Following Minkowski [1908], we consider the relative velocity to be the Minkowski space-like vector (and *not* the Minkowski bivector as it is in the Hestenes theory [Hestenes 1974]). The Lorentz boost entails the relative velocity (as a space-like Minkowski vector) to be ternary: ternary relative velocity is a velocity of a body with respect to an interior observer *as seen* by a preferred exterior-observer. The Lorentz boosts imply non-associative addition of ternary relative velocities. Within Einstein's special relativity theory, each preferred observer (fixed stars, aether, etc), determine the unique relative velocity among each pair of massive bodies. Therefore, the special relativity founded on Minkowski's axiom, that each pair of reference systems *must* be related by Lorentz isometry, needs a preferred reference system in order to have the unique Einstein's relative velocity among each pair of massive bodies. This choice-dependence of relative velocity violate the Relativity Principle that all reference systems must be equivalent.

This astonishing conflict of the Lorentz relativity group, with the Relativity Principle, can be resolved in two alternative ways. Either, abandon the Relativity Principle in favor of a preferred reference system [de Abreu 2004; de Abreu & Guerra 2005, 2006]. Or, within the Relativity Principle, replace the Lorentz relativity group by the relativity *groupoid*, with the choice-free binary relative velocities introduced by Minkowski in 1908.

An axiomatic definition of the kinematical unique binary relative velocity as the choice-free Minkowski space-like vector, leads to the groupoid structure of the set of all deduced relativity transformations (instead of the Lorentz relativity group), with the associative addition of binary relative velocities.

Observer-independence, and the Lorentz-invariance, are distinct concepts. This suggest the possibility of formulating many-body relativistic dynamics without Lorentz/Poincare invariance.

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03.30.+p Special relativity.
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1 Binary relative velocity

We formulate the relativity theory in the ancient Aristotelian spirit, that precede the Galileo and Descartes revolution, by postulating that the concept of relative velocity must be the primary concept of the theory. In Aristotelian spirit, time and space are attributes of relative velocity. Our postulate leads to a groupoid category. This formulation could be contrasted with the sacred understanding of the relativity where the primary concept is the Lorentz (Poincaré) **group** of isometries in the Felix Klein spirit: the group must be primary!

The main objective of the present short note is the concept of a ternary relative velocity. A ternary relative velocity is a velocity of a massive body with respect to an interior observer *as seen* by a preferred exterior-observer. It will be convenient to present the ternary relative velocity, in terms of the binary velocity (of a medium), relative to an observer, used by Minkowski in 1908. Following Minkowski, we identify a reference system (a massive observer) with a normalized time-like Minkowski vector field, $P^2 = -1$.

1.1 Definition (Observers and subjects-observed)**.** Each non-zero space-like Minkowski vector, $\mathbf{w} \neq \mathbf{0}$, possess the following pair of two-dimensional sub-manifolds (hyperbolises) of potential observers of $\mathbf{w}$, and of massive subjects that are potential possessors of $\mathbf{w}$,

$$O_{\mathbf{w}} \equiv \{A^2 = -1, A \cdot \mathbf{w} = 0\}, \qquad S_{\mathbf{w}} \equiv \{B^2 = -1, B \cdot \mathbf{w} = \mathbf{w}^2\}. \tag{1.1}$$

The notation, $A \in O_{\mathbf{w}}$, is equivalent to the phrase that an observer A is observing a massive body, $\sqrt{\mathbf{w}^2+1}\,A + \mathbf{w} \in S_{\mathbf{w}}$, possessing a space-like Minkowski vector $\mathbf{w}$.

Analogously, $B \in S_{\mathbf{w}}$, means that a massive body B possess a space-like Minkowski vector $\mathbf{w}$, relative to an observer, $\frac{B-\mathbf{w}}{\sqrt{\mathbf{w}^2+1}} \in O_{\mathbf{w}}$.

In what follows, for a bounded space-like Minkowski vector, such that, $\mathbf{v}^2 < c^2$, the Heaviside - FitzGerald - Lorentz scalar factor is denoted by,

$$\mathbf{v} \longmapsto \gamma_{\mathbf{v}} \equiv \frac{1}{\sqrt{1-\frac{\mathbf{v}^2}{c^2}}}, \qquad \mathbf{w} \equiv \gamma_{\mathbf{v}} \frac{\mathbf{v}}{c} \quad \Longleftrightarrow \quad \frac{\mathbf{v}}{c} \equiv \frac{\mathbf{w}}{\sqrt{1+\mathbf{w}^2}}. \tag{1.2}$$

Sometimes, for simplicity of formulas, the scalar magnitude of the light velocity is set, $c^2 = 1$.

1.2 Axiom (Binary relative velocity)**.** Let $A \in O_{\mathbf{u}}$, and $\mathbf{u}^2 < c^2$. Then, and *only* then, $\exists!$ a massive body, $B = b_{\mathbf{u}} A \equiv \gamma_{\mathbf{u}}(A + \frac{\mathbf{u}}{c}) \in S_{\mathbf{u}}$, moving with a velocity $\mathbf{u}$ *relative* to A. This imply, $\gamma_{\mathbf{u}} = -A \cdot B$. The space-like relative velocity $\mathbf{u}$ is said to be *binary*,

$$\frac{\mathbf{u}}{c} \equiv \frac{\varpi(A,B)}{c} = \frac{B}{-B \cdot A} - A \qquad \Longrightarrow \qquad 1 - \left(\frac{\mathbf{u}}{c}\right)^2 = \frac{1}{(A \cdot B)^2} = \frac{1}{\gamma_{\mathbf{u}}^2} \tag{1.3}$$

2 Ternary relative velocity

Minkowski identify a reference system with a time-like vector field, and defined the special relativity by means of the following single axiom.

2.1 Definition (Special relativity)**.** Any two reference systems must be connected by the Lorentz isometry, *i.e.*, by the isometry acting on *all* vectors, including **not** time-like vectors.

The Minkowski Definition-axiom 2.1 does not need explicitly the concept of relative velocity, and leads to the cornerstone of XX century physics: Lorentz group-covariance.

The vector space of the Grassmann bi-vectors inside of Clifford algebra, is the Lie algebra of the Lie group of isometries. Each Minkowski bi-vector, $P \wedge Q$, generate an isometry

$$P \wedge Q \quad \hookrightarrow \quad L_{P \wedge Q} \in O(1,3).$$

2.2 Definition (Ternary relative velocity)**.** Let $\{P, A, B\}$ be a three-body massive system given by a set of three time-like Minkowski vectors, and let $P \in O_{\mathbf{v}}$, be an actual observer of a bounded space-like velocity $\mathbf{v}$, $\mathbf{v}^2 < c^2$. The Minkowski velocity vector $\mathbf{v}$ of a Bob B, relative to Alice A, as seen/measured by a preferred observed P, is said to be *isometric*, or *ternary*, or *the Einstein velocity*, if it is defined in terms of the isometric Lorentz boost as follows

$$\overline{\mathbf{v}} \equiv \gamma_{\mathbf{v}} \mathbf{v}/c, \qquad L_{P \wedge \overline{\mathbf{v}}} \in O(1,3), \qquad \boxed{L_{P \wedge \overline{\mathbf{v}}}\, A = B} \tag{2.1}$$

The above definition is motivated by the following theorem.

2.3 Theorem (Isometry-link problem (Oziewicz 2005, 2006))**.** *For the massive three-body system given in terms of the three time-like vectors* $\{P, A, B\}$, *the Lorentz-boost-link equation for unknown space-like Minkowski vector* $\mathbf{w}$, $L_{P \wedge \mathbf{w}}\, A = B$, *has the unique solution,* $\overline{\mathbf{v}} = \mathbf{w}(P, A, B)$. *A ternary relative velocity, parameterizing the Lorentz boost* (2.1)*, looks like a kind of subtraction of absolute/binary velocities,*

$$\mathbf{v}(P, A, B) \;=\; P \cdot (A + B)\, \frac{(P \cdot B)\varpi(P, B) - (P \cdot A)\varpi(P, A)}{(P \cdot A)^2 + (P \cdot B)^2 - 1 - A \cdot B}, \tag{2.2}$$

$$\mathbf{v}(P, A, B) = -\mathbf{v}(P, B, A) \quad \simeq \quad i_{gP}\{P \wedge (B - A)\}, \tag{2.3}$$

$$\gamma_{ternary} = \frac{(P \cdot A)^2 + (P \cdot B)^2 - A \cdot B - 1}{2(P \cdot A)(P \cdot B) + A \cdot B + 1} \quad \neq \quad -A \cdot B = \gamma_{binary}. \tag{2.4}$$

2.4 Corollary. Consider co-planar system of massive bodies, $P \wedge A \wedge B = 0$. In this particular case a ternary relative velocity (2.2) is reduced to the expression presented by de Abreu & Guerra [2005, page 74],

$$\mathbf{v}(P, A, B)|_{P \wedge A \wedge B = 0} \;=\; \frac{\varpi(P, B) - \varpi(P, A)}{1 - \varpi(P, B) \cdot \varpi(P, A)/c^2}. \tag{2.5}$$

2.5 Theorem. *The scalar magnitudes of the binary and ternary relative velocities,* (1.3)-(2.2)-(2.4)*, coincide,* $\gamma(binary) = \gamma(ternary)$*, if and only if the three-body system is co-planar,*

$$\gamma_{\mathbf{v}(P,A,B)} = -A \cdot B \qquad \Longleftrightarrow \qquad P \wedge A \wedge B = 0. \tag{2.6}$$

The Minkowski Definition-axiom 2.1 does not need explicitly the concept of relative velocity, and leads to choice-dependent, P-dependent, ternary relative velocity among Alice and Bob. Each reference system P (that could be interpreted as the physical fixed stars, aether, etc), gives the unique Einstein's reciprocal relative velocity among each pair of massive bodies, expression (2.2),

$$\{A, B\} \quad \xrightarrow{\text{preferred } P} \quad \mathbf{v}(P, A, B). \tag{2.7}$$

Definition 2.2 and Theorem 2.3 tells that the Lorentz-boost is *not* unique. It is true that all textbooks of special relativity, define the Lorentz boost as the 'unique' basis-dependent matrix, see for example in [Ungar 1988],

$$\text{Lorentz boost} \quad \equiv \quad \begin{pmatrix} \gamma & -(v/c)\gamma & 0 & 0 \\ -(v/c)\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.8}$$

When a priori accepting the above matrix definition (2.8), it is hard to imagine the existence of the another boost, because the above matrix is fixed by velocity parameter, and there is no option for alternative boost. We like to point that our definition of basis-free P-dependent Lorentz boost (2.1), is reduced to the above 'unique-matrix', in the particular basis, when, $P \simeq (1,0,0,0)$, and $\overline{\mathbf{v}} \simeq (0,-v/c,0,0)$. This explain why someone insists that the 'Lorentz boost is unique!'.

Contrary to the matrix-statement, (2.8), a Lorentz boost-link from A to B is not unique (2.1). Lorentz boost is choice-dependent, depends on the choice of the preferred observer P. Equivalently, the Lorentz boost depends on the choice of the non-unique embedding of the rotation group as the non-normal sub-group of the Lorentz group, $O(3) \hookrightarrow O(1,3)$. Each Lorentz boost is given by bi-vector, therefore can not be parameterized by a space-like velocity vector alone. The Minkowski Definition-axiom 2.1 means, that when a preferred vector P is *not* chosen, there is a bunch of Lorentz-link transformations from A to B, generated by many different bi-vectors,

$$P \wedge \mathbf{w} \quad \neq \quad P' \wedge \mathbf{w}' \quad \neq \quad \ldots,$$

but there is **no** relative velocity among massive bodies. To have exactly one Einstein's velocity of Bob relative to Alice, we need to chose some one and only one reference system to be preferred. This choice-dependence of Einstein's relative velocity violate the relativity principle stating that *all* reference systems must be equivalent. Within the Minkowski (Lorentz-relativity-group)-Definition-axiom 2.1, an equivalence of all reference systems is not possible: the different choices of a preferred system P in Definition 2.2, lead to distinct Einstein's velocities of Bob relative to Alice. Each observer $P \in O_{\mathbf{w}}$, gives rise to his own isometric Lorentz P-boost $L_{P\wedge\mathbf{w}}$, and therefore the Lorentz boost is not unique.

We distinguish conceptually the *relativity*-group (permuting the reference systems), from the concept of a symmetry-group. The Lorentz group is a symmetry-group of the metric of the empty space-time, it is a group of isometries. The Lorentz group is also a symmetry-*sub*group of conformal symmetry of the Maxwell equations. However, a priori, a set of all relativity transformations among massive reference systems need *not* to be a group, and need not to be a symmetry-group of some other mathematical structure. We remind that, for example, the light-like vectors do not represent reference systems, therefore, a priori, they do *not* need to be in the domain of relativity transformations. The present paper deals exclusively with the Lorentz group as the relativity-group.

In fact, the main problem of relativity theory is *not* about the special relativity identified with the Minkowski axiom 2.1, it is *not* about the interpretation of the Relativity Principle, it is *not* about the clock's synchronization, it is *not* so much about time measurements, it is *not* about one-way & two-way light velocity. The main problem is the coordinate-free definition of the concept of the relative **velocity**. Textbooks devote a lot of attention to the important dynamical concepts of acceleration and force, however, following Galileo, the kinematical velocity is defined in coordinate-dependent way, such that the eventual conceptual distinction among absolute and relative velocities, among non-relativistic and relativistic velocities, is obscure in the expression like '$x = vt$'. How such 'v' depends on the choice of the reference system? how depends on the choice of mathematical coordinates? What are abstract properties of the set of all relative velocities, including the law of addition-composition of relative velocities?

The main question is about the precise axiomatic coordinate-free definition of the concept of relative velocity. The distinct theoretical conceptual definitions of relative velocity must proceeds to the experimental measurements. How to measure, without understanding what concept are you going to measure? Textbooks repeat 'a passenger sitting in a moving train is at rest in relation to the train, but it is in a motion in relation to the ground'. How one can see this in a coordinate expression '$x = vt$'? Where in these three symbols, $\{x, t, v\}$, there is a train, passenger, ground? We need to have something like at least two-variable expression

$$\mathbf{v}(S, \text{passenger}) = \begin{cases} \mathbf{0}_{\text{train}} & \text{if } S \text{ is a train,} \\ \neq \mathbf{0} & \text{if } S \text{ is a ground,} \end{cases} \qquad \mathbf{v}(\text{train}, \text{train}) \equiv \mathbf{0}_{\text{train}}. \tag{2.9}$$

From this main point of view of the precise definitions of the concept of a relative velocity, Definition 2.2, is the very precise mathematical definition: the Einstein's ternary relative velocity, expression (2.2), is coordinate-free, and basis-free. From this definition one can deduce **all** properties of such Einstein's ternary velocities that we are listening in Section 5 (including their non-associative addition), and one can easily deduce the coordinate expression for any system of coordinates (we left this to interested students). Reader do not need to like the definition of a ternary relative velocity. Each reader could invent his own different definition of the relative velocity. However, I hope that all readers agree that Definition 2.2 is the precise mathematical definition. We do not need at this moment enter to the experimental problems of the measurements of ternary velocities, nor to the question does the Nature like or dislike the mathematical definition of ternary relative velocity?

2.6 Main conclusion. If we believe (or if we postulate) that:
(i) There must be one and only one relative velocity among each pair of massive bodies.
(ii) Each pair of massive bodies must be related by an isometry, Definition 2.1.
Then, a ternary velocity needs the choice of the preferred reference system.

Definition 2.2 say: yes, there is one and only one relative velocity among each pair of massive bodies, *provided* that the preferred reference system (absolute space) was chosen. Therefore, the concept of the preferred reference system, an aether, is built in the Einstein velocities.

- Preferred reference system is attractive for explaining the non-symmetrical ageing of twins (Herbert Dingle, Rodgrigo de Abreu & Vasco Guerra, Subhash Kak).
- Preferred reference system is meaning-less for believers in Lorentz-covariance as the corner**stone** of Physics (the Lorentz relativity-group is Sacred!).
- Preferred reference system is a fault for believers in the Relativity Principle (no need for the choice of a preferred reference system in order to have a relative velocity).

Nevertheless one can accept special relativity with Lorentz-relativity-group axiom, Definition 2.1, admitting that in this case we must violate the Relativity Principle in order to have the unique Einstein's relative velocity. The uniqueness of relative velocity needs the choice of one reference system to be preferred. Therefore, the special relativity with Lorentz relativity-group, not only is perfectly consistent with a special system of reference (alias preferred system, alias Einstein's lost frame, alias aether, fixed stars, etc), as concluded independently in [de Abreu 2004; de Abreu & Guerra 2005; Guerra 2006; Kak], but, such special Lorentz-group-relativity does **not** exist at all, without preferred reference system. No choice of a preferred vector P in Definition 2.2, no velocity $\mathbf{v}$ of Bob relative to Alice.

We see the astonishing conflict! The special relativity with Lorentz-relativity-group axiom necessarily contradicts to the Relativity Principle. Not all reference systems can be considered any more as equivalent! Instead, one reference system necessarily must be chosen in order that each pair of massive bodies has the unique relative velocity.

3 Rodrigo de Abreu: restricted Relativity Principle

De Abreu proposed to abandon the Relativity Principle in favor of 'restricted Relativity Principle' that allows the absolute space with a preferred reference system, referred to as 'the Einstein's lost frame'. This idea was future developed in [De Abreu 2002, 2004; De Abreu & Guerra 2005; Guerra & de Abreu 2006]. The velocity relative to the preferred reference system is said to be the *absolute* velocity, and a velocity relative to non-preferred system is said to be the Einstein velocity [De Abreu 2004]. The starting point of De Abreu (and jointly with Guerra) is the observation that the Einstein synchronization of clocks can be made in one and only one reference system. Analysis of the clock synchronization (related to one-way versus two-ways light velocity) leads Authors to consider the abandoning of the Relativity Principle (that all reference systems are equivalent).

3.1 Axiom (De Abreu)**.** The one-way light-velocity is source-free in **one and only one** reference system. This reference system is said to be the Einstein lost-frame.

Definition 2.2 of the ternary relative velocity, implies that *all* these velocities are reciprocal, (2.3). If $\mathbf{v}$ is the Einstein velocity of Bob relative to Alice, then the Einstein velocity of Alice relative to Bob is $-\mathbf{v}$, for *each* choice of the preferred reference system P. However, this is not the case in special relativity theory in [de Abreu & Guerra 2005]. De Abreu & Guerra in their monograph introduced three different concepts of velocities. The *absolute* velocity, denoted here by $\mathbf{v}_{AG}$, is a velocity of reference system, say Bob S', relative to the preferred absolute space at rest (say Aether S), for the chosen event P [Abreu & Guerra 2005, pages 41-42, Figure 3.10],

$$\mathbf{v}_{AG}(P,S,S') \equiv \mathbf{v}_{AG}(\text{event}, \text{Aether}, \text{Bob}) = \frac{\mathbf{x}}{t} - \left(\frac{t'}{t}\right)^2 \frac{\mathbf{x}'}{t'}. \tag{3.1}$$

This *absolute velocity* is defined in terms of the coordinates of an event P, therefore, it is the 'absolute velocity' of Bob 'as seen by an event P'. It is not obvious that the choice of a spectator-event P must be irrelevant for Definition (3.1) of absolute velocity. One can guess that 'preferred spectator-event P' (denoted accidently on Figure 3.10 on page 41 by the first letter of 'preferred'), is probably assumed to be in the rest relative to the Aether (or it is in the rest relative to the Bob?). What would be if the event P will be chosen to be neither in the rest relative to Aether, nor in the rest relative to the Bob?

De Abreu and Guerra define the *Einstein speed* that is the reciprocal Einstein velocity among two frames [Abreu & Guerra 2005, page 74], and this concept coincide with our ternary relative velocity for co-planar systems, Corollary 2.4. Moreover, the Authors have also the 'Rodrigo' non-reciprocal *relative speed* defined on page 44. What seems to me to be the most essential peculiarity of the Abreu & Guerra's special relativity theory, that their absolute-velocity $\mathbf{v}_{AG}$ (3.1), is *not* reciprocal, and the velocity of the Aether as measured by Bob has much higher scalar-magnitude when compared with the absolute velocity of the Bob as measured by the Aether, [Abreu & Guerra 2005, page 42], viz.

$$\mathbf{v}_{AG}(\text{event}, \text{Bob}, \text{Aether}) = -(\gamma_{\mathbf{v}})^2\, \mathbf{v}_{AG}(\text{event}, \text{Aether}, \text{Bob}). \tag{3.2}$$

4 Relativity-groupoid as alternative

Does exist some alternative theory that is completely compatible with the Relativity Principle? The alternative philosophy is to keep Relativity Principle, however, *replace* the Lorentz relativity-group (with choice-dependent Einstein's relative velocity), by a relativity groupoid (with choice-free axiomatic binary relative velocity). The alternative is to consider the relative velocity among massive bodies as the primary concept, and then derive/deduce/*define* the transformation among reference systems in terms of this primordial, given a priori, binary relative velocity.

The Einstein special relativity theory, consider the isometry Lorentz relativity transformation, $L \in O(1,3)$, as the primordial concept, and the relative velocity as the derived concept,

$$\left.\begin{array}{c}\text{group of isometries} \equiv \\ \text{relativity transformations}\end{array}\right\} \quad \Longrightarrow \quad \text{ternary relative velocities}$$

The possible alternative is to axiomatize the concept of the unique binary kinematical relative velocity as the primordial concept, and *derive* the relativity transformation among reference systems in terms of this given choice-free binary velocity,

$$\text{binary relative velocities} \quad \Longrightarrow \quad \left\{\begin{array}{c}\text{relativity transformations} \\ \text{groupoid category}\end{array}\right.$$

Then, could we have a hope that such *derived* set of all relativity transformations, parameterized by the choice-free axiomatized relative velocities, will coincide with the *group* of Lorentz isometries (parameterized by the choice-dependent Einstein's velocities)?

The binary relative velocity-morphism is the choice-free, Definition 1.1, and Axiom 1.2. We claim that such axiomatic binary velocity can *not* parameterize the isometric Lorentz transformation. The one reason, among other, is that the domain of the action parameterized by the choice-free binary velocity, $\frac{\mathbf{v}}{c} \equiv \frac{\mathbf{w}}{\sqrt{1+\mathbf{w}^2}}$, is restricted to the two-dimensional sub-manifold of Minkowski vector fields, $O_{\mathbf{w}} \xrightarrow{\mathbf{w}} S_{\mathbf{w}}$, Definition 1.1. The set of all relativity transformations parameterized by binary relative velocities has the structure of a groupoid category (that is not a group), and the addition of binary velocities is associative.

4.1 Definition (Groupoid category)**.** A category is said to be a *groupoid category*, if and only if every morphism has a two-sided inverse. In particular a *group* is a groupoid one-object-category, with just one object, hence with universal unique neutral element-morphism. A groupoid category is said to be *connected* if there is an arrow joining any two of its objects.

We propose to formulate the physics of relativity in terms of the groupoid category of observers, keeping strictly the most democratic interpretation of the Relativity Principle that *all* reference systems are equivalent. The groupoid relativity starts with the axiomatic definition of the binary relative velocities-morphisms, that are choice-free, Axiom 1.2, and conclude that these relative-velocities can not parameterize the isometric Lorentz transformations.

This formulation is perfectly consistent with the principle of relativity, because all reference systems are equivalent, and there is no need for the choice of the preferred reference system. In particular, within the groupoid relativity, the inverse of relative velocity is not reciprocal, however this groupoid non-reciprocity is different from non-reciprocity in Abreu-Guerra theory (3.2),

$$\mathbf{v}^{-1} \cdot \mathbf{v} = \begin{cases} -\ \gamma_{\mathbf{v}}\ \mathbf{v}^2, & |\mathbf{v}^{-1}| = |\mathbf{v}| \quad \text{in groupoid relativity,} \\ -(\gamma_{\mathbf{v}})^2 \mathbf{v}^2, & |\mathbf{v}^{-1}| \neq |\mathbf{v}| \quad \text{in Abreu \& Guerra relativity.} \end{cases} \tag{4.1}$$

Within the groupoid relativity, the inverse is an involutiv operation, $(\mathbf{v}^{-1})^{-1} \equiv \mathbf{v}$, whereas this is not the case within Abreu & Guerra theory.

Zaripov consider anisotropic Finsler spacetime, and proved that the Finslerian inverse operation (of relative velocity) is non-reciprocal, $\mathbf{v}^{-1} \neq -\mathbf{v}$, and, in Zaripov's theory, it is also non-involutive, $(\mathbf{v}^{-1})^{-1} \neq \mathbf{v}$, [Zaripov 2006].

The associative $\circ$-addition of binary relative velocities is a trivial corollary that follows from two related concepts: the binary relative velocity is a categorical morphism, and a derived groupoidal boost can not be an isometry, Axiom 1.2. In the consequence, a bivector, $(\mathbf{u}^{-1}) \wedge \mathbf{u} \neq 0$, does not vanish.

The relativity-groupoid predicts exactly the same time-dilation as the relativity-Lorentz-group, however no material rod contraction. One can test experimentally the Lorentz-group relativity versus groupoid-relativity within moving media electrodynamics, and we refer to related proposals by Gladyshev et al. [2000, 2005]. Consider two reference systems given by Minkowski time-like vectors, Alice and Bob, and let $\mathbf{u}$ be a space-like Minkowski vector of binary velocity of Bob relative to Alice, *i.e.* the relative velocity as measured by Alice, $\mathbf{u} \cdot A = 0$. Let, $\mathbf{E}'$ and $\mathbf{B}'$, be electric and magnetic fields measured by Bob $= \gamma(A + \mathbf{u}/c)$. Let, $\mathbf{E}$, and $\mathbf{B}$, be the electric and magnetic fields as measured by Alice. Then,

$$\mathbf{u} \times \mathbf{B} = \mathbf{0} \quad \Longrightarrow \quad \mathbf{E}' \cdot \mathbf{E} - \gamma\, \mathbf{E}^2 = \begin{cases} -\frac{\gamma^2}{\gamma+1} \left(\frac{\mathbf{u}}{c} \cdot \mathbf{E}\right)^2 & \text{in Lorentz-group relativity,} \\ 0 & \text{in groupoid relativity.} \end{cases} \tag{4.2}$$

Some other experimental predictions of the groupoid relativity, versus the predictions of the Lorentz relativity-group, are discussed in [Oziewicz 2006, 2007].

5 Addition of ternary relative velocities

Table 1 shows the addition of relative velocities parameterizing the isometric Lorentz relativity-transformations, [Fock 1955, 1961 §16, formula (16.08), 1964].

5.1 $\oplus$-inverse

The $\oplus$-inverse is the reciprocal velocity, $\mathbf{u}^{-1} = -\mathbf{u}$, as in the case of absolute time:

$$\begin{gathered} \mathbf{v} \oplus \mathbf{u} = \mathbf{0} \quad \Longleftrightarrow \quad \mathbf{v} + \mathbf{u} = \mathbf{0}, \\ \text{that is:} \quad \oplus -\text{inverse} \; = \; (+) - \text{inverse.} \end{gathered} \tag{5.1}$$

Table 1: Nonassociative $\oplus$-addition of reciprocal ternary relative velocities, no Figure. The addition of orthogonal relative velocities, $\mathbf{v}\cdot\mathbf{u}^{-1}=0$, looks ‘the same’ for binary and ternary relative velocities [Oziewicz 2005].

$$
\begin{aligned}
&P\cdot\mathbf{u}=0,\quad P\cdot\mathbf{v}=0,\quad \mathbf{u}^{-1}=-\mathbf{u},\quad \mathbf{v}^{-1}=-\mathbf{v} \qquad \Longrightarrow\\
&\left(1-\tfrac{\mathbf{v}\cdot\mathbf{u}^{-1}}{c^2}\right)\mathbf{v}\oplus_P\mathbf{u} = \mathbf{u}+\tfrac{\mathbf{v}}{\gamma_{\mathbf{u}}}-\tfrac{\gamma_{\mathbf{u}}}{(\gamma_{\mathbf{u}}+1)}\tfrac{(\mathbf{v}\cdot\mathbf{u}^{-1})}{c^2}\,\mathbf{u}\\
&\qquad\qquad\qquad\quad\;\; = \mathbf{u}\;+\;\mathbf{v}\;+\;\tfrac{\gamma_{\mathbf{u}}}{\gamma_{\mathbf{u}}+1}\,\mathbf{u}^{-1}\cdot\tfrac{(\mathbf{v}\wedge\mathbf{u}^{-1})}{c^2}
\end{aligned}
$$

5.2 Mocanu paradox

The coincidence of the Galilean $(+)$-inverse and the Einstein $\oplus$-inverse, gives the Mocanu paradox [Mocanu 1986]: $\oplus$-inverse is $\oplus$-automorphism,

$$(\mathbf{v}\oplus\mathbf{u})^{-1}=(\mathbf{v}^{-1})\oplus(\mathbf{u}^{-1})\quad\neq\quad(\mathbf{u}^{-1})\oplus(\mathbf{v}^{-1}).\tag{5.2}$$

Whereas one would expect that the unary inverse operation is an *anti*-automorphism,

$$(f\circ g)^{-1}=(g^{-1})\circ(f^{-1}).\tag{5.3}$$

Figure 1: Addition of ternary relative velocities: Mocanu paradox

$$
\begin{tikzcd}
A \arrow[r, bend left, "\mathbf{u}"] \arrow[rr, bend left=60, "\mathbf{v}\oplus\mathbf{u}"] & B \arrow[l, bend left, "-\mathbf{u}"] \arrow[r, bend left, "\mathbf{v}"] & C \arrow[l, bend left, "-\mathbf{v}"] \arrow[ll, bend left=60, "(-\mathbf{u})\oplus(-\mathbf{v})\neq-\mathbf{v}\oplus\mathbf{u}"]
\end{tikzcd}
$$

5.3 Ungar’s discovery: nonassociativity

In this subsection we put $c^2=1$. In 1988 Ungar discovered that the $\oplus$-addition is non-associative [Ungar 1988, p. 71],

$$
\begin{aligned}
\frac{\gamma_{\mathbf{w}\oplus(\mathbf{v}\oplus\mathbf{u})}}{\gamma_{\mathbf{w}}\gamma_{\mathbf{v}}\gamma_{\mathbf{u}}}&=1+\mathbf{u}\cdot\mathbf{v}+\mathbf{v}\cdot\mathbf{w}+\mathbf{w}\cdot\mathbf{u}+\frac{\gamma_{\mathbf{u}}}{\gamma_{\mathbf{u}}+1}(\mathbf{w}\wedge\mathbf{u})\cdot(\mathbf{u}\wedge\mathbf{v}),\\
\frac{\gamma_{(\mathbf{w}\oplus\mathbf{v})\oplus\mathbf{u}}}{\gamma_{\mathbf{w}}\gamma_{\mathbf{v}}\gamma_{\mathbf{u}}}&=1+\mathbf{u}\cdot\mathbf{v}+\mathbf{v}\cdot\mathbf{w}+\mathbf{w}\cdot\mathbf{u}+\frac{\gamma_{\mathbf{u}}}{\gamma_{\mathbf{u}}+1}(\mathbf{w}\wedge\mathbf{v})\cdot(\mathbf{v}\wedge\mathbf{u}),\\
\{\mathbf{w}\oplus(\mathbf{v}\oplus\mathbf{u})\}\;\wedge\;\{(\mathbf{w}\oplus\mathbf{v})\oplus\mathbf{u}\}&=A(\mathbf{w}\wedge\mathbf{v})+B(\mathbf{v}\wedge\mathbf{u})+C(\mathbf{u}\wedge\mathbf{w})\;\neq\;0.
\end{aligned}\tag{5.4}
$$

Thus, not only are these two resulting relative velocities, $\mathbf{w}\oplus(\mathbf{v}\oplus\mathbf{u})$, and $(\mathbf{w}\oplus\mathbf{v})\oplus\mathbf{u}$, non collinear (5.4), but also their differ in their scalar *magnitude*.

Many opponents disagree with non-associative law of addition of the Einstein relative velocities. Opponents claim that if a group G is associative (think about composition of matrices), then the Lobachevski factor space G/H must be necessarily an associative group, even if H is not a normal subgroup, e.g. [Barrett 2006; Daniel Sudarsky, UNAM, ICN, 2003].

6 Addition of binary relative velocities

Consider a system of three bodies $\{A, B, C\}$, Figure 2. Clare C is moving with a binary velocity $\mathbf{v}$ relative to Bob B, and Bob B is moving with a binary velocity $\mathbf{u}$ relative to Alice A. What is the binary velocity of Clare C relative to Alice A?

Figure 2: Three body, $\{A, B, C\}$, in relative motions, $B \cdot \mathbf{u}^{-1} = 0$

$$\begin{tikzcd}
\text{Alice} \arrow[r, "\mathbf{u}"] \arrow[rr, bend right, "\mathbf{v}\circ\mathbf{u}"'] & \text{Bob} \arrow[l, bend right, "\mathbf{u}^{-1}"'] \arrow[r, "\mathbf{v}"] & \text{Clare}
\end{tikzcd}$$

Table 2: Associative $\circ$-addition of binary relative velocities, Figure 2. The addition of orthogonal relative velocities, $\mathbf{v} \cdot \mathbf{u}^{-1} = 0$, looks 'the same' for binary and ternary relative velocities [Oziewicz 2005].

$$A \cdot \mathbf{u} = 0, \quad B \cdot \mathbf{v} = 0, \quad B \cdot \mathbf{u}^{-1} = 0, \quad \mathbf{u}^{-1} \neq -\mathbf{u}, \quad \mathbf{v}^{-1} \neq -\mathbf{v} \qquad \Longrightarrow$$

$$\left(1 - \tfrac{\mathbf{v}\cdot\mathbf{u}^{-1}}{c^2}\right) \mathbf{v} \circ \mathbf{u} \quad = \quad \mathbf{u} + \frac{\mathbf{v}}{\gamma_{\mathbf{u}}} \quad + \quad \frac{\mathbf{v} \cdot \mathbf{u}^{-1}}{c} A.$$

$$(\gamma_{\mathbf{u}}^2 - 1)\left(1 - \tfrac{\mathbf{v}\cdot\mathbf{u}^{-1}}{c^2}\right) \mathbf{v} \circ \mathbf{u} = \left\{\gamma_{\mathbf{u}}^2\left(1 - \tfrac{\mathbf{v}\cdot\mathbf{u}^{-1}}{c^2}\right) - 1\right\} \mathbf{u} - \gamma_{\mathbf{u}}\, \mathbf{u}^{-1} \cdot \frac{(\mathbf{v} \wedge \mathbf{u}^{-1})}{c^2}$$

7 Final remarks

7.1 Group structure was postulated in 1905. An analysis of the derivation of the Lorentz group as the group of transformations relating observers, and the velocity $\oplus$-addition in [Einstein 1905], reveals that the inverse-velocity property (5.1) was the most important tacit independent assumption used most effectively as an axiom [Einstein 1905 p. 901], and is not related to the verbal Einstein's two postulates [1905 pp. 891-892]. The reciprocal-velocity axiom (5.1) tells that every observer measuring some velocity can measure also inverse of this velocity. It is however true that Einstein's reciprocal-velocity axiom (5.1) is absolutely necessary for the derivation of the Lorentz *group* as the one that relates two observers:

$$\text{Lorentz group relating observers} \qquad \Longrightarrow \qquad \{\oplus\text{-inverse} \quad = \quad (+)\text{-inverse}\}.$$

7.2 Thomas rotation. The Thomas rotation (Thomas in 1926, see [Ungar 1988]), means non-transitivity of the parallelism of the spatial frames. The addition of Einstein relative velocities, being non-associative, is not a group operation. It is a *loop* operation [Ungar 1988]. Non-associative $\oplus$-addition is counterintuitive and paradoxical: for a system of four or more bodies the $\oplus$-addition of three non-collinear relative velocities gives the **two** distinct velocities between two bodies. There have been attempts [Ungar 1988] to explain the non-associativity, and also Mocanu paradox, as the Thomas rotation, *i.e.* as non-transitivity of the parallelism of the spatial frames. We consider this attempt not satisfactory. Jackson [1962] argued that the Thomas rotation is *necessary* in order to explain factor '2' in the doublet separation for spin-orbit interaction. Einstein was surprised that Thomas's relativistic 'correction' could give factor '2'. Dirac in 1928 explained the same factor and the correct spin levels in terms of the Clifford algebra and the Dirac equation, without invoking the Thomas rotation. The Dirac equation conceptually ought to be understood in terms of the Clifford algebra alone. No longer did anyone need Thomas's precession except for the non-associative $\oplus$-addition of velocities.

7.3 Main conclusion. Most readers will consider three properties §5.1-5.2-5.3, of the Lobachevsky manifold of Einstein's velocities, $O(1,3)/O(3)$, as being very attractive from the point of view of mathematics and physics. Moreover, these properties are consistent with the concept of the absolute preferred space, that explain the twin paradox [Herbert Dingle, Rodrigo de Abreu, Subhash Kak]. Does these attractiveness must forbid the consideration of the alternative theories of special relativity that does not need the absolute space?

Maybe some readers would like to consider the necessity of the absolute space to be a deficient property of such special relativity theory?

$$\begin{aligned} \text{No absolute space} \quad &\Rightarrow \quad \text{no Einstein's relative velocities} \\ &\Rightarrow \quad \text{no Einstein's special relativity} \\ &\Rightarrow \quad \text{no asymmetric biological ageing of twins} \end{aligned}$$

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The problems of movement behind the light barrier in the Special theory of relativity formation and development context

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The perception of a scientific theory always passes a stage of its main positions critical judgement. The formation of the relativity theory also passed through this stage. One of the main postulates of the Special Theory of Relativity, such as the constancy of the velocity of light, arose to extensive controversies in the scientific world. Its acceptance as the main postulate, allowed refusing the ether theories and thus explaining the outcomes of Michelson and Morley's experiments.

At the moment of relativity theory developing a set of alternate theories appeared which allowed explaining Michelson and Morley's experiments, thus giving an opportunity to remain within the framework of classic physics. One of them was Ritz's ballistic hypothesis. The velocity of light c from a moving source was the vector total of velocity c_0 from a fixed source and the velocity of moving source V. According to Galilean addition theorem of velocity: $\vec{c} = \vec{c}_0 + \vec{V}$. The given theory was supported by W. Ritz (1908), Comstock (1910), Kunz (1910), R. Tolman, (1910),
Ya. Grdina (1923). Actually, they supposed that $c=3*10^8$ m\s only concerning a source of radiation. (It is necessary to say, that there was a polemic between Ritz and Einstein, concerning constancy of the velocity of light.) At present it is considered, that the given theory has only historical value. At the same time Abraham in oder to interpretation of Michelson's experiment, on the basis of the ether fixed theory, proposed another law of the velocity addition $c' = c\sqrt{1 - V^2/c^2}$. For the second time Abraham's formula was discovered by Rapier in 1961 based on a hypothesis of existence Lorentz's - invariant - completely - carried - away ether.

The prohibition of the velocity of light exceeding is frequently substantiated with the reference to Einstein's papers though the STR (Special Theory of Relativity) itself does not prohibit supraluminal motions. A. Einstein himself, in the paper of 1907, justified supraluminal velocity impossibility concluding from the character of our experience, which is not the main law. To this fact have paid attention Kirgnic D.A., Sazonov B.H. in the paper [2], which one have indicated, on the basis of the causality requirement, instead of relativistic conclusions, supraluminal moving was rejected in Pauli's «The Relativity theory» and Mandelshtam's «The lectures on the optician, relativity theory and quantum mechanics».

Further a number of physicists, such as Terleckiy Ya. [4], Feinberg G. [6], Bilaniuk O. and Sudarshan E. [5], in 60-s of the past century came to the conclusion, that supraluminal movements did not contradict the STR. The further push in the development of the theory of supraluminal motions belongs to Gerald Feinberg, who in 1967 brought in the concept of a supraluminal particle with imaginary mass (tachyon).

The question of the existence of speeds exceeding the velocity of light was discussed in the 20-s of the last century. The results of these controversies were the subject of a wide speculation on the pages of a number of scientific journals, in particular, such as “Zietschrift fur Physik”. The main attention was given to the interpretation of these experiments and their reliability.

The theoretical problems of this period touching the possibility of supraluminal velocity existence appeared to be as though less “tangible”. The new time has arisen new theoretical works and the pioneer works appeared to have already been forgotten and unknown to the wide scientific society. Foregoing in particular concerns prof. L. Strum’s scientific activities.

The main aim of the given paper is these scientists’ works, touching the problem of supraluminal velocity in the context of controversies of that time review and also the comparative analysis with the works concerning the moment of the tachyons theory development.

One of the objections against tachyons existence possibility consists of the fact that the given particles can have a negative energy. In Strum's paper 1923 [7] was formed a position, deciding the above-stated contradiction, which further in the paper [5] would be called „as a principle of re-interpretation”. Let's examine explicitly the conclusion proposed by Strum. Let at the moment of $t = 0$ from the origin of coordinates starts spreading a signal in lengthwise of axis X of the S system with the speed V > c. The system $S^{/}$ goes relatively of the system S with the speed $v < c$. Geometrically the proof was conducted this way.

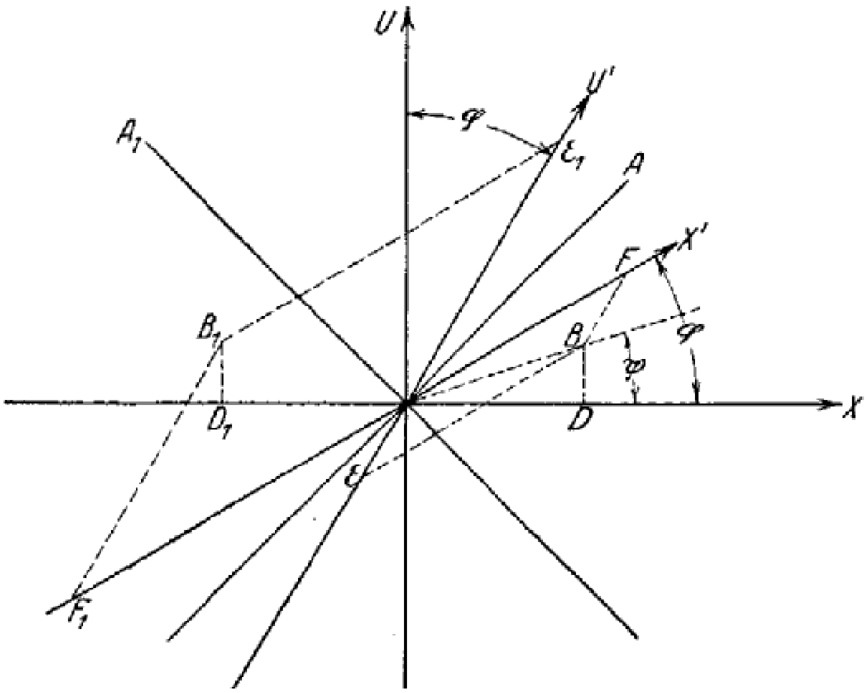

Figure 1 space-time diagram, reduced in L.Strum' papers

In the system *S*:

$$OD=x,\ BD=U=ct,\ OD_1=-x_1;\ B_1D_1=u1=ct_1$$

$$tg\varphi = \frac{v}{c};\ tg\psi = \frac{U}{x} = \frac{c}{V};\ \angle\ XOA = \frac{\pi}{4}$$

OA, OA₁ – the cone intersection with the plane XU.

At $V>c$, $\psi<\frac{\pi}{4}$, and point *B* lies in the middle of the corner $\angle XOA$, that is in "intermediate space" [7a, p. 42]. The same reasoning can be also conducted for point B_1. When $v>\frac{c^2}{V} \Rightarrow \frac{v}{c}>\frac{c}{V}$, that is $tg\varphi > tg\psi$. The point *B* lies in the middle of $\angle xOx'$ therefore has a negative coordinate U'.

As L. Strum marked the uniqueness of the given formulas consists of «not only of the fact that the coordinate of time becomes negative, but also the coordinate of time is negative only in the system S' whereas in the other system *S* the given coordinate is positive. *It demonstrates that only at the condition of the existence of processes, which spread with the speed exceeding the velocity of light in any system S, only such speed of rectilinear and uniform motion of the other system S/ relatively S, is possible, at which the direction of time in the system S' for such processes is opposite to the direction of time in the system S.* Thus, the theory of relativity reduces to new consequences as contrasted with earlier version. First of all, the possibility of speeds exceeding the velocity of light does not contradict the STR». [7, p. 86]

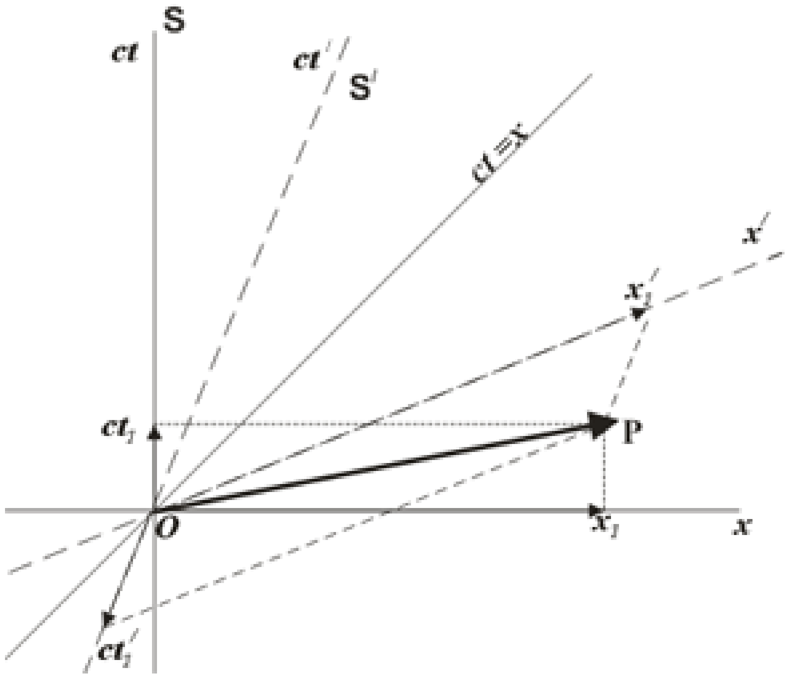

Figure 2 space-time diagram, reduced in Bilaniuk O. and Sudarshan E.'s papers

The given argument stated by Strum L. in the 20-th of the last century, became relevant again in Feinberg's G., Bilaniuk O. and Sudarshan E.'s papers. Introduced the concept of the «particles with the negative energy», they noted, that «the return sequence of events is supervised when point B lays below axis *x*. It is astonished, that it comes exactly under the same conditions, at which the product $v\omega$ (where, v - speed of a tachyon concerning the system S, ω - the movement speed of the system S' relatively *S*. The comment of the authors) exceeds the value c^2. As it will be seen hereinafter, the interpretation of this coincidence of signs change is a clue to the sequent theory of superluminal particles». The given principle, was called the principle of re-interpretation which asserts that the particles «with negative energy, at first absorbed and then emitted, are emitted and absorbed in the return order» [5].

The concept of the phase velocity used in the STR also reduces to appearance of speeds exceeding the velocity of light. However, the phase velocity exceeding *c*, can not be used for signal transfer, but as Mandelstam indicated in this case: «in order to deny (the theory of relativity, comment. Author) it is possible only if in the nature there are processes of signal character, more speedy, than light» [10].

Alongside with it there were attempts to justify the correspondence phase velocity with kinematics of the theory of relativity. The conclusion proposed by Strum [8] can be explained on the basis of the following reasons. Let us esteem the addition formula of velocities:

$$V' = \frac{V - v}{1 - \frac{Vv}{c^2}}$$

where ***v*** is the speed of the system S' relatively of the system *S*, ***V*** is the speed of the process in the system S, *V '* - counterpart of the speed in the system *S'*.

We suppose that ***V*** transmits the speed of a phase wave, which is a counterpart of material body moving with the speed *w* in the system *S*, i.e. $V = \frac{c^2}{w}$

Therefore, the scientist made the supposition that it was possible to prove, that $V^{/}$ was the speed of a phase wave, which corresponds with the same body moving in the system $S^{/}$.

Thus, on the one hand, when we substitute $V = \frac{c^2}{w}$ in the formula of speeds addition, we obtain $V' = \frac{c^2 - vw}{w - v}$

On the other hand, the speed *w* in the system S corresponds to the speed in the system S' $w^{/} = \frac{w - v}{1 - \frac{wv}{c^2}}$.

To the given speed corresponds a phase velocity $\frac{c^2}{w'} = \frac{c^2 - vw}{w - v}$ and it is equal *V'*.

As Strum wrote: « If ***V* < *c²/v***, i.e. ***w* > *u***, a body, which goes in the system S with speed ***w***, passes ahead of the system *S '*, then *V* and *V '* in both systems are rectified. If ***V* = *c²/v***, i.e. ***w* = *v***, ***V'*** = ∞**.** But in this case, if the speeds *v* and *w* are equal, the body fixed relativity of the system *S '*, and then the phase velocity of the fixed mass point is indefinitely high, since in this case the process of oscillations $\Phi = \Phi_0 \pi \upsilon_0 t_0$ is temporary, but not spatially, periodic. If ***V* > *c²/v***, i.e. ***w* < *v***, the directions *V* and *V '* in both systems are opposite. But at w < v the speed of the body relatively the system *S'* gains a negative sign, and then the speed of the appropriate phase wave directed in the system *S'* is the same. Thus, the concept of a phase wave, which is exceeding the velocity of light, completely enters kinematic of the theory of relativity, without meeting any objections». [8, p. 407]

The theory of superluminal particles is being developed, tending to enter naturally becoming classical Einstein's theory of relativity. So, there are theories in which are derived the relations of the STR and wave equations for tachyons, but already with a material mass. [9] However, as can be seen from the above mentioned the ideas expressed in the early years of new physics formation have found the reflection in development of the modern theory. Nevertheless, at that stage of science advancement, both experimental and theoretical bases, based on which one could realize courageous suppositions, were not ready yet (in particular, the fundamental particles theory did not exist). Although until present time supraluminal particles existence has not been experimentally proved, the researches in the given area will lead to more profound comprehension of physics. As it is indicated in the paper [5] «in modern physics there is an unwritten rule, frequently playfully called, general Gell - Mann's principle, stating, that in physics for certain there are things which are not forbidden»!

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About linkage of electrodynamics and gravitation

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We have considered two types of gravitons, the quantization of gravitation charge, time dependences of graviton and electron charges. After that we have considered the quantization of the gravitation orbital and internal currents moments. Also we have considered decay of graviton and other graviton interactions. We have supposed explaining of origin of dark energy and dark matter.

I. Introduction

We pay attention to the old and new aspects of the Maxwell electrodynamics and the gravitation theory linkage. Maxwell, Heaviside, Brillouin, Bridgman, Carstoiu [1], Petrov [2], Logunov [3], Rajput [4] analyzed this linkage. There is no complete symmetry between electrical charges in the Coulomb's law and gravitation masses in the Newton's law. This asymmetry is also observed in the relativistic gravitation theory [3]. Therefore it is very interesting to consider case of the complete symmetry.

2. Analysis of problem

We have considered system of two small bodies. These bodies have gravitation masses m_{G1}, m_{G2}. In order that the Newton's law had form of the Coulomb law introduce the gravitation charges by following method:

$$q_{Gs} = i(-1)^{s+1} G^{1/2} m_G\,, \quad s = 1,2; \tag{1}$$

where G is the gravitation constant and q_G is the gravitation charge. This value is an imagine number. Also we have supposed that gravitation permeability of vacuum ε_G is equal unity. Then the Newton's law has form of Coulomb's law

$$F_l = \frac{q_{G1} q_{G2}}{\varepsilon_G r^2} \left(\frac{r_l}{r}\right), \tag{2}$$

where F_l is gravitation force, r_l is radius vector connected the point charges.

So we suppose that $s = 1,2$.These values are corresponding two types of gravitation charges. The opposite gravitation charges are attracted.

Hence we have important distinction the gravitation charges from electrical charges. The distinction may be one of keys to solution such complete problems as the dark energy and dark matter. However we'll receive the real key if we know elementary processes with gravitation charges.

3. Gravitation charge quantization

We have used the similarity of electrodynamics and Newton`s gravitation theory for solution of question about gravitation charge quantization. So we know that electrical charge q has quantum spectrum and it has following form:

$$q = \pm N|e|, \ldots, N = 0,1,2,3,\ldots, \tag{3}$$

where $|e|$ is the modul of electron charge. We can write similar form for the gravitation charge

$$q_G = \pm i N_G |e_G|, \ldots, N_G = 1,2,3,\ldots, \tag{4}$$

where $|e_G|$ is modul of the gravitation charge quant. But the magnitude of $|e_G|$ is unknown. We have used the Plank's unities system $(\hbar, c, G)$ for determination of $|e_G|$. If we take the time scale and $t = 0$ is birth moment of Metagalactica, we can suppose

$$|e(t_{+0})| = |e_G(t_{+0})| \tag{5}$$

where $t_{+0} \le t_{Pl}$; $t_{Pl} = (\frac{\hbar G}{c^5})^{1/2}$ and $2\pi\hbar$ is the Plank's time and constant correspondingly, c is light velocity. Then evaluation of the magnitudes $|e(t_{+0})|$ and $|e_G(t_{+0})|$ is

$$|e(t_{+0})| = |e_G(t_{+0})| = (\hbar c)^{1/2} = q_0, \tag{6}$$

where q_0 is the Plank's charge. But now $t_{MG} >>> t_{Pl}$ and we have in the Plank's system

$$|e(t_{MG})| = |\Lambda_e|^{1/2} q_0, \tag{7a}$$

$$|e_G(t_{MG})| = |\Lambda_G|^{1/2} q_0, \tag{7b}$$

where Λ_e, Λ_G are dimensionless constants of interactions, and we have equality

$$|\Lambda_e(t_{+0})| = |\Lambda_G(t_{+0})| = 1 \ . \tag{8}$$

In order to determine $|e(t)|$ and $|e_G(t)|$ we have used similarity of the Dirac's hypothesis about the time dependence of the physical constants [5]. We have supposed that the dependence has following form:

$$|\Lambda_Q| = C_Q \tau^{-\beta_Q}, Q = \{e, G\}, \tag{9}$$

where $C_Q, \tau = \frac{t}{t_{Pl}}, \beta_Q$ are constants. We can calculate constant C_Q. Accordance to formula (8) (if $\tau = 1$, $(t = t_{Pl})$) we have $C_Q = C_e = C_G = 1$ and

$$|\Lambda_Q| = \tau^{-\beta_Q}. \tag{10}$$

Therefore we have following dependence for electron charge

$$|e(t_{MG})| = |e| = (\hbar c)^{1/2} \tau^{-\frac{\beta_e}{2}} . \tag{11}$$

Hence constant β_e is equal

$$\beta_e = -\frac{\ln(\alpha)}{\ln(\tau_{MG})} , \tag{12}$$

where $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant, $\tau_{MG} = \frac{t_{MG}}{t_{Pl}}$. We have used evaluations of time from Big Fire or birth Metagalactica $t_{MG} = (13.7 \pm 0.2) \times 10^9\, years$ [3c, 6] or $(4.323 \pm 0.063) \times 10^{17} s$ and the Plank's time $t_{Pl} = 5.3906 \times 10^{-44} s$ [7]. Then evaluation of τ_{MG} is $\tau_{MG} = (8.02 \pm 0.12) \times 10^{60}$. Supposing that constant α is equal 7.29735×10^{-3} [7] we have received following numerical result: $\beta_e = 0.035085 \pm 0.000004$ ($\frac{\beta_e}{2} = 0.017543 \pm 0.000002$).

If sign of charge is constant the time dependence of charge is determined by formula

$$\frac{e_Q(t)}{e_Q(t_{Pl})} = |\Lambda^{1/2}{}_Q| , \tag{13}$$

therefore we have formula for derivation $|\Lambda^{1/2}{}_Q|$:

$$\frac{d\left|\Lambda^{1/2}{}_Q\right|}{dt} = -\frac{\beta_Q}{2t}\left(\frac{t}{t_{Pl}}\right)^{\frac{-\beta_Q}{2}} . \tag{14}$$

We have produced evaluation the derivation and received $\frac{d\left|\Lambda^{1/2}{}_e\right|}{dt}\Big|t = t_{MG} \cong -8.6\times10^{-21} s^{-1}$ or -2.6×10^{-13} $year^{-1}$. Probably this result is out of experiment possibilities.

Now we consider ratio gravitation and the Coulomb forces between two electrons. It is equal

$$\frac{Gm_e{}^2}{e^2} = \frac{\left|q_G\right|^2}{q_e{}^2} = \left|\frac{\Lambda_G}{\Lambda_e}\right| = \tau^{\beta_e - \beta_G} \tag{15}$$

in accordance with formulas (1),(2),(7) and (10). Then we have formula for β_G

$$\beta_G = \frac{2\ln(\eta_e)}{\ln(\tau_{MG})} , \tag{16}$$

where $\eta_e = \frac{m_{Pl}}{m_e}$. Using numerical evaluations $m_e = 9.109382\times10^{-28} g$ and $m_{Pl} = 2.177\times10^{-5} g$ [7] we have received following results: $\beta_G = 0.73487\pm0.00007$ ($\frac{\beta_G}{2} = 0.36744\pm0.00004$). Now the time derivation of $\left|\Lambda_G{}^{1/2}\right|$ is equal

$$\frac{d\left|\Lambda^{1/2}{}_G\right|}{dt}\Big|(t = t_{MG}) = -\frac{\beta_G}{2t_{Pl}}\tau_{MG}{}^{-(1+\frac{\beta_G}{2})} . \tag{17}$$

Using known numerical significations of t_{Pl} and t_{MG} we have received that $\left|\left(\frac{d\left|\Lambda^{1/2}{}_G\right|}{dt}\right)_{t=t_{MG}}\right| < 10^{-40} s^{-1}$. So this numerical result is very small we have supposed that Λ_G is constant now.

Also we have considered the inflation model. Here we have supposed the following time dependence

$$\frac{q_Q(\tau)}{q_Q(\tau_{MG})} = \exp(\beta_Q(1-\tau)), \tag{18}$$

where β_Q is constant. The formulas for β_Q were deduced

$$\beta_e = -\frac{\ln(\alpha)}{\tau_{MG}-1}, \tag{19a}$$

$$\beta_G = \frac{2\ln(\eta_e)}{\tau_{MG}-1} , \tag{19b}$$

The following numerical results for β_Q were received: $\beta_e = (6.13\pm0.09)\times10^{-61}$, $\beta_G = (1.285\pm0.019)\times10^{-59}$ and for time derivation of $\left|\Lambda_Q{}^{1/2}\right|$ we have

$$\frac{d\left|\Lambda^{1/2}{}_Q\right|}{dt}\Big|(t = t_{MG}) = -\frac{\beta_Q}{t_{Pl}}\exp(\beta_Q(1-\tau_{MG})). \tag{20}$$

The next evaluations of this derivation were received:

$$\frac{d\left|\Lambda^{1/2}{}_e\right|}{dt}\Big|(t=t_{MG}) = -8.0\times10^{-21}\ s^{-1}(-2.5\times10^{-13}\ year^{-1}),$$
$$\frac{d\left|\Lambda^{1/2}{}_G\right|}{dt}\Big|(t=t_{MG}) = -4.2\times10^{-60}\ s^{-1}(-1.3\times10^{-52}\ year^{-1}).$$

We have considered the oscillation model. The model time dependence for $\frac{e(t)}{e(0)}$ (the electrical charge quant) had the following view:

$$\frac{e(t)}{e(0)} = \sin\left(\frac{2\pi}{T_{MG}}t + \varphi_o\right), \tag{21}$$

where T_{MG} is the oscillation period and φ_o is the initial phase. We have received $\varphi_o = \frac{\pi}{2}$ for positron and $\varphi_o = \frac{3\pi}{2}$ for electron. The electron charge time dependence is shown on fig. 1.

Now we have such result $\left|\frac{e(t_{MG})}{(\hbar c)^{1/2}}\right| = \alpha^{1/2} = \cos\left(\frac{2\pi\, t_{MG}}{T_{MG}}\right)$. Therefore the oscillation period is determined by valid formula

$$T_{MG} = \frac{4t_{MG}}{1-\frac{2}{\pi}\arcsin(\alpha)^{1/2}}. \tag{22}$$

Using the numerical evaluation for α we have received approximation

$$T_{MG} \cong 4\left(1+\frac{2}{\pi}\alpha^{1/2} + O(\alpha)\right)t_{MG} \tag{23}$$

Here we have supposed that the time dependences of the electrical charge quant and the gravitation charge quant were similar. Then their oscillation periods are equal and we have following numerical evaluation of $T_{MG} \cong (57.8 \pm 0.8)x10^9\ years$.

So signs of graviton and antigraviton charges are opposite their initial phases in formula (21) are equal $\varphi_o = \frac{\pi}{2}$ and $\varphi_o = \frac{3\pi}{2}$ for graviton and antigraviton correspondingly. The time dependences of the graviton (antigraviton) charge are presented on fig.2. We see graviton – antigraviton transformations.

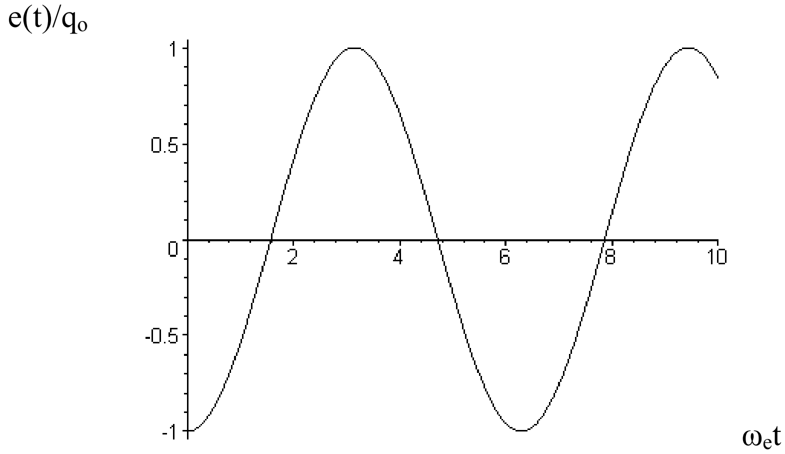
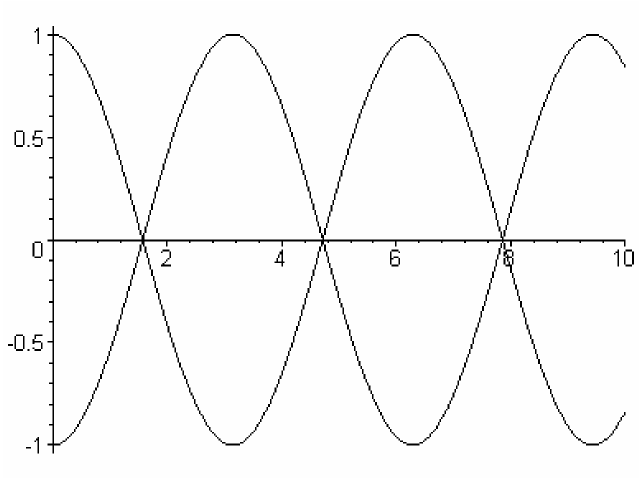


Fig.1. Charge oscillations of electron

eG/iqo



ωGt

Fig.2. Charge oscillations of graviton (antigraviton)

So the graviton charge is imagine number we can consider the more general case. Suppose that the charge is complex number or $q_Q = q'_Q + iq''_Q$ where q'_Q and q''_Q are real and imagine parts correspondingly. Then the simplest oscillation model is form

$$\frac{q_Q(t)}{q_0} = 1 + a_Q \exp(-i(\omega_Q t + \varphi_Q)), \tag{24}$$

where $q_0 = |q_Q(t=0)|$, a_Q, ω_Q, φ_Q are unknown constants. Separating real and imagine parts we have following formulas:

$$\begin{aligned} \left(\frac{q_Q(t)}{q_0}\right)' &= 1 + a_Q \cos(\omega_Q t + \varphi_0), \\ \left(\frac{q_Q(t)}{q_0}\right)'' &= -a_Q \sin(\omega_Q t + \varphi_0). \end{aligned} \tag{25}$$

and $q_Q(t) = \pm i\tilde{q}(t)$, where $\tilde{q}'' = 0$ for graviton and antigraviton correspondingly. Then valid view of the formulas (25) is for time $t = 0$

$$\begin{aligned} &1 + a_G \cos(\varphi_G) = 0, \\ &......a_G \sin(\varphi_G) = s_{\mp}, \end{aligned} \tag{26}$$

where $s_{\mp} = (-1,+1) = (graviton, antigraviton)$. We have received two equations for constants a_G and φ_G. Then we have equalities for determination a_G and φ_G

$$\begin{aligned} &ctg(\varphi_G) = -s_{\mp}, \\ &a_G = \pm(1 + s_{\mp}{}^2)^{1/2}. \end{aligned} \tag{27}$$

There are two pairs of equivalent solutions $\varphi_G = \frac{\pi}{4} \bigcup a_G = -2^{1/2}$ and $\varphi_G = \frac{5\pi}{4} \bigcup a_G = 2^{1/2}$.

Therefore now $(t = t_{MG})$ we have condition

$$\cos(\omega_G t_{MG} + \frac{\pi}{4}) = 2^{-1/2} \quad . \tag{28}$$

If $\omega_G = 0$ then it is no oscillation of the graviton charge else the oscillation period T_G is equal

$$T_G = \frac{t_{MG}}{N_G},...N_G = 1,2,3,... \qquad (29)$$

and time derivation of $\frac{q_Q(t)}{q_0}$ is $2^{1/2}\frac{2\pi N_G}{t_{MG}}\cos(\frac{2\pi N_G}{t_{MG}}t + \frac{\pi}{4})$. If N_G is equal 1 then the time derivation of the graviton charge is $4.6x10^{-10}\,year^{-1}$ approximately.

Considering the case of electron we have $\varphi_e = 0 \bigcup a_e = -2$ and $\varphi_e = \pi \bigcup a_e = 2$. Now $(t = t_{MG})$ there is condition $\sin(\omega_e t_{MG}) = 0$ and the electron charge oscillation period T_e is

$$T_e = \frac{2\pi}{\omega_e} = \frac{2t_{MG}}{N_e}, N_e = 1,2,3,... \qquad (30)$$

The time derivation of the electron charge is $2.3x10^{-10}\,year^{-1}$ approximately. So we have considered three types of time dependences of electron-graviton charges.

4. Other quantum numbers of graviton

If we consider gravitation theory as similarity of the Maxwell electrodynamics, we have conservation gravitation charge law [1]. So we have equality (1), the conservation gravitation charge law is the conservation mass law.

We have considered gravitation currents. Orbital gravitation current has magnetic gravitation moment L_{Gl} . The moment is equal

$$L_{Gl} = \frac{q_G vr}{2} e_l,$$

where v is the orbital velocity of graviton, r is the radius of orbit and e_l is the ort of normal to orbit plane. The angular moment or mechanical moment L_l is equal

$$L_l = m_G vr\ e_l.$$

Therefore orbital magnetomechanical ratio of graviton is

$$\frac{L_{Gz}}{L_z} = \frac{(-1)^{s+1} iG^{1/2}}{2},..., s = 1,2. \qquad (31)$$

According to the Bohr`s quantization rule we have

$$L_z = \pm N\hbar,..., N = 0,1,2,3,.... \qquad (32)$$

where L_z is $z-$projection of angular moment. Then $z-$projection of magnetic gravitation moment of orbital current is equal

$$L_{Gz} = \frac{(-1)^{s+1} iG^{1/2}}{2}\hbar N,..., N = 0,1,2,3,.... \qquad (33)$$

It is known that modul of $z-$projection of graviton spin moment s_z is equal 0 or 2 [8].So all the quantum numbers of the mechanical moment of graviton are defined. There are conservation laws of energy-impulse and mechanical moment in maxvelization theory of gravitation [1-4].

5. Elementary graviton processes

Now we consider some simplest elementary processes.

5.1. Birth of gravitons and antigravitons.

We have considered primitive vacuum. There are fluctuations in the vacuum. It is possible decay of fluctuation. The decay of fluctuation may be presented by the Feynman`s diagram (fig.3).

The diagram on fig.3 presents birth process of graviton and antigraviton in primitive vacuum.

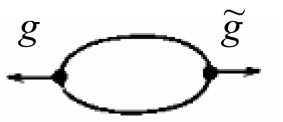


Fig.3.The Feynman`s diagram of fluctuation matter decay on graviton g and antigraviton $\tilde{g}$.

Suppose that graviton (or antigraviton) is unstable particle. The decay is presented on fig.4.

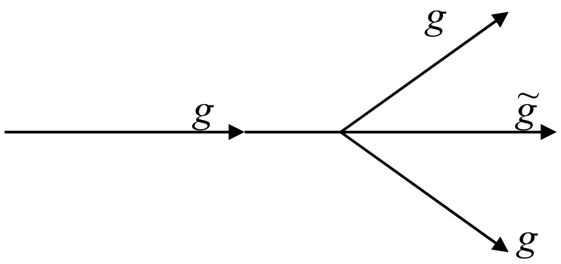


Fig.4. The Feynman`s diagram of graviton g (antigraviton $\tilde{g}$) decay on two gravitons (antigravitons) and antigraviton (graviton).

If gravitons (antigravitons) are not connected with photons we have dark matter.

5.2. Graviton (antigraviton) transformation in other particles

Decay of graviton (antigraviton) may be birth other particles also. For example, we have considered the decay is presented by the Feynman`s diagram on fig.5.

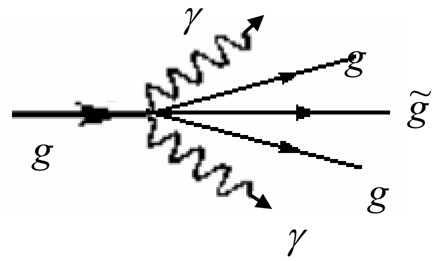


Fig.5. The Feynman`s diagram of graviton (antigraviton) decay on two gravitons (antigravitons), antigraviton (graviton) and two photons (wave lines).

Photon may be transformed in other particles. Example of such transformation is presented by the Feynman`s diagram on fig.6.

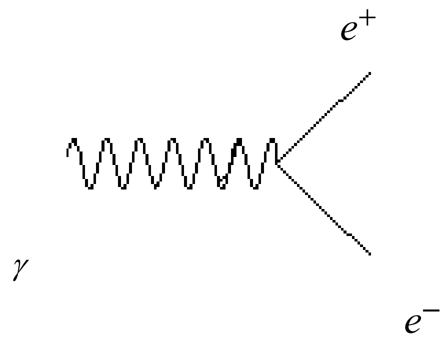


Fig.6. The Feynman`s diagram of photon decay on electron-positron pair (e^-, e^+).

Electron-electron (positron- positron) collisions may be source of mesons and more massive particles. Decay of neutron may source of electron neutrino.

Also we have considered graviton - antigraviton collision. The simplest Feynman`s diagram is presented on fig.7.

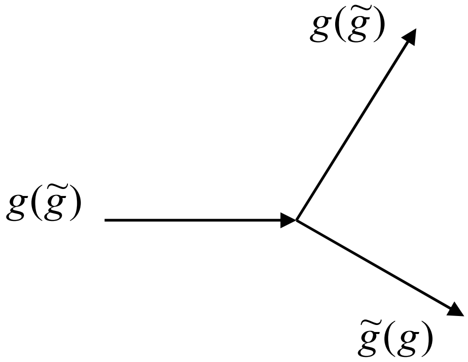


Fig.7. The Feynman`s diagram of graviton (antigraviton) scattering on rest antigraviton (graviton).

The process for graviton (antigraviton) is similarity of the Compton scattering photon with high energy. Birth of new particles is absent in this collision. But there is possible the other picture. Such elementary process is presented on fig.8.

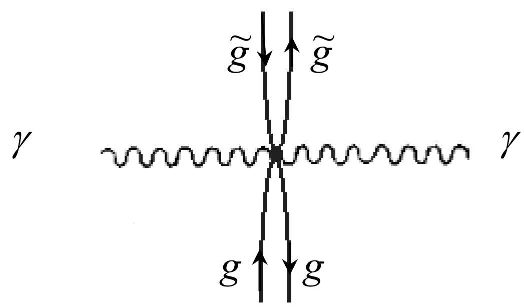


Fig.8. The Feynman`s diagram of collision of graviton (antigraviton) and antigraviton (graviton) with photons birth.

So processes presented on fig. 5, 8 may be sources of photons.

We have considered the deep unelastic scattering of graviton (antigraviton) on rest antigraviton (graviton). The process is transformation

$$g + \tilde{g} \rightarrow g + \tilde{g} + g + \tilde{g} + ... \qquad (34)$$

The threshold energy T of flying graviton (antigraviton) is determined by formula

$$T = \frac{(N^2 - 1)(m_g + m_{\tilde{g}})^2 c^4}{2m_{\tilde{g}(g)} c^2}, \qquad (35)$$

where N is number of graviton-antigraviton pairs after the process. So it is $m_g + m_{\tilde{g}} = 0$, then the process hasn't threshold and the graviton gas is cooled.

6. Discussion

We should like make remark about formula (1). One may suppose also that

$$q_{Gs} = (-1)^{s+1} G^{1/2} m_G, ..., s = 1,2; \qquad (36)$$

Then formula (2) is

$$F_l = -\frac{q_{G1}q_{G2}}{\varepsilon_G r^2}(\frac{r_l}{r}), \qquad (37)$$

But in this case in accordance to received formula (2) we must suppose that gravitation permeability of vacuum ε_G is equal

$$\varepsilon_G = -1. \qquad (38)$$

This admission is opposite usual of electrodynamics admittance. But formulas (3)-(20), (32) are right in this case also. In formulas (31), (33) difference is consist of changing i on 1.

Conclusions

We have received all following results on base of analysis of linkage electrodynamics and gravitation:

1. It is proposed new form for gravitation charge. If that gravitation permeability of vacuum is equal unity then significations of gravitation charges are imagine numbers.
2. So opposite gravitation charges are attracted. Annihilation of graviton and antigraviton is impossible.
3. Gravitons and antigravitons may be generated in case of local vacuum fluctuation.
4. Gravitons and antigravitons are generated in such pairs as graviton - antigraviton. Generation of these pairs may be cause of the expansion acceleration of Metagalactica.
5. Gravitons and antigravitons may be formed dark matter.
6. Gravitons (antigravitons) may be scattered on rest antigravitons (gravitons) as photons may be scattered on rest electrons in the Compton effect.
7. Gravitons (antigravitons) may be transformed in other particles if usual conservation laws and gravitation charge conservation law are fulfilled. This transformation forms photons, leptons and more hard particles.
8. It is proposed the time dependence of electrical and gravitation charges. It is determined constants in this dependence for both types of charges. Graviton –antigraviton and antigraviton – graviton transformations are possible.
9. We have received evaluations of Metagalactica pulsation period.
10. It is shown that it is possibility of the spontaneous multiplication of gravitons and antigravitons by scattering gravitons (antigravitons) on rest antigravitons (gravitons).

Detailed consideration of photon-graviton interactions is future problem.

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Geometrization of material wave fields and electromagnetic waves

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Within a single geometrical approach we show that the Dirac equation for material wave field and the Maxwell equations for electromagnetic waves can be considered as relations describing propagation of local distortions of the space metric— topological defects having wave–corpuscular properties. Such approach gives an opportunity to explain an appearance of probabilities in the quantum mechanics formalism and independence of light velocity on the source movement.

Introduction

We will show below, within the unified geometrical approach, that the matter field (free quantum particle) and electromagnetic waves can both be considered not as something existing into the space that plays the role of scene but as a propagating topological defects of the space itself. This defect appears to be embedded into five–dimensional space, and observable quantum object and electromagnetic wave are projections of the defect on the three-dimensional space. Their wave properties are a consequence of the periodical movement of the defect into "outer"space. The classical notions of mass, energy and momentum of considered wave fields are expressed through the parameters with dimensionality of length, and the Planck constant and the light velocity play the role of coefficients of transfer from one system of units to the other. Spin happens to be a topological invariant. It is shown that light velocity can be also considered as a topological invariant, and this is the reason for its independence of the source motion. The uncertainty of shapes of topological manifolds is the reason of the appearance of probabilistic description.

We start, at first, with the geometrization of matter field and, doing so, we have to keep in mind the exceptional accuracy of the modern quantum formalism. Therefore, we suppose that attempts to find out new geometrical description of quantum objects have to begin not with the creation of a new formalism but with finding out a geometrical interpretation of the well-known relativistic quantum equations whose validity is beyond question. This is the reason why we start with an attempt to find out geometrical interpretation of the Dirac equation. Preliminary results see at hep-th/0605060.

1. Topological interpretation of the Dirac equation

This equation has the following symbolic form (see, e.g.,[1])

$$i\gamma^{\mu}\partial_{\mu}\psi = m\psi, \tag{1}$$

where $\partial_{\mu} = \partial/\partial x_{\mu}$, $\mu = 1,2,3,4$, $\psi(x)$ is the four-component Dirac bispinor, $x_1 = t, x_2 = x, x_3 = y$, $x_4 = z$, and γ^{μ} are four-row Dirac matrices. The summation in Eq.(1) goes over the repeating indices with a signature $(1,-1,-1,-1)$. Here, $\hbar = c = 1$. For definite values of 4-momentum p_{μ}, the solution to Eq.(1) has the form of the plane wave

$$\psi = u(p_{\mu})\exp(-ip_{\mu}x^{\mu}), \tag{2}$$

where $u(p_{\mu})$ is a normalized bispinor. Substitution of (2) in Eq.(1) gives the following relation for p_{μ}

$$p_1^2 - p_2^2 - p_3^2 - p_4^2 = m^2. \tag{3}$$

Eq.(1) serves as a basis for describing of experiments with fine accuracy in the intermediate energy range where relativistic corrections can not be ignored but where we can neglect of the possibility of new particles appearance. Wave function (2) is interpreted as a description of some free wave field with mass m and spin 1/2. The wave function amplitude squared defines the probability to find particle in the corresponding point and the phase factor in (2) describes the wave–corpuscular properties of a particle using postulated relations between particle's 4-momentum p_{μ} and "particle's wavelength"λ_{μ}

$$\lambda_{\mu} = 2\pi p_{\mu}^{-1}. \tag{4}$$

Any model or any pictorial representation of the quantum object is absent, and the space-time is considered as a scene where such objects exist and interact.

In the theory of classical wave fields these different fields are described by tensors with different transformation properties: they realize different representations of the Lorentz group (group of 4-rotation). Dirac bispinor consists of two–component spinors that realizes two–dimensional representation of the Lorentz group, and this is the formal reason to classify the Dirac wave field of Eq.(1) as the object with "internal"angular momentum whose Z–projection equals to $\pm 1/2$ [2]. Let us, firstly, consider transformation properties of the Dirac bispinor from the other point of view. It is known that there may be established correspondence between every kind of tensors and some class of geometrical objects in the sense that these tensors define invariant properties of above objects. For example, coefficients in equations for planes of second order in the euclidean space are usual tensors of second rank and so on [3]. From this point of view spinors correspond to nonorientable geometrical object (see, e.g., [4]). Therefore, we suppose that spinors are used in Eg.(1), because, in particular, that this equation describes some nonorientable geometrical object and "$spin = 1/2$"is a formal expression of the nonorientable property of the object.

The above assumption have to be considered as the starting hypothesis only. To define properties of the proposed geometrical object more exactly we consider more precisely the symmetry properties of the solution of Eq.(1). Using (4), we rewrite function (2) in the form

$$\psi = u(p_\mu) \exp(-2\pi i x^\mu \lambda_\mu^{-1}). \tag{5}$$

Rewrite relation (3) in the form

$$\lambda_1^{-2} - \lambda_2^{-2} - \lambda_3^{-2} - \lambda_4^{-2} = \lambda_m^{-2}, \quad \lambda_m = 2\pi m^{-1}. \tag{6}$$

Let us now consider λ_μ as some parameters with a dimensionality of length that have nothing to do with any wave process into the space. Function (5) is an invariant with respect to coordinates transformations

$$x'_\mu = x_\mu + n_\mu \lambda_\mu, \quad n_\mu = 0, \pm 1, \pm 2, ... \quad . \tag{7}$$

We see that transformations (7) can be considered as elements of the discrete group—the group of translations operating in the 4-space where wave function (4) is defined. Then function (4) can be considered as a vector realizing this group representation.

As a bispinor, function (4) realizes representation of one more group of the symmetry transformation of 4-space that is not so obvious. Being a four-component spinor, $\psi(x)$ is related to the matrices γ^μ by the equations (see, e.g. [2])

$$\psi'(x') = \gamma^\mu \psi(x),$$

where $x \equiv (x_1, x_2, x_3, x_4)$, and $x' \equiv (x_1, -x_2, -x_3, -x_4)$ for $\mu = 1, x' \equiv (-x_1, x_2, -x_3, -x_4)$ for $\mu = 2$, and so on. This means that the matrices γ^μ are the matrix representation of the group of reflections along three axes perpendicular to the x_μ axis, and the Dirac bispinors realize this representation.

Taken together, above two groups form a discrete group—four sliding symmetries with perpendicular axes (sliding symmetry means translations plus corresponding reflections; see, e.g., [5]). The physical space-time does not have such symmetry. Therefore, this group may operate only in some auxiliary space. From the other hand, it is known in topology that discrete groups operating in some space can reflect a symmetry of geometrical objects that have nothing in common with this space. This will be the case when such space is a universal covering space of some closed topological manifold. Universal covering spaces are auxiliary spaces that are used in topology for the description of closed manifolds, because discrete groups operating in these spaces are isomorphic to fundamental groups of manifolds—groups whose elements are different classes of pathes on manifolds starting and ending at the same point (so called π_1 group [6,7]).

We assume that function (4) realizes a representation of the fundamental group of some closed nonorientable topological 4-manifold—a specific curved part of the space-time. Eq.(1) describes this manifold and imposes limitations (6) on the possible values of the fundamental group parameters λ_μ. Space-time plays also the role of an universal covering space for above manifold. How above manifold can describe something moving in the space ?. At the present time, only two-dimensional closed manifolds are classified in details, and their fundamental groups and universal covering planes are identified [6]. As it is known to author, four-dimensional manifolds with above sliding symmetry group operating in pseudoeuclidean universal covering space were not considered before. The most suitable two-dimensional example is a two-dimensional nonorientable closed manifold homeomorphic to the Klein bootle—its fundamental group is generated by two sliding symmetries with parallel axes on the euclidean universal covering space [6,7].

Therefore, we have no opportunity for rigorous consideration in details of specific properties of suggested 4-manifold. But qualitative properties, explaining main ideas of new interpretation, can be investigated using one of the advantages of geometrical approach—possibility of employment of low-dimensional analogies. Using these analogies we will show within elementary topology that above mentioned 4-manifold represents propagation of the space topological defect that can demonstrate specific properties of quantum particle represented by solution (2): stochastic behavior and wave-corpuscular dualism.

2. Space-time closed manifold represents a moving region of space points with stochastic properties

We consider in this Section the simplest example of a closed topological manifold—one-dimensional manifold homeomorphic to a circle whose perimeter length is fixed and equals λ. The closed topological manifold is representable by any of its possible deformations (without pasting) that conserve manifold's continuity, and we will see that just this property explains appearance of probabilities in quantum formalism. For simplicity we consider only plane deformations of the circle. The fundamental group of this manifold is a group isomorphic to the group of integers [6,7]. This group is isomorphic, in its turn, to the discrete group of one-dimensional translations along a straight line over a distance λ[8]. This line (0X axis) is the universal covering space of our manifold. Therefore, the universal covering space for our circle is a one-dimensional euclidean space where the above symmetry group operates.

To use simple calculations, we consider only all possible manifold's deformations that have a shape of ellipse with perimeter length λ. The equation for the ellipse on an euclidean plane has the form

$$X^2/a^2 + Y^2/b^2 = 1, \tag{15}$$

where all possible values of the semiaxes a and b are connected with the perimeter length λ by the known approximate relation

$$\lambda \simeq \pi[1,5(a+b) - (ab)^{1/2}]. \tag{16}$$

This means that the range of all possible values of a is defined by the inequality $a_{min} \leq a \leq a_{max} \simeq \lambda/1,5\pi, a_{min} \ll a_{max},$.

In the pseudoeuclidean two-dimensional "space-time,"the equation for our ellipses has the form (after substitution $Y = iT$)

$$X^2/a^2 - T^2/b^2 = 1, \tag{17}$$

and this equation defines the dependence on time T for a position of the point X of the manifold corresponding to definite a. At $T = 0, X = \pm a$; that is, our manifold is represented by the two point sets in one-dimensional euclidean space, and the dimensions of these point sets are defined by all possible values of a. So, at $T = 0$, the manifold is represented by two regions of the one–dimensional euclidean space $a_{min} \leq |X| = a \leq a_{max}$. It can easily be shown that at $T \neq 0$ these regions increase and move along the X-axis in opposite directions.

All another possible deformations of our circle will be obviously represented by points of the same region, and every such point can be considered as "quantum object"that is described by our manifold. All manifold's deformations are realized with equal probabilities (there are no reasons for another suggestion). Therefore, all possible positions of the object into the region realized with equal probabilities. So, this example shows the possibility of the consideration of above object as a point with probability description of its positions as it suggested within standard representation of quantum particles. In fact, this point is not yet a material point—it is a geometrical point only. In the next Section we will show how this point becomes the material point. Note also that parameter λ defines the minimal size of a region where the object can be localized (region at $T = 0$).

3. Space-time closed manifold represents a moving topological defect with wave-corpuscular properties.

The simple example of preceding Section does not explain what geometrical properties allows to differ points of the moving region from neighbour points of the euclidean space making them observable. To answer at this question let us consider more complex analogy of the closed 4-manifold—two-dimensional torus. In euqlidean 3-space such torus is denoted in topology as a production of two one-dimensional closed manifolds $S^1 \times S^1$. The role of different manifold's deformations as a reason for stochastic behaviour was considered in preceding Section. Therefore, now restrict our consideration to one simplest configuration when one of S^1 is a circle in the plane XY and another is a circle in the plane ZX (we denote it as S^1_t).

In pseudueuclidean space this torus looks like a hyperboloid. The hyperboloid appears if we replace the circle S^1_t by a hyperbola (as it was done in Section 2). Positions of the geometrical object described by our pseudoeucliden torus are defined by time cross-sections of the hyperboloid. These positions looks like as an expanding circle into two-dimensional euclidean plane. But we need to have in mind that two-dimensional pseudoeuclidean torus describes the object existing into two-dimensional space-time with one-dimensional euclidean "physical"space. This means that an observable part of the object is represented in our example by the points of intersections of above circle with $0X$ axis though, as a whole, the object is represented by a circle "embedded"into two-dimensional, "external"space. This circle can be considered as a topological defect of the physical one-dimensional euclidean space. Just an affiliation of the intersection points to the topological object differs these points geometrically from neighboring points of the one-dimensional euclidean space. Therefore, in pseudoeuclidean four-dimensional physical space-time the suggested object described by the Dirac equation looks like a topological defect of physical euclidean 3-space that is embedded into 5-dimensional euclidean space, and its intersection with physical space represents an observable quantum object.

Above analogy with torus does not yet demonstrate appearance of any wave-corpuscular properties of the object, represented in "physical"one-dimensional space by the moving intersection point— properties that could be expressed by wave function (5) and relation (4). In the case of considered two-dimensional "space–time"this solution has the form

$$\psi = u(p)\exp(-2\pi i x^1 \lambda_1^{-1} + 2\pi i x^2 \lambda_2). \tag{17}$$

Topological defect represented by the expanding circle does not demonstrate any periodicity when the intersection point (physical object) moves along one-dimensional euclidean $0X$–space.

We will see below that appearance of observable wave-corpuscular properties is a consequence of nonorientable character of the topological defect. Torus is a orientable two-dimensional closed manifold and, therefore, we need to use some nonorientable low-dimensional analogy. The nonorientable Klein bottle could be such two–dimensional analogy [5,6]. In the case with torus topological defect was represented by cross-sections of pseudoeuclidean torus–plane circles. The Klein bootle is a manifold that is obtained by gluing of two Mobius strips (see, e.g.[9]). Therefore, the Klein bootle cross-section is an edge of the Mobius strip. This edge can not be placed in the two–dimensional XY–plane without intersections, and it means that corresponding topological defect is now a closed curve embedded into three–dimensional XYZ–space.

In this case the position of the topological defect (closed curve) relative to its intersection with $0X$ axis (physical object) can change periodically. The parameters of this periodical movement depends on geometrical parameters λ_1 and λ_2 (there are no other parameters with corresponding dimensionality). Such periodical process can be expressed by the function (17). This gives an opportunity for the new interpretation of the wave function as a description of periodical movement of the topological defect relative to its projection on the physical space.

In this case corpuscular properties of the above periodical movement appear as a result of the definition for classical notion of 4-momentum trough the wave characteristic of the topological object, namely

$$p_\mu = 2\pi/\lambda_\mu. \tag{18}$$

Substitution of these relations into (17) leads to the Dirac solution (2)

$$\psi = u(p)\exp(-ip_1x^1 + ip_2x^2). \tag{19}$$

It is important to note that within suggested geometrical interpretation the notions of the less general, macroscopic theory (4-momentums) are defined by (18) trough the notions of more general microscopic theory (wave parameters of the defect periodical movement). This looks much more natural than the opposite definitions (4) within traditional interpretation.

4. Geometrization of electromagnetic waves

It is known that the Maxwell equations for electromagnetic waves in vacuum, when they are written in the Majorana form, looks like the Dirac equations (1)[10]. Therefore, it seems reasonable to use for their geometrization the same arguments as we used for the Dirac equation geometrization. As for topological interpretation of solution (2) of the Dirac equation, we suggest that wave solution of Maxwell equations does not define any wave process into the space but describes the movement of the space topological defect. Beeing bivector (not spinor), solution of the Maxwell equation does not realize the representation of reflections along three different axes of 4-space as it was for bispinors. This solution is transformed into itself only in result of the reflection of space axes. Therefore, this solution realizes representation of a sliding symmetry in 4-space only along time–axis. This distinguishes the supposed fundamental group from the fundamental group considered in previous Sections (four sliding symmetries along four perpendicular axes).

There is no investigation in topology (as it is known to author) where 4-manifolds with above fundamental group were considered. Therefore, we again can establish connections between geometrical properties of the manifold and observable physical properties of electromagnetic waves only using low-dimensional analogies. Wave-corpuscular dualism of electromagnetic waves and possibility of stochastic behavior can be demonstrated in the same manner as in Sections 2 and 3. But electromagnetic waves have some additional important property—their velocity does not depend on the source motion. We will show below how geometrical properties of the closed topological 4-manifold can explain this fact.

Suppose that 4-manifold corresponding to electromagnetic wave has the form of topological product $M^3(\mathbf{r}) \times M^1(\mathbf{r}, t)$, that is it can be represented as a production of nonorietable three-dimensional euclidean closed manifold $M^3(\mathbf{r})$ and one-dimensional pseodoeuclidean manifold $M^1(\mathbf{r}, t)$ homeomorphic to a pseudoeuclidean circle. The formal reason for such representation is the distinguished role of the euclidean space within fundamental group: only into euclidean subspace translation group is combined with reflections. Consider now a low-dimensional analogy that explains an independence of light velocity on the source motion.

Instead of four-dimensional manifold $M^3(\mathbf{r}) \times M^1(\mathbf{r}, t)$ we consider, as in previous Section, two-dimensional analogy—manifold $S^1 \times S^1_t$, where S^1 is a one-dimensional euclidean circle and S^1_t is a pseudoeuclidean circle. This manifold was considered in Section 3, and it looks like a hyperboloid. For electromagnetic waves $m = 0, E = cp$. Within our notation it leads to relation $p_1 = p_2 = p$ or $\lambda_1 = \lambda_2 = \lambda$. Therefore, there have to be only one parameter with dimensionality of length, and this will be the case if S^1_t is a pseudoeuclidean circle of zeroth radius. Equation for such circle has the form $x^2 - t^2 = 0$, and hyperboloid is transformed into a cone. This means that the points representing in this example electromagnetic wave move with velocity equals ± 1 ($\pm c$–in chosen units system), and this velocity does not depend on coordinate frame rotations (does not depend on transfer from one moving inertial frame to another). From topological point of view this result is a consequence of the fact that the zeroth radius can be considered as topological invariant. Therefore, within geometrical approach light velocity appears to be topological invariant of the manifold representing electromagnetic wave, and this is the reason of its independence of the source motion.

Geometrical interpretation of the Dirac equation for a hydrogen atom is considered at hep-th/0605060.

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Выбор космологической модели Фридмана

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1. Модель Фридмана, «стандартный» подход

Известно, что геометрию вселенной можно аппроксимировать изотропной метрикой Фридмана, где компоненты метрического тензора пропорциональны квадрату радиуса кривизны пространства $a(t)$ — это произвольная функция, зависящая лишь от временнóй координаты $x^0 = c\,t$. При выборе космологической модели для получения конкретного вида функции $a(t)$ решают уравнения Эйнштейна, которые в рамках *закрытой* изотропной модели они сводятся к *двум* обыкновенным дифференциальным уравнениям второго порядка относительно *трех* неизвестных функций от t:

$$\dot{a}^2 + 1 = 8\pi \cdot a^2 T_0^0 \ ; \quad \ddot{a} = 4\pi \cdot a \cdot \left(T_\alpha^\alpha - T_0^0 \right) \ .$$

Для пополнения системы уравнений обычно используют тензор энергии-импульса, построенный еще в механике специальной теории относительности (СТО) для математического моделирования законов сохранения энергии и импульса сплошных сред:

$$T_{ik} = (p + \varepsilon)\cdot u_i\, u_k - \ p \cdot g_{ik} \qquad (1)$$

Для «холодной» модели ($T_\alpha^\alpha = -3p \ = 0$; $T_0^0 = \varepsilon = \rho c^2$) получают «стандартное» решение, которое совпадает с финитным движением гравитирующего шара из холодной материи:

$$R(t) = 1 - \cos\eta\,; \quad t = \eta - \sin\eta\,; \quad d\eta = dt \big/ R(t)$$

Следует отметить, что выбор, так называемой, «стандартной» модели Фридмана далеко не однозначен. Кроме закрытой возможны открытая, плоская, а также инфляционная модели. Нетрудно убедиться, что все варианты «стандартной» модели противоречат простым *соображения размерности*. Действительно, переход от реальной вселенной к изотропной модели заключается, по существу, в некоторой процедуре усреднения плотности всех, без исключения, материальных структур равномерно по пространству. Получить же определенный эталон длины в рамках абсолютно однородной модели невозможно.

Следовательно, функциональная зависимость радиуса кривизны от времени $a(t)$ для изотропной модели реальной вселенной, не может содержать *локальную* константу размерности длины. Подобному требованию удовлетворяет только линейная зависимость «$a = c\,t$». Но для такого решения уравнения Эйнштейна с T_i^k из механики СТО приводят к нефизическому уравнению состояния вещества с отрицательным давлением : $p = -\rho c^2 \big/ 3$. В свое время (1977) это обстоятельство вызвало у автора *удивление* и побудило, в конечном счете, отказаться, следуя Эйнштейну, от «*специального выбора тензора энергии импульса*» T_i^k из механики СТО.

В ОТО корректное физическое обоснование имеет, строго говоря, лишь временнáя компонента: $T_0^0 = \varepsilon = \rho c^2$. К такому выводу можно прийти, анализируя физический смысл соотношения $2\sqrt{-g} \cdot T_{i;k}^k \equiv 2\partial\left(\sqrt{-g}\,T_i^k\right) \big/ \partial x^k - \sqrt{-g} \cdot g_{kl,i}\, T^{kl} = 0$, аналогично тому, как это проделывается в механике СТО. Для этого запишем это уравнение в синхронной системе отсчета $g_{00} = 1,\ g_{0\alpha} = 0$, где временная и пространственная части метрики разделены. Затем проинтегрируем соотношения по 4-объему Ω, ограниченному с двух сторон гиперповерхностями $(t = t_1,\ t = t_2)$, используя четырехмерную теорему Гаусса:

$$\oint_{\Sigma} T_i^k \sqrt{-g}\, dS_k = \frac{1}{2} \int_{\Omega} T^{kl} g_{kl,i} \sqrt{-g}\, d\Omega .$$

Подставляя сюда элемент гиперповерхности Σ в виде $dS_k = (dV, 0, 0, 0)$, получим равенство: $P_i(t_2) - P_i(t_1) = (1/2c) \cdot \int_{\Omega} T^{kl} g_{kl,i} \sqrt{-g}\, d\Omega.$ Здесь: $c \cdot P_i = \int_{V} T_i^0 \sqrt{-g}\, dV$ — интеграл по гиперповерхности $t = \text{const}$. В СТО всегда можно выбрать глобальную галилееву метрику, где правая часть соотношения обращается в нуль. Это дает основание отождествить сохраняющуюся величину с 4-импульсом замкнутой механической системы: $P^i = \left(E/c, \vec{P}\right)$. Отсюда, а также из интегрального выражения для и следует известная физическая трактовка компонент $T_0^0 = \varepsilon \ldots \text{плотность энергии}; \ c^{-1} \cdot T_0^{\alpha} \ldots \text{плотность импульса}$. Её можно сохранить и в ОТО, чтобы сохранить преемственную связь с механикой СТО. Следуя известной процедуре (см. [2], § 33), проинтегрируем эти же соотношения по некоторому объему пространства *V*, используя при этом трёхмерную теорему Гаусса:

$$\frac{\partial}{\partial t} \int_{V} T_0^0 \sqrt{-g}\, dV = -\oint c T_0^{\beta} \sqrt{-g}\, df_{\beta} + \int_{V} \left(\frac{1}{2} T^{\alpha\beta} \frac{\partial g_{\alpha\beta}}{\partial t} \right) \sqrt{-g}\, dV$$

$$\frac{\partial}{\partial t} \int_{V} \frac{1}{c} T_{\alpha}^0 \sqrt{-g}\, dV = -\oint T_{\alpha}^{\beta} \sqrt{-g}\, df_{\beta} + \int_{V} \left(\frac{1}{2} T^{\beta\nu} \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} \right) \sqrt{-g}\, dV$$

Контурный интеграл в правой части первого интегрального равенства берётся по поверхности Σ, охватывающей трёхмерный объем. В галилеевой системе отсчета СТО интегралы по объему в правой части этих же уравнений обращаются в нуль. Поэтому оставшиеся интегральные соотношения допускают трактовку величин cT_0^{β} — как плотности потока энергии и cT_{α}^{β} — как плотности потока импульса.

В ОТО ситуация меняется. Здесь правая часть уравнений, помимо поверхностных, содержит интегралы по объему, что делает невозможным строгое сохранение прежней трактовки пространственных компонент тензора энергии-импульса. Теперь изменение энергии в объеме *V* обусловлено не только потоком величины $c\,T_0^{\beta}$ через его границу. Имеется ещё и второе слагаемое в правой части уравнения, где подынтегральное выражение пропорционально пространственным компонентам $T^{\alpha\beta}$.

Особенно отчетливо несостоятельность прежней трактовки T_{α}^{β} проявляется в рамках изотропной модели Фридмана, где в сопутствующей системе отсчета компоненты, пропорциональные плотности импульса, равны нулю. Вместе с ними обращается в нуль и контурный интеграл по поверхности в первом интегральном соотношении. Отсюда непреложно следует, что в модели Фридмана пространственные компоненты тензора энергии-импульса не имеют никакого отношения к плотности потока импульса, то есть, к давлению. Следовательно, использование формулы $\boxed{T_i^k = (p + \varepsilon)\, u_i u^k - p\,\delta_i^k}$ в правой части уравнений Эйнштейна, как это делается при выборе «стандартной» космологической модели, ничем не обосновано.

2. Локальный принцип соответствия

Для пополнения системы уравнений предлагается использовать локальный принцип соответствия (ЛПС) релятивистской теории тяготения Эйнштейна механике Ньютона. Поскольку ОТО Эйнштейна является, строго говоря, локальной теорией, соответствие также следует установить *локальное*. Законы Ньютона применимы при выполнении *двух условий*.

Макроскопическая скорость вещества должна быть мала по сравнению со скоростью света, а безразмерное значение характерного потенциала φ в окрестности рассматриваемой точки — мало по сравнению с единицей: $v/c \ll 1$; $\varphi/c^2 \ll 1$. Для выполнения первого условия достаточно выбрать локально-сопутствующую систему отсчета, где $v = 0$. Второе условие выполняется, если перейти в свободно падающий «лифт Эйнштейна», где 4-сила тяготения близка к нулю, а метрика слабо отличается от галилеевой.

Переход к локально-инерциальным координатам $\tilde{x}^i$ осуществляется в малой окрестности начала координат посредством следующего преобразования:

$$x^i = \tilde{x}^i - \frac{1}{2}\Gamma^i_{kl}\tilde{x}^k\tilde{x}^l + \frac{1}{3}a^i_{klm}\tilde{x}^k\tilde{x}^l\tilde{x}^m + \cdots. \tag{2}$$

Здесь и в последующих выражениях данного раздела коэффициенты разложения по $\tilde{x}^i$ содержат производные от исходного метрического тензора, вычисленные в начале координат C_0. Запишем метрический тензор $\tilde{g}_{ik}(\tilde{x}) = \dfrac{\partial x^l}{\partial \tilde{x}^i}\dfrac{\partial x^m}{\partial \tilde{x}^k}g_{lm}(x)$ в виде разложения: $\tilde{g}_{ik}(\tilde{x}) = g^{(0)}_{ik} + (g_{ik,l} - \Gamma_{i,kl} - \Gamma_{k,il})\tilde{x}^l + A_{iklm}\tilde{x}^l\tilde{x}^m + \cdots$

Неизвестные коэффициенты A_{iklm} связаны с величинами a^i_{klm} и g_{ik} в точке C_0. Вследствие тождества, связывающего частную производную от метрического тензора и символы Кристоффеля $g_{ik,l} \equiv \Gamma_{i,kl} + \Gamma_{k,il}$, в разложении метрического тензора исчезают слагаемые, линейные по $\tilde{x}^i$. В результате получаем:

$$\tilde{g}_{ik}(\tilde{x}) = g^{(0)}_{ik} + A_{iklm}\tilde{x}^l\tilde{x}^m + \cdots. \tag{3}$$

Матрица A_{iklm} симметрична к перестановке в парах (ik) и (lm). Никакими другими свойствами симметрии, вследствие произвольности величин $\tilde{x}^i$ в разложении (3), коэффициенты A_{iklm} обладать не могут. Этому требованию удовлетворяет выражение:

$$A_{iklm} = \frac{1}{2}(g_{ik,l,m} + \Gamma_{n,ik}\Gamma^n_{lm}) + N_{iklm}\ . \tag{4}$$

Здесь $N_{iklm}(x^n)$ — геометрический объект, также симметричный к перестановке индексов в парах (ik) и (lm). С помощью величин N_{iklm} тензор кривизны можно записать в удобной для данного рассмотрения форме:

$$R_{iklm} = N_{iklm} + N_{klim} - N_{ilkm} - N_{kmil}\ . \tag{5}$$

Для обоснования соотношения (4) используем равенство исходного и преобразованного тензоров кривизны в начале координат:

$$R_{iklm}(0) = \tilde{R}_{iklm}(0). \tag{6}$$

Представляя соотношение (6) в развернутом виде, получим следующее равенство:

$$N_{imkl} + N_{klim} - N_{ilkm} - N_{kmil} = A_{imkl} + A_{klim} - A_{ilkm} - A_{kmil}\,. \tag{7}$$

Соотношению (7) удовлетворяет равенство (4).

В итоге получаем окончательный вид формулы (3):

$$\tilde{g}_{ik}(\tilde{x}) = g^{(0)}_{ik} + N_{iklm}\tilde{x}^l\tilde{x}^m + \cdots. \tag{8}$$

Теперь все готово для установления локального соответствия ОТО механике Ньютона. Эта цель достигается посредством сравнения уравнения движения частицы в поле тяготения и уравнения геодезической в кривом пространстве-времени:

$$\frac{d^2X^\alpha}{dt^2} = -\nabla^\alpha\varphi \Leftrightarrow \left\{\frac{d^2X^i}{ds^2} = -\Gamma^i_{kl}u^k u^l\right\}\ . \tag{9}$$

Осталось приравнять правые части уравнений (9), записанных вблизи начала локально-геодезической системы отсчета:

$$\nabla^{\alpha}\varphi = c^2\tilde{\Gamma}^{\alpha}_{00} = g^{\alpha m}\left(2\,A_{0m0l} - A_{00ml}\right)\cdot\tilde{x}^{l} \tag{10}$$

Вычислив с помощью выражения (8) величины $\tilde{\Gamma}^{\alpha}_{00}$, можно получить искомое соответствие между ньютоновским гравитационным потенциалом и метрикой риманова пространства-времени в локально-геодезической системе отсчета произвольной гравитирующей механической системы. Однако для практических целей удобно преобразовать полученное соответствие к трехмерному скалярному соотношению.

Возьмем трехмерную дивергенцию от обеих частей равенства (10), а также используем дифференциальную форму закона всемирного тяготения в виде уравнения Пуассона:

$$\Delta\,\varphi = 4\pi\,G\cdot\rho\,. \tag{11}$$

В итоге получим трехмерно-скалярную форму локального принципа соответствия:

$$\frac{\partial\tilde{\Gamma}^{\alpha}_{00}}{\partial\tilde{x}^{\alpha}} = \frac{4\pi\,G}{c^2}\,\rho\,. \tag{12}$$

Вычисляя дивергенцию $\tilde{\Gamma}^{\alpha}_{00,\alpha}$ с помощью используя формулы (8), запишем соотношение (12) в окончательном виде

$$2N^{0\alpha}_{\alpha 0} - N^{0\alpha}_{0\alpha} = \frac{4\pi\,G}{c^2}\cdot\rho\,. \tag{13}$$

Равенство (13), вообще говоря, не совпадает с аналогичным временным уравнением Эйнштейна:

$$\frac{1}{2}\left(R^{0}_{0} - R^{\alpha}_{\alpha}\right) = \frac{8\pi\,G}{c^2}\cdot\rho\,. \tag{14}$$

Исключая плотность из уравнений (13) и (14), получим искомое соотношение:

$$4\left(2N^{0\alpha}_{\alpha 0} - N^{0\alpha}_{0\alpha}\right) = R^{0}_{0} - R^{\alpha}_{\alpha}\,. \tag{15}$$

Оно представляет собой дополнительное ограничение, которое накладывает на метрику риманова пространства-времени требование выполнимости локального соответствия ОТО теории Ньютона.

Уравнение инвариантно (в классе синхронных систем отсчета) по отношению к преобразованиям координат, не затрагивающих времени. Следовательно, принцип локального соответствия в форме (15) может быть использован для выбора космологической модели из класса изотропных, где сопутствующая глобальная система отсчета всегда является синхронной.

3. Выбор космологической модели Фридмана

После усреднения по областям пространства, имеющим размеры более 100 Мпс, вещество звезд можно считать *равномерно* распределенным во вселенной со средней плотностью $\rho_{*} \approx 10^{-31}$ г/см^3. Косвенные астрофизические данные свидетельствуют о наличии «скрытой массы», примерно на два порядка превышающей массу звезд. Поэтому следует ожидать, что $\rho_{\mathrm{W}} \approx 10^{-29}$ г/см^3. Измерения свойств «реликтового» излучения свидетельствуют в пользу *изотропии* вселенной. Физические предположения об однородности и изотропии — *основные космологические постулаты* [6] — положены в основу математической модели Фридмана, которая аппроксимирует реальную вселенную.

Свойствам однородности и изотропии отвечают аналогичные свойства метрики пространства космологической модели. В каждый момент мирового времени t все точки пространства можно характеризовать одним скалярным параметром — радиусом кривизны $a(t)$. Математический аппарат теории допускает альтернативу открытой и закрытой моделей.

Квадрат интервала и метрика *закрытой* изотропной модели имеют вид:

$$ds^2 = c^2dt^2 - a^2(t)\cdot\left\{d\chi^2 + \sin^2\chi\cdot\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right\}$$

$$x^i = \left(x^0, x^1, x^2, x^3\right) = \left(ct, \chi, \theta, \varphi\right); \qquad (16)$$

$$g_{00} = 1; \quad g_{11} = -a^2(t); \quad g_{22} = -a^2(t)\cdot\sin^2\chi; \quad g_{33} = -a^2(t)\cdot\sin^2\chi\cdot\sin^2\theta.$$

Здесь: θ и φ — обычные угловые сферические координаты; χ — дополнительная «угловая» координата, определяющая расстояние $l = a\cdot\chi$ от начала координат ($\chi = 0$). Метрика *открытой* модели получается из (16) формальной заменой: $\chi \Rightarrow i\chi$; $a \Rightarrow ia$.

Уравнения Эйнштейна для изотропной метрики Фридмана сводится к *двум* обыкновенным дифференциальным уравнениям второго порядка относительно *трех* неизвестных функций от времени ($a(t)$, $T_0^0(t)$, $T_\alpha^\alpha(t)$):

$$\frac{\dot{a}^2 + \delta}{a^2} = \frac{8\pi G}{3c^4}\cdot T_0^0\ ; \quad \frac{\ddot{a}}{a} = \frac{4\pi G}{3c^4}\cdot\left(T_\alpha^\alpha - T_0^0\right). \qquad (17)$$

Здесь: $\dot{a} = \dfrac{da}{c\,dt}$; $\ddot{a} = \dfrac{d^2a}{c^2dt^2}$; $\delta = \begin{cases} +1 \text{ ... закрытая модель} \\ -1 \text{ ... открытая модель} \end{cases}$.

Так как мы отказались от (1), этого недостаточно полноты системы. В модели Фридмана определенный смысл имеют лишь следующие компоненты тензора энергии-импульса:

$$T_0^0 = \varepsilon = \rho c^2\ ; \ T_0^\alpha = 0 \qquad (18)$$

(*плотность импульса в сопутствующей системе равна нулю*).

Необходимо обратиться к локальному принципу соответствия (13), который для метрики Фридмана принимает вид:

$$\frac{\dot{a}^2}{a^2} = \frac{4\pi G}{3c^2}\cdot\rho, \qquad (19)$$

Это уравнение пополняет систему (17).

Исключая величины T_0^0 и ρ из соотношений (17), и (19), получим дифференциальное уравнение относительно искомой функции $a(t)$: $\dot{a} = \delta$. (20)

В классе положительных значений радиуса кривизны имеем *однозначное* решение уравнения (20) для $\delta = +1$ (закрытая модель): $a(t) = c\,t$. (21)

Подставляя (21) во второе уравнение (17), получим равенство временной и суммы пространственных компонент тензора энергии-импульса: $T_0^0 = T_\alpha^\alpha$. (22)

Отсюда, если придерживаться формулы (1), следует уравнение состояния $\varepsilon = -3p$, не имеющее физического смысла для реальной материи. Это наглядно подтверждает необоснованность традиционной физической трактовки (1) *пространственных компонент* тензора энергии-импульса в уравнениях ОТО.

Таким образом, отказ от традиционной трактовки правой части уравнений Эйнштейна вместе с использованием локального принципа соответствия позволило осуществить *однозначный* выбор закрытой космологической модели «$a = c\,t$» из широкого класса изотропных.

3.1. Специфика новой космологической модели

Закрытая модель Фридмана — это всюду однородное физическое тело, каждая точка которого является *равноправным центром симметрии*. В природе не существует других физических объектов с подобными свойствами. Так, например, однородный шар, погруженный в пустое плоское пространство, имеет лишь один центр симметрии, к которому направлено его радиально неоднородное гравитационное поле. Однородность шара нарушается на его границе. Однородность модели Фридмана сохраняется всюду в течение всей ее эволюции.

К закрытой космологической модели в целом не применима обычная форма закона тяготения Ньютона. Однако можно утверждать, что в каждой точке модели равнодействующая всех сил тяготения обращается в нуль. Таким образом, локальные внутренние силы тяготения, понимаемые как градиент некоторого потенциала, в модели Фридмана отсутствуют (в противном случае нарушалась бы ее изотропия). Подобная система не может расширяться с *ускорением*, как обычное механическое тело конечных размеров, движущееся под действием собственного поля тяготения. Эти общие соображения по динамике механической системы далее подтверждаются конкретным и однозначным выбором космологической модели, радиус кривизны которой, по сути дела, вообще остается *постоянным*.

Вселенная, аппроксимируемая закрытой моделью, имеет *конечный* объем V и массу M. Наружная граница и внешнее асимптотически плоское пространство, разумеется, отсутствует. Любой точке закрытой модели соответствует максимально удаленная от нее точка противоположного полюса пространства:

$$V = 2\pi^2 a^3 \;\; ; \quad M = V \cdot \rho \;\; ; \;\; l_{max} = \pi a \;\; .$$

3.2. Соображения размерности

В последующем будем иногда использовать «релятивистски-гравитационную» систему единиц (RG-система), полагая $c = G = 1$. Уравнения Эйнштейна в RG-системе не содержат никаких физических констант. Все размерные физические величины здесь можно выразить *единообразно*, например, в сантиметрах: 1 г $\cong 0.74247 \cdot 10^{-28}$ см ; 1 сек $\cong 3 \cdot 10^{10}$ см. Отношение массы космологической модели к ее радиусу кривизны — величина постоянная, равная в RG-системе: $M / a = 3\pi / 2$

Можно предположить, что и *третья* мировая «константа» $\hbar = 1.0545887 \cdot 10^{-27}$ (г·см^2·сек$^{-1}$) — «постоянная» Планка — также претендует на статус фундаментальной, то есть отражают *универсальные* свойства материи — *квантовые*. Они проявляются в дискретности микроструктуры материального мира. На данном этапе эволюции вселенной численное значение $\hbar$, имеющей в RG-системе размерность квадрата длины, равна: $\hbar \approx 2.6 \cdot 10^{-66}$ см^2 ; $\sqrt{\hbar} \approx 1.6 \cdot 10^{-33}$ см .

Используя набор из *трех* «констант» со взаимно независимыми размерностями $\{c, G, \hbar\}$, можно ввести *фундаментальные* планковские эталоны длины r_0, времени t_0 и массы m_0:

$$r_0 = \sqrt{\hbar G / c^3} \cong 1.616 \cdot 10^{-33} \;(\mathrm{cm}) \; ; \quad t_0 = r_0 / c = 0.539 \cdot 10^{-43} \;(\mathrm{s}) \;\; ;$$
$$m_0 = \frac{3\pi}{2} \sqrt{\hbar c / G} \cong 1.029 \cdot 10^{-4} \;(\mathrm{g}) \, . \qquad (23)$$

В RG-системе планковские единицы пропорциональны корню квадратному из «постоянной» Планка: $r_0 = t_0 = \dfrac{2}{3\pi} m_0 = \sqrt{\hbar}$.

В определение планковской массы (23) дополнительно введен множитель $\dfrac{3\pi}{2} = \dfrac{V_{\mathrm{Fr}}}{V_{\mathrm{Dec}}} = \dfrac{2\pi^2 \cdot R^3}{(4\pi/3) \cdot R^3}$, равный отношению объема закрытой модели Фридмана, имеющей радиус кривизны R, к объему шара такого же радиуса в плоском декартовом пространстве. Это упрощает запись соотношений космологической модели:

$$a / r_0 = t / t_0 = M / m_0 = N \; ;$$
$$N = e^{\eta} \; , \; d\eta = dt / a(t) \; ; \;\; \eta = \ln \frac{t}{t_0} \; . \qquad (24)$$

Здесь η — *безразмерное* мировое время. Функциональная зависимость радиуса кривизны *космологической модели* от безразмерной временной координаты $a(\eta) = e^{\eta}$ простирается в неограниченном интервале безразмерного мирового времени η: $[-\infty < \eta < +\infty]$. Последнее обстоятельство снимает традиционно трудный для космологии вопрос о начальном моменте эволюции.

Величину N будем называть фундаментальным параметром микроструктуры. Для того чтобы понять физический смысл нового параметра N и получить представление о фундаментальной структуре материи в самом общем плане, воспользуемся *соображениями размерности*.

Имеются три фундаментальных закона природы, которым подчиняются все, без исключения формы материи: релятивизм (предельная скорость распространения возмущений), гравитация и квантовые законы. Им соответствует набор трех универсальных мировых констант со взаимно независимой размерностью: $\{c, G, \hbar\}$. Эквивалентный ему набор — планковские единицы длины, времени и массы: $\{r_0, t_0, m_0\}$. По сути метода размерностей набор величин характеризует состояние рассматриваемой физической системы (в данном случае – материи), управляемой тремя фундаментальными законами природы. А именно, *основная* часть материи во вселенной *постоянно* находится в областях пространства, имеющих размеры r_0 где в течение интервала времени t_0 концентрируется масса m_0 с плотностью $\rho_0 \approx m_0/(r_0)^3 \approx 10^{95}$ г/см^3. Кратковременное искривление пространства здесь достигает предельного значения (радиус кривизны пространства сравним с размерами объекта), представляя собою локальный, предельно короткий и сильный *всплеск кривизны*, который для определенности назовем — планкеон, поскольку его появление обязано «постоянной» Планка. Столь сильное локальное искривление пространственно-временной метрики, разумеется, не может быть стабильным. Как следует из соображений размерности и общих физических принципов, оно расплывается за время $\sim t_0$, превращаясь, очевидно, в расходящуюся волну кривизны с шириной $\sim r_0$. Пройдя расстояние $l = \pi \cdot a$, волна сойдется на ее противоположном полюсе. В соответствии с соотношениями (24), интервал безразмерного мирового времени при этом составит $\Delta\eta = \pi$. Таким образом, через равные интервалы мирового времени последовательно происходит отражение возмущения метрики от противоположных полюсов пространства. Получаем своего рода *универсальные мировые часы*, ход которых не зависит от выбора единиц измерения.

4. Выход за рамки космологической модели

Совокупность планкеонов, в силу своей фундаментальности, должна лежать в основе всех наблюдаемых материальных структур. Так, например, следует ожидать, что в объеме нуклона фундаментальный всплеск кривизны появляется в среднем один раз за $\Delta t_n \approx t_0 \cdot (m_0/m_n) \approx 10^{-23}$ сек. В результате плотность нуклона оказывается на ~ 80 порядков меньше фундаментальной планковской плотности $\rho_0 \approx m_0/(r_0)^3 \approx 10^{95}$ г/см^3. Непосредственно планкеоны, разумеется, не наблюдаемы. Приведенные выше общие *соображения размерности* в дальнейшем будут конкретизированы на основе новой космологической модели.

В каждый момент мирового времени во вселенной находится N планкеонов – всплесков кривизны, в которых сосредоточена основная масса вселенной. Численное значение радиуса кривизны вселенной и ее массы, выраженное в единицах, фундаментального параметра $\sqrt{\hbar}$ экспоненциально растет с ходом безразмерного мирового времени η. При $\eta = 0$ во вселенной условно находился один планкеон, а при $\eta < 0$ — доли планкеона.

Таким образом, сам планковский параметр $r_0 = \sqrt{\hbar}$ также является функцией η: $r_0(\eta) \equiv \sqrt{\hbar(\eta)} = r_{0\mathrm{p}} \cdot e^{-(\eta-\eta_\mathrm{p})}$, где $r_{0\mathrm{p}} = r_0(\eta_\mathrm{p})$— значение планковского параметра на данном (present), наблюдаемом нами, этапе эволюции, когда $\eta = \eta_\mathrm{p}$. При этом параметры вселенной остаются постоянными в течение всей ее эволюции:

$$a(\eta) = M(\eta) = N(\eta) \cdot r_0(\eta) = e^{\eta} \cdot r_{0\mathrm{p}} \cdot e^{-(\eta-\eta_\mathrm{p})} = r_{0\mathrm{p}} \cdot e^{\eta_\mathrm{p}} = a_\mathrm{p} = \mathrm{const}. \tag{25}$$

Следовательно, эволюция реальной вселенной заключается, по сути дела, в постоянном увеличении фундаментального параметра микроструктуры *N*. Величина $\sqrt{\hbar}$ в процессе подобной *структурной* эволюции вселенной монотонно *уменьшается*.

Свидетельством этого является наблюдаемый астрофизиками эффект «красного смещения» Хаббла, согласно которому длина волны определенного участка спектра увеличивается (смещается в «красную» сторону) по мере удаления источника (звезды, галактики) от наблюдателя

$$\frac{\Delta\lambda}{\lambda_\mathrm{p}} \equiv \frac{\lambda(\eta) - \lambda_\mathrm{p}}{\lambda_\mathrm{p}} = H \cdot \Delta l. \tag{26}$$

Здесь: $H \cong 55 \frac{\mathrm{km/sec}}{\mathrm{Mps}} \cong 1.6 \cdot 10^{-18}\ \mathrm{sec}^{-1}$ — «постоянная» Хаббла;

$\Delta l = a_p \cdot \Delta\chi \equiv a_p \cdot \Delta\eta$, где $\Delta\eta = \eta_\mathrm{p} - \eta$.

Собственное время фотона, движущегося со скоростью света, от момента излучения $\eta < \eta_\mathrm{p}$ до регистрации его земным наблюдателем, равно нулю. Следовательно, его длина волны за время полета не может измениться. Отношение эталона длины в момент η к моменту наблюдения η_p равно $e^{-(\eta-\eta_\mathrm{p})} = e^{\Delta\eta}$. Таково же, очевидно, и соотношение между соответствующими длинами волн:

$$\lambda(\eta) = \lambda_\mathrm{p} \cdot e^{\Delta\eta}. \tag{27}$$

Поэтому выражение (26) можно записать в виде

$$e^{\Delta\eta} - 1 = H \cdot a_\mathrm{p} \cdot \Delta\eta . \tag{28}$$

Для малых значений величины $\Delta\eta$, равенство можно (28) упростить:

$$a_\mathrm{p} = \frac{e^{\Delta\eta} - 1}{H \cdot \Delta\eta} \approx \frac{1}{H} \text{ если } \Delta\eta \ll 1.$$

Отсюда получаем численные значения основных параметров наблюдаемой вселенной — ее размер (радиус кривизны пространства), средняя плотность, характерное время эволюции, масса (все это в единицах CGS), а также фундаментальный параметр микроструктуры и безразмерное время эволюции:

$$a_\mathrm{p} = \frac{c}{H} \cong 2 \cdot 10^{28}\,\mathrm{cm}\ ; \quad \rho_\mathrm{p} = \frac{3H^2}{4\pi G} \cong 10^{29} \frac{\mathrm{g}}{\mathrm{cm}^3};$$

$$t_\mathrm{p} = \frac{1}{H} \cong 6.25 \cdot 10^{17}\ \mathrm{sec}(20\ \text{mlrd. years});\ M_\mathrm{p} = \frac{3\pi c^3}{2GH} \cong 10^{57}\,\mathrm{g}; \tag{29}$$

$$N_\mathrm{p} = \frac{c}{r_0 \cdot H} \approx 1.16 \cdot 10^{61}\ ; \quad \eta_\mathrm{p} = \ln(N_\mathrm{p}) \approx 140.6$$

Примечательно, что численное значение плотности ρ_{P} в соотношениях (29) примерно на два порядка выше средней плотности наблюдаемого светящегося вещества звезд. Это соответствует имеющимся представлениям о наличии темной «скрытой» массы во вселенной.

4.1. Фокусировочные состояния

Осуществим выход за рамки космологической модели, используя, прежде всего ее специфику — пространственную замкнутость и возможность неограниченного числа отражений произвольного возмущения от противоположных полюсов пространства через одинаковые промежутки безразмерного времени: $\Delta\eta = \pi$.

Модель « $a = c\,t$ » была получена, прежде всего, из простых *соображений размерности*. Воспользуемся этим же приемом для того, чтобы получить представление о фундаментальной структуре материи в самом общем плане. Имеются три фундаментальных закона природы, которым подчиняются все, без исключения формы материи: релятивизм (предельная скорость распространения возмущений), гравитация и квантовые законы. Им соответствует набор трех универсальных мировых констант со взаимно независимой размерностью: $\{c, G, \hbar\}$. Эквивалентный ему набор — планковские единицы длины, времени и массы: $\{r_0, t_0, m_0\}$. По сути метода размерностей набор величин характеризует состояние рассматриваемой физической системы (в данном случае – материи), управляемой тремя фундаментальными законами природы. А именно, *основная* часть материи во вселенной *постоянно* находится в областях пространства, имеющих размеры r_0 где в течение интервала времени t_0 концентрируется масса m_0 с плотностью $\rho_0 \approx m_0/(r_0)^3 \approx 10^{95}$ г/см^3. Кратковременное искривление пространства здесь достигает предельного значения (радиус кривизны пространства сравним с размерами объекта), представляя собою локальный, предельно короткий и сильный *всплеск кривизны*, который для определенности назовем — планкеон, поскольку его появление обязано «постоянной» Планка.

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В каждый момент мирового времени во вселенной находится $N{\cdot}$ планкеонов – всплесков кривизны. Столь сильное локальное искривление пространственно-временной метрики, разумеется, не может быть стабильным. Как следует из соображений размерности, оно существует лишь короткое время $\sim\ t_0 = r_0$ (RG-system), превращаясь, по-видимому, в расходящуюся волну кривизны с шириной $\sim r_0$. Основанием для последнего утверждения, кроме общих теоретических соображений о поведении волнового пакета, является следующее дифференциальное соотношение:

$$u'' - \ddot{u} + \frac{2\cos\chi}{\sin\chi}u' = 2\left[\left(u'\right)^2 - \left(\dot{u}\right)^2 - \left(u'' - \ddot{u} + \frac{2\cos\chi}{\sin\chi}u'\right)\cdot u\right]$$

$$u = u\left(t,\chi\right)\ ;\ \dot{u} = \frac{\partial u}{\partial t}\quad;\quad u' = \frac{\partial u}{\partial \chi};\quad 0 \le \chi \le \pi\ . \tag{30}$$

Оно получается из самого простого соотношения предполагаемой системы уравнений ЕТП

$$R_0^0 = 0 \tag{31}$$

для метрического тензора следующей формы:

$$g_{ik}=\begin{pmatrix} 1+2u(t,\chi) & 0 & 0 & 0 \\ 0 & -1-2u(t,\chi) & 0 & 0 \\ 0 & 0 & -\sin^2\chi & 0 \\ 0 & 0 & 0 & -\sin^2\chi\cdot\sin^2\theta \end{pmatrix} \tag{32}$$

Здесь $u(t,\chi)$ это — некоторое «сферическое» возмущение в метрике закрытой статической модели Фридмана с единичным радиусом кривизны $a=1$. При $\chi\to 0$ получаем обычное волновое уравнение с нелинейной правой частью, которую можно рассматривать как плотность источника.

Пройдя расстояние $l=\pi\cdot a$, волна сфокусируется на ее противоположном полюсе, вновь создав сильное локальное искривление 4-пространства, аналогичное исходному. Схематически это изображено на рис.1, где по оси абсцисс — время эволюции η, а по оси ординат — расстояние между полюсами пространства. Как показывают наблюдения в локальных областях пространства имеют место относительно устойчивые по времени скопления вещества (нуклоны, например). Им соответствует такая же *устойчивая* цепочка фокусировочных состояний (Plankeons). Она помечена жирной буквой **P** в левом нижнем углу рис.1. От нее вверх направо идет цепочка расходящихся волн кривизны. По прохождении экватора пространства ($\chi=\pi/2$), фронты цепочки элементарных волн выгибаются в сторону противоположного полюса, и цуг волн фокусируется в левой верхней точке F. Так исходный материальный объект P породил второй объект F (правда, на другом полюсе пространства космологической модели). Из точки F с момента $\eta_F=\eta_P+\pi$ направо вниз начинает распространяться следующий цуг волн, порождающий второй объект P. В этом же районе будет находиться и первоначальный объект P, переместившийся по временной координате на величину $\eta_{PP}=\eta_P+2\pi$. Локальный наблюдатель зафиксирует в данном районе «спонтанное рождение» второй стабильной частицы: $P\Rightarrow PP$. Однако *глобальный* анализ ситуации показывает, что вторая «частица» — отраженное «эхо» первичного объекта. Таким образом, через равные интервалы безразмерного мирового времени $\Delta\eta=\pi$ число устойчивых «частиц» во вселенной удваивается.

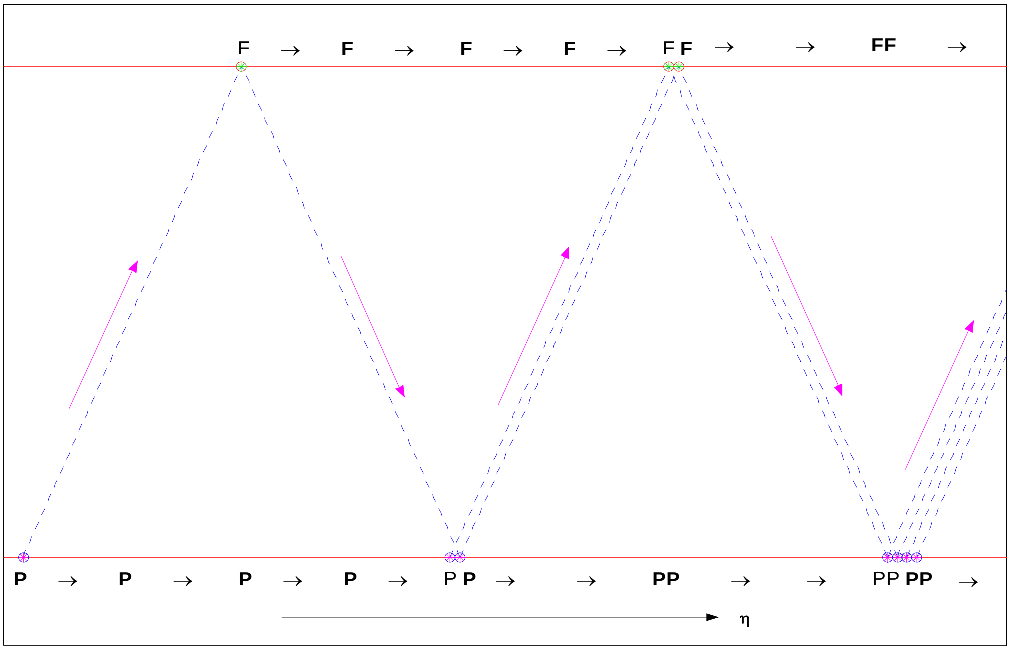

Рис. 1. Эволюция устойчивой цепочки элементарных волн кривизны

Фактически же процесс фокусировок и, соответственно, увеличение N происходит не дискретно, как в упрощенном предыдущем примере, а непрерывно. Из общих соображений ясно, что приращение dN должно быть пропорционально как числу N источников элементарных волн кривизны, так и приращению безразмерного мирового времени $d\eta$, поскольку процесс последовательных фокусировок происходит *равномерно* именно в интервале безразмерного времени. Далее следует цепочка равенств:

$$dN \sim N{\cdot}d\eta\,; \;\Rightarrow\; N \sim e^{\eta} \sim e^{\ln t} \sim t\,. \tag{33}$$

Одинаковый результат в форме соотношений (21) и (33), полученных в разных предположениях, может рассматриваться как дополнительное взаимное подтверждение обоснованности выбора космологической модели и представлений о фокусировочных состояниях волн кривизны.

До сих пор мы рассматривали распространение волн кривизны независимо друг от друга. На самом деле дополнительное искривление 4-пространства, создаваемое остальными ФС и волнами кривизны, неизбежно должно сказаться на их движении и форме волнового фронта. Кроме *слабых* плавных искажений элементарного слоя кривизны, вызванных макроскопическими элементами иерархической структуры, в нем могут возникать *сильные* (локальные) планковские искривления, в тех участках пространства, где слой элементарной волны проходит непосредственно через фокусировочное состояние. Сделаем соответствующие оценки.

Слой элементарной волны вблизи «экватора» вселенной имеет объем (*единицы планковские*) N^2, что составляет $N^2/N^3 = 1/N$ от объема вселенной, содержащей N фокусировочных состояний. В течение планковского (единичного) интервала времени в теле элементарной волны кривизны образуется, в среднем, одно сильное локальное искажение метрики. Это, своего рода, планковская «оспина» на гладкой поверхности волны. Всего таких «оспин» на элементарной волне за время ее движения набирается N. По достижении предфокусировочного радиуса $r_{\text{bit}} = N^{1/2} \approx 10^{-3}$ см, на поверхности волны, имеющей площадь $(N^{1/2})^2 = N$, все «оспины» окажутся *плотно* прижатыми друг к другу. Искривление пространства-времени в пределах «оспины» является сильным, хотя и двумерным. Между плотно прижатыми «оспинами» может возникнуть *нелинейное* взаимодействие, которое начнет стягивать волну. В результате сгущения «оспин» взаимодействие нарастает. Это приводит, в конечном счете, к образованию сильного трехмерного искривления пространства. Можно предположить далее, что «сила» трехмерного сжатия пропорциональна числу «оспин» N. В таком случае размер нового фокусировочного состояния, как и его масса, со временем уменьшаются $\sim 1/N$. При этом, в соответствии с выражениями (23), снижается «эффективность» постоянной Планка: $\hbar \sim 1/N^2$. Учтем далее дифракционные явления, возникающие при прохождении элементарной волны через препятствие.

На пути элементарной волны встретится N планкеонов, образующих дифракционную пространственную решетку. Предположим сначала, что планкеоны вспыхивают и исчезают в пространстве вселенной равномерно со средним шагом $d = a \cdot N^{-1/3} = r_0 \cdot N^{2/3}$. Ширина основного дифракционного пятна, где группируются фокусировочные состояния волн кривизны (новые планкеоны), для такой решетки составляет: $\Delta r = \dfrac{r_0}{d} \cdot a = r_0 \cdot \sqrt[3]{N} \approx 10^{-13}$ cm $\approx r_n$. Это дает основание отождествить, подобные пятна с нуклонами, основными (по массе) элементарными частицами во вселенной. Из релятивистского соотношения неопределенности $r_n \cdot m_n = \hbar/c$, получим массу нуклона $m_n = m_0 \cdot N^{-1/3} \approx 10^{-24}$ g, а также их полное число во вселенной на данном этапе ее эволюции

$N_n = M / m_n = N^{4/3} \approx 10^{81}$. Макроскопическое (суммарное) сечение нуклонов равно сечению вселенной: $r_n^2 \cdot N_n = \left(r_0 \cdot N\right)^2 = a^2$. Совокупность материальных объектов, удовлетворяющих двум условиям $\{ N_i \cdot m_i = M;\ N_i \cdot r_i^2 = a^2 \}$, назовем *i*- тым полным уровнем структурной иерархии вселенной.

4.2. Иерархическая структура вселенной

Последующее рассмотрение проводится в приближении *геометрической оптики*. Волна кривизны, порожденная фокусировочным состоянием, ометает пространство вселенной, проходя через каждую из N_i частиц *i*-го уровня иерархии, и фокусируется затем на противоположном полюсе. При этом на поверхности волны образуются N_i «вмятин» площадью $\sim (r_i)^2$ каждая Отклонение лучей, нормальных к фронту волны соответствует углу отклонения фотонов α_i вблизи тела массы m_i : $\alpha_i = r_g / r_i$, где $r_g = Gm_i / c^2$.

По прохождении расстояния порядка радиуса кривизны вселенной $\sim a$ отклонение луча от места идеальной фокусировки составит $d_i = \alpha_i \cdot a$:

$$d_i = a \cdot \frac{Gm_i}{c^2 r_i} = a \cdot \frac{a}{r_i} \cdot \frac{m_i}{M} = \frac{\left(N_i \cdot r_i^2\right)}{r_i \cdot N_i} = r_i \,.$$

Таким образом, в результате рассеяния волны кривизны на всех элементах *полного i*-го уровня иерархии размер дефокусировки вблизи противоположного полюса пространства сравнивается с величиной отдельной частицы данного уровня: $d_i = r_i$. К этому моменту вся поверхность волны кривизны, ометающей вселенную, в среднем равномерно и плотно покрывается «вмятинами» от совокупности частиц данного уровня иерархии, в чем и проявляется его *полнота*.

По мере приближения к точке $(\eta = \pi)$ в процессе кумуляции вмятины на постоянно уменьшающейся поверхности фронта волны становятся, по-видимому, все более рельефными. На расстоянии $\sim r_i$ от места фокусировки, равном размеру частицы *i*-го уровня, очень сложная и разветвленная *двумерная* поверхность трансформируется в *трехмерное* многообразие с N_i фрагментами. Следовательно, каждому *i*-му множеству из N_i элементов, вообще говоря, соответствует структура более глубокого $(i-1)$-го уровня иерархии с полным числом частиц: $N_{i-1} = (N_i)^2$. Соответствующие размеры частиц получим из соотношений:

$$N_{i-1} \cdot r_{i-1}^2 = N_i \cdot r_i^2 = a^2 = r_0^2 \cdot N^2$$

На основе этих соотношений можно составить таблицу иерархической структуры вселенной. Исследование иерархической лестницы начнем с нижнего, *нуклонного* уровня $i = 1$. На основе предыдущих соотношений построим рекуррентные формулы

$$\begin{array}{lll} N_n \equiv N_1 = N^{4/3} & N_{n-1} = N_n^{1/2} & N = c / \left(r_0 \cdot H\right) \approx 10^{61} \\ r_1 = r_0 \cdot N^{1/3} & r_{n-1} = r_n \cdot N_n^{1/4} & r_0 = 1 \ \left(\approx 10^{-33}\ \text{cm}\right) \\ m_1 = m_0 \cdot N^{-1/3} & m_{n-1} = m_n \cdot N_n^{1/2} & m_0 = 1 \ \left(\approx 10^{-4}\ \text{g}\right). \end{array}$$

Отсюда можно получить следующие выражения для числа, радиуса и массы структурных элементов *i*–го уровня

$$3 \cdot \lg N_i = 2^{3-i} \cdot \lg N;\ 3 \cdot \lg r_i = \left(3 - 2^{2-i}\right) \cdot \lg N;\ 3 \cdot \lg m_i = \left(3 - 2^{3-i}\right) \cdot \lg N$$

Нуклоны, полное число которых на данном этапе эволюции составляет $N_n \equiv N_1 = N^{4/3} \approx 10^{81}$, порождены более высоким, вторым уровнем с числом компонент $N_2 = \sqrt{N_1} = N^{2/3} \approx 10^{40}$. Они, в свою очередь, порождены всей совокупностью объектов третьего уровня $N_3 = \sqrt{N_2} = N^{1/3} \approx 10^{20}$. В результате возникает иерархическая структура, которую назовем главной.

В Таб. 1 приведена схема главных уровней современной иерархической структуры. Из них только пять можно достаточно определенным образом связать с наблюдаемыми астрофизическими объектами. Здесь N_i — полное число «частиц» i–го уровня.

Таблица 1. Главные уровни иерархической структуры вселенной

I	N_i	$\sim r_i$ (см)	$\sim m_i$ (г)	$\sim\bar{\rho}_i$ (г/см^3)	Тип структуры
…	$N_\infty = N^0 = 1$	10^{28}	10^{57}	10^{-29}	Вселенная
…	…	…	…	…	…
9	$N_9 \approx N^{1/192} \approx 2$	—	—	—	*
8	$N_8 \approx N^{1/96} \approx 4$	—	—	—	*
7	$N_7 \approx N^{1/48} \approx 19$	—	—	—	*
6	$N_6 \approx N^{1/24} \approx 350$	—	—	—	*
5	$N_5 \approx N^{1/12} \approx 1.2{\cdot}10^5$	10^{25}	10^{51}	10^{-24}	«Ячейки Эйнасто»
4	$N_4 \approx N^{1/6} \approx 1.5{\cdot}10^{10}$	10^{22}	10^{46}	10^{-20}	Протогалактики
3	$N_3 \approx N^{1/3} \approx 2.3{\cdot}10^{20}$	10^{17}	10^{36}	10^{-15}	Протозвезды
2	$N_2 \approx N^{2/3} \approx 5.1{\cdot}10^{40}$	10^7	10^{16}	10^{-5}	Протокометы
1	$N_n \approx N^{4/3} \approx 2.6{\cdot}10^{81}$	10^{-13}	10^{-24}	10^{15}	Нуклоны

Здесь N_i — полное число «частиц» i–го уровня. Нетрудно убедиться, что n_i — число элементов (i-1)-го уровня в одном элементе i–го уровня совпадает с величиной N_i: $n_i = N_{i-1}/N_i = N_i$. Далее для упрощения будем опускать приставку прото- в названиях структурных элементов 3÷4 уровней.

Таким образом, вместо традиционной концепции *Большого взрыва* получаем альтернативный вариант — постепенное эволюционное изменение структурных элементов вселенной в сторону их усложнения. Именно это приводит к кажущемуся расширению вселенной. Эффект Хаббла в новой трактовке является следствием постепенного эволюционного уменьшения всех структурных элементов вселенной, включая субатомные. Сама же вселенная оказывается, по сути дела, статической, как в начале и предполагал А. Эйнштейн.

5. ОТО Эйнштейна как Единая теория поля

Физики теоретики ищут некое универсальное физическое взаимодействие, объединяющее четыре известных — электромагнитное, слабое, сильное и гравитационное. Но гравитация сама по себе является универсальным взаимодействием (в отличие от остальных трех), одинаково применимым ко всем формам материи. Вряд ли в природе имеется два универсальных взаимодействия. Более того, утверждение о наличии двух универсальных взаимодействий на наш взгляд означает, что ни одно из них не является полностью универсальным. Отсюда следует вывод, что именно ОТО Эйнштейна и является, по сути дела, той самой Единой теорией, к созданию которой стремятся теоретики. Конкретная реализация этого утверждения была найдена на пути физического осмысления эмпирического тождества инертной m_i и тяжелой m_g масс: $m_i = m_g$, положенного Эйнштейном в основу ОТО. Не тривиальность тождества заключается в том, что входящие в него величины имеют принципиально *разную физическую природу*. Инертная масса — мера инерции, количественная характеристика *материальности* тела, проявляющаяся в его взаимодействии с окружающими физическими объектами. Тяжелая масса — «*гравитационный заряд*» тела в законе всемирного тяготения Ньютона. Отсюда, в частности, следует вывод о единственности ОТО Эйнштейна как *релятивистской теории гравитации*. Действительно, все материальные объекты, обладают «гравитационным зарядом», поэтому их пространственно-временные траектории в неоднородном и всюду проникающем поле тяготения в той или иной степени искривлены. Причем в глобальных космических масштабах искривление пространства может стать столь значительным, что приведет к его полному замыканию.

Из эмпирического тождества с необходимостью следует соответствие между компонентами тензора кривизны и компонентами тензора энергии-импульса. Иначе говоря, материальный мир допускает *альтернативу* своего математического описания.

Прежде всего, это общепринятое описание с помощью физических параметров, которые, в конечном счете, можно выразить через распределение плотности в пространстве-времени — совокупность компонент тензора энергии-импульса. Вторая возможность — рассматривать мир как искривление римановой метрики в виде совокупности компонент тензора кривизны.

Уравнения Эйнштейна отражают связь g_{ik} и $\rho(x^l)$ на уровне свернутых форм тензора кривизны — тензора Риччи и скалярной кривизны, хотя полная связь должна быть на уровне тензоров 4-го ранга. При этом определенный физический смысл можно придать лишь первому, временному уравнению Эйнштейна:

$$2E_0^0 \equiv 2R_0^0 - \delta_0^0 \cdot R \equiv \left(R_0^0 - R_\alpha^\alpha\right) = 16\pi \cdot T_0^0 \equiv 16\pi \cdot \rho; \quad c = G = 1.$$

Связь остальных компонент тензора энергии-импульса с плотностью в общем случае анализу не поддается.

Можно предположить, что существует равенство $R_{ik}^{lm} = 8\pi \cdot T_{ik}^{lm}$, где T_{ik}^{lm} — некий «тензор материи», свойства симметрии которого совпадают с R_{ik}^{lm}, а инвариантами являются алгебраические выражения $\mathrm{P}^i(\rho)$, содержащие только плотность $\rho(x^k)$: $J^i = 8\pi \cdot \mathrm{P}^i(\rho)$.

Здесь J^i — 14 инвариантов (I = 1,2,…,14) тензора кривизны R_{ik}^{lm} .

Инварианты Петрова для статической метрики Фридмана $\{ g_{00} = 1; \quad g_{11} = -a^2; \quad g_{22} = -a^2 \cdot \sin^2\chi; \quad g_{33} = -a^2 \cdot \sin^2\chi \cdot \sin^2\theta \}$ равны:

$$J_4 = J_5 = J_{11} = J_{12} = J_{13} = 0,$$

$$J_1 = -6/a^2,\ J_2 = 6/a^4,\ J_3 = -6/a^6,\ J_6 = 3/a^4,\ J_7 = 3/a^6,$$

$$J_8 = 24/\left(4a^8\right),\ J_9 = 3/\left(2a^6\right),\ J_{10} = 3/a^8,\ J_{14} = -3/a^{10},$$ или

$$E_0^0 = 3/a^2$$

$$\boxed{\begin{gathered} J_1 = -2E_0^0;\ \ J_4 = J_5 = J_{11} = J_{12} = J_{13} = 0; \\ 3J_2 = 2\left(E_0^0\right)^2;\ 9J_3 = -2\left(E_0^0\right)^3;\ 3J_6 = \left(E_0^0\right)^2;\ 9J_7 = \left(E_0^0\right)^3; \\ 108J_8 = 7\left(E_0^0\right)^4;\ 18J_9 = \left(E_0^0\right)^3;\ 27J_{10} = \left(E_0^0\right)^4;\ 81J_{14} = -\left(E_0^0\right)^5 \end{gathered}}$$

Есть основания предположить, что и в реальной вселенной, где $\rho = \rho(x^k)$, с метрикой $g\left(x^k\right)$ соотношения в рамке также будут выполняться. Иначе будет трудно объяснить, как в процессе усреднения плотности материальных объектов по всему пространству (именно в этом и заключается переход к модели Фридмана) система уравнений в рамке сохраняет свой вид на любом этапе структурной эволюции. Таким образом, получим набор из 14 дифференциальных уравнений относительно глобальной метрики $g_{ik}(x^l)$. По своей сути этот набор полностью определяет геометрическую и, соответственно, физическую структуру реальной вселенной. Обратим внимание на то, что, в отличие от обычных задач теоретической физики, здесь начальные и граничные условия задачи отсутствуют. Их отсутствие компенсируется увеличением числа уравнений.

Вычислив все десять функций $g_{ik}(x^l)$, получим плотность $\rho(x^l)$ из временного уравнения Эйнштейна: $E_0^0 = 8\pi\rho$. Вследствие равенства инварианта J_1 скалярной кривизне R, первое уравнение $J_1 = -2E_0^0$ системы принимает простой вид $R_0^0 = 0$. Ранее были получены приближенные параметры основной *элементарной* частицы — нуклона. Это — *главное* дифракционное «пятно», порождаемое глобальной *дифракционной решеткой* из N планкеонов. Остальным элементарным частицам соответствуют, очевидно, «пятна» следующих порядков. Элементы структурной иерархии, приведенные в таблице можно рассматривать как набор *макроскопических* объектов, составленных из элементарных частиц. Очевидно, что все эти структуры должны содержаться в решениях полной системы уравнений. Однако строгое решение этой системы на уровне планковских структур, разумеется, невозможно. Остается искать приближенные способы решения уравнений.

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Космологические решения в релятивистской теории гравитации

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Рассмотрены различные космологические решения уравнений РТГ. Показано, что космологический член с отрицательной космологической постоянной в уравнениях РТГ приводит к существованию решений этих уравнений, описывающих материю, не имеющую гравитационного поля. Сформулированы модифицированные уравнения РТГ, учитывающие наличие в пространстве событий материи и физического вакуума с постоянной плотностью энергии. Вычислен параметр замедления расширения современной Вселенной и сформулировано условие на величину космологической постоянной, при котором происходит ускоренное расширение Вселенной.

1 Уравнения РТГ

Релятивистская теория гравитации (РТГ) представляет собой теорию гравитационного поля в четырехмерном псевдоевклидовом пространстве Минковского, полевые функции которого выражаются через компоненты g_{ij} метрического тензора четырехмерного псевдориманова пространства V. Предполагается, что компоненты γ_{ij} метрического тензора пространства Минковского и компоненты метрического тензора g_{ij} определены на одном и том же многообразии x^i $(i = 1,\ 2,\ 3,\ 4)$. Динамические уравнения РТГ получаются при помощи вариационного принципа с лагранжианом $\Lambda = \Lambda_{(m)} + \Lambda_{(g)}$, где лагранжиан материи (идеальной жидкости) $\Lambda_{(m)}$ определен как функция от инвариантной массовой плотности ρ идеальной жидкости $\Lambda_{(m)} = \Lambda_{(m)}(\rho)$; лагранжиан гравитационного поля $\Lambda_{(g)}$ определяется следующим образом [1,2]

$$\Lambda_{(g)} = \frac{1}{2\varkappa} g^{ik}\left(G^j_{ik}G^n_{jn} - G^n_{ij}G^j_{kn}\right) - \frac{m^2}{2\varkappa}\left(\frac{1}{2}g^{ij}\gamma_{ij} - 1 - \sqrt{\frac{\gamma}{g}}\right).$$

Здесь $g = \det \|g_{ij}\|$, $\gamma = \det \|\gamma_{ij}\|$; компоненты тензора третьего ранга G^s_{ij} определяются равенством $G^s_{ij} = \Gamma^s_{ij} - \gamma^s_{ij}$, в котором Γ^k_{ij} — символы Кристоффеля псевдориманова пространства V, γ^k_{ij} — символы Кристоффеля пространства Минковского. Постоянные m, $\varkappa$ выражаются через массу гравитона m_g и ньютоновскую гравитационную постоянную G: $m = m_g c/\hbar$, $\varkappa = 8\pi G/c^4$, c — скорость света, $\hbar$ — постоянная Планка.

Динамические уравнения РТГ можно записать в виде [1, 2]

$$R_{ij} - \frac{m^2}{2}\left(g_{ij} - \gamma_{ij}\right) = \varkappa\left(T_{ij} - \frac{1}{2} g_{ij} T_s{}^s\right), \tag{1}$$

$$D_j\left(\sqrt{-g}\, g^{ij}\right) = 0, \tag{2}$$

где R_{ij} — компоненты тензора Риччи псевдориманова пространства V; D_i — символ ковариантной производной в пространстве Минковского; T_{ij} — симметрические компоненты тензора энергии-импульса идеальной жидкости, определяемые соотношениями

$$T_{ij} = (p+e)u_i u_j - p g_{ij},$$
$$p = -\rho^2 \frac{d(\Lambda_{(m)}/\rho)}{d\rho}, \quad e = -\Lambda_{(m)}, \tag{3}$$

в которых u_i — компоненты единичного вектора скорости индивидуальных точек жидкости $u_i u^i = 1$. Массовая плотности ρ удовлетворяет уравнению неразрывности

$$\partial(\rho u^i \sqrt{-g}\,)/\partial x^i = 0. \tag{4}$$

Различные канонические тензоры энергии–импульса, соответствующие уравнениям (1), (2), рассмотрены в [3, 4].

2 Однородное изотропное пространство

Далее будем рассматривать решения уравнений (1) — (4) с однородным изотропным пространством V, когда метрика g_{ij} и γ_{ij} определяются соотношениями [5]:

$$ds^2 = d\tau^2 - \beta^4 a^2(dx^2 + dy^2 + dz^2),$$
$$d\sigma^2 = a^{-6} d\tau^2 - dx^2 - dy^2 - dz^2. \tag{5}$$

Здесь ds — интервал псевдориманова пространства V, $d\sigma$ — интервал пространства Минковского; x, y, z — пространственные переменные системы координат, $\tau = \boldsymbol{c}t$, t — время, β — произвольная безразмерная постоянная, $a = a(\tau)$ — искомая функция. Из принципа причинности следует [5], что величины a, β должны удовлетворять неравенству $a^4 \leq \beta^4$.

Уравнение (2) для этой метрики выполняется тождественно, а уравнение неразрывности (4), в предположении, что идеальная жидкость в пространстве событий неподвижна $u_4 = 1$, $u_1{=}u_2{=}u_3{=}0$, имеет следующее решение

$$\rho = \frac{\mathrm{const}}{a^3}. \tag{6}$$

Из уравнения (1) в рассматриваемом случае следует

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{\varkappa}{6}(e+3p) + \frac{m^2}{6}\left(\frac{1}{a^6} - 1\right), \tag{7}$$

$$\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 = \frac{\varkappa}{3}e - \frac{m^2}{12}\left(2 - \frac{3}{\beta^4 a^2} + \frac{1}{a^6}\right). \tag{8}$$

Если $a \neq$ const, то уравнение (7) является следствием уравнений (6), (8).

Рассмотрим случай, когда идеальная жидкость (пыль) определяется уравнениями состояния

$$p = 0, \quad e = \rho c^2 = \frac{B}{a^3}, \tag{9}$$

где B — постоянная. Нетрудно убедиться, что уравнения (6)— (9) имеют точное решение [6]

$$a^{-3} = \frac{1 + \sqrt{1+4\eta^2}}{2\eta}, \quad \beta^4 a^2 = \frac{2\eta^2}{2\eta^2 - 1 - \sqrt{1+4\eta^2}}, \tag{10}$$

где постоянный параметр η определяется равенством $\eta = m^2/\varkappa B$. Условие положительности величины $\beta^4 a^2$ во второй формуле выполняется при $\eta^2 > 2$. Для этого решения при $\eta^2 > 2$ плотность энергии жидкости e удовлетворяет неравенству $0 < e < m^2/\varkappa$ и выполняется принцип причинности [2].

Очевидно, что метрика g_{ij}, определяемая решением (10), псевдоевклидова. Таким образом, уравнения РТГ (1), (2) допускают решения, описывающие материю, не имеющую гравитационного поля. Нетрудно убедиться, что существование таких решений связано с наличием в уравнении (1) космологического члена $-\frac{m^2}{2}g_{ij}$ с отрицательной космологической постоянной.

Для того, чтобы оценить вклад этого космологического члена в решения уравнений (1), (2), можно сравнить известные решения этих уравнений с решениями уравнений (1), (2) без космологического члена. Можно сделать также некоторые качественные выводы из структуры этих уравнений. Прежде всего, из условия положительности правой части уравнения (8) следует, что при наличии космологического члена решение меняется от $a_{min} \neq 0$ до некоторого максимального значения a_{max}. Поэтому в такой теории отсутствуют черные дыры и получается пульсирующая Вселенная [1,2]. В уравнениях (1), (2) без космологического члена таким же образом получается $a_{min} < a < \infty$, но черные дыры тоже отсутствуют.

Рассмотрим теперь точные решения уравнения (8) без космологического члена с ультрарелятивистским уравнением состояния, выполняющимся для раннего этапа эволюции Вселенной:

$$e = 3p = -\Lambda_{(m)} = Q\rho^{\frac{4}{3}}, \quad Q = \text{ const}.$$

Учитывая уравнение (6) находим в этом случае $e = A/a^4$ ($A =$ const) и уравнение (8) принимает вид

$$\left(\frac{da}{d\tau}\right)^2 = \frac{m^2}{4\beta^4} + \frac{\varkappa A}{3}\frac{1}{a^2} - \frac{m^2}{12}\frac{1}{a^4}. \tag{11}$$

При $m = 0$ уравнение (11) переходит в уравнение Фридмана для плоского пространства.

Точное решение уравнения (11) при $m \neq 0$ выражается через эллиптические интегралы первого и второго рода $F(\varphi, k)$, $E(\varphi, k)$ [6]:

$$\pm\frac{m}{2\beta^2}\tau = \int\limits_{a_{\min}}^{a} \frac{a^2 da}{\sqrt{(a^2+\theta^2)(a^2-a_{\min}^2)}} =$$
$$= \frac{a_{\min}^2}{\sqrt{a_{\min}^2+\theta^2}}\, F(\varphi,k) - \sqrt{a_{\min}^2+\theta^2}\, E(\varphi,k) + \frac{1}{a}\sqrt{(a^2+\theta^2)(a^2-a_{\min}^2)}\,. \tag{12}$$

Здесь обозначено

$$\varphi = \arccos\frac{a_{\min}}{a}, \quad k = \sqrt{\frac{\theta^2}{a_{\min}^2+\theta^2}} \equiv \left[1 + 3\left(\frac{a_{\min}}{\beta}\right)^4\right]^{-\frac{1}{2}}.$$

Постоянные коэффициенты $a_{\min}$ и θ определяются следующими соотношениями

$$a_{\min}^2 = \frac{2\varkappa A\beta^4}{3m^2}\left(\sqrt{1+\frac{3m^4}{4\varkappa^2A^2\beta^4}} - 1\right),$$
$$\theta^2 = \frac{2\varkappa A\beta^4}{3m^2}\left(\sqrt{1+\frac{3m^4}{4\varkappa^2A^2\beta^4}} + 1\right).$$

При малых $a - a_{\min}$ с учетом малости постоянной m из формулы (12) получаем

$$a = a_{\min}\left(1 + \frac{\varkappa}{6}e_{\max}\tau^2\right),$$

где $e_{\max} = A/a_{\min}^4$, $a_{\min}^2 = m^2/4\varkappa A$. Эта формула совпадает (с точностью до обозначений) с соответствующей формулой в теории с отрицательной космологической постоянной, полученной в [2] при помощи приближенного решения уравнений (8). Таким образом, качественное поведение решения уравнения (8) в малой окрестности $a \sim a_{\min}$ не отличается от поведения соответствующего решения уравнения (11).

Если $a \gg a_{\min}$, то из (12) следует

$$a^2 = \frac{m^2}{4\beta^4}(\tau \pm \tau_0)^2 - \theta^2, \tag{13}$$

где τ_0 — постоянная.

3 Модифицированные уравнения РТГ.

Как известно [2], если ограничиться случаем наличия в пространстве только радиации и нерелятивистской материи, то окажется, что наблюдаемое сейчас расширение Вселенной, описываемое уравнениями (1), (2), должно замедляться, тогда как опытные данные показывают [7, 8], что оно ускоряется. Для описания наблюдаемого ускорения расширения современной Вселенной в РТГ вводится [5] так называемая квинтэссенция, которая представляет собой материю со специальным уравнением состояния $p = \nu e$, причем $-1 < \nu < -1/3$. Отметим, что скорость звука для квинтэссенции оказывается мнимой.

Использование квинтэссенции для описания ускоренного расширения Вселенной вызывает некоторые возражения [9]; кроме того, использование квинтэссенции усложняет интегрирование уравнений — сейчас, по-видимому, не имеется даже приближенных решений уравнений РТГ с учетом квинтэссенции.

Далее будет показано, что для описания ускоренного расширения современной Вселенной в рамках биметрической теории, как и в общей теории относительности, достаточно использовать в динамических уравнениях космологический член с положительной космологической постоянной λ. Использование в уравнениях космологического члена с положительной космологической постоянной $\lambda > 0$ не приводит к появлению рассмотренных здесь решений, описывающих материю без гравитационного поля. Ниже рассмотрим такую теорию.

Рассмотрим систему уравнений

$$R_{ij} + \frac{m^2}{2}\gamma_{ij} = \varkappa\left(T_{ij} - \frac{1}{2}g_{ij}T_s{}^s - \frac{\lambda}{\varkappa}\,g_{ij}\right),$$
$$D_j\left(\sqrt{-g}\,g^{ij}\right) = 0, \tag{14}$$

соответствующую лагранжиану

$$\Lambda = \frac{1}{2\varkappa}g^{js}\left(G^q_{js}G^n_{qn} - G^n_{jq}G^q_{sn}\right) - \frac{m^2}{4\varkappa}g^{jn}\gamma_{jn} + \Lambda_{(m)} - \frac{\lambda}{\varkappa} + \nabla_i\Omega^i. \tag{15}$$

Дивергентный член $\nabla_i\Omega^i$ в лагранжиане (15) не влияет на динамические уравнения и вводится здесь для возможности изменения определения канонического тензора энергии–импульса и соотношений на поверхности разрыва определяющих параметров жидкости и поля.

Канонический тензор энергии–импульса, соответствующий лагранжиану (15), определяется соотношением

$$P_i^{\,k} = T_{ij}g^{jk} + \frac{1}{2\varkappa\sqrt{-g}}\left[G^k_{sn}D_i\left(\sqrt{-g}\,g^{sn}\right) - G^n_{sn}D_i\big(\sqrt{-g}\,g^{sk}\big)\right] -$$
$$- \frac{1}{2\varkappa}g^{js}\left(G^q_{js}G^n_{qn} - G^n_{jq}G^q_{sn}\right)\delta^k_i + \frac{m^2}{4\varkappa}g^{jn}\gamma_{jn}\delta^k_i + \frac{\lambda}{\varkappa}\delta^k_i +$$
$$+ \frac{1}{\sqrt{-g}}\left[D_i\left(\sqrt{-g}\,\Omega^k\right) - D_s\left(\sqrt{-g}\,\Omega^s\right)\delta^k_i\right],$$

в котором компоненты тензора энергии–импульса идеальной жидкости T_{ij} определены равенствами (3). Если положить $\Omega^i = 0$, то в декартовой системе координат пространства Минковского компоненты $P_i^{\,k}$ при $m = \lambda = 0$ совпадают с компонентами псевдотензора энергии-импульса Эйнштейна. Если величины Ω^i определить соотношением

$$\sqrt{-g}\,\Omega^i \;= \frac{1}{3\varkappa}g^{ij}D_j\sqrt{-g} \equiv \frac{1}{3\varkappa}\sqrt{-g}\,g^{ij}G^s_{sj},$$

то получим тензор энергии–импульса, след которого определяется соотношением

$$P_i^{\,i} = \frac{m^2}{2\varkappa}\gamma_{ij}g^{ij}.$$

Для безмассового гравитационного поля след такого тензора, как и для всех известных безмассовых полей, равен нулю.

Согласно современным воззрениям космологический член в полевых уравнениях описывает реальный физический вакуум. В соответствии с такой интерпретацией и для согласования с РТГ первое уравнение в (14) можно записать следующим образом

$$R_{ij} - \frac{m^2}{2}\big(g_{ij} - \gamma_{ij}\big) = \varkappa\left(\overset{*}{T}_{ij} - \frac{1}{2}g_{ij}\overset{*}{T}_s{}^s\right),$$

где $\overset{*}{T}_{ij}$ — компоненты полного тензора энергии–импульса материи и вакуума:

$$\overset{*}{T}_{ij} = T_{ij} + \frac{1}{\varkappa}\left(\lambda + \frac{1}{2}m^2\right)g_{ij} = \big(\overset{*}{p} + \overset{*}{e}\big)u_iu_j - \overset{*}{p}g_{ij},$$

а полное давление $\overset{*}{p}$ и объемная плотность энергии $\overset{*}{e}$ материи и вакуума имеют вид

$$\overset{*}{p} = p - \frac{1}{\varkappa}\left(\lambda + \frac{1}{2}m^2\right), \quad \overset{*}{e} = e + \frac{1}{\varkappa}\left(\lambda + \frac{1}{2}m^2\right). \tag{16}$$

Из определения (16) следует, что вакуум обладает эффективной энергией с объемной плотностью $\varkappa^{-1}(\lambda + \frac{1}{2}m^2)$. При таком рассмотрении уравнения (14) имеют решение с псевдоевклидовой метрикой g_{ij} (что означает отсутствие гравитационного поля), если $\overset{*}{T}_{ij} = 0$, т. е. если в пространстве событий нет ни материи, ни вакуума. В РТГ принимается по определению [1, 2] $\lambda = -m^2/2$ и эффективная энергия вакуума оказывается равной нулю.

Рассмотрим случай, когда метрика определена соотношениями (5), жидкость неподвижна в пространстве событий, а уравнения состояния взяты в виде (9) и соответствуют пыли. В этом случае уравнения (14) принимают вид

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = \frac{m^2}{6}\left(\frac{1}{a^6} - \frac{\varkappa B}{m^2}\frac{1}{a^3} + \frac{2\lambda}{m^2}\right),$$
$$\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 = \frac{m^2}{12}\left(-\frac{1}{a^6} + \frac{4\varkappa B}{m^2}\frac{1}{a^3} + \frac{3}{\beta^4}\frac{1}{a^2} + \frac{4\lambda}{m^2}\right).$$

При помощи этих уравнений легко вычислить параметр q замедления расширения современной Вселенной ($a \gg 1$):

$$q \stackrel{\text{def}}{=} -\frac{a\ddot{a}}{\dot{a}^2} \simeq \frac{1}{2}\left(1 - \frac{\lambda c^2}{H^2}\right). \tag{17}$$

Здесь $H^2 = c^2\dot{a}^2/a^2$ — современное значение постоянной Хаббла, точка над буквой означает дифференцирование по τ. Из уравнения (17) следует, что за счет выбора космологической постоянной λ всегда можно получить требуемое значение параметра q. В частности, из (17) следует, что $q < 0$ (т. е. происходит ускорение расширения), если $\lambda > H^2/c^2$. Полагая $H \simeq 2,1 \cdot 10^{-18}\text{с}^{-1}$, для критического значения $\lambda = \lambda_*$, при котором $q = 0$, находим $\lambda_* = 0,49 \cdot 10^{-56}\text{см}^{-2}$. Если $\lambda \simeq 10^{-56}\text{см}^{-2}$, то $q \simeq -0,52$.

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Undisturbed Kepler's motion in the Universe

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1. Introduction

Movement of primary planets in the Solar system, movement of stars in constellations, of secondary and minor planets, comets, etc. all follow the trajectory of motion of material points mutually attracted according to Newton's law. Consider a well-known problem of movement of two material bodies in space:

$$\frac{d^2x}{dt^2} = -\frac{\mu x}{r^3}, \qquad \frac{d^2y}{dt^2} = -\frac{\mu y}{r^3}, \qquad \frac{d^2z}{dt^2} = -\frac{\mu z}{r^3}, \tag{1}$$

where μ is a constant value which is a factor of masses M and m of these material bodies, while

$$r = \sqrt{x^2 + y^2 + z^2}$$

is distance between a moving object and the origin of coordinates. The movement described by (1), takes place in the same plane and obeys Kepler's laws. Any of the above equations, for instance, for coordinate y, is a partial equation of the second order.

$$\frac{d^2y}{dt^2} + A\frac{dy}{dt} + By = 0,$$

where A = 0, and B > 0. A characteristic equation of the second order differential equation can have three types of square roots: 1) one real double root; 2) two different real roots; 3) a pair of complex-conjugated roots. In 1) and 2), assuming that A = 0, we obtain either B = 0, or B < 0, which means that it is not possible to obtain the equation of motion (1). Equations (1) can be obtained in the case of a pair of complex-conjugated roots, i.e. in the case of one pair out of the three, consequently, the probability of this event equals one third. Consequently, we witness an essential violation of the probability theory since there seem to be no objective reason to prevail for this pair of roots in the second order equation. We can also state an essential violation of mathematical statistics laws since there is an enormous number of different motions in the Universe which obey the equations (1).

This paper proves that neither the probability theory, nor the laws of mathematical statistics are violated if we deal with solutions of the second order equations which are positive definite [1].

Definition. A continuous function $h(\tau)$ of the real argument τ is called *positive definite* in the range $-\infty < \tau < \infty$, if, whatever real values are $\tau_1, \tau_2, \ldots, \tau_n$, $\zeta_1, \zeta_2, \ldots, \zeta_n$ and the integer n,

$$\sum_{k=1}^{n}\sum_{j=1}^{n} h(\tau_j - \tau_k)\zeta_j\zeta_k \geq 0.$$

For the function $v(\tau)$ to meet this inequality it is necessary and sufficient to have in the inverse Fourier transform

$$V(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-i\omega\tau)\, v(\tau)d\tau$$

function $V(\omega) \geq 0$ for all ω [1]. This property $V(\omega)$ is used in the proof of the theorem given in the Appendix.

The property of the function to be positive definite is well known in the theory of probability, it has not been known in the theory of deterministic systems which includes the theory of differential equations [2].

The paper also proves that a positive definite function is a sum of harmonic oscillations of any frequency whose argument is a time interval between two arbitrary sections of a stationary random process, while the propagation velocity of such oscillations is not constant. Other issues discussed include the movement of the planet of Mercury, and the anomaly of the planet's motion in the light of the theory of differential equations which was developed by the great I. Newton.

2. Positive definite function as a solution of differential equation

According to theorem (see Appendix) a solution of the second order differential equation for certain initial conditions can be presented as the Fourier integral

$$v(\tau) = 2\int_0^\infty V(\omega)\cos\omega\tau d\omega, \quad (-\infty < \tau < \infty)$$

where functions v(τ) and V(ω) have indices which depend on the type of the characteristic equation's roots. For further discussion the specific type of these functions is not relevant, hence, we will skip them. Since function V(ω) ≥ 0 and V(ω) = V(−ω) for any type of the roots (see Appendix for the details), then one can extend the limits of integration in function $v(\tau)$, as

$$v(\tau) = \int_{-\infty}^{\infty} V(\omega)\cos\omega\tau d\omega, \quad (-\infty < \tau < \infty) \tag{2}$$

From theorem 1 we know that v(0) = 1, hence

$$v(0) = 1 = \int_{-\infty}^{\infty} V(\omega)d\omega,$$

consequently, V(ω) is a normalized density of the probability distribution of a random value Ω with mathematical expectation

$$E\Omega = \int_{-\infty}^{\infty} \omega V(\omega)d\omega = 0,$$

because the integrand function is odd.

Most important from the point of view of application of this are two theorems – direct and inverse [1]. According to these theorems the functions V(ω) and v(τ) are in reciprocal relation. To study this relationship we will substitute the integration variable in V(ω):

$$\omega = \frac{\omega_1}{\alpha}, \quad (\alpha > 0) \tag{3}$$

and present V(ω) as

$$V(\omega) = V(\frac{\omega_1}{\alpha}) = \overline{V}(\omega_1).$$

Let us see how the function v(τ) changes, but, before doing this we will find out the physical implication of the substitution (3), which is <u>of utmost importance</u> for what will follow. The abscissa axis in the coordinate system (ω,V(ω)) expands for α > 1 or contracts for α < 1, which means a change in the scale of ω, which is the integration variable in (2). Since the area under the distribution curve does not change and remains equal to unity, then V(ω) should change becoming either more flat or more acute depending on the specific value of α. We receive thereby a new density of probability distribution $\overline{V}(\omega_1)$, with a different dispersion, i.e. the substitution (2) has quite a <u>real-valued meaning</u>. The above qualitative considerations of the substitution will be supported with the proof in what will follow.

To understand the change of function v(τ), we will differentiate (2) twice under the sign of integration with respect to τ as a parameter (this is possible as it is known from [1,2]) and substituting in the second-order equation

$$\frac{d^2v}{d\tau^2}+a\frac{dv}{d\tau}+bv=0\,,\qquad (\tau>0) \tag{4}$$

whose characteristic equation can be written as $\eta^2+a\eta+b=0$,
we obtain identity

$$-\int_{-\infty}^{\infty}\omega^2V(\omega)\cos\omega\tau d\omega-a\int_{-\infty}^{\infty}\omega V(\omega)\sin\omega\tau d\omega+b\int_{-\infty}^{\infty}V(\omega)\cos\omega\tau d\omega\equiv 0. \tag{5}$$

Let me mention in advance that experts in differential equations are used to argument τ in (4) being time. In the case discussed here this is not so, however. To prove this statement let us once again discuss the Fourier integral (2) and note one more its property. It follows from the well-known Khinchine' theorem [2]: for the function $v(\tau)$ to be correlation function of a continuous stationary random process it is necessary and sufficient to be able to present it as (2), where $V(\omega)$ is the probability distribution density. $V(\omega)$ meets this requirement, which means that $v(\tau)$ is the correlation function of a random stationary process $\xi(t)$. In its turn it means that τ is the time interval between sections of $\xi(t)$ at arbitrary times t_1 and t_2:

$$\tau = t_1 - t_2. \tag{6}$$

We will come back to the result obtained, and now let us substitute (3) in the identity (5):

$$-\int_{-\infty}^{\infty}\omega_1^2\overline{V}(\omega_1)\cos(\frac{\omega_1}{\alpha}\tau)d\omega_1-\alpha a\int_{-\infty}^{\infty}\omega_1\overline{V}(\omega_1)\sin(\frac{\omega_1}{\alpha}\tau)d\omega_1+\alpha^2b\int_{-\infty}^{\infty}\overline{V}(\omega_1)\cos(\frac{\omega_1}{\alpha}\tau)d\omega_1\equiv 0. \tag{7}$$

To find a change in identity (7) compared to (5), we will assume in these identities that $\tau = 0$. We obtain the following equations:

$$D\Omega=\int_{-\infty}^{\infty}\omega^2V(\omega)d\omega==b\int_{-\infty}^{\infty}V(\omega)d\omega=b,\qquad D\Omega_1=\int_{-\infty}^{\infty}\omega_1^2\overline{V}(\omega_1)d\omega_1=\alpha^2b\int_{-\infty}^{\infty}\overline{V}(\omega_1)d\omega_1=\alpha^2b.$$

Thus, in the light of the theory of probability the substitution (3) caused a change in the dispersion $D\Omega$ of the random value Ω to make it equal to the value of $D\Omega_1$, as it had been shown earlier in the qualitative discussion. Changes, however, have also taken place from the point of view of differential equations as well: the coefficient b in (4) has changed to become equal to α^2b. The change in the coefficients of (4) leads to a change of its characteristic equation, i.e. a change in the roots of that equation. To find out the real change the roots have undergone, we will assume that prior to the substitution (3) in the characteristic equation we had complex-conjugated roots $-\chi \pm i\lambda$. It follows from the Vieta's theorem that the roots product should be equal to the coefficient b: $\chi^2+\lambda^2=b$.

Denote the root values after the substitution as $-\chi_1 \pm i\lambda_1$, then the product of the new roots by virtue of the same theorem is $\chi_1^2+\lambda_1^2=\alpha^2b$.

Comparing the results we obtain: $\chi_1=\alpha\chi$, $\lambda_1=\alpha\lambda$.

The change in the roots should lead not only to the change in the coefficient b in equation (4), but also to a change in the coefficient a, which is the case. This can be easily seen assuming that in identity (7) $\tau = \alpha\tau_1$ and reduce it to the following form

$$-\int_{-\infty}^{\infty}\omega_1^2\overline{V}(\omega_1)\cos(\omega_1\tau_1)d\omega_1-\alpha a\int_{-\infty}^{\infty}\omega_1\overline{V}(\omega_1)\sin(\omega_1\tau_1)d\omega_1+\alpha^2b\int_{-\infty}^{\infty}\overline{V}(\omega_1)\cos(\omega_1\tau_1)d\omega_1\equiv 0.$$

Now note that we will obtain the same identity by substituting the Fourier integral

$$\overline{v}(\tau_1)=\int_{-\infty}^{\infty}\overline{V}(\omega_1)\cos(\omega_1\tau_1)d\omega_1$$

in the second order differential equation of type (3), whose characteristic function, however, will have the following form

$$\eta^2+\alpha a\eta+\alpha^2b=0\,.$$

Consequently, the substitution (3), i.e. a change in the scale of the integration variable ω, can be treated also as a change in the scale of the roots of the characteristic equation in the differential equation (4). The change in the coordinate system $(\omega, V(\omega))$ as a result of the substitution (3), looks quite natural since the change affected the abscissa axis as has already been mentioned. The change in the roots of the characteristic equation is difficult to understand. The substitution (3) cannot cause all this at least because the equation (4) describes the motion of a system which would not change as a result of the substitution done. Hence, in equation (4) the substitution (3) should inevitably result in the substitution of the argument τ for τ_1:

$$\frac{\tau}{\alpha} = \tau_1,$$

which would make up for the change effected as shown below:

$$\frac{d^2v}{d\tau^2} + a\frac{dv}{d\tau} + bv = 0 \;\rightarrow\; \frac{d^2v}{d\left(\frac{\tau^2}{\alpha^2}\right)} + \alpha a\frac{dv}{d\left(\frac{\tau}{\alpha}\right)} + \alpha^2 bv = 0.$$

In an ordinary homogeneous differential equation the above transformation does not have any meaning since this is just a re-designation of the roots and time. In our case, however, the substitution (3) has a clear meaning as already mentioned above. Note that an old argument τ has changed to a new argument τ_1, but in such a way that the product $\omega\tau$ remained unchanged:

$$\omega\tau = \left(\frac{\omega_1}{\alpha}\right)(\alpha\tau_1) = \omega_1\tau_1 = \text{const}. \tag{8}$$

Totally unexpected one-to-one relation between the density $V(\omega)$ and the function $v(\tau)$: the frequency ω in the Fourier integral (2) and the time interval τ between sections of a stationary process $\xi(t)$ are related between each other in such a way that the expansion ω leads to the contraction τ and, vice versa, the contraction ω results in the expansion τ, their product remaining unchanged.

Relationship (8), first, is scale-based, because the scale of variable ω and τ really changes due to the substitution (3). Second, in (8), values of ω and τ obviously can be chosen arbitrarily, however, having done so, we remain referred for ever to the constant that we have chosen. Let us keep this result in mind and use it in what follows for theoretical proof of the following statement the velocity of harmonic oscillation $\cos\omega\tau$ is not constant.

Let's consider two equations

$$\frac{d^2v}{d\tau_1^2} + \alpha a\frac{dv}{d\tau_1} + \alpha^2 bv = 0, \quad (\alpha > 0, \tau_1 > 0), \qquad \frac{d^2v}{d\tau_1^2} - \alpha a\frac{dv}{d\tau_1} + \alpha^2 bv = 0. \quad (\tau_1 < 0).$$

Differentiating the Fourier integral

$$\overline{v}(\tau_1) = \int_{-\infty}^{\infty} \overline{V}(\omega_1)\cos(\omega_1\tau_1)d\omega_1 \quad (\tau_1 = \frac{\tau}{\alpha},\ \omega_1 = \alpha\omega.)$$

twice under under the sign of integration, and, substituting in the above second order equations, we obtain identities

$$-\int_{-\infty}^{\infty} \omega_1^2\overline{V}(\omega_1)\cos\omega_1\tau_1 d\omega_1 - \alpha a\int_{-\infty}^{\infty} \omega_1\overline{V}(\omega_1)\sin\omega_1\tau_1 d\omega_1 + \alpha^2 b\int_{-\infty}^{\infty} \overline{V}(\omega_1)\cos\omega_1\tau_1 d\omega_1 \equiv 0, \quad (\tau_1 > 0)$$

$$-\int_{-\infty}^{\infty} \omega_1^2\overline{V}(\omega_1)\cos\omega_1\tau_1 d\omega_1 + \alpha a\int_{-\infty}^{\infty} \omega_1\overline{V}(\omega_1)\sin\omega_1\tau_1 d\omega_1 + \alpha^2 b\int_{-\infty}^{\infty} \overline{V}(\omega_1)\cos\omega_1\tau_1 d\omega_1 \equiv 0. \quad (\tau_1 < 0)$$

In these identities two left integrals for $\tau_1 > 0$ and $\tau_1 < 0$ are equal to each other exactly like the two extreme right integrals for $\tau_1 > 0$ and $\tau_1 < 0$ are equal to each other. Hence, the intermediate integral, i.e.

$$\alpha a\int_{-\infty}^{\infty} \omega_1\overline{V}(\omega_1)\sin\omega_1\tau_1 d\omega_1 \equiv 0,$$

and since $\alpha \neq 0$ and $a \neq 0$, we obtain

$$\int_{-\infty}^{\infty} \overline{V}(\omega_1)\sin\omega_1\tau_1 d\omega_1 \equiv 0.$$

The above result may seem unrealistic because it is as follows: the first derivative of function $\overline{v}(\tau_1)$ turned out to be identical to zero, while the function itself and its second derivative are not equal to zero. The matter of fact is that the function $\overline{v}(\tau_1)$ was substituted in the wrong equation. Let us substitute cosωt in the second order equation which already has the first derivative. Then we will have to admit that the first derivative of the function cosωt is identical to zero. The same holds for function $\overline{v}(\tau_1)$: an even function $\overline{v}(\tau_1)$, similarly to cosωt, is substituted in the second order equation which already has the first derivative. This is how the above results are to be interpreted.

As a corollary of the result obtained the above two identities can be rewritten in the form of one identity

$$-\int_{-\infty}^{\infty} \omega_1^2 \overline{V}(\omega_1)\cos\omega_1\tau_1 d\omega_1 + \alpha^2 b\int_{-\infty}^{\infty} \overline{V}(\omega_1)\cos\omega_1\tau_1 d\omega_1 \equiv 0$$

or as

$$\int_{-\infty}^{\infty} [-\omega_1^2 \overline{V}(\omega_1)\cos\omega_1\tau_1 + \alpha^2 b\overline{V}(\omega_1)\cos\omega_1\tau_1]d\omega_1 \equiv 0.$$

Having chosen integrands at any specified frequency we can write a new identity

$$-\omega_1^2 \overline{V}(\omega_1)\cos\omega_1\tau_1 + \alpha^2 b\overline{V}(\omega_1)\cos\omega_1\tau_1 \equiv 0,$$

since due to arbitrary α we can assume that

$$\omega_1^2 = \alpha^2 b.$$

Thus we approach the second order equation

$$\frac{d^2v}{d\tau_1^2} + \alpha^2 bv = 0, \qquad (9)$$

where

$$\tau_1 = \frac{\tau}{\alpha},$$

but its solution is equal to $\cos\omega_1\tau_1$.

Note several important points concerning equation (9):

1. if $\alpha = 1$, and $\tau_1 = t$, then we have one of the motion equations for two material bodies in space (1), given in the beginning of the paper;
2. for arbitrary α and argument τ – this is the wave equation since only in the case of the wave the oscillation frequency can change so easily.

Consequently, first, this confirms a well-known experimental fact of wave-corpuscle duality introduced to physics by Lois de Broigle. Second, this corrects the mistake discussed earlier in this paper since Kepler's motion in the Universe described by (9), holds for any type of roots of the characteristic equation in (4) (three pairs of the roots out of three possible!).

3. Velocity of propagation of harmonic oscillation cosωτ and the movement of the planet of Mercury

From (1) for the movement of two material bodies in space discussed earlier in this paper we will use only the first two. Treating the solution of these two equations in the parametrical form

$$x = A\sin\omega t, \quad y = B\cos\omega t,$$

we can present the movement of a celestial body in the plane x0y as

$$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1. \qquad (10)$$

This is the equation of a circle if A = B, or the equation of an ellipse if A ≠ B. The movement of the planet of Mercury follows the trajectory of an elongated ellipse, i.e. A ≠ B. Projecting the trajectory of the movement of this planet to axes x and y, respectively, and treating the intervals obtained (−A, A) and (−B, B) as the intervals where two random values are distributed with the same probability density and have mathematical expectation equal to zero, we see that these two values have different dispersions along axes x and y.

Consider now identities of type (5) along axes x and y respectively, with the required indices, including the arguments:

$$-\int_{-\infty}^{\infty}\omega^2 V_x(\omega)\cos\omega\tau_x d\omega - a_x\int_{-}^{\infty}\omega V_x(\omega)\sin\omega\tau_x d\omega + b_x\int_{-\infty}^{\infty}V_x(\omega)\cos\omega\tau_x d\omega \equiv 0, \quad (\tau>0) \qquad (11)$$

$$-\int_{-\infty}^{\infty}\omega^2 V_y(\omega)\cos\omega\tau_y d\omega - a_y\int_{-\infty}^{\infty}\omega V_y(\omega)\sin\omega\tau_y d\omega + b_y\int_{-\infty}^{\infty}V_y(\omega)\cos\omega\tau_y d\omega \equiv 0. \quad (\tau>0) \qquad (12)$$

Assume that $\tau = 0$ in identities (14) and (15) and, as already noted above, we obtain:

$$D\Omega_x = b_x, \qquad D\Omega_y = b_y,$$

hence an inequality

$$b_x \neq b_y, \qquad (13)$$

since

$$D\Omega_x \neq D\Omega_y.$$

For axes x and y we can write down equations of type (9) as

$$\frac{d^2v_x}{d\tau_x^2}=-b_x v_x, \qquad \frac{d^2v_y}{d\tau_y^2}=-b_y v_y \qquad (14)$$

or as

$$\frac{d^2v_x}{d\tau_x^2}=-b_x v_x, \qquad \frac{d^2v_y}{d\tau_{\alpha y}^2}=-\alpha^2 b_y v_y, \qquad (15)$$

taking into account that the substitution (2) can change the argument τ, for instance, in the function $v_y(\tau)$, changing at the same time the value of the constant b_y, as already noted above. If the parameter α in (15) is chosen so that we have an equation $\omega_x^2 = b_x = \alpha^2 b_y = \omega_y^2$, then it is obvious that in (15) we will also have an equation $\tau_x = \tau_{\alpha y}$, which follows from (1): the equations (1) and (15) differ only by designations. Consequently, due to the inequality (13) in the equations (14)

$$\tau_x \neq \tau_y. \qquad (16)$$

If in (14) we have $b_x = b_y$, i.e. the dispersions of random values along axes x and y are equal, then obviously we obtain an equation of type (10) for A = B, i.e. the circumference equation.

If, to the contrary, in (14) $b_x \neq b_y$, i.e. $\omega_x \neq \omega_y$, then from equations (14) also follows an equation of type (10) for A ≠ B, i.e. the ellipse equation, since by virtue of (8) we have an equation

$$\sin^2\omega_x\tau_x + \cos^2\omega_y\tau_y = 1,$$

though the oscillation frequencies are not identical but $\omega_x\tau_x = \omega_y\tau_y$.

Consequently, equations of motion for any celestial body in the Solar system are written as (1) or (15), if the motion of the body follows the circumferential trajectory and in both cases we have equalities

$$b_x = b_y, \qquad \tau_x = \tau_y.$$

Consequently, systems described by equations (1) and (15) are identical.

If the motion of a celestial body in the Solar system follows an elliptical trajectory then we can correctly map it by a motion equation only in the form of the system (15), and in this case we have inequalities

$$b_x \neq b_y, \qquad \tau_x \neq \tau_y. \tag{17}$$

Hence, it immediately follows that the system (15) describes the movement of the celestial body more precisely than the system (1) does, since it includes the movement ignored by the system (1). Inequality (13) in equations (1) is absolutely not possible, because the body rotation along axes x and y is not the same, which means that we will not obtain the equation (10), and hence, this will not coincide with the observed trajectory of the movement of the planet of Mercury.

Let us now show that in case of the elliptical movement the speed of harmonic oscillation propagation along axes x and y is not the same. Consider the oscillation velocity v_x along axis x using the example of the equations (11), ascribing indices also to the oscillation frequency ν_x: $v_x = \lambda\nu_x$.
The wavelength does not have any index because it is known that the velocity of a wave of any length is the same. Multiplying both sides in the velocity formula by $2\pi\tau_x$ and rewrite it as

$$2\pi v_x\tau_x = \lambda\,\omega_x\tau_x = C_1\lambda, \qquad C_1 = \omega_x\tau_x, \qquad \omega_x = \sqrt{b_x}\,.$$

The arbitrary constant value C_1, as already noted above is done by the researcher, hence

$$v_x = \frac{\lambda C_1}{2\pi\tau_x} = C\frac{\lambda}{\tau_x}\,, \qquad \left(C = \frac{C_1}{2\pi}\right). \tag{18}$$

The velocity of oscillation propagation along the axis y is specified by the formula, similar to (18):

$$v_y = C\frac{\lambda}{\tau_y} \tag{19}$$

Note that the constant value C in formulas (17) and (18) is the same since

$$C_1 = \omega_x\tau_x = \omega_y\tau_y\,, \quad C = \frac{C_1}{2\pi}, \quad \omega_y = \sqrt{b_y}\,.$$

According to (16) $\tau_x \neq \tau_y$, hence, the velocity of the harmonic wave propagation along axis x and axis y is not identical: $v_x \neq v_y$.

Obviously, the resultant velocity in the plane x0y for $v_x \neq v_y$ (or, which is the same, for $b_x \neq b_y$) will be different from the resultant velocity which we obtain in the case when $v_x = v_y$ (or $b_x = b_y$). The difference in velocity should result (and it does) in the rotation of Mercury's perihelion, observed in the Universe different from the value which could be calculated based on the Newtonian law of universal gravitation. Так как скорость вращения Меркурия высока и орбита этой планеты есть ярко выраженный эллипс, то эта разница приводит к расхождению, составляющему 42 угл. сек за столетие, что требовало объяснения. Автор считает, что такоет объяснение дано на основе теории, которая создана великим И.Ньютоном.

4. Conclusions

1. Assuming that the equations (15):

a) provide a more precise description of the movement of any celestial body;

b) are the equations of the correlation function of a stationary process;

c) have double application: describe the movement of a material body and the movement of a wave;

d) can be obtained for any pair of the characteristic equation as it has been proved by the theorem,

and that all harmonic oscillations are of the same nature, then almost surely, one can state that both, electromagnetic and light oscillations have $\tau = t_1 - t_2$ as an argument, i.e. the time interval between two arbitrary sections of a stationary random process at times t_1 and t_2. It follows from the above that the Universe is stationary and, consequently, Euclidean.

2. In a stationary Universe no process of runaway of Galaxies observed by astronomers can take place. Consequently, physics in the Universe differs from that on Earth and requires other methods of investigation, different from the ones used at present. For instance, the method of analyzing phenomena on the basis of the Doppler effect is not acceptable, because it is a method of non-stationary environment.

3. Law of universal gravitation is a numerical representation of the universal field law which describes the motion of bodies in the stationary environment when the motion is circular. This specification of the law of universal gravitation merely changes the accents and views advocated by the great I. Newton.

A numerical representation of a more general law of nature when the motion of bodies in the stationary environment is not circular is still to be found and explained.

4. Relationship between the frequency ω of the harmonic oscillation $\cos\omega\tau$ and the argument τ, has to be taken into account when analyzing phenomena of nature. For instance much is written about atomic oscillations as ideal natural clock. Almost surely the argument of such oscillations is τ. Consequently, the assumption is not just slightly incorrect, but most likely is absolutely wrong.

Appendix

Theorem. Let $y_i(t)$ be a solution of an ordinary homogeneous asymptotically stable differential equation with constant real-valued coefficients, which is determined:

1) by any pair of complex-conjugated roots for $i = 1$,

2) by any pair of different real-valued roots for $i = 2$,

3) by one real-valued root of double multiplicity for $i = 3$

with initial conditions

$$\frac{dy_i(t)}{dt}\Big|_{t=0} = 0, \; y_i(t)\Big|_{t=0} = 1.$$

Then the function $v_i(\tau)$, consisting of $y_i(t)$ for $t \geq 0$ and $y_i(t)$ continued in the even manner to $t < 0$, is positive definite for all $i = 1, 2, 3$.

Proof.

1. Let the solution $y_1(t)$ be determined by complex-conjugated numbers $-\chi \pm i\lambda$, i.e.

$y_1(t) = (A\cos\lambda t + B\sin\lambda t)\exp(-\chi t)$. $(\chi > 0, \lambda > 0, t > 0)$

For initial conditions we obtain:

$$y(t)\Big|_{t=0} = A = 1, \qquad \frac{dy(t)}{dt}\Big|_{t=0} = \frac{B}{A} = \frac{\chi}{\lambda}.$$

Hence, we rewrite $y_1(t)$ in view of the initial conditions as

$$y_1(t) = (\cos\lambda t + \frac{\chi}{\lambda}\sin\lambda t)\exp(-\chi t).$$

Obtain the even function $v_1(t)$ as

$v_1(t) = y_1{}^*(t)$ при $t \leq 0$, $\qquad v_1(t) = y_1(t)$ при $t \geq 0$.

The Fourier transformation $V_1(i\omega)$ of the function $v_1(t)$ is equal to

$$V_1(i\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_1(t)\exp(-i\omega t)dt = \frac{1}{2\pi}[\int_0^{\infty} y_1^*(-s)\exp(i\omega s)ds + \int_0^{\infty} y_1(t)\exp(-i\omega t)dt].$$

Since according to our designations $y_1{}^*(-s) = y_1(s)$, the intervals in $V_1(i\omega)$ differ one from the other only by the sign of ω, hence, their sum is a real-valued function, which permits writing the Fourier transformation of the function $v_1(t)$ as

$$V_1(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_1(t)\exp(-i\omega t)dt = \frac{1}{\pi}\int_0^{\infty} y_1(t)\cos\omega t dt.$$

Substituting in $V_1(\omega)$ the function $y_1(t)$ and calculating this intergral, we obtain:

$$V_1(\omega) = \frac{1}{\pi}\int_0^{\infty}(c\text{os}\,\lambda t + \frac{\chi}{\lambda}\sin\lambda t)\exp(-\chi t)\cos\omega t dt = \frac{2\chi(\chi^2+\lambda^2)}{\pi[\chi^2+(\lambda+\omega)^2][\chi^2+(\lambda-\omega)^2]} \geq 0$$

for all $\omega \geq 0$.

2. Let solution $y_2(t)$ be determined by different real-valued numbers $-\chi_1$ and $-\chi_2$, i.e.

$$y_2(t) = A\exp(-\chi_1 t) + B\exp(-\chi_2 t),$$

which assuming the initial conditions can be written as

$$y_2(t) = \frac{\chi_2}{(\chi_2-\chi_1)}[\exp(-\chi_1 t) - \frac{\chi_1}{\chi_2}\exp(-\chi_2 t)].$$

Obtain the even function $v_2(t)$ as

$$v_2(t) = y_2(-t) = y_2^*(t) \text{ for } t \leq 0, \qquad v_2(t) = y_2(t) \text{ for } t \geq 0.$$

The Fourier transformation $V_2(i\omega)$ of the function $v_2(t)$ is equal to

$$V_2(\omega) = \frac{1}{\pi}\int_0^{\infty} y_2(t)\cos\omega t dt = \frac{1}{\pi}\int_0^{\infty}\frac{\chi_2}{(\chi_2-\chi_1)}[\exp(-\chi_1 t) - \frac{\chi_1}{\chi_2}\exp(-\chi_2 t)]\cos\omega t dt =$$

$$= \frac{\chi_1\chi_2(\chi_1+\chi_2)}{\pi(\chi_1^2+\omega^2)(\chi_2^2+\omega^2)} \geq 0$$

for all $\omega \geq 0$.

3. Let solution $y_3(t)$ be determined by a real-valued- number $-\chi$ of double multiplicity, i.e.
$y_3(t) = A\exp(-\chi t) + Bt\exp(-\chi t)$,
which assuming the initial conditions is given as
$y_3(t) = \exp(-\chi t) + \chi t\exp(-\chi t)$.
In this case too we can obtain an even function $v_3(t)$ in accordance with the above rule. The Fourier transformation $V_3(i\omega)$ of the function $v_3(t)$ is equal to

$$V_3(i\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_3(t)\exp(-i\omega t)dt = \frac{1}{\pi}\int_0^{\infty}[\exp(-\chi t) + \chi t\exp(-\chi t)]\cos\omega t dt = \frac{2\chi^3}{\pi(\chi^2+\omega^2)^2} \geq 0$$

for all $\omega \geq 0$.

The Fourier transformation $V_i(\omega)$ of the function $v_i(\tau)$ $(i = 1, 2, 3)$ for all types of roots which can be found in the second order equation is a non-negative function. Consequently, $V_i(\omega)$ is the probability distribution density of a random value Ω_i. Since $v_i(\tau)$ is a continuous function and

$$\int_{-\infty}^{\infty} |v_i(\tau)|\,d\tau < \infty,$$

then $v_i(\tau)$, as known, can be presented as a Fourier integral

$$v_i(\tau) = \int_{-\infty}^{\infty}\exp(i\omega\tau)V(\omega)d\omega = 2\int_0^{\infty} V_i(\omega)\cos\omega\tau\, d\omega.$$

Function $v_i(\tau)$ for all $i = 1, 2, 3$ is positive definite by virtue of the Bochner - Khinchin's theorem [1]. This proves Theorem.

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Index of refraction of a gravity wave

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On the basis of a finiteness of a velocity of a gravitational interaction the new properties of a gravitational field are obtained: reflection and refraction of gravity waves by mass distribution. The method is similar to a method in optics. The method is connected to a velocity of light in matter [1]. In that paper is shown, that the index of refraction of gravity waves in matter is possible to introduce. The index of refraction of gravity waves in matter can be more and less than 1. There can be parameters, when the effect of full external reflection of gravity waves from mass distribution can be took place.

1. Model of a harmonic oscillator

The calculation of index of refraction of a gravity wave will be made on the basis of model of a harmonic oscillator. Body (planet or satellite) is rotated around of center of a gravitational attraction (star or planet, respectively).

Let's record an equation of a balance of the forces:

$$m\omega_0^2 r_0 = \gamma \frac{mM}{r_0^2}. \tag{1}$$

Here *m*- mass of a body, *M* - mass of a center of gravity, γ a gravitation constant, r_0 - radius of rotation of a body around of a center of gravity.

We receive, that

$$\omega_0 = \sqrt{\gamma \frac{M}{r_0^3}}. \tag{2}$$

The forced vibrations of a body arise under operation of a gravity wave.

We receive, that the operation of a gravity wave is determined by strength of a gravitational field. The force acts on a body:

$$F = mG, \tag{3}$$

Where G - field of a wave:

$$G = G_0 \cos \omega t. \tag{4}$$

2. Equation of a dispersion

We record an equation of motion for a body

$$mr'' = mG - m\omega_0^2 r \tag{5}$$

We divide on *m* and is received:

$$r'' + \omega_0^2 r = G_0 \sin \omega t \tag{6}$$

This equation represents an equation of motion at forced vibrations. The solution of this equation can be written to a kind

$$r = A \sin \omega t . \tag{7}$$

Here amplitude of forced vibrations is set by an equation:

$$A = \frac{G_0}{\left(\omega_0^2 - \omega^2\right)} . \tag{8}$$

The analog of polarization P is determined by expression

$$P = \rho r , \tag{9}$$

Here ρ is the density of the mass *m*.

Then *P* is connected with *G* as follows:

$$P = \frac{\chi G}{4\pi\gamma} \tag{10}$$

Here χ - susceptibility, which one we shall calculate below. We shall substitute expression for *G* (4) in (10):

$$P = \rho G_0 \frac{\sin \omega t}{\left(\omega_0^2 - \omega^2\right)} \tag{11}$$

We equate (11) and (9), in which one have substituted (7) and (8) and is received:

$$\rho G_0 \frac{\sin \omega t}{\left(\omega_0^2 - \omega^2\right)} = \frac{\chi G_0 \sin \omega t}{4\pi\gamma} . \tag{12}$$

From expression (12) is discovered, that the susceptibility is set by expression:

$$\chi = \frac{4\pi\rho}{\left(\omega_0^2 - \omega^2\right)} . \tag{13}$$

The analog of a permittivity ε for a gravitational field will look like:

$$\varepsilon = n^2 = 1 + \frac{4\pi\gamma\rho}{\left(\omega_0^2 - \omega^2\right)} . \tag{14}$$

Thus index of refraction is determined by the formula:

$$n = \sqrt{1 + \frac{4\pi\gamma\rho}{\left(\omega_0^2 - \omega^2\right)}} . \tag{15}$$

We see form this formula, that the index of refraction depends on frequency of an external field ω, i.e. the retrieved formula transmits a phenomenon of a dispersion of a gravitational field.

We express ω_0 through density of a center of gravity ρ_1

$$\omega_0 = \sqrt{\gamma \frac{4\pi\rho_1 R^3}{3r_0^3}}, \tag{16}$$

Here R - radius of a center of gravity.

We assume for a simplicity, that the densities of a body and center of gravity are equal each other. Let's enter parameter a:

$$a = 4\pi\gamma\rho. \tag{17}$$

We receive for identical densities of a body and center of gravity:

$$\omega_0 = \sqrt{a \frac{R^3}{3r_0^3}}. \tag{18}$$

The parameter a is set by expression:

$$a = \frac{3\omega_0^2 r_0^3}{R^3}. \tag{19}$$

Therefore we can simplify expression for an index of refraction using (19):

$$n = \sqrt{1 + \frac{3\omega_0^2 r_0^3}{\left(\omega_0^2 - \omega^2\right) R^3}}. \tag{20}$$

We divide numerator and denominator of the fraction in (20) on ω_0^2. We receive following expression:

$$n = \sqrt{1 + \frac{3r_0^3}{\left(1 - \frac{\omega^2}{\omega_0^2}\right) R^3}}. \tag{21}$$

Let's define x as:

$$x = \frac{\omega}{\omega_0}. \tag{22}$$

We assume that dependence of the radiuses for a simplicity equal

$$\frac{3r_0^3}{R^3} = 10,$$

That corresponds to that the body goes outside of a center of gravity, i.e.:

$$r_0 > R$$

Though the dependence of the radiuses can be taken other.

We receive following expression for *n* at these assumptions:

$$n = \sqrt{1 + \frac{10}{\left(1 - x^2\right)}}\,. \tag{23}$$

We designate the radicand as *N*. Thus:

$$N = 1 + \frac{10}{\left(1 - x^2\right)}\,. \tag{24}$$

The dependence *N* from x is shown in a fig. 1 and 2.

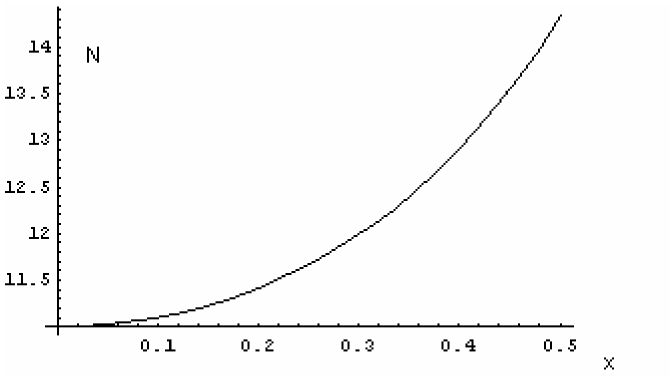

Fig. 1 Dependence N from x at x < 1.

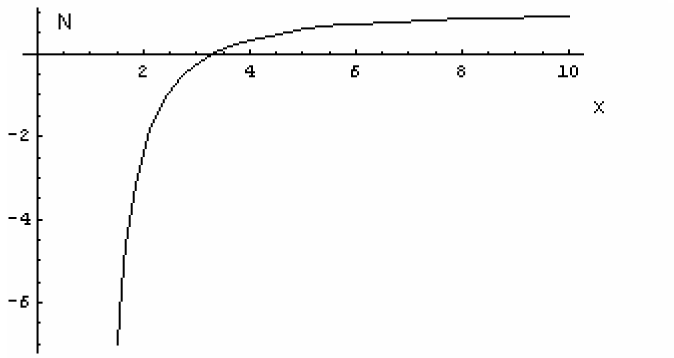

Fig. 2. Dependence *N* from *x* at *x* > 1.

We see on fig. 1 , that at $x < 1$ index of refraction of a gravity wave is positive, more than 1 and grows. When *x* limit from the left to 1, *N* limit to plus infinity. When *x* limit from the right to 1, *N* limit to minus infinity. We see on fig.2 , that there are two areas, in which values of an index of refraction distinguish qualitatively. In the first area he becomes imaginary ($N < 0$). In the second area ($0 < N < 1$)

the index of refraction of a gravity wave is positive and everywhere less than 1. The boundary between these areas corresponds to value x equal x_k. For our parameters x_k is equal: $x_k = \sqrt{11}$.

If x greater, than x_k the gravity waves are completely reflected from a body. It corresponds to effect of full external reflection in optic. In this area the velocity of gravity waves in matter is more than a velocity of gravity waves in vacuum. In this area the effect of full external reflection for gravity waves is watched.

3. Conclusion

On the basis of a finiteness of a velocity of a gravitational interaction the new properties of a gravitational field are obtained: reflection and refraction of gravity waves by mass distribution. In that paper is shown, that the index of refraction of gravity waves in matter is possible to introduce. The index of refraction of gravity waves in matter can be more and less than 1. There can be parameters, when the effect of full external reflection of gravity waves from mass distribution can be took place.

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Закон Тициуса-Боде и целочисленные последовательности

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Характеризующий Солнечную систему закон Тициуса-Боде впервые был сформулирован как целочисленная последовательность планетных расстояний. Рассмотрена связь этого закона с последовательностями Архимеда и последовательностью Шарковского.

1. Введение

Немецкий астроном Иоганн Даниель Тициус (1729-1796) в 1766 г. предложил эмпирический закон для средних расстояний планет от Солнца в виде целочисленной последовательности

$$r_n = 4 + 3 \cdot 2^n .$$

Расстояние Меркурия принималось равным четырем единицам длины. Каждая планета имела свой порядковый номер $n : n = -\infty$ (Меркурий), $n = 0$ (Венера), $n = 1$ (Земля), $n = 2$ (Марс), $n = 4$ (Юпитер), $n = 5$ (Сатурн). К середине 19 века были добавлены номера: $n = 3$ (пояс астероидов) и $n = 6$ (Уран). Нептун из этой зависимости выпал. Допустимыми являются такие значения n , которым соответствуют целочисленные значения степени двойки. Закон запрещает планетам приближаться к Солнцу ближе, чем Меркурий. Позднее закон стали связывать также с именем немецкого астронома Иоганна Элерта Боде (1747-1826), который включил закон Тициуса в текст своей книги.

Недавно в астероидном поясе Койпера (обширной зоне, лежащей за орбитой Нептуна) было открыто несколько планетообразных тел: 136199 Эрида (Эрис, Ксена, 2003 UB313), 136108 Санта (2003 EL61), 90337 Седна (2003 VB12), 136472 Истербанни (2005 FY9), 90482 Оркус (2004 DW), 50000 Квавар (2002 LM60) и так далее. Кроме того, в поясе Койпера находятся еще два значительных по размерам планетообразных тела, открытые в 20 веке: 134340 Плутон и 134340 I Харон. Эти два тела образуют двойную систему, барицентр (общий центр масс) которой находится в открытом космосе (вне Плутона и вне Харона). Ещё только предстоит систематизировать все эти тела и осмыслить их роль в рамках закона Тициуса-Боде. Трудно также понять имеет ли какое-либо отношение к закону Тициуса-Боде кометное облако Оорта, важный объект Солнечной системы.

2. Статус Плутона и его роль в законе Тициуса-Боде

76 лет Плутон считался девятой планетой Солнечной системы с момента его открытия 18 февраля 1930 года Клайдом Томбо. Но 24 августа 2006 года в Праге 26-я Генеральная ассамблея Международного астрономического союза лишила Плутон статуса планеты. Одновременно ассамблея приняла следующее определение планеты в Солнечной системе: «Планета − это небесное тело, которое (a) обращается вокруг Солнца, (b) имеет достаточную массу, для того, чтобы самогравитация превосходила твердотельные силы и тело могло принять гидростатически равновесную (близкую к сферической) форму, и (c) очищает окрестности своей орбиты (т.е. рядом с планетой нет других сравнимых с ней тел)». Плутон не соответствует этому определению, поэтому число планет в Солнечной системе сократилось с девяти до восьми. Аналогично, с момента, когда 22 июня 1978 года был открыт Харон, он считался спутником планеты Плутон до 24 августа 2006 года. Сегодня

Харон перестали считать спутником Плутона и рассматривают его как часть двойной системы Плутон-Харон. Когда Плутон был планетой, в законе Тициуса-Боде ему приписывался номер $n = 7$. Новый статус Плутона формально исключает его из закона планетных расстояний.

3. Последовательности Архимеда

В своей работе «Измерение круга» Архимед (287-212 до н.э.) использовал последовательности правильных многоугольников, в которых каждый многоугольник имеет в два раза больше сторон, чем предыдущий многоугольник. Будем считать, что многоугольник задан, если указано число его сторон (число 3 обозначает треугольник, 4 − квадрат, 2 − двуугольник, 1 − точку, 5 − пятиугольник и так далее). С помощью последовательности $\{2^n\}$ Архимед нашел формулу площади круга, для этого он последовательно удваивал число сторон квадрата. С помощью последовательности $\{3 \cdot 2^n\}$ Архимед получил оценку значения числа π ($223/71 < \pi < 22/7$), для этого он последовательно удваивал число сторон шестиугольника. В общем случае процесс удвоения числа сторон произвольного многоугольника характеризуется следующими последовательностями (будем называть их последовательностями Архимеда):

$$1 \to 2 \to 2^2 \to 2^3 \to 2^4 \to 2^5 \to 2^6 \to 2^7 \to \ldots ;$$
$$3 \to 3 \cdot 2 \to 3 \cdot 2^2 \to 3 \cdot 2^3 \to 3 \cdot 2^4 \to 3 \cdot 2^5 \to 3 \cdot 2^6 \to 3 \cdot 2^7 \to \ldots ;$$
$$5 \to 5 \cdot 2 \to 5 \cdot 2^2 \to 5 \cdot 2^3 \to 5 \cdot 2^4 \to 5 \cdot 2^5 \to 5 \cdot 2^6 \to 5 \cdot 2^7 \to \ldots ;$$
$$7 \to 7 \cdot 2 \to 7 \cdot 2^2 \to 7 \cdot 2^3 \to 7 \cdot 2^4 \to 7 \cdot 2^5 \to 7 \cdot 2^6 \to 7 \cdot 2^7 \to \ldots ;$$
$$9 \to 9 \cdot 2 \to 9 \cdot 2^2 \to 9 \cdot 2^3 \to 9 \cdot 2^4 \to 9 \cdot 2^5 \to 9 \cdot 2^6 \to 9 \cdot 2^7 \to \ldots ;$$

…………………………………………………………………………..

Знак $\to$ (стрелка) указывает на удвоение числа сторон.

Все последовательности Архимеда осуществляют разбиение множества натуральных чисел, так как каждое натуральное число встречается один и только один раз среди всех чисел, образующих эти последовательности. Таким образом, последовательность Тициуса-Боде $\{4 + 3 \cdot 2^n\}$ является суммой постоянной последовательности $\{4\}$ и второй последовательности Архимеда $\{3 \cdot 2^n\}$.

4. Последовательность Шарковского

Сосуществование циклов непрерывного отображения прямой в себя характеризуется последовательностью всех натуральных чисел, расположенных в определенном порядке:

$$1 \lhd 2 \lhd 2^2 \lhd 2^3 \lhd 2^4 \lhd 2^5 \lhd 2^6 \lhd 2^7 \lhd \ldots$$
$$\ldots \lhd 13 \cdot 2^4 \lhd 11 \cdot 2^4 \lhd 9 \cdot 2^4 \lhd 7 \cdot 2^4 \lhd 5 \cdot 2^4 \lhd 3 \cdot 2^4 \lhd \ldots$$
$$\ldots \lhd 13 \cdot 2^3 \lhd 11 \cdot 2^3 \lhd 9 \cdot 2^3 \lhd 7 \cdot 2^3 \lhd 5 \cdot 2^3 \lhd 3 \cdot 2^3 \lhd \ldots$$
$$\ldots \lhd 13 \cdot 2^2 \lhd 11 \cdot 2^2 \lhd 9 \cdot 2^2 \lhd 7 \cdot 2^2 \lhd 5 \cdot 2^2 \lhd 3 \cdot 2^2 \lhd \ldots$$
$$\ldots \lhd 13 \cdot 2 \lhd 11 \cdot 2 \lhd 9 \cdot 2 \lhd 7 \cdot 2 \lhd 5 \cdot 2 \lhd 3 \cdot 2 \lhd \ldots \lhd 13 \lhd 11 \lhd 9 \lhd 7 \lhd 5 \lhd 3 .$$

Цикл обозначается натуральным числом. Знак $\lhd$ определяет порядок взаимного расположения натуральных чисел или взаимного расположения циклов. Пусть натуральные числа p и q расположены в порядке $p \lhd q$. Это означает, что из существования цикла q

следует существование цикла p. Рассмотренные здесь последовательность и ее порядок принято называть последовательностью Шарковского и порядком Шарковского. Последовательность Шарковского распадается на бесконечное число серий (подпоследовательностей), разделенных многоточием. Все серии Шарковского осуществляют разбиение множества натуральных чисел.

Первая серия Шарковского $\left\{2^n\right\}$ совпадает с первой последовательностью Архимеда $\left\{2^n\right\}$. Построим бесконечную матрицу, строками которой являются остальные (кроме первой) серии Шарковского. Столбцами такой матрицы являются остальные (кроме первой) последовательности Архимеда. Последовательность чисел $\left\{3 \cdot 2^n\right\}$, с которых начинаются остальные серии Шарковского, совпадает со второй последовательностью Архимеда $\left\{3 \cdot 2^n\right\}$, а следовательно, в сумме с постоянной последовательностью $\{4\}$, дает последовательность Тициуса-Боде.

5. Заключение

Закон Тициуса-Боде в простой математической форме отражает сложные процессы самоорганизации планет, происходящие в Солнечной системе. Физический смысл этих процессов пока не удается понять.

В настоящей работе указана математическая связь последовательности Тициуса-Боде со второй последовательностью Архимеда и с расположением числовых серий, образующих последовательность Шарковского. Можно предположить, что эта связь не является случайной, а отражает закономерности устройства Солнечной системы.

Отметим также, что последовательность Тициуса-Боде попала в замечательную энциклопедию целочисленных последовательностей Н. Слоэна, содержащую более 130000 последовательностей (2007 г.).

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Influence of interstellar gas and increasing of speed of light onto movement of Pioneer 10

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In the frame of the simplest six-dimensional treatment of the expanding Universe, an influence of an increasing with the time in speed of light onto movement of spacecraft Pioneer 10 is considered. It is shown that the increase with the time in speed of light increases the speed of spacecraft with acceleration proportional to its speed. For spacecraft moving in the direction from the observer, this increase causes a red anomalous shift of the frequency of received signal and therefore can not be responsible for the observed blue anomalous shift of frequency. It is shown that the transfer of a momentum from the spacecraft to molecules of interstellar gas, as a result of elastic collisions, may present such reason. It refers to movement both inside and outside of the Kuiper belt.

Analysis of radio tracking data received from Pioneer 10 and 11, shows that their signals did arrive some faster, than it was expected. It is considered that Pioneers move some closer to the Earth than according to account [1-3]. Among others, two possible explanations of these anomalies were specified. Firstly, by a source of braking can be dark matter. Secondly, it is asserted that the gradual increase in speed of light, as time go on, leads to apparent slowing of spacecraft: if the speed of light is increased, the telemetry signal comes faster, than it is expected. Then a question not only about the reason of increase in speed of light arises, but also: whether it influences speed of movement of spacecraft. If such influence were absent, for relativistic particles it would result in gradual decrease in their energy, as if the energy disappears, it is not known where, contrary to the principle of energy. Obviously, the first reason partly reduces to the second one, if the solar system is submerged into a cloud of dark matter by the size about several tens of astronomical units, because inside a massive cloud the speed of light is less than beyond its limits.

In [3] the braking of the device on its path by a dust and gas was considered and is obtained that, when the density of medium is equal to $1.38 \cdot 10^{-19}\ g/cm^3$, the Pioneer 10 is acted by the observed anomalous Sunward acceleration of $a_p = (8.74 \pm 1.33) \cdot 10^{-8}\ g/cm^2$. The account is carried out for inelastic collisions. It is pointed out that the Kuiper belt may have such density.

Substantially, that the speed of light may be varying with time only if the total space is multidimensional one but not three-dimensional one. Let us consider at first the influence of an increase of speed of light, according to six-dimensional treatment of the expanding Universe [4-6], on the speed of movement of a spacecraft. This treatment is the simplest six-dimensional one of three-dimensional expanding Universe in the form of three-dimensional sphere, that appeared as a result of the intersection of three simplest geometrical objects of finite size in the six-dimensional Euclidean space – of three uniformly expanding five-dimensional spheres. A scenario in which the energy of each elementary particle in the six-dimensional space is constant with time is considered.

The treatment is based on a principle of simplicity and similarity of the basic properties of substance and light. To this principle is referred the assumption of movement of particles of substance with speed of light in multidimensional space in the Compton vicinity of three-dimensional sphere. Thus it is supposed, that in additional space there are cosmological forces holding each elementary particle on its Compton distance from three-dimensional sphere. If such forces there did not exist, the formation of macroscopic bodies would be impossible.

In six-dimensional Euclidean space, the speed of elementary particles is supposed constant. The considered approach goes back to the idea of F. Klein about movement of particles with speed of light in multidimensional space [7-10] and Einstein's statement that "the nature saves on principles". The first substantiation of six-dimensionality of space is given by di Bartini [11].

It is well known that the light and as well particles of substance have corpuscular and as well wave properties. Examples are diffraction of electrons, when they are represented as a wave, and photoelectric emission, when photon is represented as a particle. Therefore, following a principle of simplicity, it is natural assume, that some basic properties of light and particles of substance are identical. The basic property of light is that it propagates with the same speed in any frame of reference. Then elementary particles of substance should move also with the same speed. It is impossible in three-dimensional space, but it is possible in multidimensional one, if the observer registers positions of particles in a projection onto three-dimensional space.

The particle that is at rest from the point of view of the three-dimensional observer, moves with speed of light in the simplest case on a circle located in one of planes of additional three-dimensional space. In any other inertial frame of reference the considered particle moves along a helical line with an axis belonging to three-dimensional space.

Natural measure of the proper time of a particle is the number of its revolutions in additional space about of this axis. Therefore the proper time of a particle is proportional to this number or length of path in additional space. The energy of photon is equal to $h\nu$, where ν is the frequency of light, h is Plank's constant. By virtue of a principle of similarity of the basic properties of substance and light being a concrete definition of a principle of simplicity, the rest energy mc^2 of a particle of mass m also should be represented as quantum of energy $h\nu$, so $mc^2 = h\nu$, where c is speed of light. By unique and natural frequency ν for a particle of substance is the frequency of its revolutions in additional space. On the other hand, the particle is moving along the additional space with speed of light. Hence the frequency ν and radius a of a helical line obey the relation $2\pi a = c/\nu$. Eliminating the frequency ν from this relation and formula $mc^2 = h\nu$, one finds $a = \hbar/mc$ [4,12,13].

In six-dimensional cosmology, the speed of light changes according to the formula $c_*(r_u) = c\,\mathrm{Re}\sqrt{\frac{1-wr_u^{-2}}{1-w}}$, where $c_*(r_u)$ is speed of light at an instance when the ratio of radius of three-dimensional sphere to its today's magnitude was equal to r_u, c is the speed of light today, $w = q/p$, $p = (\tau+2)\tau$; q and τ are parameters of the theory. Preferable parameters, for which the theoretical and observed data are satisfactorily fitted, including the distributions of galaxies and gamma-bursts in dependence upon redshift, are equal to $\tau = -4.9$, $q = 0.9$. By $r_u \geq w$, relative derivative of the square of speed of light with respect to time on the light clock is equal to $\varepsilon(r_u) = \frac{1}{c_*^2(r_u)}\frac{dc_*^2(r_u)}{dt} = \frac{2H_0 q}{r_u(pr_u^2 - q)}\sqrt{\frac{p-q}{pr_u^2-q}\cdot\frac{pr_u^2+1}{p+1}}$, where H_0 is Hubble constant. Whence at the present time $\varepsilon(1) = \frac{2w}{1-w}H_0$. By $H_0 = 65\,km/s\cdot Mpc$, it equals $\varepsilon(1) = 2.85\cdot 10^{-19}\,s^{-1}$. It corresponds to an increase of speed of light with acceleration $\varepsilon c/2 = 4.27\cdot 10^{-9}\,cm/s^2$; it is hereinafter designated: $\varepsilon = \varepsilon(1)$.

Not only velocities of particles along their helical trajectories are increasing, but also projections of these velocities, in the same proportion. Therefore three-dimensional speeds of particles also grow, namely, are growing with acceleration (on intervals of time, small in comparison with an interval, on which the speed of light essentially changes, *i.e.* in comparison with the age of the uni-universe). Therefore spacecraft moves a little bit faster than in the case when speed of light would be constant.

The energy of each elementary particle, including photon, increases proportionally to the square of speed of light c_*^2. This effect is caused by a constancy of total energy of elementary particles in six-dimensional space in the case, when the expansion of three-dimensional sphere is slowing down. According to a principle of similarity of the basic properties of substance and light, the energy of photon $h_* \upsilon_*$ is proportional to c_*^2 also. But the frequency of radiation ν_* is proportional to c_*. From here, one finds the relation $h_* = h c_* / c$. From constancy of the fine structure constant, $\alpha_* = e_*^2 / h_* c_*$, in course of time, it follows that the charge of electron is proportional to the speed of light also: $e_* = e c_* / c$. The gravitation energy as well as energy of each elementary particle, and, therefore, gravitational constant G_*, are proportional to c_*^2, too: $G_* = G c_*^2 / c^2$. At the instant t_*, in linear approximation on ε one has $G_* = G(1 - \varepsilon\tau)$, where gravitational constant G refers to the instant of sending of a signal by the device, $\tau = t - t_*$. Therefore, taking into account the influence of gravitation onto movement of devices along radial coordinate r from the center of gravitation,

$$\frac{dv}{dt_*} = -\frac{2GM}{r^2}(1 - \varepsilon\tau) + \varepsilon v, \qquad (1)$$

where M is mass of the Sun together with planets. Multiplying both sides of equality (1) by v, after integration with respect to time, using relation $v dt_* = dr$, one finds

$v^2(r) = v_0^2 + 2GM\left(\frac{1}{r} - \frac{1}{r_0}\right) + \varepsilon J$, where $v_0 = v(r_0)$ is the speed of device in the instant t_0, when its radial coordinate was equal to r_0,

$$J = \int_{r_0}^{r} \left[v(x) - \tau(x,r) \frac{d}{dx}\left(\frac{2GM}{x}\right) \right] dx, \qquad (2)$$

$$\tau(x,r) = \int_{x}^{r} \frac{dy}{v(y)} \cong \cong \frac{1}{v_0 B}\left[\sqrt{(Br + A)r} - \sqrt{(Bx + A)x} - \frac{A}{\sqrt{B}} \ln\left(\frac{\sqrt{Br + A} + \sqrt{Br}}{\sqrt{Bx + A} + \sqrt{Bx}} \right) \right]. \qquad (3)$$

Here $\tau(x,r)$ represents the time interval of movement of the device from a point with radial coordinate x up to a point r (in the right side of (3), a term proportional to ε is omitted), $B = 1 - (A/r_0)$, $A = 2GM/v_0^2$. Integrating in (2) in parts with account of equality $v d\tau = -dr$ and that, in considered approximation, $v(y) \cong \sqrt{v_0^2 + 2GM\left(\frac{1}{y} - \frac{1}{r_0}\right)}$, one obtains $J = \tau(r_0, r) v_0^2$, whence follows:

$$v(r) = \sqrt{v_0^2 + 2GM\left(\frac{1}{r} - \frac{1}{r_0}\right)} + \frac{\varepsilon}{2v(r)} \tau(r_0, r) v_0^2,$$

$$v_0^2 \cong v^2(r) - 2GM\left(\frac{1}{r} - \frac{1}{r_0}\right), \tag{4}$$

$$\tau(r_0, r) = \frac{1}{v_0 B}\left[\sqrt{(Br + A)r} - r_0 - \frac{A}{\sqrt{B}} \ln\left(\frac{\sqrt{Br + A} + \sqrt{Br}}{\sqrt{r_0} + \sqrt{Br_0}}\right)\right] - \delta\tau(r_0, r), \tag{5}$$

$\delta\tau(r_0, r) = \frac{\varepsilon}{2} v_0^2 \int_{r_0}^{r} \frac{\tau(x, r)}{v^3(x)} dx$. Here $\varepsilon\tau^2(r_0, r)/4 \le \delta\tau(r_0, r) \le \varepsilon\tau^2(r_0, r) v_0^2 / 4v^2(r)$.

The minus sign of latter member in the formula (5) points out, that the device arrives to any given point earlier, than in the case of constant speed of light, and the anomalous amendment is proportional to a square of time of flight of the device.

The contribution into Doppler shift of frequency of received signal due to anomalous increase in the speed of device is equal to $\Delta\nu = -\nu\varepsilon\tau(r_0, r) v_0^2 / 2v(r)c$, where v_0^2 is expressed through $v^2(r)$ by the formula (4). According to data [1,2], for Pioneer 10 $\nu = 2292 \cdot 10^6$ Hz, the distance from the Sun at the time of last contact with the device 27.04.02 is equal to 80.421 astronomical units (AU). The distance and the speed 27.04.06 were equal to 90.49 AU. and 12.16 km / s, respectively. 10.02.76 the distance was equal to 9.54 AU. (radius of the orbit of Saturn). According to these data, for the portion of a trajectory between 10.02.76 and 27.04.02 one finds $\Delta\upsilon = -2.31 \cdot 10^{-5}$ Hz, $\delta\tau = 0.088$ s.

An increment in speed of the signal on its path from spacecraft to an observer does not give a contribution in the shift of observed frequency. Because, during a time of propagation of a signal, the characteristic frequencies that are using for definition of a time unit, are changing in the same proportion. This means that the ratio of frequency of an accepted signal to characteristic frequency obtained at the laboratory does not vary, for given v/c, with any time of the signal's propagation. Thus, caused by increase in speed of light the anomalous displacement of frequency of a signal, received from the device moving in the direction from the observer, is red and consequently can not be responsible for observed blue anomalous displacement of frequency.

Increase in kinetic energy of particles due to an increase in speed of light, as well as the increase in potential energy caused by increase in the gravitational constant, seems to be inconsistent with the law of conservation of energy. However, at drawing up of balance of energy in multidimensional space it is necessary to take into account of energy of all kinds of movement. This includes movement in additional space around of a three-dimensional projection of a trajectory of a particle on Compton distance $a = \hbar/mc$ from this projection, as well as the movement in additional space due to expansion of three-dimensional universe. In six-dimensional cosmology, the total speed of a particle in six-dimensional space remains constant with the time, irrespective of a form of a trajectory in three-dimensional space. Due to this the principle of conservation of energy in total space is not violated. When expansion of three-dimensional universe slow down, the speed of light is increasing, and when the expansion accelerate, the speed of light is decreasing. Note that the recent conclusion about the accelerated expansion of the universe is made in the implicit assumption of applicability of Einstein's equations to cosmology. In six-dimensional treatment of expansion of the Universe this assumption is not used.

The growth of speed of light is limited. Limit of the square of speed of light is equal to $c^2/(1-w)=1.068\cdot c^2$. Thus there is an energy reserve, not comparable with any other, which should yet come in the Universe in the future. It is 6.8 % from all forms of energy, including propre energy of a particle, which equals energy of its movement with speed of light $E=mc_*^2$ in additional space. Trajectory of a particle pass at Compton distance from its projection onto three-dimensional space [4,12,13]. The corresponding energy inflow, which mainly determines the rate of observed continuous star formation in galaxies during all of their history, is proportional to $\frac{dc_*^2(r_u)}{dt}=c^2\frac{2H_0q}{r_u^3(p-q)}\sqrt{\frac{p-q}{pr_u^2-q}\cdot\frac{pr_u^2+1}{p+1}}$. It means that, when radius of the Universe will be doubled, the inflow will decrease in 8.4 times. At epoch, when radius was equal to half of the today's one, that corresponds to redshift $z=1.084$, the inflow was in 9.8 times larger than the present one. To redshift $z=\infty$ correspond $r_u=0.416$ and inflow in 19.3 times larger than the today inflow.

In [4, 12-14], the geometrical treatment of Lorentz transformations, interval of the theory of relativity, relativistic mechanics, spin and isospin, proper magnetic moment, the fine structure formula, distinction between particles and antiparticles, length, phase and speed of de Broglie waves, Klein – Gordon equation, CPT-symmetry, quark model of nucleons, and six-dimensional treatment of gravitation are given.

In addition to the result of work [3], the following is noteworthy. At the centers of typical molecular cold gas clouds (10 K) concentration of molecules of hydrogen is $n\approx 10^5 cm^{-3}$ [15], that corresponds to density of the medium $\rho=2M_pn\approx \approx 3.3\cdot 10^{-19}\, g/cm^3$, where $M_p=1.66\cdot 10^{-24}g$ is the mass of an atom of hydrogen. For such concentration of molecules, the free path length is of order of $10^5 km$. In the case of large free path length in comparison with the sizes of the device the transfer of amount of momentum from the device to everyone participant of collision can be considered separately from other collisions with subsequent summation of loss of momentum over all collisions. At elastic collision of a particle with a surface of device moving with speed v, the molecule of mass $m_1=2M_p$, motionless prior to collision, gets a momentum $2m_1v$ in the direction of movement of the device. The average number of collisions per second is equal to vnS, where S is effective cross-section of scattering. The braking force is equal to $F=2m_1v^2nS=2\rho v^2S$. On dividing this by mass of the device $M_a=258$ kg, one finds acceleration $a=2m_1v^2n\,S/M_a=2\rho v^2\,S/M_a$. From this it is evident, the reason of observed slowing down of movement of the device, not only inside but also outside the Kuiper belt, can be transfer of amount of momentum to molecules of interstellar gas.

The force of braking can be reduced by means of increasing mass of the device and/or reducing its cross-section of scattering. But the supervision of movement of asteroids is already carry out. Their masses are so great that influence of their braking on measurement of parameters of motion are small in comparison with influence of increase of speed of light.

Conclusions. It is shown that the increase with the time in speed of light increases the speed of spacecraft with acceleration proportional to the its speed. Caused by increase in speed of light, the anomalous displacement of frequency of a signal, received from the device moving in the direction from the observer, is red and, consequently, can not be responsible for observed blue anomalous displacement of the frequency. This conclusion concerns any theory giving positive derivative of speed of light with respect to time. The appropriate quantitative data are given for six-dimensional treatment of expansion of the Universe. Is shown, that, for moving of Pioneer 10, not only inside but also outside the Kuiper belt, the observed blue anomalous displacement of frequency of an received signal can be caused by transfer of amount of momentum to molecules of clouds of interstellar gas in their collisions with the device.

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Релятивистские модели анизотропно жестких сред

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Relativistic models of anisotropic rigid media
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Analysis of possible material symmetries of relativistic anisotropic rigid media is developed. New description of rigidity is proposed.

Проводится анализ возможных материальных симметрий релятивистских анизотропно жестких сплошных сред. Предлагается новая формулировка условий жесткости.

1. В рамках специальной теории относительности рассмотрим свойства уравнений, определяющих закон движения сплошных сред $\xi^\alpha = \xi^\alpha(x^i)$, где ξ^α, $\alpha = 1,2,3,$ – лагранжевы и x^i, $i = 0,1,2,3,$ – эйлеровы координаты (инерциальные и декартовы), при наличии геометрических связей вида

$$I^k(\gamma^{\alpha\beta}) = I^k(\gamma_0^{\alpha\beta}), \qquad \gamma^{\alpha\beta} = -\eta^{ij}\xi_i^\alpha \xi_j^\beta, \qquad \xi_i^\alpha = \partial\xi^\alpha/\partial x^i \tag{1}$$

Тензор Минковского $(\eta^{ij}) = \mathrm{diag}(1,-1,-1,-1)$, индекс $k = 1,\dots,N \le 6$. Здесь используется определение жесткости по М. Борну [1]. Уравнения движения в случае адиабатического процесса отвечают лагранжевой форме

$$\Lambda = \Big(\rho U(I^s,\xi^\alpha) + \lambda_k(I^k - I_0^k)\Big)\, dx^0 dx^1 dx^2 dx^3 \tag{2}$$

Часть инвариантов I^s входит в удельную внутреннюю энергию U, часть – в связи. Варьируются лагранжевы переменные ξ^α и множители λ_k. Плотность массы покоя $\rho = \rho_0\sqrt{\gamma/\gamma_0}$, $\gamma = \det(\gamma^{\alpha\beta})$. Нулем обозначено начальное состояние, зависящее от ξ^α.

Уравнения движения имеют вид

$$\frac{\partial\Lambda}{\partial\xi^\alpha} - \nabla_i\left(\frac{\partial\Lambda}{\partial\xi_i^\alpha}\right) = 0 \tag{3}$$

2. Пусть $p_\alpha^\beta = -\xi_i^\beta \partial\Lambda/\partial\xi_i^\alpha$ – тензор напряжений, и имеет место равенство

$$p_\alpha^\beta\, a_\beta^\alpha = 0 \tag{4}$$

где $(a_\beta^\alpha(\xi^\gamma))$ – не зависящая от времени матрица. Тогда множество матриц a, как алгебра Ли, локально определяет линейную непрерывную группу нечувствительности среды G (или группу материальной симметрии) в данной материальной точке ξ^γ.

Предельными случаями являются абсолютно твердое тело (группа нечувствительности – единичная грппа с нулевой алгеброй Ли, $N = 6$) и пыль – среда без напряжений (группа нечувствительности есть GL_3). Идеальная несжимаемая жидкость имеет одну связь вида $\gamma = \gamma_0$ с группой нечувствительности $SL_3 \ni A$, $\det A = 1$. Внутренняя энергия в этом случае не зависит от I.

На этом пути, опуская зависимость внутренней энергии U от I^s, можно дать описание множества всех возможных связей и, следовательно, соответствующих сплошных сред, а также их фазовых переходов, основываясь на классификации непрерывных подгрупп полной линейной группы GL_3 трехмерных преобразований тензора $\gamma^{\alpha\beta}$, как групп материальной симметрии, и их инвариантов $I^k(\gamma^{\alpha\beta})$ [2].

Таких групп с точностью до сопряженности 99 типов, считая по одной непрерывные серии и отдельные группы. См. таблицу 1, где приведены алгебры Ли в каноническом базисе матриц $(e^i_j)^k_l = \delta^{ik}\delta_{jl}$ и инарианты, составленные из компонент произвольного симметричного тензора $\mathbf{q}$.

В нашем случае $q^{\alpha\beta} = \gamma^{\alpha\beta}$. Здесь $(q_{\alpha\beta}) = (q^{\alpha\beta})^{-1}$ как матрицы и $q = \det(q^{\alpha\beta})$. Оператор $d = e^1_1 + e^2_2 + e^3_3$

Подобного рода среды могут быть также использованы для решения внутренних задач для тел в общей теории относительности [3].

3. Условие жесткости М. Борна (1) с точки зрения кинематики твердого тела не вполне удовлетворительно. В частности, уже при одномерном поступательном движении твердого тела оно допускает произвольное начальное распределение скорости по частицам (правда, деленное на скорость света c). Чтобы избежать такого рода перекосов теории (дискуссия по этому поводу опубликована также в сб. [1], с. 341-350), лучше использовать вместо тензора $\gamma^{\alpha\beta}$ тензор

$$g^{\alpha\beta} = (l^i l^j - \eta^{ij})\xi^\alpha_i \xi^\beta_j \tag{5}$$

связанный с системой отсчета инерциального наблюдателя l^i, который полностью совпадает с ньютоновским определением фундаментального тензора, служащего для построения тензора деформаций $\varepsilon^{\alpha\beta} = 1/2(g^{\alpha\beta} - g^{\alpha\beta}_0)$. Хотя при фиксированном постоянном l^i,

$$\nabla_j l^i = 0 \tag{6}$$

нарушается лоренцева инвариантность формы действия (2), для ее восстановления можно присоеденить соотношение (6) к Λ с некоторыми множителеями Лагранжа μ^j_i, считая l^i искомым.

Заметим, что такого рода модернизация делается только в аргументах функции $U(I^s(g^{\alpha\beta}), \xi^\gamma)$, зависящих или не зависящих от $\mathbf{g}$, и при формулировке связей. Определение плотности $\rho = \rho_0\sqrt{\gamma/\gamma_0}$, связанное с релятивистскими эффектами движения, сохраняется. Причем имеет место соотношение

$$\gamma = (1 - |\mathbf{v}|^2/c^2)g \tag{7}$$

где в обычных обозначениях трехмерная скорость равна $v^k = -\xi^\alpha_t x^k_\alpha$, здесь $k = 1, 2, 3$; соответственно, дисторсия есть $(x^k_\alpha) = (\xi^\alpha_k)^{-1}$.

4. Коснемся вопроса интегрирования связей. Решение уравнений связей (1), в которых используется тензор $\mathbf{g}$ (5) является теперь независимой чисто кинематической проблемой, в которую не входит скорость движения сплошной среды $\mathbf{v}$. Рассмотрим случай однородной среды, когда матрицу a можно считать постоянной. Тогда уравнения связей переписываются в виде

$$\frac{\partial \xi^\alpha}{\partial x^i} = A^\alpha_\beta(a^r(x^i))O^\beta_i(\theta^m(x^i)), \qquad A \in G \subset GL_3, \qquad O \in SO_3 \tag{8}$$

где a^r – параметры группы нечувствительности G и θ^m – пераметры группы вращений SO_3, $m = 1, 2, 3$. Время $t = x^0/c$ входит здесь только как параметр.

Исследование совместности системы (8) приводит к отысканию функций $a^r(x^i)$ и $\theta^m(x^i)$. Например, если группа нечувствительности есть группа всестороннего растяжения, то в результате получим 10-параметрическое множество конформных преобразований, на котором со временем происходит движение сплошной среды.

В частности, рассмотрим плоскую задачу с однопараметрической группой G. В этом случае можно привести полное решение уравнений связи

$$\begin{cases} \xi^\alpha = A^\alpha_\beta(a)O^\beta_i(\theta)x^i + r^\alpha(a, \theta), \\ 0 = dA^\alpha_\beta/da\, O^\beta_i\, x^i + \partial r^\alpha/\partial a, \\ 0 = A^\alpha_\beta\, dO^\beta_i/d\theta\, x^i + \partial r^\alpha/\partial\theta \end{cases} \tag{9}$$

Исключая алгебраически x^1, x^2, получим систему линейных уравнений первого порядка для величин r^1, r^2. Например, для группы двумерных растяжений $D \ni \mathrm{diag}(a, a)$ имеем систему уравнений Коши-Римана, которая, в свою очередь, решается с помощью функций комплексного переменного.

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Таблица 1. Непрерывные подгруппы полной линейной группы

dim	N^0	Алгебра Ли	Инварианты
1	1	$x(e_1^1+e_2^2)+ye_3^3+e_2^1-e_1^2$, $x\geq 0$	$q_{33}(q_{11}+q_{22})^2q, q_{33}q^{33}$, $(q_{11}+q_{22})\exp(-x\mathrm{arctg}\frac{2q_{12}}{q_{11}-q_{22}})$, $(q_{11}+q_{22})^y q_{33}^{-x}, (q_{11}+q_{22})(q^{11}+q^{22})$
	2	$x(e_1^1+e_2^2)+ye_3^3+e_2^1$, $x=0;1$	$q_{33}q_{11}^2q, q_{11}q_{13}^2q, q_{13}q^{23}$, $q_{11}\exp(-\frac{2xq_{12}}{q_{11}}), q_{33}\exp(-\frac{2yq_{12}}{q_{11}})$,
	3	$e_1^1+xe_2^2+ye_3^3$, $1\geq x\geq y$	$q_{11}q_{22}q_{12}^{-2}, q_{11}q_{33}q_{13}^{-2}, q_{22}q_{33}q_{23}^{-2}$, $q_{22}q_{11}^{-x}, q_{33}q_{11}^{-y}$,
	4	$xd+e_2^1+e_3^2$, $x=0;1$	$q_{11}^3q, q_{11}q^{33}, (q_{22}-2q_{13})q^{33}$, $q_{11}q^{23}+q_{12}q^{33}, q^{33}\exp(\frac{2xq_{12}}{q_{11}})$
2	1	$x(e_1^1+e_2^2-2e_3^3)+e_2^1-e_1^2$, $d, \qquad x\geq 0$	$q_{33}(q_{11}+q_{22})^2q, q^{33}(q^{11}+q^{22})^2q^{-1}$, $q_{33}q^{33}, q_{33}^3q\exp(6x\mathrm{arctg}\frac{2q_{12}}{q_{11}-q_{22}})$,
	2	$x(e_1^1+e_2^2-2e_3^3)+e_2^1, d$, $x=0;1$	$q_{33}q_{11}^2q, q_{11}q_{13}^2q^{1/2}, q_{13}q^{23}$, $q_{11}^3q\exp(-\frac{6xq_{12}}{q_{11}})$
	3	$e_1^1+xe_2^2-(1+x)e_3^3, d$, $-1/2<x\leq 1$	$q_{11}q_{23}^2q, q_{22}q_{13}^2q, q_{11}q_{22}q_{33}q$, $(q_{33}q_{11}^{1+x})^3q^{2+x}$,
	4	$e_2^1+e_3^2, d$	$q_{11}^3q, q^{-1}(q^{33})^3, (q_{22}-2q_{13})^3q$, $q_{11}q^{23}+q_{12}q^{33}$
	5	$e_1^1+e_2^2-2e_3^3+yd$, $e_2^1-e_1^2+zd$	$q_{33}(q_{11}+q_{22})^2q, q^{-1}q^{33}(q^{11}+q^{22})^2$, $q_{33}q^{33}, q_{33}^{3y}q^{y-2}\exp(-6z\mathrm{arctg}\frac{2q_{12}}{q_{11}-q_{22}})$,
	6	$e_1^1-e_2^2+yd$, $e_1^1-e_3^3+zd$	$q_{11}q_{23}^2q, q_{22}q_{13}^2q, q_{11}q_{22}q_{33}q$, $q_{11}^{-3y}q_{33}^{3(z-y)}q^{-1-2y+z}$
	7	$e_2^1+e_3^2+yd, e_3^1+zd$, $y=0;1, z=0;1$	$q_{11}^3q, q^{-1}(q^{33})^3, q_{11}q^{23}+q_{12}q^{33}$ $q^{33}\exp(\frac{2yq_{12}+z(2q_{13}-q_{22})}{q_{11}})$
	8	$e_1^1+e_2^2-2e_3^3+yd, e_2^1+zd$, $z=0;y$	$q_{33}q_{11}^2q, q_{11}q_{13}^2q, q_{13}q^{23}$, $q_{11}^{3y}q^{1+y}\exp(\frac{6zq_{12}}{q_{11}})$,
	9	e_2^1+yd, e_3^1+zd, $y=0;1, z=0;1$	$q_{11}^3q, q^{-1}(q^{22})^3, q^{-1}(q^{23})^3$, $q^{11}\exp(-\frac{2(yq_{12}+zq_{13})}{q_{11}})$
	10	e_3^1+yd, e_3^2+zd, $y=0;1, z=0;1$	$q_{11}^3q, q_{12}^3q, q_{22}^3q$, $q^{11}\exp(-\frac{2yq_{13}}{q_{11}}+\frac{2zq^{23}}{q^{33}})$

Таблица 1 (продолжение).

dim	N^0	Алгебра Ли	Инварианты
2	11	$e_2^1+e_3^2, e_1^1-e_3^3+yd$	$(q^{33})^3q_{11}^{-3}q^{-2}, (q_{22}-2q_{13}))^2q,$ $q_{11}^{3y}q^{1+y}, q^{-1}q_{11}^{-3}(q_{11}q^{23}+q_{12}q^{33})^2$
	12	$e_2^1, xe_1^1+(1+x)e_2^2-$ $-(1+2x)e_3^3+yd$	$q_{13}^2q_{11}^{-1}q_{33}^{-1}, (q^{23})^2q_{11}^{-1}q^{-1},$ $(q_{13}q^{23})^{6y}q^{-1}, q_{33}^xq_{11}^{1+2x}q^{x+1/3}$
	13	$e_3^1, e_1^1+e_2^2-2e_3^3+e_2^1+yd$	$(q^{23})^3q_{11}^{-3}q^{-2}, q^{33}q_{11}^{-2}q^{-1}, q_{11}^{3y}q^{1+y},$ $q_{11}^3q\exp(-\frac{6q_{12}}{q_{11}})$
	14	$e_3^1, -2e_1^1+e_2^2+e_3^3+e_3^2+yd$	$q_{11}q_{22}^2q, q_{12}^4q_{11}^{-1}q, q_{11}^{3y}q^{y-2},$ $q^{-1}(q^{33})^3\exp(-\frac{6q^{23}}{q^{33}})$
3	1	$e_1^1+e_2^2-2e_3^3,$ $e_2^1-e_1^2, d$	$q_{33}(q_{11}+q_{22})^2q, q_{33}q^{33},$ $q^{33}(q^{11}+q^{22})^2q^{-1}$
	2	e_1^1, e_2^2, e_3^3	$q_{11}q_{22}q_{33}q, q_{11}q_{23}^2q, q_{22}q_{13}^2q$
	3	$e_2^1+e_3^2, e_3^1, d$	$q_{11}^3q, q^{-1}(q^{33})^3, q_{11}q^{23}+q_{12}q^{33}$
	4	$e_1^1+e_2^2-2e_3^3, e_2^1, d$	$q_{11}^2q_{33}q, q_{11}q_{13}^2q^{-1}, q_{13}q^{23}q$
	5	e_2^1, e_3^1, d	$q_{11}^3q, q^{-1}(q^{22})^3, q^{-1}(q^{23})^3$
	6	e_3^1, e_3^2, d	$q_{11}^3q, q_{12}^3q, q_{22}^3q$
	7	$e_1^1-e_3^3, e_2^1+e_3^2, d$	$(q_{33})^3q_{11}^{-3}q^{-2}, (q_{22}-2q_{13})^3q,$ $q^{-1}(q_{11}q^{23}+q_{12}q^{33})q_{11}^{-3}$
	8	$xe_1^1+(1+x)e_2^2-(1+2x)e_3^3,$ e_2^1, d	$q_{11}q_{33}q_{13}^{-2}, q_{33}^xq_{11}^{1+2x}q^{x+1/3},$ $(q^{23})^2q_{11}^{-1}q^{-1}$
	9	$e_1^1+e_2^2-2e_3^3+e_2^1,$ e_3^1, d	$(q^{23})^3q_{11}^{-3}q^{-2}, q^{33}q_{11}^{-2}q^{-1},$ $q_{11}^3q\exp(-\frac{6q_{12}}{q_{11}})$
	10	$-2e_1^1+e_2^2+e_3^3+e_3^2,$ e_3^1, d	$q_{11}q_{22}^2q, q_{12}^4q_{11}^{-1}q,$ $q^{-1}(q^{33})^3\exp(-\frac{6q^{23}}{q^{33}})$
	11	$e_2^1+e_3^2, e_3^1,$ $e_1^1-e_3^3+yd$	$q_{11}^{3y}q^{1+y}, q_{11}^{y-1}(q^{33})^{y+1},$ $(q_{11}q^{23}+q_{12}q^{33})^{2(1+y)}q_{11}^{-3}$
	12	$e_2^1, e_1^1-e_2^2+yd,$ $e_1^1-e_3^3+zd$	$q_{11}q_{33}q_{13}^{-2}, (q^{23})^2q_{11}^{-1}q^{-1},$ $(q_{13}q^{23})^{6y}q_{33}^{3(y-2z)}q^{2+y-2z}$

Таблица 1 (продолжение).

dim	N^0	Алгебра Ли	Инварианты
3	13	$-(1+x)e_1^1+xe_2^2+e_3^3+yd,$	$(q^{23})^2 q_{11}^{-1} q^{-1}, q_{11}^{3y} q^{-1-x+y},$
		$e_2^1, e_3^1, \quad \lvert x\rvert \leq 1, x \neq -1/2$	$(q^{22})^{3(1+x)} q_{11}^{-3x} q^{-(1+2x)}$
	14	$e_1^1+e_2^2-2e_3^3+zd,$	$(q^{23})^2 q_{11}^{-1} q^{-1}, q_{11} q^{22},$
		$e_2^1+yd, e_3^1, \quad y=0;1$	$q_{11}^{3z} q^{1+z} \exp(\frac{6yq_{12}}{q_{11}})$
	15	$e_3^1, e_2^1+yd, e_3^2+zd,$	$q_{11}^3 q, (q^{33})^3 q^{-1},$
		$y=0;1, z=0;1$	$q_{11} \exp(-\frac{2yq_{12}}{q_{11}}+\frac{2zq^{23}}{q^{33}})$
	16	$-2e_1^1+e_2^2+e_3^3+e_3^2+yd,$	$(q^{33})^2 q_{11}^{-1} q^{-1}, q_{11}^{3y} q^{y-2},$
		e_2^1, e_3^1	$q^{-1}(q^{33})^3 \exp(-\frac{6q^{23}}{q^{33}})$
	17	$x(-2e_1^1+e_2^2+e_3^3)+$	$(q^{22}+q^{33})^2 q_{11}^{-1} q^{-1}, q_{11}^{3y} q^{-2x+y},$
		$+e_3^2-e_2^3+yd, e_2^1, e_3^1$	$(q_{11})^3 q \exp(6x\mathrm{arctg}\frac{2q^{23}}{q^{22}-q^{33}})$
	18	$xe_1^1+e_2^2-(1+x)e_3^3+yd,$	$q_{11}q_{22}q_{12}^{-2}, q_{11}^3 q_{22}^{-3x} q^{1-x},$
		$e_3^1, e_3^2, \quad \lvert x\rvert \leq 1, x \neq -1/2$	$(q_{11}q_{12}^2)^{3y} q^{1+2x+3y}$
	19	$e_1^1-2e_2^2+e_3^3+zd,$	$q_{11}q_{22}q_{12}^{-2}, q_{11}^2 q_{22} q,$
		e_3^1+yd, e_3^2	$q_{11}^{3z} q^{1+z} \exp(\frac{6yq_{13}}{q_{11}})$
	20	$e_1^1+e_2^2-2e_3^3+e_2^1+yd,$	$q^{33} q_{11}^{-2} q^{-1}, q_{11}^{3y} q^{1+y},$
		e_3^1, e_3^2	$q_{11}^3 q \exp(-\frac{6yq_{12}}{q_{11}})$
	21	$x(e_1^1+e_2^2-2e_3^3)+$	$q^{33}(q_{11}+q_{22})^{-2} q^{-1}, (q^{33})^{3y} q^{2x-y},$
		$+e_2^1-e_1^2+yd, e_3^1, e_3^2$	$q^{-1}(q^{33})^3 \exp(-6x\mathrm{arctg}\frac{2q_{12}}{q_{11}-q_{22}})$
	22	$e_2^1+e_1^2, e_3^1+e_1^3, e_3^2-e_2^3$	$q, q_{11}-q_{22}-q_{33}, q^{11}-q^{22}-q^{33}$
	23	$e_1^1-e_2^2, e_2^1, e_1^2$	q, q_{33}, q^{33}
	24	$e_2^1-e_1^2, e_3^2-e_2^3, e_1^3-e_3^1$	$q, q_{11}+q_{22}+q_{33}, q^{11}+q^{22}+q^{33}$
4	1	$e_2^1+e_1^2, e_3^1+e_1^3, e_3^2-e_2^3, d$	$(q_{11}-q_{22}-q_{33})^3 q, q^{-1}(q^{11}-q^{22}-q^{33})^3$
	2	$e_1^1-e_2^2, e_2^1, e_1^2, d$	$(q_{33})^3 q, q^{-1}(q^{33})^3$
	3	$e_2^1-e_1^2, e_3^2-e_2^3, e_1^3-e_3^1, d$	$(q_{11}+q_{22}+q_{33})^3 q, q^{-1}(q^{11}+q^{22}+q^{33})^3$
	4	$e_2^1+e_3^2, e_3^1, e_1^1-e_3^3, d$	$(q^{33})^3 q^{-2} q_{11}^{-3}, q^{-1}(q_{11}q^{23}+q_{12}q^{33})^2 q_{11}^{-3}$
	5	$e_2^1, e_1^1, e_2^2, e_3^3$	$q_{11}q_{33}q_{13}^{-2}, (q^{23})^2 q_{11}^{-1} q^{-1}$
	6	$-(1+x)e_1^1+xe_2^2+e_3^3, e_2^1,$	$(q^{23})^2 q_{11}^{-1} q^{-1}, (q^{22})^{3(1+x)} q_{11}^{-3x} q^{-(1+2x)}$
		$e_3^1, d, \quad x \leq 1$	

Таблица 1 (продолжение).

dim	N^0	Алгебра Ли	Инварианты
4	7	$x(-2e_1^1+e_2^2+e_3^3)+e_3^2, e_2^1,$ $e_1^3, d \quad x=0;1$	$(q^{33})^2 q_{11}^{-1} q^{-1}, q^{-1}(q^{33})^3 \exp(-\frac{6xq^{23}}{q^{33}})$
	8	$x(-2e_1^1+e_2^2+e_3^3)+$ $+e_3^2-e_2^3, e_2^1, e_3^1, d$	$(q^{22}+q^{33})^2 q_{11}^{-1} q^{-1},$ $(q_{11})^3 q \exp(6x\mathrm{arctg}\frac{2q^{23}}{q^{22}-q^{33}})$
	9	$xe_1^1+e_2^2-(1+x)e_3^3,$ $e_3^1, e_3^2, d \quad x\le 1$	$q_{11}q_{22}q_{12}^{-2}, q_{11}^3 q_{22}^{-3x} q^{1-x}$
	10	$e_1^1+e_2^2-2e_3^3+e_2^1, e_3^1, e_3^2, d$	$q^{33}q_{11}^{-2}q^{-1}, q_{11}^3 q \exp(-\frac{6q_{12}}{q_{11}})$
	11	$x(e_1^1+e_2^2-2e_3^3)+e_2^1-e_1^2$ e_3^1, e_3^2, d	$q^{33}(q_{11}+q_{22})^{-2}q^{-1},$ $q^{-1}(q^{33})^3 \exp(-6x\mathrm{arctg}\frac{2q_{12}}{q_{11}-q_{22}})$
	12	$e_1^1-e_2^2+yd, e_2^2-e_3^3+zd,$ e_2^1, e_3^1	$(q^{23})^2 q_{11}^{-1} q^{-1}, (q^{33})^{3z} q_{11}^{3y} q^{1+y-z}$
	13	$-2e_1^1+e_2^2+e_3^3+yd,$ $e_3^2-e_2^3+zd, e_2^1, e_3^1$	$(q^{22}+q^{33})^2 q_{11}^{-1} q^{-1},$ $(q_{11})^{3y} q^{y-1} \exp(-3z\mathrm{arctg}\frac{2q^{23}}{q^{22}-q^{33}})$
	14	$e_1^1-e_2^2+yd, e_2^2-e_3^3+zd,$ e_3^1, e_3^2	$q_{11}q_{22}q_{12}^{-2}, q_{11}^{3z} q_{12}^{6y} q_{22}^{3(z-y)} q^{1+y+2z}$
	15	$e_1^1+e_2^2-2e_3^3+yd,$ $e_2^1-e_1^2+zd, e_3^1, e_3^2$	$(q_{11}+q_{22})^2 (q^{33})^{-1} q,$ $(q^{33})^{3y} q^{2-y} \exp(6z\mathrm{arctg}\frac{2q_{12}}{q_{11}-q_{22}})$
	16	$e_1^1-e_2^2+yd, e_2^1, e_3^1, e_3^2$	$(q^{33})^3 q^{-1}, q_{11}^{3y} q^{1+y}$
	17	$-(1+x)e_1^1+xe_2^2+e_3^3+yd,$ $e_2^1, e_3^1, e_3^2 \quad x<1, x\ne -1/2$	$(q^{33})^{3(1+x)} q_{11}^{-3} q^{-(2+x)}, q_{11}^{-3y} q^{1+x-y}$
	18	$e_1^1+e_2^2-2e_3^3+yd,$ e_2^1+zd, e_3^1, e_3^2	$q^{33}q_{11}^{-2}q^{-1}, (q_{11})^{3y} q^{1+y} \exp(6z\frac{q_{12}}{q_{11}})$
	19	$-2e_1^1+e_2^2+e_3^3+yd, e_2^1,$ e_3^1, e_3^2+zd	$(q^{33})^2 q_{11}^{-1} q^{-1}, q_{11}^{3y} q^{y-2} \exp(\frac{12zq^{23}}{q^{33}})$
	20	$e_1^1-e_2^2, e_2^1, e_1^2, e_2^2-e_3^3+yd$	$q_{33}q^{33}, q_{33}^{3y} q^{y-1}$

Таблица 1 (продолжение).

dim	N^0	Алгебра Ли	Инварианты
5	1	$e_2^1, e_3^1, e_1^1, e_2^2, e_3^3$	$(q^{23})^2 q_{11}^{-1} q^{-1}$
	2	$e_2^1, e_3^1, -2e_1^1 + e_2^2 + e_3^3,$ $e_3^2 - e_2^3, d$	$(q^{22} + q^{33}))^2 q_{11}^{-1} q^{-1}$
	3	$e_3^1, e_3^2, e_1^1, e_2^2, e_3^3$	$q_{11} q_{22} q_{12}^{-2}$
	4	$e_3^1, e_3^2, e_1^1 + e_2^2 - 2e_3^3,$ $e_2^1 - e_1^2, d$	$q^{33}(q_{11} + q_{22})^{-2} q^{-1}$
	5	$e_2^1, e_3^1, e_3^2, e_1^1 - e_2^2, d$	$(q^{33})^3 q^{-1}$
	6	$-(1+x)e_1^1 + xe_2^2 + e_3^3,$ $e_2^1, e_3^1, e_3^2, d, \quad x \leq 1$	$(q^{33})^{3(1+x)} q_{11}^{-3} q^{-(2+x)}$
	7	$e_2^1, e_1^2, e_1^1, e_2^2, e_3^3$	$q_{33} q^{33}$
	8	$e_2^1, e_3^1, e_3^2, e_1^1 - e_2^2 + yd$ $e_2^2 - e_3^3 + zd$	$(q^{33})^{3z} q_{11}^{3y} q^{1+y-z}$
	9	$e_2^1, e_1^2, e_3^1, e_3^2, e_1^1 - e_2^2$	q, q^{33}
	10	$e_2^1, e_3^1, e_3^2, e_2^3, e_2^2 - e_3^3$	q, q_{11}
6	1	$e_2^1, e_3^1, e_3^2, e_1^1, e_2^2, e_3^3$	1
	2	$e_2^1, e_3^1, e_3^2, e_1^2, e_1^1 - e_2^2, d$	$q^{-1}(q^{33})^3$
	3	$e_2^1, e_3^1, e_3^2, e_2^3, e_2^2 - e_3^3, d$	$q_{11}^3 q$
	4	$e_2^1, e_3^1, e_3^2, e_1^2, e_1^1 - e_2^2, e_3^3 + yd$	$q^{-(1+y)}(q^{33})^{1+3y}$
	5	$e_2^1, e_3^1, e_3^2, e_2^3, e_2^2 - e_3^3, e_1^1 + yd$	$q^{1+y} q_{11}^{1+3y}$
7	1	$e_2^1, e_3^1, e_3^2, e_1^2, e_1^1, e_2^2, e_3^3$	1
	2	$e_2^1, e_3^1, e_3^2, e_2^3, e_1^1, e_2^2, e_3^3$	1
8	1	$e_2^1, e_1^2, e_3^1, e_1^3, e_3^2, e_2^3, e_1^1 - e_2^2,$ $e_2^2 - e_3^3$	q
9	1	$e_1^1, e_2^2, e_3^3, e_2^1, e_1^2, e_3^1, e_1^3, e_3^2, e_2^3$	1

Non-Euclidean method of the generalized geometry construction and perspectives of further geometrization of physics

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Abstract

The space-time geometry is considered to be a physical geometry, i.e. a geometry described completely by the world function. All geometrical concepts and geometric objects are taken from the proper Euclidean geometry. They are expressed via the Euclidean world function σ_{E} and declared to be concepts and objects of any physical geometry, provided the Euclidean world function σ_{E} is replaced by the world function σ of the physical geometry in question. The set of physical geometries is more powerful, than the set of Riemannian geometries, and one needs to choose a true space-time geometry. In general, the physical geometry is multivariant (there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$,... which are equivalent to vector $\mathbf{P}_0\mathbf{P}_1$, but are not equivalent between themselves). The multivariance admits one to describe quantum effects as geometric effects and to consider existence of elementary particles as a geometrical problem, when the possibility of the physical existence of an elementary geometric object in the form of a physical body is determined by the space-time geometry. Multivariance admits one to describe discrete and continuous geometries, using the same technique. A use of physical geometry admits one to realize the geometrical approach to the quantum theory and to the theory of elementary particles.

1 Introduction

Geometrization is the principal direction of the contemporary theoretical physics development. It began in the nineteenth century. One can list the following stages of the physics geometrization:

1. Conservation laws of energy-momentum and angular momentum

2. The first modification of the space-time geometry (geometrization of the space-time, the concept of simultaneity, geometrization of particle motion, problem of high velocities)

3. The second modification of the space-time geometry (existence of nonhomogeneous space-time geometry, influence of the matter distribution on the space-time geometry)

4. Geometrization of charge and electromagnetic field. (Kaluza, O. Klein)

5. The third modification of the space-time geometry (the new space-time geometry of microcosm, the concept of multivariance, geometrization of mass, existence of geometrical objects in the form of physical bodies, geometrical approach to the elementary particles theory).

Now the theoretical physics stands before the third modification of the space-time geometry, connected with investigation of microcosm.

Necessity of the third modification appeared in the thirtieth of the twentieth century, when diffraction of electrons on the small hole was discovered. Motion of a free particle depends only on the space-time geometry, and one needs such a space-time geometry, where the free particle motion be multivariant, and the multivariance intensity depend on the particle mass. Neither physicists, nor mathematicians could imagine such a space-time geometry. As a result the problem of the particle motion multivariance has been solved in the framework of dynamics (but not on the level of geometry). Classical principles of dynamics in microcosm were replaced by quantum ones.

Impossibility of the multivariance problem solution on the geometric level was connected with imperfection of the method of the geometry construction. It does not admit one to construct multivariant geometries, which possess properties, necessary for explanation quantum effects and other properties of microcosm.

In the end of the twentieth century a more perfect method of the space-time geometry construction has been suggested [1]. This method is known as the deformation principle. Geometries, constructed by this method, are known as tubular geometries (T-geometries) This method is simpler and more general, than the conventional Euclidean method, because it does not use such a constraint of the conventional method, as absence of multivariance. In particular, in the framework of the Riemannian geometry there is only one plane uniform isotropic space-time geometry: the Minkowski geometry, whereas in the framework of T-geometries there is a set of plane uniform isotropic space-time geometries, labelled by a function of one argument. All geometries of this set (except for the Minkowski one) are multivariant with respect to timelike vectors.

As far as there exist many uniform isotropic space-time geometries, we are to choose the true space-time geometry from this set. We are to make the best of agreement of the space-time geometry with the experimental data. It appears that the parameters of the space-time geometry can be chosen in such a way, that the classical principles of dynamics describe correctly both quantum and classical motion of a free particle.

Such an expansion of the space-time geometry capacities is connected with the non-Euclidean method of the geometry construction. This method of the geometry construction may be qualified as the deformation principle, because any physical geometry can be obtained as a result of a deformation of the proper Euclidean geometry. Capacities of the space-time geometries constructed by means of the deformation principle do not exhausted by explanation of quantum effects. Structure of elementary particles, their masses, appearance of short-range force fields in microcosm and such an enigmatic phenomenon as confinement can be easily explained in terms of the space-time geometry and its particularity. At any rate the mathematical technique of T-geometry admits this. In this paper we shall show only that mathematical capacity of microcosm geometry are larger, than that of the contemporary theory of elementary particles.

Thus, in this paper we demonstrate only mathematical capacity of space-time geometry in explanation of the microcosm phenomena.

2 Approaches to geometry

There are two approaches to geometry. According to the conventional approach a geometry (axiomatic one) is constructed on a basis of some axiomatics. All propositions of the axiomatic geometry are obtained from several primordial propositions (axioms) by means of logical reasonings. Examples of axiomatic geometries: Euclidean geometry, affine geometry, projective geometry etc. The main defect of the axiomatic geometry: impossibility of axiomatization of nonhomogeneous geometries.

Axiomatization of geometry means that from the set $\mathcal{S}$ of all geometry propositions one can separate several primordial propositions $\mathcal{A}$ (axioms) in such a way that all propositions $\mathcal{S}$

can be obtained from axioms $\mathcal{A}$ by means of logical reasonings. Possibility of axiomatization is a hypothesis. Its validity has been proved only for the proper Euclidean geometry [2]. For nonhomogeneous geometries (for instance, for the Riemannian one) a possibility of axiomatization was not proved. In general, a possibility of axiomatization for nonhomogeneous geometries seems to be doubtful. Felix Klein [3] assumed that the Riemannian geometry (nonhomogeneous one) is rather a geography or a topography, than a geometry.

According to another approach a geometry (physical geometry) is a science on mutual position of geometrical objects in the space or in the space-time. (Euclidean geometry, metric geometry). All relations of the metric geometry are finite, but not differential, and the metric geometry may be given on an arbitrary set of points, but not necessarily on a manifold. It is supposed that the mutual position of geometrical objects is determined, if the distance (metric) between any two points of the set is given. The simplicity of the geometry characteristic and absence of constraints on the set of points (on the space) is the principal advantage of the metric geometry. The main defect of metric geometry is its poverty. Such important concepts of Euclidean geometry as scalar product of vectors and concept of linear dependence are absent in the metric geometry. However, the proper Euclidean geometry is a special case of the metric geometry. It means that in the case of Euclidean geometry the scalar product, concept of linear dependence of vectors and other concepts and objects of Euclidean geometry can be expressed via Euclidean metric. These expressions of the Euclidean concepts via metric are declared to be valid for any metric geometry. Replacing Euclidean metric by the metric of the metric geometry in question, we obtain a system of geometrical concepts in any metric geometry. As a result we obtain the metric geometry, equipped by all concepts of Euclidean geometry [4].

Besides, one removes such constraints on the metric, as the triangle axiom and positivity of metric ρ. They are not necessary, if concepts of the Euclidean geometry are introduced in the metric geometry. Instead of metric ρ we use the world function $\sigma = \frac{1}{2}\rho^2$, which is real even in the geometries with indefinite metric (for instance, in the geometry of Minkowski). We shall refer to such geometries as the tubular geometries (T-geometry). Such a name of geometry is connected with the fact, that the straight in T-geometry is a tube, but not a one-dimensional line. The tubular character of straights in T-geometry is conditioned by the property of multivariance. Multivariance of a T-geometry means, that there exist many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$, $\mathbf{Q}_0\mathbf{Q}_1''$, ...which are equivalent (equal) to vector $\mathbf{P}_0\mathbf{P}_1$, but are not equivalent between themselves.

The multivariance is very important in application of geometry to physics. For instance, the real space-time geometry appears to be multivariant. In particular, multivariance of the space-time geometry explains freely quantum effects as geometrical effects. Besides, the multivariance admits one to set the problem of existence of geometrical objects in the form of physical bodies. This problem cannot be set in framework of the Riemannian geometry. The statement of this problem is important, to obtain a simple geometrical approach to the elementary particles theory.

To overcome defects of physical and axiomatic geometries we use the fact, that the proper Euclidean geometry is the axiomatic and physical geometry simultaneously. The proper Euclidean geometry has been constructed as an axiomatic geometry, and consistency of its axioms has been proved. On the other side, the proper Euclidean geometry is a physical geometry and, hence, it is to be described completely in terms of the metric ρ. Indeed, such a theorem has been proved [4].

As soon as the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is a known geometry, all propositions P_{E} of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ can be presented in terms of the world function σ_{E} of the proper Euclidean geometry: $P_{\mathrm{E}} = P_{\mathrm{E}}(\sigma_{\mathrm{E}})$. Replacing the world function σ_{E} of $\mathcal{G}_{\mathrm{E}}$ by the world function σ of another physical geometry $\mathcal{G}$ in all propositions $P_{\mathrm{E}}(\sigma_{\mathrm{E}})$ of the Euclidean geometry: $P_{\mathrm{E}}(\sigma_{\mathrm{E}}) \rightarrow P_{\mathrm{E}}(\sigma)$, one obtains all propositions of the physical geometry $\mathcal{G}$. Replacement of the world function σ_{E} by other world function σ means a deformation of the Euclidean geometry (Euclidean space). It may be interpreted in the sense, that any physical geometry is a result of a deformation of the proper Euclidean geometry.

Thus, to construct a physical geometry one needs to express all propositions of the proper Euclidean geometry in terms of the Euclidean world function σ_{E}.

3 Non-Euclidean method of the physical geometry construction (deformation principle)

Any physical geometry is described by the world function and obtained as a result of deformation of the proper Euclidean geometry. The world function is described by the relation

$$\sigma: \qquad \Omega \times \Omega \rightarrow \mathbb{R}, \qquad \sigma(P,Q) = \sigma(Q,P), \qquad \sigma(P,P) = 0, \qquad \forall P,Q \in \Omega \tag{3.1}$$

The vector $\mathbf{PQ} \equiv \overrightarrow{PQ}$ is the ordered set of two points $\{P,Q\}$, $P,Q \in \Omega$. The length $|\mathbf{PQ}|_{\mathrm{E}}$ of the vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}|_{\mathrm{E}}^2 = 2\sigma_{\mathrm{E}}(P,Q) \tag{3.2}$$

where index "E" means that the length of the vector is taken in the proper Euclidean space.

The scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}}$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{P}_2$ having the common origin P_0 is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}} = \sigma_{\mathrm{E}}(P_0,P_1) + \sigma_{\mathrm{E}}(P_0,P_2) - \sigma_{\mathrm{E}}(P_1,P_2) \tag{3.3}$$

which is obtained from the Euclidean relation

$$|\mathbf{P}_1\mathbf{P}_2|^2 = |\mathbf{P}_0\mathbf{P}_2 - \mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{P}_0\mathbf{P}_2|^2 + |\mathbf{P}_0\mathbf{P}_1|^2 - 2(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}} \tag{3.4}$$

The scalar product of two remote vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ has the form

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = \sigma_{\mathrm{E}}(P_0,Q_1) + \sigma_{\mathrm{E}}(P_1,Q_0) - \sigma_{\mathrm{E}}(P_0,Q_0) - \sigma_{\mathrm{E}}(P_1,Q_1) \tag{3.5}$$

The necessary and sufficient condition of linear dependence of n vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,...$\mathbf{P}_0\mathbf{P}_n$ is defined by the relation

$$F_n(\mathcal{P}^n) = 0, \qquad \mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \tag{3.6}$$

where $F_n(\mathcal{P}^n)$ is the Gram's determinant

$$\begin{aligned} F_n(\mathcal{P}^n) &\equiv \det||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)_{\mathrm{E}}|| = \det||\sigma_{\mathrm{E}}(P_0,P_i) + \sigma_{\mathrm{E}}(P_0,P_k) - \sigma_{\mathrm{E}}(P_i,P_k)|| \\ i,k &= 1,2,...n \end{aligned} \tag{3.7}$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are parallel, if

$$\mathbf{P}_0\mathbf{P}_1 \upuparrows_{\mathrm{E}} \mathbf{Q}_0\mathbf{Q}_1: \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}} \cdot |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{E}} \tag{3.8}$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent (equal) $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$, if

$$\mathbf{P}_0\mathbf{P}_1 eqv \mathbf{Q}_0\mathbf{Q}_1 : \qquad (\mathbf{P}_0\mathbf{P}_1 \upuparrows \mathbf{Q}_0\mathbf{Q}_1) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|) \tag{3.9}$$

or

$$\mathbf{P}_0\mathbf{P}_1 eqv \mathbf{Q}_0\mathbf{Q}_1 : \qquad \left((\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1|^2\right) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|) \tag{3.10}$$

The property of the equivalence of two vectors in the proper Euclidean geometry is reversible and transitive.

In general case of physical geometry the equivalence property is intransitive. The intransitivity of the equivalence property is connected with its multivariance, when there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$, $\mathbf{Q}_0\mathbf{Q}_1''$,...which are equivalent to the vector $\mathbf{P}_0\mathbf{P}_1$, but they are not equivalent between themselves. Multivariance of the equivalence property is conditioned by the fact, that the system of equations for determination of the point Q_1 (at fixed points P_0, P_1, Q_0) has, many solutions, in general. It is possible also such a situation, when these equations have no solution.

4 Construction of geometrical objects in T-geometry

Geometrical object $\mathcal{O} \subset \Omega$ is a subset of points in the point set Ω. In the T-geometry the geometric object $\mathcal{O}$ is described by means of the skeleton-envelope method. It means that any geometric object $\mathcal{O}$ is considered to be a set of intersections and joins of elementary geometric objects (EGO).

The finite set $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ of parameters of the envelope function $f_{\mathcal{P}^n}$ is the skeleton of elementary geometric object (EGO) $\mathcal{E} \subset \Omega$. The set $\mathcal{E} \subset \Omega$ of points forming EGO is called the envelope of its skeleton $\mathcal{P}^n$. The envelope function $f_{\mathcal{P}^n}$

$$f_{\mathcal{P}^n} : \qquad \Omega \to \mathbb{R}, \tag{4.11}$$

determining EGO is a function of the running point $R \in \Omega$ and of parameters $\mathcal{P}^n \subset \Omega$. The envelope function $f_{\mathcal{P}^n}$ is supposed to be an algebraic function of s arguments $w = \{w_1, w_2, ...w_s\}$, $s = (n+2)(n+1)/2$. Each of arguments $w_k = \sigma(Q_k, L_k)$ is the world function σ of two points $Q_k, L_k \in \{R, \mathcal{P}^n\}$, either belonging to skeleton $\mathcal{P}^n$, or coinciding with the running point R. Thus, any elementary geometric object $\mathcal{E}$ is determined by its skeleton $\mathcal{P}^n$ and its envelope function $f_{\mathcal{P}^n}$. Elementary geometric object $\mathcal{E}$ is the set of zeros of the envelope function

$$\mathcal{E} = \{R | f_{\mathcal{P}^n}(R) = 0\} \tag{4.12}$$

Characteristic points of the EGO are the skeleton points $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\}$. The simplest example of EGO is the segment $\mathcal{T}_{[P_0P_1]}$ of the straight line between the points P_0 and P_1, which is defined by the relation

$$\mathcal{T}_{[P_0P_1]} = \{R | f_{P_0P_1}(R) = 0\}, \tag{4.13}$$

$$f_{P_0P_1}(R) = \sqrt{2\sigma(P_0, R)} + \sqrt{2\sigma(R, P_1)} - \sqrt{2\sigma(P_0, P_1)} \tag{4.14}$$

Definition. Two EGOs $\mathcal{E}(\mathcal{P}^n)$ and $\mathcal{E}(\mathcal{Q}^n)$ are equivalent, if their skeletons are equivalent and their envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ are equivalent. Equivalence of two skeletons $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ and $\mathcal{Q}^n \equiv \{Q_0, Q_1, ..., Q_n\} \subset \Omega$ means that

$$\mathbf{P}_i\mathbf{P}_k eqv \mathbf{Q}_i\mathbf{Q}_k, \qquad i, k = 0, 1, ...n, \quad i < k \tag{4.15}$$

Equivalence of the envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ means that

$$f_{\mathcal{P}^n}(R) = \Phi\left(g_{\mathcal{P}^n}(R)\right), \qquad \forall R \in \Omega \tag{4.16}$$

where Φ is an arbitrary function, having the property $\Phi(0) = 0$.

5 Existence of geometrical objects as physical objects

By definition an elementary geometric object $\mathcal{O}_{\mathcal{P}^n}$ exists at the point $P_0 \in \Omega$ in the space-time as a physical object, if it exists at any time moment at any place of the space-time. Mathematically it means, that at any point $Q_0 \in \Omega$ there exists a geometrical object $\mathcal{O}_{\mathcal{Q}^n}$ with the skeleton $\mathcal{Q}^n \text{eqv} \mathcal{P}^n$. The relation $\mathcal{Q}^n \text{eqv} \mathcal{P}^n$ means that

$$\mathbf{P}_i\mathbf{P}_k eqv \mathbf{Q}_i\mathbf{Q}_k, \qquad i,k = 0,1,...n, \qquad i<k \tag{5.17}$$

According to definition of equivalence (3.10) the equivalence equation $\mathbf{P}_i\mathbf{P}_k \text{eqv} \mathbf{Q}_i\mathbf{Q}_k$ means two relations

$$(\mathbf{P}_i\mathbf{P}_k.\mathbf{Q}_i\mathbf{Q}_k) = |\mathbf{P}_i\mathbf{P}_k|^2, \qquad |\mathbf{P}_i\mathbf{P}_k| = |\mathbf{Q}_i\mathbf{Q}_k| \tag{5.18}$$

There are $n(n+1)$ equations for determination of $4n$ coordinates of points $Q_1, Q_2, ...Q_n$ in the 4-dimensional space-time. The skeleton $\mathcal{P}^n$ and the point Q_0 are supposed to be given.

If the number of the skeleton points increases, the number $n(n+1)$ of constraints increases faster, than the number $4n$ of coordinates, to be determined.

In the simplest case, when all $n(n+1)/2$ vectors $\mathbf{P}_i\mathbf{P}_k$ of the skeleton are timelike, the relation between $n(n+1)$ constraints and $4n$ coordinates is given by the following table

n	$n(n+1)$	$4n$	$diff$
2	6	8	2
3	12	12	0
4	20	16	-4

We see that existence of complicated elementary geometrical objects is impossible.

6 Evolution of geometrical object

In some cases skeletons of equivalent geometrical objects may form a chain of identical skeletons. In such cases we shall speak on temporal evolution of the geometrical object. For instance, let skeletons $\left\{P_0^{(l)}, P_1^{(l)}, ...P_n^{(l)}\right\}$, $l = ...0, 1, ...$are equivalent in pairs

$$\mathbf{P}_i^{(l)}\mathbf{P}_k^{(l)} eqv \mathbf{P}_i^{(l+1)}\mathbf{P}_k^{(l+1)}, \qquad i,k = 0,1,...n; \qquad l = ...1,2,... \tag{6.19}$$

and besides

$$P_1^{(l)} = P_0^{(l+1)}, \qquad l = ...1,2,... \tag{6.20}$$

If vectors $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$ are timelike $\left|\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}\right| > 0$, one may speak on the temporal evolution of the geometrical object $\mathcal{O}(\mathcal{P}^n)$, which is described by the chain, consisting of equivalent skeletons $\mathcal{P}^n$. One may not speak on a temporal evolution of a geometrical object, if skeletons of the chain are not equivalent.

We consider some simple examples of temporal evolution of the skeleton, consisting of two points, in the flat homogeneous isotropic space-time $V_{\mathrm{d}} = \{\sigma_{\mathrm{d}}, \mathbb{R}^4\}$, described by the world function

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + d \cdot \operatorname{sgn}(\sigma_{\mathrm{M}}), \qquad d = \lambda_0^2 = const > 0 \tag{6.21}$$

$$\operatorname{sgn}(x) = \begin{cases} 1, & if \quad x > 0 \\ 0, & if \quad x = 0 \\ -1, & if \quad x < 0 \end{cases}, \tag{6.22}$$

where σ_{M} is the world function of the 4-dimensional space-time of Minkowski. λ_0 is some elementary length.

The distorted space V_{d} describes the real space-time better, than the Minkowski space-time. Description by means of V_{d} is better in the sense, that the space-time (6.21) describes quantum effects, if the distortion constant d is chosen in the form [5]

$$d = \frac{\hbar}{2bc} \tag{6.23}$$

where $\hbar$ is the quantum constant, c is the speed of the light and b is the universal constant, coupling the geometrical length μ of the vector $\mathbf{P}_i\mathbf{P}_{i+1}$ in the chain of skeletons with the conventional mass m of the particle, described by this chain

$$m = b\mu \tag{6.24}$$

Consideration of distortion taken, in the form (6.23) means a consideration of the quantum constant as a parameter of the space-time.

The space-time is discrete in the space-time model (6.21). The space-time is discrete in the sense that there are no timelike vectors $\mathbf{P}_0\mathbf{P}_1$ with $|\mathbf{P}_0\mathbf{P}_1|^2 \in (0, \lambda_0^2)$ and there are no spacelike vectors $\mathbf{P}_0\mathbf{P}_1$ with $|\mathbf{P}_0\mathbf{P}_1|^2 \in (-\lambda_0^2, 0)$ However, the space-time model (6.21) is not a final space-time geometry [5]. The fact is that the relation

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + \frac{\hbar}{2bc} \tag{6.25}$$

may be not valid for all $\sigma_{\mathrm{M}} > 0$. For explanation of quantum effects, it is sufficient, that the relation (6.25) be satisfied for $\sigma_{\mathrm{M}} > \sigma_0$, where the constant σ_0 is determined by the geometrical mass of the lightest massive particle (electron) by means of relation

$$\sqrt{2\sigma_{\mathrm{d}}} = \sqrt{2\sigma_0 + \frac{\hbar}{bc}} \leq \mu_{\mathrm{e}} = \frac{m_{\mathrm{e}}}{b} \tag{6.26}$$

where m_{e} is the electron mass.

Let we have two connected timelike vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$. If $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{P}_1\mathbf{P}_2$ and vector $\mathbf{P}_0\mathbf{P}_1$ is given, the vector $\mathbf{P}_1\mathbf{P}_2$ can be determined. Let coordinates of points P_0, P_1, P_2 in the inertial coordinate system be

$$P_0 = \{0, 0, 0, 0\}, \qquad P_0 = \{s, 0, 0, 0\}, \qquad P_2 = \{2s + \alpha_0, \gamma_1, \gamma_2, \gamma_3\} \tag{6.27}$$

where the quantity s is given and the quantities $\alpha_0, \gamma_1, \gamma_2, \gamma_3$ are to be determined.

Vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ have coordinates

$$\mathbf{P}_0\mathbf{P}_1 = \{s, 0, 0, 0\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \{s + \alpha_0, \gamma_1, \gamma_2, \gamma_3\} \tag{6.28}$$

According to (3.5) in the space-time V_{d}

$$
\begin{aligned}
(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2) &= \sigma(P_0,P_2)-\sigma(P_0,P_1)-\sigma(P_1,P_2) \\
&= \sigma_{\mathrm{M}}(P_0,P_2)-\sigma_{\mathrm{M}}(P_0,P_1)-\sigma_{\mathrm{M}}(P_1,P_2)+w \\
&= (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\mathrm{M}}+w
\end{aligned} \tag{6.29}
$$

where

$$
w=\lambda_0^2\left(\operatorname{sgn}(\sigma_{\mathrm{M}}(P_0,P_2))-\operatorname{sgn}(\sigma_{\mathrm{M}}(P_0,P_1))-\operatorname{sgn}(\sigma_{\mathrm{M}}(P_1,P_2))\right) \tag{6.30}
$$

and σ_{M} means the world function of the Minkowski space-time.

Note that in the space-time (6.21)

$$
\operatorname{sgn}(\sigma_{\mathrm{M}}(P_0,P_2))=\operatorname{sgn}(\sigma(P_0,P_2)) \tag{6.31}
$$

If the vector $\mathbf{P}_0\mathbf{P}_1$ is timelike, $|\mathbf{P}_0\mathbf{P}_1|^2=s^2+2\lambda_0^2>0$, The equivalence equations of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ take the form

$$
|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2=|\mathbf{P}_1\mathbf{P}_2|_{\mathrm{M}}^2, \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\mathrm{M}}+w=|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2+2\lambda_0^2 \tag{6.32}
$$

where

$$
w=\lambda_0^2\left(\operatorname{sgn}(\sigma_{\mathrm{M}}(P_0,P_2))-2\right) \tag{6.33}
$$

Scalar products with a subscript "M" are usual scalar products in the Minkowski space-time. In the coordinate form the relations (6.32) are written as follows

$$
(s+\alpha_0)^2-\gamma_1^2-\gamma_2^2-\gamma_3^2=s^2 \tag{6.34}
$$

$$
s(s+\alpha_0)+\lambda_0^2\left(\operatorname{sgn}(\sigma_{\mathrm{M}}(P_0,P_2))-2\right)=s^2+2\lambda_0^2 \tag{6.35}
$$

The vector $\mathbf{P}_0\mathbf{P}_2=\{2s+\alpha_0,\gamma_1,\gamma_2,\gamma_3\}$ have the length

$$
|\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2=(2s+\alpha_0)^2-\gamma_1^2-\gamma_2^2-\gamma_3^2 \tag{6.36}
$$

Using relations (6.34) and (6.35), we eliminate α_0 and $\gamma_1^2+\gamma_2^2+\gamma_3^2$ from the relation (6.36). We obtain

$$
|\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2=4s^2+8\lambda_0^2-2\lambda_0^2\operatorname{sgn}(\sigma_{\mathrm{M}}(P_0,P_2))>0 \tag{6.37}
$$

It means that $\operatorname{sgn}(\sigma_{\mathrm{M}}(P_0,P_2))=1$, and the relation (6.35) has the form

$$
s\alpha_0=3 \tag{6.38}
$$

Solution of equations (6.34) and (6.38) has the form

$$
\alpha_0=\frac{3\lambda_0^2}{s}, \qquad \gamma_\alpha=\frac{\lambda_0}{s}\sqrt{6s^2+9\lambda_0^2}\frac{\beta_\alpha}{\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}}, \qquad \alpha=1,2,3 \tag{6.39}
$$

where β_1,β_2,β_3 are arbitrary real quantities.

Let us consider now two null connected equivalent vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ ($|\mathbf{P}_0\mathbf{P}_1|^2=|\mathbf{P}_1\mathbf{P}_2|^2=0$). We take the coordinate representation for vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$

$$
\mathbf{P}_0\mathbf{P}_1=(s,s,0,0), \qquad \mathbf{P}_1\mathbf{P}_2=(s+\alpha_0,s+\alpha_1,\gamma_2,\gamma_3) \tag{6.40}
$$

Using the same method, as for timelike vectors, we obtain

$$
\alpha_1=\alpha_0, \qquad \gamma_2=\gamma_3=0 \tag{6.41}
$$

Thus, in the case of two connected equivalent null vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ we have

$$\mathbf{P}_0\mathbf{P}_1 = (s, s, 0, 0)\,, \qquad \mathbf{P}_1\mathbf{P}_2 = (s+\alpha_0, s+\alpha_0, 0, 0) \tag{6.42}$$

where α_0 is an arbitrary real number. The result does not depend on the elementary length λ_0. It takes place in the Minkowski space-time also.

Let we have two connected spacelike equivalent vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$. If $\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{P}_1\mathbf{P}_2$ and vector $\mathbf{P}_0\mathbf{P}_1$ is given, the vector $\mathbf{P}_1\mathbf{P}_2$ can be determined. Let coordinates points P_0, P_1, P_2 in the inertial coordinate system be

$$P_0 = \{0,0,0,0\}\,, \qquad P_1 = \{0,l,0,0\}\,, \qquad P_2 = \{\alpha_0, 2l+\gamma_1, \gamma_2, \gamma_3\} \tag{6.43}$$

where the quantity l is given and the quantities $\alpha_0, \gamma_1, \gamma_2, \gamma_3$ are to be determined.

Using the same method as for calculation of timelike vectors, we obtain

$$\gamma_1 = \frac{3\lambda_0^2}{l}, \qquad \alpha_0 = \sqrt{\gamma_2^2 + \gamma_3^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}} \tag{6.44}$$

where γ_1 and γ_2 are arbitrary real numbers. Thus

$$\mathbf{P}_0\mathbf{P}_1 = \{0,l,0,0\}\,, \qquad \mathbf{P}_1\mathbf{P}_2 = \left\{\sqrt{\gamma_2^2 + \gamma_3^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}}, l, \gamma_2, \gamma_3\right\} \tag{6.45}$$

Let us imagine now that there is an infinite chain of connected equivalent vectors $...\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_1\mathbf{P}_2, ...\mathbf{P}$ the vectors are timelike, the chain may be interpreted as a multivariant "world line" of a free particle. The vector $\mathbf{P}_k\mathbf{P}_{k+1}$ may be interpreted as the geometric particle momentum, and $|\mathbf{P}_k\mathbf{P}_{k+1}|$ may be interpreted as the geometric mass μ. To obtain the conventional particle mass m, one needs to use the relation (6.24), where b is some universal constant. Statistical description of the multivariant particle motion leads to quantum description in terms of the Schrödinger equation [5]. However, this correspondence between the geometrical description and the quantum one admits one to determine only production $\lambda_0^2 b = \hbar/(2c)$. The universal constants λ_0 and b are not determined asunder from this relation. Thus, in the case of timelike vector $\mathbf{P}_k\mathbf{P}_{k+1}$ we obtain dynamics of free particle from the pure geometrical consideration (dynamics is a corollary of geometry).

If the vectors $\mathbf{P}_k\mathbf{P}_{k+1}$ of the chain are null, it is difficult to speak about temporal evolution. Although the chain of null vectors is single-variant, but vectors of the chain may change their direction, because the constant α_0 in (6.42) may have any sign and module.

At first sight, any temporal evolution in the chain of spacelike vectors $\mathbf{P}_k\mathbf{P}_{k+1}$ is impossible. It is true, provided there are no additional constraints on the chain of spacelike vectors. However, if the skeleton contains more than two points, for instance, $P_0, P_1, Q_1, ...$ the chain, determined by the points P_0, P_1, may contain additional constraints, generated by additional points $Q_1, ...$ of the skeleton. These constraints may be such, that the spacelike "world line" forms a helix with a timelike axis. If so, the helix may be interpreted as a world line of a particle, moving with the superlight velocity along some circle. Such a world line associates with the classical Dirac particle, whose world line is also a helix [6, 7]

We are not sure, that such a situation may appear in the distorted space-time (6.21). However, there may exist such a space-time geometry, where the spatial evolution leads to the temporal evolution.

In reality, there exists a very important strategic problem. What is the starting point for investigation of microcosm phenomena? Now this investigation is produced on the basis of supposition, that elementary particles are described by some enigmatic wave functions.

After construction of multivariant geometries it became possible to explain quantum properties as an appearance of the multivariance of the space-time geometry. Besides, it becomes possible to set the problem of existence of geometrical objects in the form of physical bodies. At such conditions it seems more reasonable at first to investigate capacities of the geometric approach to the elementary particle theory.

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Наблюдения рентгеновского излучения от массивного ядра Галактики заставляют пересмотреть статические модели черных дыр

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In June 2004 in the astronomical World was a sensation.(Science, 13 August 2004, p.934). At conference in Dublin has asked S.W.Hawking, which is the founder the traditional concept on black holes. We think that it has buried all fashionable science about black holes. Author since 1966 developed the no conventional concept on evolution of massive heavenly body. A last stage of evolution there comes gravitational collapse – fall of substance on the center. We consider, that collapse has oscillatory character, therefore the gravitational tomb for substance and energy is not formed. In present paper we are going no tell about observant consequences from the non-conventional concept. We shall try to understand physics of occurrence of x-ray radiation in massive heavenly bodies.

В июне 2004 в астрономическом мире произошла сенсация. О ней написал популярный американский журнал «Наука» (Science, 13 August 2004, c.934). Журнал опубликовал короткую статью о конференции в Дублине. На этой конференции выступил выдающийся британский физик теоретик Стефан Хокинг – «генерал науки о черных дырах», который является создателем этой науки. Генерал покинул поле сражения - он отказался от своих работ по черным дырам. Напомню, что эти работы были опубликованы за последние 30 лет. Он сказал, что эти работы приводят к трудностям, которые противоречат основам физики. Думаю, что он похоронил всю модную науку о черных дырах. Эта сенсация, о которой сообщили все телевизионные каналы новостей. К сожалению, статья самого Хокинга еще не опубликована и поэтому мы не можем о ней написать. В научных журналах надо обсуждать конкретные работы, а не свои мысли по этому поводу. После выступления прошло два года, почему не появилась его статья – я не знаю. Нет статей и его учеников**.** Поэтому я вынужден был ограничиться написанием популярной статьи в «Техника Молодежи»[1].

По математической стороне вопроса в «НТР» была дискуссия.[2-5]. До этого у меня была серия статей, начиная с 1966, в научных журналах [6-9].

Я хотел бы остановиться на человеческом факторе. Пионерские работы Хокинга вышли 30 лет тому назад. Его авторитет среди астрономов, физиков и математиков был колоссален, я лично ожидал, что он получит Нобелевскую премию за эти работы. Но после его выступления в Дублине этого не произошло в 2004 году и вряд ли произойдет в будущем! Почему же он все - таки выступил? Я думаю, что здесь два фактора – высокая научная порядочность, как ученого, и нежелание того, чтобы его последователи столкнулись с этими трудностями. Я считаю, что поведение Хокинга заслуживает восхищения и одобрения.

Автор почти 40 лет придерживается нетрадиционной точки зрения, что модная теория черных дыр, предполагающая необратимый характер гравитационного коллапса – только падение - содержит математическую ошибку. Наука о черных дырах вся была основана на статической модели - метрика стационарна $\frac{\partial}{\partial t} g_{ik} = 0$, как следствие существуют горизонты. Однако для больших масс холодной материи статического решения просто не существует, и необходим переход к динамике. Реально происходят колебания, и поэтому свойства реальных небесных тел большой массы не соответствует тому, что об них пишут фантазеры-астрофизики. В статье [1] было также сопоставление с наблюдениями, которые однозначно говорят против традиционной точки зрения. Таким образом, я был еретиком в науке о черных дырах. Должен сказать, что я был не один, но горжусь тем, что я был самый последовательный. Частные недостатки теории отмечались многими, я пришел к выводу о необходимости пересмотра основных положений. Самому А.Эйнштейну центральное точечное тело решения Шварцшильда тоже не нравилось [10], в 1939 году он искал другие варианты.

Фраза о рассмотрении горизонта как полупроницаемой мембраны, которая пропускает свет и материю только в одном направлении, определяемом физической предысторией, есть в книге [3]. Однако существование такой мембраны в течение длительного времени противоречит термодинамике. Низкотемпературный источник будет нагревать черную дыру в течение длительного времени до сколь угодно высоких температур, а обратное излучение мембрана не пропустит. Таким образом, возникает нарушение термодинамики, о котором говорил С. Хокинг. В наших работах [4-8] полупроницаемые мембраны возникают только на короткое время.

Источник решения Шварцшильда статический: линия $r = 0$ тахион, что было отмечено в 1949 году Сингом.[11].Таких частиц не существует в природе. Поэтому мне источник – тахион – не нравится. Думаю, что сегодня это не устраивает и Хокинга. Вопрос ставится так: что происходит с веществом после его ухода под сферу Шварцшильда - ухода под гравитационный радиус? Традиционный ответ – материя падает на сингулярность r=0 (что это такое?), затем останавливается (почему?). Но задавать вопросы нельзя – сигналы из внутренности сферы Шварцшильда к нам не доходят – ответ мы принципиально не узнаем никогда! Но если это наука, то вопросы задавать можно и нужно! Вопрос о природе центрального тела решения Шварцшильда был поставлен Сингом, ответ, что это тахион, его не устраивал, он написал статью, в которой говорилось об этой проблеме [11]. Другой вариант ответа был сформулирован И.Д.Новиковым в 1966 [15 §4.6] году – материя уходит в другое пространство! Этот вариант ответа он не считал удачным, что видно из примечания на стр. 175 книги [15]. И.Д.Новиков занимался другими вопросами и больше не возвращался к этой проблеме. В 1968 проблемой центрального тела метрики Шварцшильда заинтересовался Нобелевский лауреат J.Bardeen (Дж.Бардин), в его докладе на гравитационной конференции GR-5 в Тбилиси он анализировал характер статического источника. Из формул Бардина следует, что источник не удовлетворяет условиям физической реализации. Подчеркнем, что в перечисленных работах проблема рассматривалась как физическая – использовались физические условия на источник поля и на уравнения состояния материи.

Мой ответ следующий – материя под сферой Шварцшильда есть, покоиться в сильном гравитационном поле она не может, и поэтому продолжает двигаться. В задаче есть две математические особенности.

1. Координатная особенность – горизонт - полюс $r=r_g$. Формально она устраняется сдвигом полюса на комплексную плоскость. Это было введено нами в 1966 [8], затем Хокингом [12] в 1975 и сегодня общепринято. Если материя внутри сферы Шварцшильда двигается, то свет может выходить наружу. Свойства горизонта зависят от времени, направление пропускания меняется во времени

2. Физическая сингулярность - точка бесконечной плотности, r=0. Она устраняется регуляризацией решения электрическим зарядом. При этом удается построить пример частного решения без физической сингулярности.[9] Наличие рентгена независимо показывает, что излучение выходит из-под горизонта.

К сожалению, сообщение Хокинга очень краткое и абстрактное. Поэтому сформулируем его на наглядном физическом языке. В литературе [3] рассматривается модель, в которой вокруг черной дыры на горизонте имеется полупрозрачная мембрана с вентильными свойствами, пропускающая свет только в одном направлении – внутрь. Направление пропускания определяется физической историей [15]. К сожалению, существование такой статической мембраны в течение длительного интервала времени противоречит термодинамике. Широкополосный сигнал с температурой T_0 проникает только в одном направлении – внутрь. И внутри объема скапливается реактивная энергия излучения. И в течение длительного интервала времени плотность и температура излучения растет, и возникает ситуация, когда будет $T_1 >> T_0$. Создается ситуация, когда сравнительно холодный источник температурой T_0 ученые создает область с более высокой температурой T_1. А это противоречит термодинамике. Именно об этом противоречии говорил S.W. Hawking на конференции в Дублине. Он пришел к противоречию, анализируя следствия, мы их получили более наглядным способом, анализируя свойства мембраны. Противоречия не получается, если направление пропускания мембраны периодически меняется, в зависимости от движения материи внутри черной дыры, которая тем самым периодически превращается в белую дыру и обратно. Именно такое решение и было построено. Я назвал его колебательным коллапсом. В области $r_g < r < 2,6\ r_g$ материя не может за что ни будь зацепиться и остановиться. Падение происходит до горизонта.

Я хотел бы сказать еще об одном новом наблюдательном факте – массивные тела в центре нашей Галактики, которые считаются черными дырами, излучают жесткое электромагнитное излучение в рентгеновской области.[12-14] Наблюдается переменность излучения во времени. Это означает, что все обще принятые статические модели черных дыр надо менять! Черная дыра, как считается сегодня, есть статическая сингулярность - (не ясно, какая!), окруженная горизонтом. Горизонт – сфера Шварцшильда – это координатная, а не физическая особенность. Это некая граница, которой в реальном мире нет, и которая есть только в математике на определенной координатной карте. И не ясно, каким образом эта граница может осуществлять «космическую цензуру» - не пропускать к нам свет. Равным образом, при пересечении горизонта материей нет физических причин появления оптического или жесткого излучения. Рентген может образоваться только при столкновении заряженных быстрых частиц материи и при возбуждении глубоких электронных уровней, и, может быть даже уровней ядер. Для возбуждения глубоких электронных уровней нужна либо высокая температура, либо столкновения движущейся материи с большими скоростями. При этом существование покоящейся материи в сильных гравитационных полях во многих случаях запрещено расположением световых конусов. Следовательно, в черной дыре есть двигающаяся материя, и поэтому лучи света и частицы могут проходить горизонт в любом направлении. Просто горизонт меняет свои свойства во времени. К выводу о необходимости пересмотра также приводит анализ наблюдательных данных по нейтринному излучению от сверхновой 1987А. Реакция нейтронизации - с рождением нейтрино происходит при высокой плотности вещества [15], которая возникает при коллапсе. После коллапса возникают колебания, импульсы нейтрино возникают в моменты появления высокой плотности. Чем выше энергия нейтрино, тем больше требуемая плотность. Поэтому я думаю, что нейтрино возникают под гравитационным радиусом.

Источником энергии является аккреция.[15 гл. 12] Напомним, что аккрецией называется падение вещества – межзвездного газа на массивный объект. Частицы вещества отдают свою энергию тому массивному телу, на который они падают. Традиционное рассмотрение аккреционного диска как отдельного долгоживущего объекта неправильно. Круговые орбиты существуют только на расстояниях $r > 2{,}6rg$, и любая частица материя, вектор скорости которой не касателен к круговой орбите, падает на горизонт. Это процесс быстрый - время порядка $4rg/c$.[15] Падают на черную дыру все частицы, кроме тех, скорости которых касательные – к круговым орбитам. Фазовый объем падающих частиц больше, чем у частиц, остающихся в окрестности круговых орбит. Поэтому эта большая часть фазового объема в пространстве скоростей быстро уходит из аккреционного диска, падая на черную дыру. Рассмотрим теперь меньшую часть фазового объема, которая лежит в окрестности круговых орбит. Обмен частицами между различными частями фазового объема обусловлен диффузией, причем имеет место не классическая температурная диффузия, а более быстрая. В нашей работе [14] считалось, что время жизни аккреционного диска определяется диффузией, на самом деле имеет место быстрый уход материи из аккреционного диска. Это приводит к тому, что плотность материи в диске невелика, и число столкновений, при которых возбуждаются рентгеновские кванты, мало. Сравнительно быстро упавшие частицы и их энергия оказываются внутри черной дыры. Значительная часть энергии падающих частиц приобретается на участке траектории около горизонта. Межзвездный газ это в основном водород, поэтому и основной компонентой аккреционного диска тоже является водород. Однако водород не имеет рентгеновских уровней. Энергии частиц, падающих на аккреционный диск, велики и вопрос о том, уцелеют ли тяжелые ядра, остается открытым. Условия для образования более тяжелых элементов существуют в плотном веществе внутри черной дыры. Прежде всего, это процессы нейтронизации вещества – поглощение ядром электрона, протон превращается в нейтрон и излучается нейтрино, эти процессы при достаточно высокой плотности идут последовательно.

Нейтринное излучение из космоса было впервые обнаружено на Земле во время вспышки Сверхновой Звезды 1987А. Теория предсказывала один импульс нейтрино [15 §11.4] длительностью 0,9 с, полной энергии нейтрино 1052 эрг. Результаты были неожиданными - было приняты 2 серии импульсов с интервалом между ними около 5 часов. Полная энергия оказалась слишком большой – 1054 эрг.

Большое расхождение измеренных и ожидаемых значений послужило причиной недоверия к результатам измерений, которые нельзя повторить. Поэтому я тогда старался не приводить эти результаты. Сверхновая – это взрыв звезды на небольшом расстоянии от Земли, далеких звезд больше и события происходят чаще. И они регистрируются как гамма всплески. Значение энергии 10^{54} эрг проверено по нагреву окружающих близких звезд. Доверие к предсказаниям теории упало после выступления Хокинга.

Принципиальной трудностью является то, что мы почти не знаем динамических решений с материей внутри черной дыры для физически интересных случаев. Должно быть выполнено условие физической реализуемости источника. Я нашел простейшее динамическое решение, оно пока не обще признано. Что является источником гравитационного поля при наличии вращения - в решении Керра - до конца не исследовано.
Мы хотели бы подчеркнуть принципиальную ценность наблюдательных исследований типа [12-14], без учета этих данных построить физическую модель черной дыры просто невозможно. Рентгеновские линии дают информацию о химическом составе и ядерных реакциях, а переменность во времени и сдвиг частоты линий их уширение связано - с движением материи.

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Small effects of low-energy quantum gravity

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Abstract

Small effects of quantum gravity on the scale $\sim 10^{-3}eV$ and their cosmological consequences are discussed and compared with observations of supernovae 1a, gamma-ray bursts and galaxies.

Our knowledge of the nature is restricted for many reasons, but sometimes we attack it with a view of victors being sure that we know enough to go ahead namely in the given way. An attempt to introduce dark energy to rescue the picture of expanding universe seems to me to be such the case.

I would like to show here that small effects of very-low-energy quantum gravity (on the scale $\sim 10^{-3}eV$) [1] can give an alternative explanation of supernovae 1a, gamma-ray bursts and galaxy number counts observations. The new picture has the very dramatic consequence: nor dark energy nor any expansion of the universe exist in it.

There are two small effects in the sea of super-strong interacting gravitons [1]: average energy losses of a photon due to forehead collisions with gravitons and an additional relaxation of a photonic flux due to non-forehead collisions of photons with gravitons. The first effect leads to the geometrical distance/redshift relation: $r(z) = ln(1 + z) \cdot c/H$, where H is the Hubble constant. The both effects lead to the luminosity distance/redshift relation: $D_L(z) = c/H \cdot \ln(1 + z) \cdot (1 + z)^{(1+b)/2}$, where the "constant" b belongs to the range 0 - 2.137 [2] ($b = 2.137$ for a very soft radiation, and $b \to 0$ for a very hard one). For an arbitrary source spectrum, a value of the factor b should be still computed. It is clear that in a general case it should depend on a rest-frame spectrum and on a redshift. Because of this, the Hubble diagram should be a multivalued function of a redshift: for a given z, b may have different values for different kinds of sources. Further more, the Hubble diagram may depend on the used procedure of observations: different parts of rest-frame spectrum will be characterized with different values of the parameter b.

In Figure 2 of my paper [1], the Hubble diagram $\mu_0(z)$ with $b = 2.137$ is shown; observational data (82 points) are taken from Table 5 of [3]. The predictions fit observations very well for roughly $z < 0.5$. It excludes a need of any dark energy to explain supernovae dimming. Improved distances to nearby type Ia supernovae (for the range $z < 0.14$) can be fitted with the function $\mu_c(z)$ for a flat Universe with the concordance cosmology with $\Omega_M = 0.30$ and $w = -1$ [4]. The difference $\mu_c(z) - \mu_0(z)$ between this function and distance moduli in the considered model for $b = 1.52$ has the order of ± 0.001 in the considered range of redshifts [2]. Results from the ESSENCE Supernova Survey together with other known supernovae 1a observations in the bigger redshift range $z < 1$ can be best fitted in a frame of the concordance cosmology in which $\Omega_M \simeq 0.27$ and $w = -1$ [5]; the function $\mu_c(z)$ for this case is almost indistinguishable from distance moduli in the considered model for $b = 1.405$: the difference is not bigger than ± 0.035 for redshifts $z < 1$.

Theoretical distance moduli $\mu_0(z) = 5 \log D_L + 25$ are shown in Fig. 1 for $b = 2.137$ (solid), $b = 1$ (dot) and $b = 0$ (dash). If this model is true, all observations should lie in the stripe between lower and upper curves. Theoretical distance moduli $\mu_c(z)$ for a flat Universe with the concordance cosmology with $\Omega_M = 0.27$ and $w = -1$, which give the best fit to gamma-ray bursts observations [6], are very close to the Hubble diagram $\mu_0(z)$ with $b = 1.1$ of this model. GRB observational data (+, 69 points) are taken from Table 6 (μ^a) of [6] by Schaefer.

The galaxy number counts/magnitude relation in this model $f_3(m)$, m is a magnitude, in this model (for more detail, see [7]), which takes into account the Schechter luminosity function, is based on the same two small effects. To compare this function with observations by Yasuda et al. [8], we can choose the normalizing factor from the condition: $f_3(16) = a(16)$, where $a(m) \equiv A_\lambda \cdot 10^{0.6(m-16)}$ is the function giving the best fit to observations

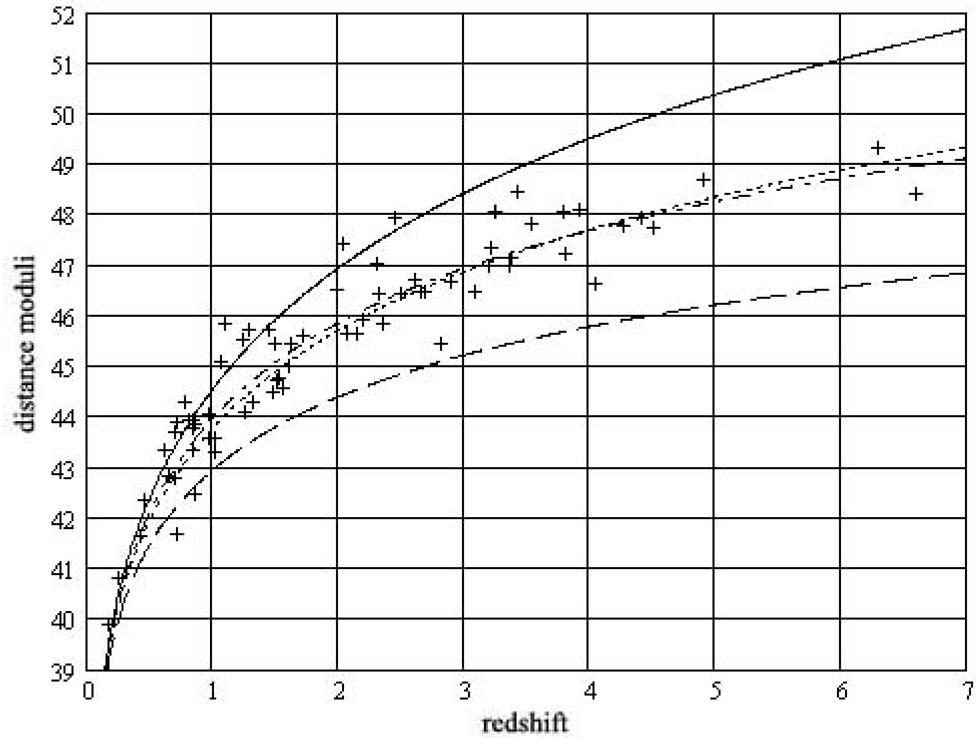

Figure 1: Hubble diagrams $\mu_0(z)$ with $b = 2.137$ (solid) and $b = 0$ (dash); the Hubble diagrams $\mu_0(z)$ with $b = 1.1$ of this model (dot) and the one of the concordance model (dadot) which is the best fit to GRB observations [6]; GRB observational data (+, 69 points) are taken from Table 6 (μ^a) of [6] by Schaefer.

[8], $A_\lambda = const$. The ratio $\frac{f_3(m)-a(m)}{a(m)}$ is shown in Fig. 2 for different values of the constant $A_1 \simeq 5 \cdot 10^{17} \cdot L_\odot/L_*$ by $\alpha = -2.43$ and $b = 2.137$. If we compare this figure with Figs. 6,10,12 from [8], we see that the considered model provides a no-worse fit to galaxy observations than the function $a(m)$ if the same K-corrections are added.

The considered effects of low-energy quantum gravity are very small on micro level, but they may be the basic ones for cosmology. The ones are beyond the general relativity, and astrophysical observations seem to stay an unexpected tool of quantum gravity laboratory.

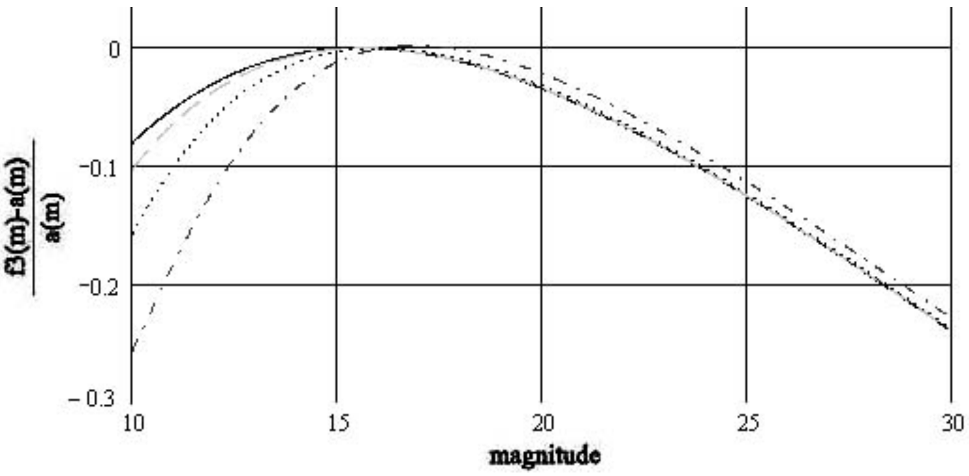

Figure 2: The relative difference $(f_3(m) - a(m))/a(m)$ as a function of the magnitude m for $\alpha = -2.43$ by $10^{-2} < A_1 < 10^2$ (solid), $A_1 = 10^4$ (dash), $A_1 = 10^5$ (dot), $A_1 = 10^6$ (dadot).

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Active pulse laser system for visual heterogeneities diagnostics in photonic crystals

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This work presents optical absorption method for heterogeneities diagnostics and investigations of photonic crystals[1] and light-scattering samples in near infrared (IR) regions with low level probe irradiation. Main features of the method are IR pulse laser lightening the investigated optical sample and gated 3rd Generation image intensifier application to eliminate scattering light and view suitable object image. Technical parameters of used equipment: laser radiation pulse duration – 10 ns, gating time – 0.2...50 ns, monochromatic current sensitivity of image intensifier (λ=830nm) - 100 $mA \cdot W^{-1}$, amplification - $2 \cdot 10^4$ $W \cdot W^{-1}$, resolution – 500 TV lines (depends on CCD array). Small overall dimensions of optical measurement unit in combination with easiness of handling and high spatial resolution make it as indispensable instrument for some heterogeneities investigation of different optical samples, particularly – photonic crystals.

1. INTRODUCTION

Synthesis of some photonic crystals from Si, SiO_2, GaAs and other optical materials transparent to V or near IR light is important modern technologic task. When IR image-converter tubes were created, they opened the possibility to see IR images in real time[7]. Some researches used it for diagnostics of different crystals heterogeneities. In some of these cases they used tomography-like treatment: the sample was scanning layer by layer by means of gating photodetector. Received images were processed by computer in order to emphasize sample tomogram. The main problem of such control methods is light scattering in samples. Because of this reason some observations are impossible even after electronic processing of images.

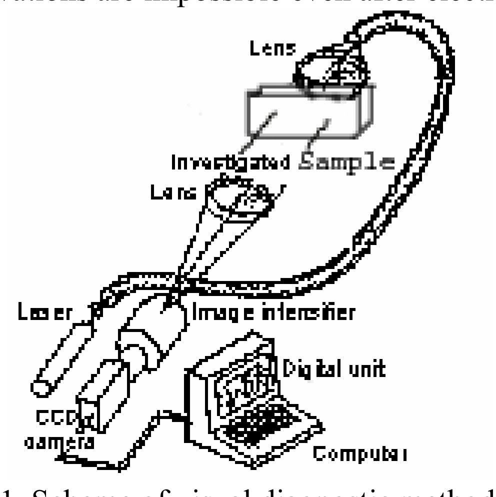

Fig.1. Scheme of visual diagnostic method with laser

Usually, researches use halogen lamps with band-pass filters as V or IR-sources for visual diagnostics. Such sources provide suitable power of radiation. Today in practice V and IR lasers are used - both solid-state and semiconductor (fig.1).

This paper is devoted to improvement of visual diagnostic method due to reduction of back scattering light because of using pulse laser radiation as highlight source and gated image-converter tube with CCD camera as radiation detector.

2. EXPERIMENTAL EQUIPMENT DESCRIPTION

Experimental equipment scheme with laser source is shown on fig.2. Observed object is illuminated by short light pulses. These pulses duration is shorter than time of light spreading to object and back. During this time the object is observed by means of image device supplied by fast-operating gate that is opened in time of light pulses with specified delay.

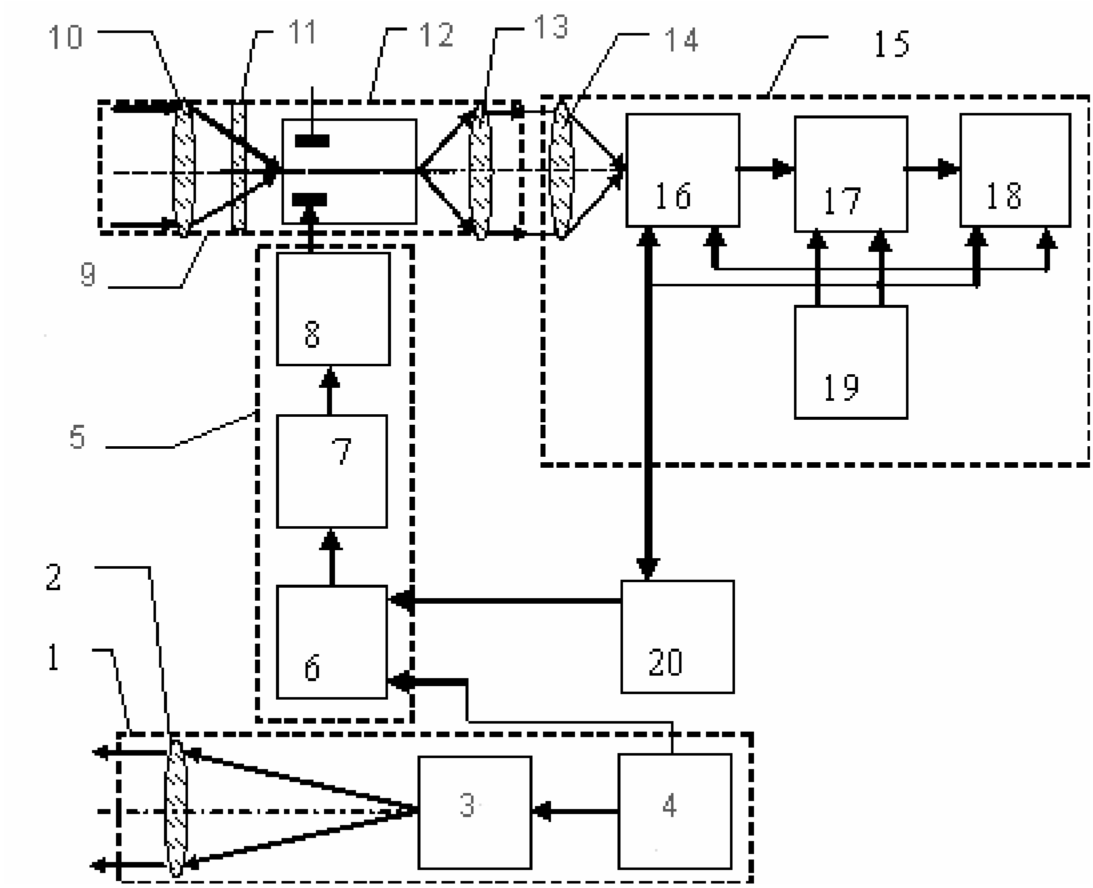

Fig.2. Scheme of experimental equipment
1 – pulse laser lighter; 2 – laser objective lens; 3 – pulse semiconductor laser diode (PLD); 4 – PLD power supply; 5 – control and synchronization unit; 6 – driving pulse generator; 7 – delay pulse unit; 8 - gating pulse former; 9 – observation unit; 10 – objective lens; 11 - band-pass filter; 12 – pulse image-converter tube; 13, 14 – relay lenses; 15 – TV channel; 16 – CCD array; 17 – video-amplifier; 18 – TV monitor; 19 – synchronizing generator; 20 – string frequency dividing unit

If time delay between light pulse beginning and moment of gate opening is equal to double time of light propagation to object and back observer sees only the object and some space around it. This spatial depth depends on the gate opened time and the light pulse time. The main parts of experimental equipment depicted on fig. 2 are the 3rd Generation image intensifier (12) and pulse laser lighter (3).

2.1.3rd Generation image intensifier

The physical principles of 3rd Generation intensifier[2,3] are relatively simple. It is vacuum tube which converts V and IR light in visible with some amplification (fig. 3). IR light that reflected and scattered from object 6 falls through the objective lens 4 on the gallium arsenide photocathode 1 producing proper image of object on it. Illuminated photocathode emits photoelectrons which move in electric field forward to microchannel plate (MCP) 2. MCP multiplies photoelectron flow about 1000 times. These photoelectrons fall on phosphor screen 3 and produce visible image appropriate of V or IR image on the photocathode 1. This visible image is viewing by sight or CCD – camera[6] via relay optics 5. Standoff between internal side of photocathode and MCP input is 0,15-0,20 mm, channels diameter – 6-8 μm. These parameters define intensifier space resolution 45-48 mm^{-1}. Photosensitivity spectral range of cathode 1 is 0,6-0,9 μm. It corresponds to semiconductor PLD radiation very good. High quantum yield (about 40%), low electric resistance (some tens ohms) provide, first, high image quality of low alight objects, and, second, image shutting in the space between photocathode and MCP without image blurring. Pulse regime does not overload photocathode of image intensifier because it is worked with low illumination as a rule. The most perfectly image quality is characterized by modulation transfer function (MTF) of image intensifier which defined by electronic shutter type and guided pulse form for pulse regime of image intensifier. In comparison with statical regime pulse one is resulted in image displacement and motion-blur on the intensifier screen. These effects make worse image in pulse regime in comparison with optimal statical one. The image worses conditioned by its motion-blur and displacement are described by MTF and phase-response characteristic (FRC), respectively. We have measured MTF of image intensifier on special arrangement both for pulse and statical regimes. Intensifier was controlled through the inlet chamber «photocathode – MCP». MTF of intensifier for pulse and statical regimes are shown on fig. 4. From MTF comparison one can resume that contrast decrease is insignificant on

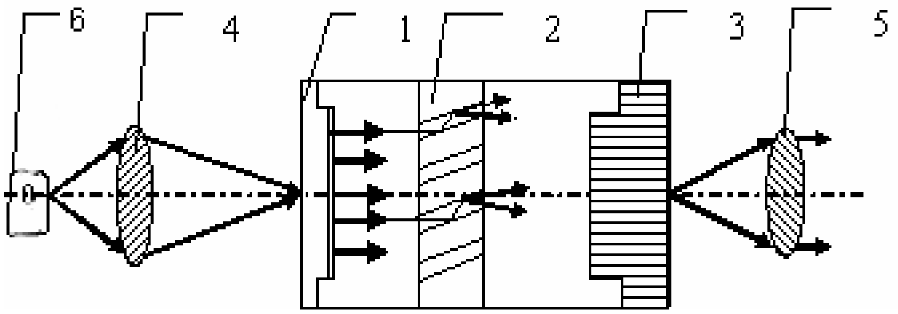

Fig 3. 3rd Generation image intensifier.
1 - GaAs–photocathode; 2 – microchannel plate; 3 - fiber lens; 4,5 – matched optical elements; 6 – observed sample

the spatial frequencies 5, 10, 15 mm^{-1} which correspond to viewing objects.

These investigations have shown that pulse operating intensifier has spatial resolution and MTF much the same as in statical regime. Furthermore, the cathode resistance of 3rd Generation intensifier is rather small so

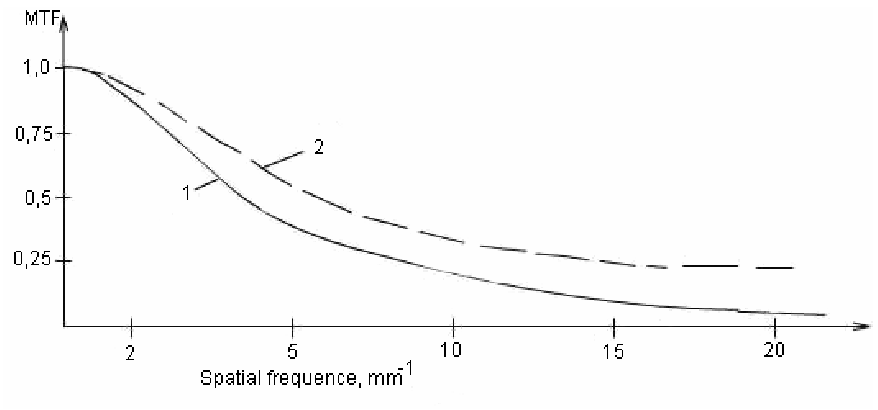

Fig. 4. MTF of intensifier: 1 – for pulse regime; 2 – statical regime

spatial resolution in the center and along the edges of the screen is practically identical. Synchronous intensifier and MCP control make them the most perspective for this application.

3rd Generation image intensifier is the most suitable for light-scattering objects visual diagnostics because of its photocathode (with negative electron affinity) monochromatic current integral characteristic (fig.5) has more narrow zone of sensitivity then traditional multi-alkaline photocathodes.

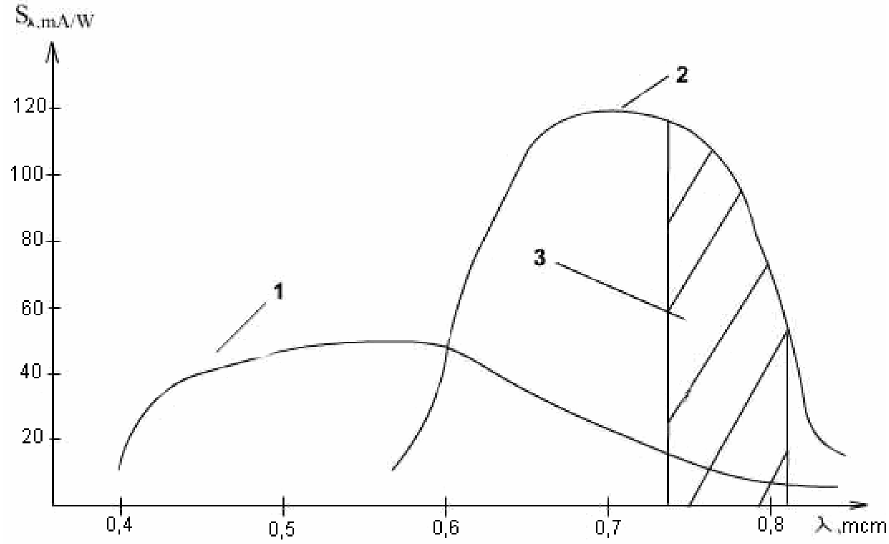

Fig.5. Monochromatic current sensitivities of photocathodes: 1 – multi-alkaline; 2 – photocathode with negative electron affinity; 3 – pass-band of inlet optical filter

Therefore, there is no need for interference filters with very narrow bandwidth which have small transparency coefficient (about 10%) for laser radiation filtration on the intensifier optical inlet. The filter we used (see hatched region on fig.5) has transparency coefficient 40%. Yet, the integral sensitivity of photocathode with negative electron affinity is 3 times more then traditional multi-alkaline one has. If one compares these sensitivities in the filter passband then a greater difference will be resume.

2.2.Technological trick for image contrast increase

3rd Generation intensifier has cathode unit construction with noticeable drawback. Monocrystalline photocathode 2 with negative electron affinity (fig.6) connected to the window glass by thermocompression welding then inlet window 1 thickness is about 6 mm. Glass surface 3 is covered by contact chrome layer. The light rays 4 incoming into cathode unite from object are reflected from contact layer and falls on the photocathode 2 making background that decreases image quality (contrast) on the low spatial frequencies (smeared picture). For contrast improvement the light-absorbent cover was placed under chrome layer. We processed special coating which does not have sharp borders between glass surface and bulk. So, it absorbed all rays falling on this border. The contrast increase was obtained about 10%.

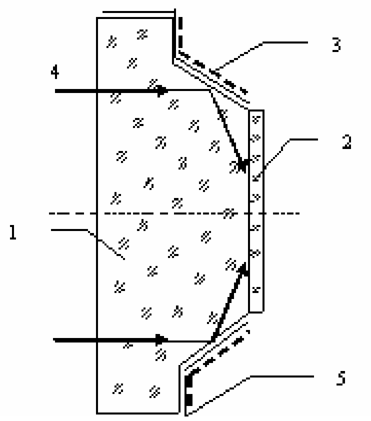

Fig. 6. The photocathode unite of 3rd Generation intensifier: 1-inlet window; 2 – photocathode; 3- chrome contact layer; 4 –light rays from object; 5 – «black» absorbent layer

2.3.Pulse laser lighter

Pulse semiconductor laser (five-diodes array) was used as lighter to provide super-short light pulses with wavelength about 840-860 nm which is coincident with photosensitivity range of 3rd Generation image intensifier. Semiconductor pulse laser[6,7] sources (SPLS) have a number of advantages as compared with solid-state and gas lasers, namely: high efficiency, minimal power consumption, long durability, mechanical loads resistance. The choice of semiconductor laser is connected, too, with its small overall dimensions, sufficiently high radiation power and small-dimensions feeble current pump source. We used laser power supply manufactured on the base of all-solid state circuitry[5,6]. It provides pump current pulses with different controlled forms to modulate laser radiation pulse. Radiating element not requires any forced cooling and high-voltage power supply. It provides shot radiation pulses suited for viewing of rather shallow space in sample material that allows suitable image contrast with respect to background. SPLS and power supply unit may be placed separately, and possibility of lightguided transference allows inputting laser radiation in necessary directions. The drawback of SPLS is high output radiation divergence (up to 40 °). To remove this drawback we processed special large aperture optics for radiation beam forming.

2.4.Optical system

Optical elements of equipment provide image projection on the photocathode, laser radiation flux forming and image transfer from phosphor screen to CCD camera. We used small catadioptric optical system with narrow-band filter to project PLD IR radiation on the tube photocathode.

For lighting optical system we used integral projecting system. Such system principle of operation is connected with spot of light projection on some surface, then reflected and transferred light indicatrix has in advance adjusted brightness distribution. At the same time, repeated spot shape may be set by integrator form. In our case, for example, it was rectangular to illuminate proper sample area. The scheme of integral projecting system is shown on fig.7. Laser array 1 radiation propagating along integrator 2 is reflected from its internal surface. An uniform brightness distribution is produced on the outlet integrator surface owing to integration of radiations of individual laser diodes composing array 1. The outlet integrator face plane is projected on the sample surface. We used dilative fiber-optic integrator (FOI) with circular inlet and rectangular outlet. Lens 3 was a flat-convex lens bonded on FOI outlet surface. It formed radiation beam. A convex glass surface was aspherical one.

PLD output was supplied by fiber-optic braid to bring laser light to the sample.

Relay lens provides image projection from image intensifier screen to CCD camera (see part 2.5).

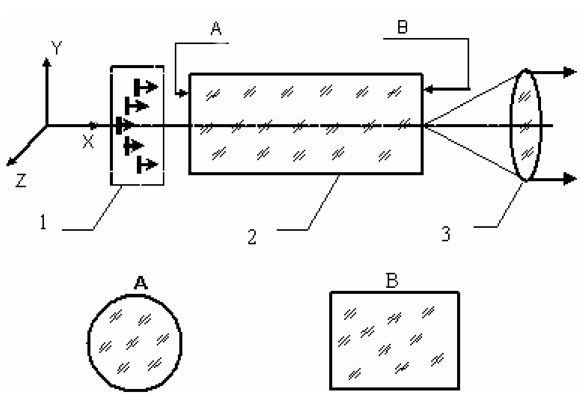

Fig.7. The scheme of integral optical projecting system. 1- laser diode array; 2 – integrator; 3 – objective lens; A, B – view of inlet and outlet integrator face planes

2.5. TV camera and electronics

TV camera[4,7] has ½ inch CCD array with 756x588 elements and provides high quality image. Computer image processing provides contrast enhancement, object outline detachment, different parts of image separation, electronic zoom, and image storage in memory.

Electronics[4] was developed in order to synchronize all system elements – PLD, image intensifier gating, pulse and delay units, etc.

The rational method of the image intensifier screen and light-sensitive cell of TV camera, i.e. CCD array, mating is the great importance for television version system design. In order to require this compliance relay optics as projecting lens or fiber-optic element (FOE) may be used. In its turn, FOE may be of constant section square or compound conically shaped concentrator. Cone concentrator provides complete image transmission from the whole of image intensifier screen on CCD array. In that way, there are no field of view losses which occur with FOE of constant section square because of conventional CCD array dimensions are smaller then image intensifier screen.

The FOE drawbacks are transmitted energy losses, decrease of image contrast and spatial resolution limitation. There are perceptible fiber defects in FOE, too. The FOE and CCD coupling is obtained by optical glue attaching. The FOE-CCD unit is mated with image intensifier screen through immersion oil. So, in a case of CCD death FOE has be thrown away, too.

The projecting lens relay optics allows exclude all drawbacks of FOE with constant section square but leads to increase of optical unit dimensions and power losses. Nevertheless, image quality benefits in combination with high CCD photo-sensitivity compensate all these losses. Moreover, the relay optics in the form of optical system with variable magnification provides quick image scale change. The latter circumstance is the great importance for visual diagnostics.

We used two variants of relay optical system: the first, with unstructured cone concentrator with scale 2.5:1 and, second, projection lens system. Parameters of relay optical system for these variants listed in table 1.

The very lens optical system was used in subsequent experiments because of its merits.

Table 1

Parameters	Cone concentrator	Lens
Magnification	0.4	0.4
Aperture, degrees	24	44
Linear field of view, mm	17.5	23
Transparency coefficient, relative units	0.63	0.72
Modulation transfer function (MTF), % on spatial frequencies, line·mm^{-1}:		
5.0	0.92	0.96
11.5	0.80	0.75
25.0	0.50	0.63
50.0	0.10	0.48
Limit of spatial resolution, line·mm^{-1}	40	75
Unit length, mm	40	110

Table 2

Parameter	Value
Strobe duration, μs	0.2...10
Build-up time, μs (not more)	0.1

Decay time, μs (not more)	0.1
Output capacitance, pF	10
Pulse amplitude, V	900
Dimension, mm	227x100x30
Power consumption, W (not more)	5

For image intensifier time strobbing (pulse control) we used strobe unit with parameters[3] indicated in tabl.2.

3. EXPERIMENTAL RESULTS

As it was noted above, the advantage of this diagnostic method is the image receiving from scattering space when object is irradiated by super-shot laser pulses. Scattering light is eliminated during image intensifier gate time. System has next parameters: laser pulse time – 10 ns, strobe gating time – 0,2...50 ns, spectral sensitivity (λ=830 nm) – 100 mA/W, amplification – 20,000 W/W, resolution – 500 TV lines. Obviously, to realize all possibilities of the method it is necessary to apply femtosecond pulse duration but we obtained suitable quality images in nanosecond regime, too (fig.8). These images comparison with X-rays photos has confirmed the correctness of research direction.

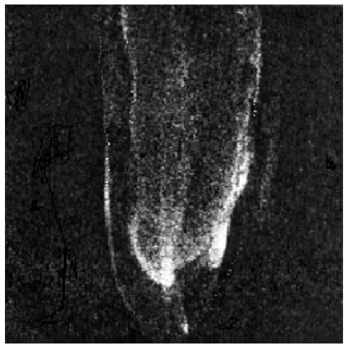

Fig.8. IR image of light-scattering sample

4. CONCLUSION

Experimental visual diagnostic method using active pulse image intensifier mode with laser pulse lighting is presented. Method provides quality images of different light-scattering samples. To apply this method in photonic crystals it is necessary to improve it by decreasing of strobe pulse duration, however, presented equipment is sufficient for some optical crystals heterogeneities diagnostics.

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Рассудочное и разумное мышления в физике: пространства Минковского и Финслера как суть формы бесконечного в философии

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При изучении микро- и мега миров мы сталкиваемся с другой, необычной физикой. Например, принцип неопределенности в квантовой физике и теория относительности в релятивистской физике действительно необычны для нашей логики. Хотим мы этого или нет, однако, нам пришлось согласиться согласно эксперименту с тем, что не может идти речь о точной локализации микрочастиц, что свет во всех инерциальных системах отсчета движется с одной и той же скоростью и потому время вовсе не абсолютная величина. Но не означает ли наше согласие с этими далеко не логичными с точки зрения обыденного сознания выводами, что мы переходим на другой уровень сознания – необыкновенному сознанию? Необыкновенному не потому, что не наше, а потому, что непривычное. Непривычно потому, что такое сознание имеет дело суть другими формами - формами бесконечного. Согласимся, что мир – вещь бесконечная и поэтому совсем другая, не обычная логика может иметь место при его познании. Для того, чтобы понять, что в этом случае происходит с обычной логикой мы должны обратиться к философии. Как писал Гегель [1], логику надо изучать не ради пользы, а ради ее самой. Логика рассматривает мышление в его деятельности и в его продукте. Все науки, кроме философии используют только продукт. Философия же, подчеркивая, что задача логики – различать суть формы конечного от бесконечного, говорит о необходимости рассматривать мышление в его деятельности. Эта деятельность есть духовная деятельность, подобно чувственному восприятию, созерцанию. Поэтому содержание логики есть сверхчувственный мир и, занимаясь ею, хотим мы этого или нет пребываем в этом мире.

Пребывая в этом мире, мы находим всеобщее. Это всеобщее – необычное всеобщее, – например, законы движения планет не начертаны на небе, т.е. не видимы, не слышимы, – существует только для размышления, для духа. Лозунг Гегеля таков: «Что разумно, то действительно», тем самым он поднимает статус мышления до истины предметного. Гегель подчеркивает, что в свое время Аристотеля не правильно поняли, приписывая ему слова, что нет ничего такого в разуме, чего не было бы в опыте. Наоборот, Аристотель говорил, что нет ничего такого в опыте, в чувствах, чего не было бы в разуме. Иначе говоря, если что-то есть в разуме, тогда это есть и в реальности. Поэтому можно не только предполагать, что наш реальный мир имеет прямую связь с финслеровой геометрией, но это действительно так. Потому что, то, что разумно, то действительно. С этой точки зрения можно согласиться со многими математиками [2-6], что Евклид мог бы направить естествознание и по-другому пути, если взял бы в качестве первичного понятия не пространство, а время. Наверное, древние жрецы Египта так и поступили, и поэтому имели совсем необычное для древних греков и для нас, принципы миросозерцание. Тому свидетельство последние работы ученых [7] о загадках строительства древних пирамид.

Человек познает мир, при этом складывается непосредственное отношение его сознания к нему – созерцание. Если проследить развитие человеческой мысли с древних времен до наших дней, то можно отметить следующие главные этапы [1] :

1) Период до Аристотеля,
2) От Аристотеля до 16 столетия,
3) От Галилея, Декарта, Ньютона до начала 20 столетия,
4) Теперь переживаемый кризис, начатый Эйнштейном, Бором, Гейзенбергом

Аристотель понял, что понятия пространства, времени и движения связаны друг с другом. Мы можем измерить пространства двумя способами:

5) 1) пространство подчиняется «движению», т.е. его длительности, и
6) 2) пространство подчиняется одновременности двух происшествий (фиксируем две точки А и В пространства).

В обоих случая время играет большую роль [9], и нужно определить отношение к нему сознания человека. Это сделал Аристотель, провозгласив общий принцип миросозерцания: «для движений, происходящих одновременно, время измеряется одинаково, независимо от скоростей этих движений, даже если одно тело находится в покое, а другое движется» [10]. Таким образом, длительность какого нибудь явления не зависит от состояния покоя или движения тела, на котором наблюдается это движение, т.е. не зависит от наблюдателя. Итак, время становится абсолютной величиной. Наверное, у Аристотеля были основания возвести время в ранг необычного. Как Аристотель пишет: «Вот несколько соображений, которые можно привести, чтобы доказать, что время совсем не существует, а если и существует, то лишь образом, мало ощутимым и весьма неясным. Так, одна из частей времени была и ее более нет, другая должна быть и ее еще нет. Однако лишь из этих элементов слагается и бесконечное время и то время, которое мы считаем в непрестанной последовательности. Но то, что составлено из элементов не существующих, представляется и само не обладающим истинным существованием.

К этому надо добавить, что для всякого делимого предмета необходимо, чтобы существовали и некоторые его части и даже все его части. Но для времени, хотя оно делимо, одни части были, другие будут, но ни одной нет в настоящем. Настоящее – момент или мгновение не есть часть времени, ибо часть какой-либо вещи служит мерою этой вещи; с другой стороны, целое должно слагаться из соединения частей, между тем время не состоит из последовательных моментов настоящего. Кроме того, самый момент, само настоящее, разграничивающее прошедшее и будущее – единое ли оно или нет, остается ли оно всегда тождественным и неизменным или же оно постоянно изменяется и постоянно различно?». Конечно, Аристотель не мог найти ответа на свой вопрос. Время, являясь необычным, с необходимостью требовал к себе определенного отношения со стороны человека, его сознания. Поэтому Аристотель, глубоко все это понимая, сформулировал именно такой общий принцип миросозерцания. Этот принцип смог очень долгое время удовлетворять потребности человека в познании мира. Почему? Потому что, то, что разумно, то действительно. В разуме есть все, что можно найти в опыте. «Нет ничего такого существующего в опыте, чего не было бы в разуме» именно так говорил Аристотель и с ним соглашается Гегель. В разуме существует много конструкций, которые можно подогнать к опыту. В те времена, а точнее до начала 20 столетия, такая конструкция, такой принцип миросозерцания был достаточен для понимания опыта. Ведь тогда не было опыта Майкельсона и Морли по измерению скорости света. Появился этот опыт (появились и другие опыты, например, по нахождению траектории микрочастицы) и стал необходим новый принцип миросозерцания. Этот принцип стал провозглашать чудовищные с точки зрения рассудка вещи. Не время, а скорость света – абсолютная величина. Длительность времени для наблюдателя зависит от его движения или покоя. Таким образом, восприятие времени зависит от состояния наблюдателя. Понимание этого факта произошло не от рассудка, а от разума, сознания наблюдателя.

Дело в том, что рассудок, который может объяснять конечные вещи оказывается недостаточным мышлением. Разумное мышление – бесконечное мышление приходит ему на смену, когда речь идет о таких бесконечных вещах, например, как мир, время, пространство. Только разумом мы можем понять вопрос Аристотеля о том, что само настоящее время, разграничивающее прошедшее и будущее – единое ли оно или нет, остается ли оно всегда тождественным и неизменным или же оно постоянно изменяется и постоянно различно? Нет одного утвердительного ответа на этот вопрос. Хотя рассудок протестует, но разумом мы можем ответить, что ответ зависит от уровня размышления, т.е. от сознания наблюдателя. Можно возразить и сказать, что не от уровня мышления, а от уровня опыта, который мы проводим. Но этот опыт зависит от уровня нашего знания, а значить в конечном итоге зависит от сознания. Любой принцип миросозерцания существует в нашем разуме. Его выбирает наш разум, наше сознание – для конкретного случая в познаваемом мире существует свой принцип миросозерцания. Поистине, наш разум – бесконечен. Действительно, как можем мы представить себе конечное мышление. Это невозможно. Наше мышление бесконечно! Наверное Аристотель знал, что ответа нет на его вопрос. Поэтому он говорит о момент времени, как о числе, о числе необыкновенном. Как он пишет: «... невозможно, чтобы моменты следовали слитно один за другим, как невозможно, чтобы на линии точки располагались слитно одна за другою...» Здесь, Аристотель уподобляет моменты времени обыкновенным числам на прямой. И можно было бы, поэтому, абсолютировать время. Но в то же время Аристотель пишет, что время связано с движением: «Мы представляем себе однако, что время не может быть постигаемо без изменений, ибо мы сами, если не испытываемв наших мыслях никакого изменения или если изменение, которое в них происходит от нас ускользает, то считаем, что не протекло и никакого времени... Итак, не подлежит сомнению, что без движения время не возможно и что время не есть движение». Аристотель, возвращаясь к тому, что время есть число, а точнее число движения (но все равно число), в то же время говорит, что это – отвлеченное, т.е. необыкновенное число: «Итак, время есть число движения, но это число не относится к одной и той же точке, которая была бы вместе с тем и началом и концом, как это имеет место на линии, границу частей которой она составляет, но сама не есть часть линии.

Таким обраом, настоящий момент, рассматриваемый, как граница между прошедшим и будущим, не есть время, это есть лишь признак времени, которое он разграничивает и определяет. Но, поскольку он служит для счета движения и времени, он есть число, с тою однако разницею, что границы непременно принадлежат к этому предмету, который они ограничивают, тогда, как отвлеченное число может служить для счета чего угодно и число десять, например, после того, как оно приложено к этим десяти лошадям перед нашими глазами, может совершенно также может быть приложено и ко множеству других предметов, число коих тоже десять». Если бы Аристотель жил бы в наше время и знал бы об обобщениях обычного числа, то конечно, он, наверное, сказал бы, что настоящий момент времени есть комплексное или гиперкомплексное число. Ведь комплексное число, имея действительную и мнимую части, благодаря мнимой части несет в себе признак числа, но все равно, благодаря своей действительной части остается числом. Как известно, существуют такие финслеровые пространства, точками которых являются гиперкомплексные числа. Поэтому время можно представить себе и понять с помощью финслерового пространства. Действительно, во многих своих работах некоторые ученые [4,5] с помощью линейных финслеровых представляет себе время и вводит в этих пространствах понятие двухмерного, трехмерного и даже четырехмерного времени. Такой подход к проблеме пространства и времени себя оправдывает, потому что измерение расстояния, длины в в обычном метрическом пространств требует в любом случае, как мы отмечали выше, определения времени. Как известно, после установления опытного факта постоянства скорости света во всех инерциальных системах, с появленим теории относительности наше отношение к реальному миру изменилось. Геометрия Минковского сыграла большую роль в понимании структуры физической реальности. Это было победой разума над рассудком. Теория относительности, геометрия Минковского, являясь суть формами бесконечного есть результат разума. Но будет ли разум останавливаться на достигнутом? Конечно, нет. Поэтому работы в этом напрвлении продолжаются.

Например, концепция многомерного времени предполагается как альтернатива пространству теории относительности [4,5]. В этих работах говорится, что можно вначале задать время в обобщенном пространстве, а затем в этом обобщенном пространстве увидеть метрическое пространство, соответствующее конкретному времени. Учитывая, что интервалы времени – это единственные величины, которые по определению предполагаются измеримыми в финслеровом многообразии, выделить среди непрерывного спектра наклонных мировых линий такие, которые характеризуются равенством интервалов, задача всегда разрешимая. Исходя из этого, вводится понятие эталонных сигналов. Благодаря набору эталонных сигналов определяется геметрия физического пространства. Например, если мировые линии эталонных сигналов почти параллельны линии наблюдателя, то в представлениях наблюдателя возникает геометрия Евклида. Но в общем случае, мировые линии эталонных сигналов в достаточной степени не параллельны мировой линии наблюдателя, и поэтому получающееся пространство имеет уже совсем другую геометрию – геометрию Финслера.

Но почему в представления наблюдателя нет этой геометрии? Это объясняется тем, что в жизни, в экксперименте мы имеем дело с малыми скоростями и поэтому отличия двух геометрий невелики. Поэтому очень вероятно, что мы можем их спутать. И мы действительно путаем! Гегель писал, что в явлении то, что для духа – это внутреннее (размышления, что за этим стоит), а то, что для чувств – это внешнее. Поэтому, ломая явление на два, мы познаем вещи в отношении причина и ее действие, сила и ее обнаружение. Во многих конечных вещах мы находим одно и тоже отношение. Это почему-то дает нам основание считать, что во всем будет проявляться то же самое отношение. Гегель пишет, что это наивный образ мышления. Все науки живут в этой вере. Как он говорит, наше мышление приступает прямо к предметам, репродуцирует из себя содержание ощущений и созерцаний. Это содержание ощущений и созерцаний мы считаем содержанием мысли и удовлетворяемся этим содержанием, видя в нем истину [1].

Приведем простой пример. Видя в опыте, что тело движется в результате действия силы, наш рассудок решил, что во всех вещах это будет выполняться. В частности, если на тело не действует сила, то тело находится в покое. Причиной движения является сила. Это содержание ощущений и созерцаний мы переводим на мысли. Это действительно было наивным образом мышления. Первый закон Ньютона положил ему конец, провозгласив, что тело может без действия силы равномерно и прямолинейно двигаться. И таких примеров множество.

К этому множеству примеров относится и то, что в нашем представлении имело бы место геометрия Финслера, если бы не наивный образ мышления. Павлов эту ситуацию описывает следующим образом [4]. Имея дело в опыте с малыми скоростями, мы определяем расстояния с помощью линейки. Поэтому линейка относится к медленным способам определения расстояния. Но можем ли мы использовать эту самую линейку, когда речь идет о больших скоростях. Конечно, нет. Но рассудок, при получении аномальных результатов будет как угодно их трактовать, но только не в направлении пересмотра «очевидных» геометрических свойств. Таким образом, если мы сможем преодолеть сопротивление рассудка и пересмотреть «очевидные» вещи, тогда наше мышление, не сразу приступая к предметам, сможет репродуцировать из себя новое содержание ощущений и созерцаний. Например, многомерное время. Существует множество параллельных во времени миров, подобно тому, как существует множество возможных состояний тела в евклидовом пространстве. Заметим, что наш разум совершенно не сопротивляется к этому новому содержанию ощущений. В опыте с очень большими скоростями нам нужно прибегнуть к этим новым ощущениям и сложить новое созерцание. Действительно, для теории относительности была в этом необходимость. Многомерное время, как нельзя лучше подходит для нового созерцания в опыте с очень большими скоростями. В работах [4-6], что в теории относительности целесообразно использовать геометрию Финслера (именно при больших скоростях видно различие между геометриями Евклида и Финслера). Отметим, что одновременность событий, которые имеют большое значение в теории относительность естественным образом находят свое решение в геометрии Финслера. Интервалы времени, как об этом писалось выше, являются измеримыми в финслеровом пространстве.

Часто спрашивают, действительно ли это так, что наш реальный мир имеет связь с финслеровой геометрией. В этом вопросе есть желания увидеть опыт, подтверждающий его идею многомерного времени. В философии Гегеля, идея – это истина разума. Возникает вопрос: может ли идея ученых о многмерном времени претендовать на идею по определению Гегеля? Да, может, потому что многомерное, в частности трехмерное время, как представитель необычного класса (линейных) финслеровых пространств имеет очень важное свойство, а именно связь с наиболее фудаментальным понятием математики – числом, самым обычным числом. Как известно, Пифагор строил свою философию на числах и считал основным определением вещей число. «Все есть число», так говорил Пифагор. Обыденное сознание никак не может согласиться с этим, считает сказанное Пифагором безумием. Рассудочное мышление, являясь конечным, говорит нам: «Смотри, как далеко зашел Пифагор, считая вещь числом, а значит и число вещью». Гегель отвечает на это, что число есть не просто вещь, а более того, число есть мысль. Поэтому многомерное время, имеющая связь с числом, имеет связь с мыслью. Идея о многомерном времени может претендовать на истину разумного. Если, однако, все равно концепция многомерного времени хочет подтверждения в опыте, то на это можно ответить следующим образом, опираясь еще раз на философию Гегеля. Гегель говорил, что опыт служит для познания явления, а не для познания истины. Опыта мало для познания истины. Эмпирическое наблюдение дает нам многочисленные одинаковые восприятия. Однако, всеобщность есть нечто другое, чем простое множество. Это всеобщее находится с помощью разума.

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Gravitational self-lensing of light in a visible disk of star α Ori (Betelgeuse)

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Photos of a star α Ori (Betelgeuse) which image is resolved in the form of a disk in angular diameter nearby 44 *mas* are known for received by a telescope "Hubble". According to data of the catalogue “Hipparcos”, this star is characterized by a parallax 7,63±1,64 *mas* and therefore, it settles down on distance from the Sun not less than in 200 *l.y*.

In the present work the assumption that the visible disk of star Betelgeuse is generated as a result of gravitational self-lensing of images of a star by its own gravitational field is put forward. If near to a disk the removed stars were visible, images of these stars would be displaced from the valid position on heavenly sphere owing to Einstein's effect. Thus the lights which have been let out by edge of Betelgeuse, are displaced on size in half of effect of Einstein. As the visible angular sizes of Betelgeuse are comparable to the named displacement, it is possible to assume, that the sizes of an observable disk and distribution of brightness on its surface in an essential measure is generated by gravitation field of Betelgeuse. It is the described phenomenon of the gravitational self-lensing.

For check of the put forward assumption of distribution of brightness along the visible image for the removed shone disk with constant brightness is calculated. However, real stars cannot be considered as shone disks with constant brightness owing to effect of dimness of the image to edge. The account of influence of this effect at modelling the phenomenon can be executed by introduction of weight factors for various points on brightness of a modelling shone disk. In particular, at research of the eclipse-variable stars, similar distribution of brightness on a visible disk of a star it is modelled by trigonometrical functions.

The result of such modelling can contain two phenomena of dimness in the integrated form - usual and gravitational which is caused by non-uniformity of a deflection of light along radius at edge of a star. For separate research of these phenomena usual dimness of the image to edge was modelled by dimness of the Sun, as unique resolved star at which gravitational dimness is very small because of greater visible geometrical sizes. The weight factors received at it have been used for specification of model of the phenomenon of gravitational self-lensing.

As a result of modelling distribution of brightness under the observable image of starlike object in view of the phenomenon of gravitational self-lensing has been received. Its parameters correlate with parameters of similar distribution for the image of star Betelgeuse.

Расширение тела комплексных чисел

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1. Действия над векторным комплексом

В работе рассматривается математическое описание скаляров, псевдоскаляров, векторов и псевдовекторов не прибегая к созданию новых сущностей и новых алгебр, как это принято в теории гиперкомплексных чисел. Показано, что, используя хорошо известные классические правила действий над полем комплексных чисел и матриц, можно построить 2+6 мерную ассоциативную алгебру с делением, объединяющую в одно тело комплексные числа, векторный анализ, кватернионы, 4-векторы и октеты (ассоциативный аналог октав Кэли-Диксона, бикватернионов и дикватернионов [1]).

Рассмотрим множество всех квадратных матриц второго порядка с комплексными элементами

$$\overset{\Rightarrow}{K} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} z_0 + z_2 & z_1 + iz_3 \\ z_1 - iz_3 & z_0 - z_2 \end{pmatrix} = \qquad \text{...(1a)}$$

которые можно представить также в виде векторного комплекса, т.е. суммы скаляра, псевдоскаляра, вектора и псевдовектора;

$$= \sum_0^3 z_k \vec{e}_k = a_0 + b_0^{\bullet} + \vec{A} + \vec{B}^* = \qquad \text{... (1в)}$$

или в экспоненциальном (тригонометрическом) виде - (обобщённая формула Эйлера)

$$= Z\exp(\Psi\vec{e}) = Z(Ch\Psi + \vec{e}Sh\Psi), \qquad \text{... (1c)}$$

где введены следующие обозначения:

$$z_k = a_k + ib_k, \ \text{k=0,1,2,3.} \quad b_0^* = i \cdot b_0, \ \vec{B}^* = i\vec{B}. \qquad \text{... (2)}$$

$$z_0 = \frac{z_{11} + z_{22}}{2}, \ z_1 = \frac{z_{12} + z_{21}}{2}, \ z_2 = \frac{z_{11} - z_{22}}{2}, \ z_3 = \frac{z_{12} - z_{21}}{2i}, \qquad \text{... (3)}$$

$$\begin{aligned}
a_0 &= \frac{1}{2}\mathrm{Re}(z_{11} + z_{22}), \quad b_0 = \frac{1}{2}\mathrm{Im}(z_{11} + z_{22}),\\
a_1 &= \frac{1}{2}\mathrm{Re}(z_{12} + z_{21}), \quad b_1 = \frac{1}{2}\mathrm{Im}(z_{12} + z_{21}),\\
a_2 &= \frac{1}{2}\mathrm{Re}(z_{11} - z_{22}), \quad b_2 = \frac{1}{2}\mathrm{Im}(z_{11} - z_{22}),\\
a_3 &= \frac{1}{2}\mathrm{Im}(z_{12} - z_{21}), \quad b_3 = \frac{1}{2}\mathrm{Re}(z_{21} - z_{12}),
\end{aligned} \qquad (4)$$

$$Z = \sqrt{\det(\overset{\Rightarrow}{K})} = \sqrt{z_{11}z_{22} - z_{12}z_{21}} = \sqrt{z_0^2 - z^2}, \quad \Psi = \theta + i\eta = \frac{1}{2}\ln\frac{z_0 + z}{z_0 - z}$$

$$z^2 = z_1^2 + z_2^2 + z_3^2, \quad \det\overset{\Rightarrow}{K} = \det\overset{\Rightarrow}{K}^{-1} = \det\overset{\Leftarrow}{K} = \det\overset{\Leftrightarrow}{K}, \quad Ch\Psi = \frac{z_0}{Z}., \quad Sh\Psi = \frac{z}{Z},$$

$$\vec{e} = \frac{z_1}{z}\vec{e}_1 + \frac{z_2}{z}\vec{e}_2 + \frac{z_3}{z}\vec{e}_3 = \frac{1}{z}\begin{pmatrix} z_2 & z_1 + iz_3 \\ z_1 - iz_3 & -z_2 \end{pmatrix} \qquad \text{.. (5)}$$

$i=\sqrt{-1}$ - стандартная мнимая единица скалярного типа, $\vec{A}=a_{\mu}\vec{e}_{\mu}$, и $\vec{B}=b_{\mu}\vec{e}_{\mu}$, обычные трёхмерные векторы (по дважды повторяющимися латинскими индексами происходит суммирование от ноля до трёх, по греческим индексам – от единицы до трёх). Объект $\overset{\mapsto}{q}_a=b_0^*+\vec{A}$, представляет собой хорошо известный в теоретической физике 4-вектор [2], объект $\overset{\mapsto}{k}_b=a_0+\vec{B}^*$ - изоморфен кватерниону [3], но фактически им не является, так как нам нет необходимости вводить новые сущности, а именно новые мнимые орты $\vec{i},\vec{j},\vec{k},\vec{e}_4,\vec{e}_5,...$ и т.д. и правила действия над ними, а именно это обстоятельство и приводит к потере ассоциативности при процедуре удвоения. Именно на этом пути и стоит шлагбаум теоремы Фробениуса [4], утверждающей, что при n>4 не существует ассоциативных алгебр. Но путь этот не единственный, что мы и демонстрируем в данной работе. В силу выше изложенного будем называть объект $\overset{\mapsto}{k}_b$ - кватером.

Сложение любых объектов, имеющихся во множестве $\overset{\Rightarrow}{K}$, происходит покомпонентно, умножение - по стандартным, классическим, обычным, естественным правилам всё на всё:

$$\sum_{i=1}^{n} M_i \cdot \sum_{j=1}^{m} N_j = \sum_{i,j}^{n,m} M_i N_j \quad . \qquad ...(6)$$

как это и делается в теории гиперкомплексных чисел. Таким образом, данное множество обладает свойствами по сложению: коммутативным, ассоциативным, дистрибутивным и обладает противоположным элементом. По умножению: скалярная часть – коммутативна, векторная часть – антикоммутативна, всё вместе – ассоциативно. Имеется обратный элемент, следовательно, в целом мы имеем ассоциативную алгебру с делением, т.е. – тело. Деление на ноль исключено.

$$\overset{\Rightarrow}{K}_1 \circ \left(\overset{\Rightarrow}{K}_2 \circ \overset{\Rightarrow}{K}_3\right) = \left(\overset{\Rightarrow}{K}_1 \circ \overset{\Rightarrow}{K}_2\right) \circ \overset{\Rightarrow}{K}_3$$

Умножение 4-вектора на псевдоскаляр превращает его в кватер, умножение кватера на псевдоскаляр превращает его в 4-вектор. Другими словами векторные комплексы, также как и любую комбинацию элементов из этого множества, можно вычитать, складывать, умножать, делить, возводить в степень и извлекать корни, оставаясь в том же множестве. Так как обычные векторы, так же как и любые невырожденные квадратные матрицы второго порядка, являются частью этого множества, то все эти действия можно совершать и с ними. И всё это можно делать любым из трёх разных способов: 1) Векторная алгебра, 2) Экспоненциально тригонометрический способ, 3) Матричный метод, (см. прил.№1).
Не очень то разумно было лишать векторы возможности умножаться, делиться, извлекаться и возводиться. Не мешает отметить, что часто встречаемая фраза – «единственным расширением комплексных чисел является теория кватернионов, октав и т.д. …», а также утверждение о том, что поле комплексных чисел – С, является частью множества кватернионов - Н, не соответствует действительности. Теория кватернионов является расширением поля действительных чисел. Комплексных чисел в теле Н нет вообще. Мнимая единица, присутствующая в С, есть величина скалярного типа, то есть коммутативная, мнимые единицы из Н, есть величины векторного типа, то есть антикоммутативные. Да и геометрическая интерпретация комплексных чисел не имеет ничего общего с геометрической интерпретацией кватернионов [5]. Векторы также не являются частью тела Н, так как это мнимые вектора. Напишем обычный классический вектор: $\vec{A}=a_1\vec{e}_1+a_2\vec{e}_2+a_3\vec{e}_{3,}$, из этой формы записи неизвестно в правой или левой системе координат - (л.с.к.) мы находимся; неизвестно вектор это или псевдовектор (полярный или аксиальный) и поэтому невозможно написать вектор ему со-

пряжённый. Как отличить скаляр от псевдоскаляра? Информация об этом содержится в предыстории этого вектора или скаляра, нам неизвестной. Прилагать к каждому вектору его родословную – это уже не математика, а генеалогия. Для устранения этих недостатков введём следующие обозначения: В правой с.к. $\vec{A} = a_{\mu}\vec{e}_{\mu}$, в левой с.к. $\overleftarrow{A} = a_{\mu}\overleftarrow{e}_{\mu}$. Величины псевдоскалярного и псевдовекторного типа будем обозначать: m - скаляр, m^* - псевдоскаляр, $\vec{A}$ - вектор, $\vec{A}^*$ - псевдовектор. Такая форма записи - «звёздочка», удобна для раскрытия физического смысла объекта, но не позволяет проводить над ними математические операции, так как символ «звёздочка» не несёт в себе никакой математической нагрузки и поэтому является временным. Чтобы найти математический эквивалент этому символу, выпишем основные свойства этих объектов исходя из того, что и скаляры и векторы не меняют своей сущности при наблюдении их как в п.с.к., так и в л.с.к. то есть $m_{пр} = m_{л}$, $\vec{A} = \overleftarrow{A}$. Можно сказать, что это инварианты по отношению к правому и левому. Что касается псевдоскаляров и псевдовекторов, то, будучи положительными (отрицательными) как в правой, так и в левой с.к. они представляют собой разные физические объекты, несовместные друг с другом. Другими словами это идентичные, но не тождественные объекты. Никакими движениями невозможно превратить правый объект в левый и наоборот. Математически это выглядит так: $m^*{}_{пр} = -m^*{}_{л}$, $\vec{A}^* = -\overleftarrow{A}^*$. Хотя на первый взгляд кажется, что путём соответствующего поворота с.к. можно изменить знак вектора на противоположный, это неверно. Правое (левое) нельзя превратить в левое (правое) никакими вращениями. Правую перчатку нельзя превратить в левую и наоборот. Итак, выпишем некоторые свойства этих объектов (выражения (7) носят не количественный, а качественный характер):

$$m \cdot m^* = m^* m = m^*,\ \ m^* \cdot m^* = m,\ \ m \cdot \vec{A}^* = \vec{A}^*,\ m^* \cdot \vec{A}^* = \vec{A},$$
$$m^* \cdot \vec{A} = \vec{A}^*,\ \vec{A} \cdot \vec{A} = m,\ \ \vec{A}^* \cdot \vec{A}^* = m,\ \ \vec{A} \cdot \vec{A}^* = \vec{A}^* \cdot \vec{A} = m^*,$$
$$(\vec{A} \times \vec{B}) = \vec{C}^*,\ \ (\vec{A}^* \times \vec{B}^*) = \vec{C}^*,\ \ (\vec{A} \times \vec{B}^*) = (\vec{A}^* \times \vec{B}) = \vec{C}$$
$$Rot\vec{A} = \vec{B}^*,\quad Rot\vec{A}^* = \vec{B},\quad Div\vec{A} = m,\quad Div\vec{A}^* = m^*,$$
$$(M + N)^* = M^* + N^*,\quad (M \circ N)^* = M^* \circ N = M \circ N^*,\quad \ldots(7)$$

где M и N, два произвольных элемента скалярного или векторного типа. Говоря «скалярного типа» - мы не отличаем скаляр от псевдоскаляра, то же относится и к словам «векторного типа». Ещё важно отметить, что скаляр и псевдоскаляр складывать нельзя, также как и вектор и псевдовектор. Однако записывать их в одну строчку, ставя между ними знак плюс или минус, как это делается в комплексных числах или с различными компонентами векторов, естественно можно.

Из анализа соотношений (7) видно, что «звёздочка» это оператор скалярного типа, коммутативный, который можно приписать к любому элементу в произведении из нескольких элементов. И такой оператор существует в математике уже 450 лет. Но не получил должной оценки, не занял достойного места в современном здании математики. Пора открыть перед ним более широкое поле деятельности. Это мнимая единица - i. С учётом этого все соотношения (7), выполняются автоматически. Мнимая единица это такой же основополагающий элемент математики, как и действительная единица. Искусственно вводимые и математике требования о «положительности», «действительности» и т.д. уже заранее отрицают сам факт существования псевдоскаляров и псевдовекторов. Поэтому все искусственные ограничения подобного рода в данной работе сняты. Комплексное число $Z = X + iY$, представляет теперь сумму скаляра и псевдоскаляра. Если $\vec{A}$-вектор, то $\vec{A}^* = i\vec{A}$ -псевдовектор, m - скаляр, то $m^* = im$ - псевдоскаляр. Запишем комплексное число Z и ему сопряжённое в правой с.к. – $Z_{пр} = X_{пр} + iY_{пр}$, $\overline{Z}_{пр} = X_{пр} - iY_{пр}$, и в левой с.к. - $Z_{л} = X_{л} + iY_{л}$, учитывая, что скаляр $X_{л} = X_{пр}$, а псевдоскаляр $iY_{л} = -iY_{пр}$, имеем $Z_{л} = X_{пр} - iY_{пр} = \overline{Z}_{пр}$. Аналогично получаем $Z_{пр} = \overline{Z}_{л}$. Другими словами мы видим, что операция сопряжения в комплексных числах – это переход из правой системы координат в левую или из левой в правую. Обобщая полученный результат, можно ввести определение: Операция сопряжение для комплексных чисел это переход из левой с.к. в правую или из правой в левую. При этом становятся очевидными два основных свойства этой операции: $\overline{\overline{Z}} = Z$ - инволюция, и $\overline{\vec{A} \times \vec{B}} = \overline{\vec{B}} \times \overline{\vec{A}} = \overleftarrow{B} \times \overleftarrow{A}$ - антиавтоморфизм. $\vec{A}$ и $\vec{B}$ переставлены местами, так как в п.с.к. вращение идёт от $\vec{A}$ к $\vec{B}$, а в левой с.к. наоборот, от $\vec{B}$ к $\vec{A}$. Нелишне отметить, что распространённое утверждение о том, что функция $W(x, y) = U(x, y) + iV(x, y) = f(z)$ является аналитической, а $f(\bar{z})$, не является аналитической, не полно. Следует продолжить фразу: $\dots = f(z)$ является аналитической в правой с.к., но не является аналитической в л.с.к.. В то время как функция $\overline{W(x, y)} = U(x, y) - iV(x, y) = f(\bar{z})$ - аналитична в левой с.к. и не аналитична в правой с.к. Условие Коши-Римана не меняется. Отметим заодно, что понятие «левое» и «правое» на самом деле шире, чем просто инверсия координат (так называемая CPT инвариантность, см. прил. №2). Чтобы формулы (1) можно было записать в матричном виде, необходимо уточнить понятие « ОРТЫ », ибо из классической записи орт в виде векторов - e1=(1,0,0), e2=(0,1,0), e3=(0,0,1), никоим образом не следует ни антикоммутативность этих объектов ни их цикличность, т.е. невозможно получить формулы

$$\vec{e}_\alpha \cdot \vec{e}_\beta + \vec{e}_\beta \cdot \vec{e}_\alpha = 2\delta_{\alpha\beta}, \quad \vec{e}_\alpha \cdot \vec{e}_\beta = \vec{e}_\gamma, \qquad \dots (8)$$

которые приходиться просто постулировать [6]. Вторая формула в (8) несёт в себе внутреннее противоречие, так как это фактически есть векторное произведение двух полярных векторов, которое даёт псевдовектор, но поскольку это выражение обладает цикличностью, то неизвестно какая из трёх орт является псевдовектором или все три орта должны быть псевдовекторами (см. ф-лы (7)). На всё это можно не обращать внимания, пока мы игнорируем существование псевдовекторов в математическом смысле, но как только мы желаем иметь в явном виде и вектора и псевдовектора, необходимо устранить это противоречие, изменив вторую часть ф-лы (8) на (9), (в соответствии с (7))

$$\vec{e}_\alpha \cdot \vec{e}_\beta = \vec{e}_\gamma^{\,*} = i\vec{e}_\gamma \qquad \dots (9а)$$

или, в общем виде, $\vec{e}_\alpha \circ \vec{e}_\beta = \delta_{\alpha\beta} \cdot e_0 + \varepsilon_{\alpha\beta\gamma} \cdot i\vec{e}_r$ . . .(9в)

где $\delta_{\alpha\beta}$ и $\varepsilon_{\alpha\beta\gamma}$ символы Кронекера и Леви-Чивита.

Правило умножения векторов, с учётом (6) и (9), теперь имеет вид:

$$\vec{A} \circ \vec{B} = (\vec{A} \cdot \vec{B}) + i(\vec{A} \times \vec{B}) \qquad \dots (10)$$

где $(\vec{A} \cdot \vec{B})$ - обычное скалярное произведение, и $(\vec{A} \times \vec{B})$ - обычное векторное произведение, являющиеся частными случаями общего выражения (10). Если вектора параллельны, то остаётся только первое слагаемое, если перпендикулярны, то только второе, а в общем случае сохраняется и то и другое. Записав выражение (10) в л.с.к. и перемножив их, получаем инвариант - известное тождество Эйлера-Лагранжа

$$a^2 \cdot b^2 = (\vec{A} \cdot \vec{B})^2 + (\vec{A} \times \vec{B})^2 \qquad \dots (11)$$

подтверждающее их родственную связь, где a и b длины векторов $\vec{A}$ и $\vec{B}$. Умножение двух векторов или двух псевдовекторов, приводит к появлению скаляра и псевдовектора, т.е. кватера, а умножение вектора на псевдовектор даёт 4-вектор. Если раньше вообще не было умножения, то теперь $(\vec{A} \circ \vec{B}) \circ \vec{C} = \vec{A} \circ (\vec{B} \circ \vec{C})$.

Вполне естественный результат. И чтобы его получить, необходимо всего лишь сохранить знак «+» в середине правой части формулы (6), при перемножении векторов, погибший в борьбе «Векторы против Кватернионов» более 100 лет назад, не без помощи О.Хэвисайда. Скалярное и векторное произведения это близнецы, разлучённые в детстве и, наконец, встретившиеся.

Выражение (9в), хорошо известно (без нашей поправки – мнимой единицы, превращающей вектор в псевдовектор), однако эту формулу можно назвать дискретной, так как все орты в ней взаимно ортогональны и могут перемещаться только циклически. Формула (10) позволяет обобщить этот выражение на непрерывный случай и сделать её более простой и понятной, (без специальных тензоров $\delta_{\alpha\beta},...u...\varepsilon_{\alpha\beta\gamma}$):

$$\vec{e}_\alpha \circ \vec{e}_\beta = \cos\varphi_{\alpha\beta} \cdot e_0 + i \sin\varphi_{\alpha\beta} \cdot \vec{e}_\gamma \qquad ...\ (9c)$$

где $\varphi_{\alpha\beta}$, произвольный угол между ортами, отсчитываемый против часовой стрелки в п.с.к., и по часовой стрелке в левой. Символ мнимой единицы, рассматриваемый как оператор поворота на 90 градусов, автоматически говорит нам, что вектор $\vec{e}_\gamma$ перпендикулярен плоскости векторов $\vec{e}_\alpha$ и $\vec{e}_\beta$. Если в выражении (9с) сделать замену орт $\vec{e}_\mu = -\vec{e}_\mu,.....\mu = 1,2,3$ и, получившееся выражение перемножить с (9с), то получим тождество - $\cos^2\varphi + \sin^2\varphi = 1$.

Американский математик Ч.С.Пирс доказал, что все линейные ассоциативные алгебры могут быть выражены в матричной форме [7]. В частности, кватернионы тождественны матричной алгебре с матрицами четвёртого порядка, называемые также матрицами Дирака (с точностью до перестановки). Чтобы найти орты, автоматически удовлетворяющие уравнениям (8) и (9), решим два матричных уравнения

$$M_{22}^2 = \pm 1, \qquad ...\ (12)$$

бесконечное множество решений которых, имеет вид

$$\begin{pmatrix} a & b \\ \frac{\pm 1 - a^2}{b} & -a \end{pmatrix}, \text{для } \forall a.b \qquad ..\ (12a)$$

Но, если в качестве элементов этого решения использовать только числа $0, \pm 1, \pm i$, то остаётся всего четыре линейно независимых решения - одно скалярного типа, т.е. коммутативное и три векторного типа, т.е. антикоммутативные, которые и назовём, соответственно: $e_0, \vec{e}_1, \vec{e}_2, \vec{e}_3.$ и $e_0^\bullet, \vec{e}_1^\bullet, \vec{e}_2^\bullet, \vec{e}_3^\bullet$, где $\vec{e}_j^\bullet = i \cdot \vec{e}_j$.

$$e_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \vec{e}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \vec{e}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \vec{e}_3 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}. \qquad ..(13)$$

Имеющиеся отрицательные решения, представляют собой решения в левой с.к. с точки зрения правой с.к. и наоборот, т.е. имеем очевидное соотношение - $\overleftarrow{e}_\alpha = -\vec{e}_\alpha$. Всё это приводит нас к тому, что $\overline{\overline{A}} = \overleftarrow{A} = -\vec{A}$, независимо от типа вектора (см. прил. №3). Можно найти подобное решение и на основе матриц четвёртого порядка, однако, следуя принципу, чем проще, тем лучше, ограничимся матрицами второго порядка. Описываемый обычно в литературе способ отличить полярный вектор от аксиального рассматривая его в зеркале абсолютно бессмыслен, т.к. при этом молчаливо предполагается, что мы видим там историю его происхождения, т.е. всю тройку векторов. Но если нам известна его история, то в зеркале уже нет необходимости. Истинное же отличие полярного вектора от аксиального (псевдовектора) состоит в том, что в природе существует всего два вида движений и полярный вектор описывает поступательное движение, а псевдовектор – вращательное. Знак минус, стоящий перед этими векторами имеет совершенно разный физический смысл. У вектора этот знак означает изменение направления поступательного движения, а у псевдовектора – изменение направления вращения. (Поэтому то их и нельзя складывать). В общем случае при операции сопряжение мы записываем объект, существующий в левой с.к., аналогичный правому, но не обязательно равный ему ($\overline{z}_{пр} = z_л \neq z_{пр}$). Условия (8) с поправкой (9) выполняются автоматически, как в левой, так и в правой с.к.. Действительно - $\overline{\vec{e}_1 \circ \vec{e}_2} = \overline{i\vec{e}_3} \Rightarrow \vec{e}_2 \circ \vec{e}_1 = -i\vec{e}_3 \Rightarrow \vec{e}_1 \circ \vec{e}_2 = i\vec{e}_3$. Интересно также, что условие цикличности (с поправкой (9)) и условие антикоммутативности совпадают и имеют вид: $(b_1 \cdot a_2 - a_1 \cdot b_2)^2 = \pm(b_1^2 + b_2^2)$, где a_k, b_k, элементы матриц (12а) и для орт (13) выполняются автоматически. Матрицы (13) давно и хорошо известны под названием – спиновые матрицы Паули (1925г.), но ещё раньше они были открыты А.Кэли (1845 г.)[1]. Поскольку эти единичные матрицы образуют единственный в своём роде ортогональный базис, естественно было бы назвать его «абсолютным четырёхмерным базисом». Комбинацию $z_0 \cdot \vec{e}_0 + z_1 \cdot \vec{e}_1 + z_2 \cdot \vec{e}_2 + z_3 \cdot \vec{e}_3 = \vec{\vec{K}}$ назовём «векторным комплексом» - (В.К.), (см. ф-ы (1-5)). В.К. отличается от комплексного вектора тем, что у последнего все компоненты векторного типа, а у В.К. одна компонента скалярного типа, а три - векторного типа. Можно конечно рассматривать В.К. в двойном базисе $(\vec{e}, \vec{e}^{\bullet})$ с восемью действительными координатами, но проще оставить только основной базис (13) с комплексными координатами, тем более что переход от одного базиса к другому в любой момент осуществляется элементарно (см. прил. №3).

Известно, что умножение матриц не всегда возможно, однако аналогичный факт, касающийся деления матриц почему-то умалчивается. Любая матрица размера m на n делиться на квадратную невырожденную матрицу размера m на m слева и размера n на n справа, а квадратная матрица второго порядка делиться на такую же двумя способами - слева и справа, что эквивалентно получению результата в л.с.к. и в п.с.к.. Учитывая, что любую квадратную матрицу второго порядка можно записать в экспоненциальном виде (формула (1с)), становиться очевидным, что эти матрицы можно не только возводить в целую степень, но и извлекать из них корни, т.е. возводить в любую степень – целую, дробную, отрицательную, мнимую и т.д. (см. прил. №1).

Учитывая матричную запись В.К. и его свойства можно дать следующие новые определения:
опр.1 - Любая коммутативная величина есть величина скалярного типа.
Опр.2 - Любая антикоммутативная величина есть величина векторного типа.
Опр.3 – Объект, квадрат которого положителен, является скаляром или вектором.
Опр.4 – Объект, квадрат которого отрицателен, является псевдоскаляром или псевдовектором.

Собственные числа матрицы (1а) равны $\lambda = z_0 \pm \sqrt{z_1^2 + z_2^2 + z_3^2}$, и находятся из уравнения - $\lambda^2 - 2z_0\lambda + z_0^2 - z^2 = 0$ откуда следует, что величины $z_0, \ldots z_0^2 - z^2 = \det\left(\overset{\Leftrightarrow}{K}\right), \ldots z^2$,

являются инвариантами.
Опр.5 – Если собственные числа - (с.ч.) матрицы (1) кратные и действительные, объект скаляр.
Опр.6 – Если с.ч. матрицы (1) кратные и чисто мнимые, объект псевдоскаляр.
Опр.7 – Если с.ч. отличаются только знаком, то объект векторного типа. (Действительные с.ч. – вектор, мнимые с.ч. – псевдовектор, комплексные с.ч. – сумма вектора и псевдовектора).
Опр.8 – Квадратом длины В.К. назовём взятый с обратным знаком детерминант В.К. т.е.
$l_k^2 = -\det(\overset{\Leftrightarrow}{K}) = \sqrt{z^2 - z_0^2} = -\left|\overset{\Leftrightarrow}{K}\right|^2$, в этом случае, если $\overset{\Rightarrow}{K} = \vec{A} = a \cdot \vec{e}_a$, то $l_k = a$. Длина псевдовектора – чисто мнимая. В общем случае длина В.К. есть комплексное число. Если $\det(\overset{\Leftrightarrow}{K}) = \pm 1,$ то назовём такой В.К. единичным.
Отметим некоторые свойства В.К.: 1) Сумма диагональных элементов матрицы (1), т.е. след, равна удвоенной скалярной части В.К. - $Sp\left(\overset{\Rightarrow}{K}\right) = 2z_0$; 2) $\left(\overset{\Leftarrow}{K}\right) = \left(\overset{\Rightarrow}{K}\right)_{прис}$ - левый В.К. равен присоединённой матрице от правого В.К., в алгебраической форме это означает просто изменение знака у векторной части $\overset{\Leftarrow}{K} = Z\exp(-\Psi \vec{e}_k) = \overline{\overset{\Rightarrow}{K}}$, (это называется – центральное сопряжение [8]);
3) $\overline{\overset{\Rightarrow}{K}_1 \circ \overset{\Rightarrow}{K}_2} = \overline{\overset{\Rightarrow}{K}_2} \circ \overline{\overset{\Rightarrow}{K}_1} = \overset{\Leftarrow}{K}_2 \circ \overset{\Leftarrow}{K}_1$; 4) $\left(\overset{\Rightarrow}{K}_1 \circ \overset{\Rightarrow}{K}_2\right)_{тр} = \left(\overset{\Rightarrow}{K}_2\right)_{тр} \circ \left(\overset{\Rightarrow}{K}_1\right)_{тр}$;
5) $\overset{\Rightarrow}{K} \circ \overset{\Leftarrow}{K} = \det\left(\overset{\Rightarrow}{K}\right) = \det\left(\overset{\Leftarrow}{K}\right) = \det\left(\overset{\Leftrightarrow}{K}\right) = \left|\overset{\Rightarrow}{K}\right|^2 = K^2 = z_0^2 - z^2 = z_{11}z_{22} - z_{12}z_{21}$,
6) $\overset{\Rightarrow}{K}^{-1} = \dfrac{\overset{\Leftarrow}{K}}{\det\left(\overset{\Leftrightarrow}{K}\right)}$; 7) Если В.К. чисто векторного типа, т.е. след матрицы равен 0 (z0=0), то $\overset{\Leftarrow}{K} = -\overset{\Rightarrow}{K}$.
Из формулы (1с) и свойства 6) видно, что символический индекс минус один, обозначающий обратную матрицу, теперь становится действительной минус первой степенью (см. прил. №1). Исходя из свойства (5) можно было бы назвать детерминант векторного комплекса его нормой или квадратом модуля, хотя в общем случае, эта величина теперь комплексная, так как она содержит в себе и скаляр и псевдоскаляр. Но зато в ней нет векторной части. Очевидно, что такая норма мультипликативная. При необходимости или желании иметь норму или модуль как положительную действительную величину, её можно получить как модуль произведения $\left|K^2 \cdot \overline{K}^2\right| = (a_0^2 - a^2 - b_0^2 + b^2)^2 + 4(a_0b_0 - a_1b_1 - a_2b_2 - a_3b_3)^2 = \|N\|^4$, однако никакого применения эта норма пока не нашла.

Совсем нелишне отметить, что все элементы, входящие в В.К., представляют собой реальные различные физические объекты. Например – масса, заряд, спин, импульс или количество движения, момент количества движения, скалярный и векторный потенциалы, плотность тока и плотность заряда, электромагнитное поле, плотность энергии электромагнитного поля и вектор Умова-Пойнтинга, Интервалы, функция Лагранжа и так и далее список можно продолжать долго. В этом и состоит принципиальное отличие В.К. от бикватернионов, октав и других гиперкомплексных чисел, создаются которые путём внедрения в математику новых сущностей с заданными по определению свойствами и не имеющими прямых физических аналогов в природе. Гиперкомплексные числа это в первую очередь чисто математические объекты и поэтому вызывает удивление и восхищение, когда полученные результаты совпадают с уже известными ранее законами и теориями, и их называют замечательные «кватернионные совпадения» [1].(Уравнения Максвелла, кватернионная теория относительности, поля Янга-Миллса и т.д.)

Прежде чем перейти к общему рассмотрению некоторых частных случаев, выпишем в явном виде несколько полезных формул, легко доказываемых с помощью рядов.

Пусть $\vec{A} = a\cdot\vec{e}_a$, где $\vec{e}_a^{\,2} = 1$, тогда

$$\exp(\pm\vec{A}) = Cha \pm \vec{e}_a Sha, \quad \exp(\pm i\vec{A}) = Cosa \pm i\vec{e}_a Sina,$$

$$Cos\vec{A} = Cosa, \quad Sin\vec{A} = \vec{e}_a Sina, \quad Ch\vec{A} = Cha, \quad Sh\vec{A} = \vec{e}Sha,$$

$$Cos\vec{C} = Cos(\vec{A} + i\vec{B}) = Cosa\cdot Chb + iSina\cdot Shb\cdot(\vec{e}_a \circ \vec{e}_b),$$

$$\ln\vec{A} = \ln a + \ln\vec{e}_a, ...a^2 \ge 1, \quad \ln\vec{e}_a = i\frac{\pi}{2}(\vec{e}_a - 1),$$

$$\ln(1+\vec{A}) = \frac{1}{2}\left[\ln(1-a^2) + \vec{e}_a \ln\left(\frac{1+a}{1-a}\right)\right], \quad a^2 < 1.$$

Рассмотрим некоторые частные случаи, предоставляемые нам векторным комплексом

$$\overset{\Rightarrow}{K} = a_0 + a_1\vec{e}_1 + a_2\vec{e}_2 + a_3\vec{e}_3 + ib_0 + b_1\vec{e}_1^{\bullet} + b_2\vec{e}_2^{\bullet} + b_3\vec{e}_3^{\bullet} = Z\exp(\Psi_0\vec{e}_k),$$

где $a_j, b_j \in R, ...Z, \Psi_j, z, z_\mu, \alpha_\mu \in C.....\mu = \overline{1,3}.....j = \overline{0,3}$,

$$\vec{e}_\mu^{\bullet} = i\vec{e}_\mu......\vec{e}_k = \cos\alpha_1\cdot\vec{e}_1 + \cos\alpha_2\cdot\vec{e}_2 + \cos\alpha_3\cdot\vec{e}_3, \quad \cos\alpha_\mu = \frac{z_\mu}{z}$$

$$z_j = a_j + ib_j......z = \sqrt{z_1^2 + z_2^2 + z_3^2}, \quad Z = \sqrt{z_0^2 - z^2}, \quad (Z = |\overset{\Rightarrow}{K}|^2)$$

$$\Psi_j = \theta_j + i\eta_j, ...ch\Psi_0 = \frac{z_0}{Z}, ...sh\Psi_0 = \frac{z}{Z}.....sh\Psi_\mu = \frac{z_\mu}{Z} = sh\Psi_0\cdot\cos\alpha_\mu.$$

Скаляр: $a_0 \ne 0, ...a_\mu, b_j, z_\mu, z, \Psi = 0, ...Z = a_0. \quad \overset{\Rightarrow}{K} = a_0$

Псевдоскаляр: $b_0 \ne 0, ...a_j, b_\mu, z_\mu, z, \Psi = 0, ...\overset{\Rightarrow}{K} = Z = ib_0 = b_0\cdot e^{i\frac{\pi}{2}}$

Вектор: $a_0, b_j, z_0 = 0, ...z_\mu = a_\mu, ...iz = \pm i\sqrt{a_1^2 + a_2^2 + a_3^2} = \pm ia$

$$ch\Psi = ch\theta\cdot\cos\eta + ish\theta\cdot\sin\eta = 0...\Rightarrow ...\eta = \pm\frac{\pi}{2}, ...\theta = 0,$$

$$sh\Psi = sh\theta\cdot\cos\eta + i\sin\eta\cdot ch\theta = \mp i, ...\Rightarrow ...\theta = 0, ...\eta = \mp\frac{\pi}{2},$$

$$\overset{\Rightarrow}{K} = \pm ia\cdot\exp(\mp i\frac{\pi}{2}\vec{e}_a) = a\vec{e}_a = \vec{A}, \quad (\overset{\Rightarrow}{K})^n = (ia)^n\cdot\exp(-i\frac{n\pi}{2}\vec{e}_a),$$

$$\sqrt[n]{\overset{\Rightarrow}{K}} = \sqrt[n]{a}\cdot\exp[-\frac{i\pi}{2n}(\vec{e}_a + 4m - 1)], \text{ m=0,1,...(n-1).}$$

при n=2 имеем: $\sqrt{\vec{A}} = \pm\frac{\sqrt{a}}{2}(1+i)\cdot(1-i\vec{e}_a)$.

Псевдовектор: $\overset{\Rightarrow}{K} = -b\cdot\exp(-i\frac{\pi}{2}\vec{e}_b) = i\vec{B},....a_j, b_0, \theta = 0,....\eta = -\frac{\pi}{2}$.

Кватер(нион): $\overset{\Rightarrow}{K} = a_0 + i\vec{B} = \overset{\mapsto}{q}$

$$a_\mu, b_0, \theta = 0,....\cos\eta = \frac{a_0}{\sqrt{a_0^2+b^2}},.....Z = \sqrt{a_0^2+b^2},....\overset{\Rightarrow}{K} = \sqrt{a_0^2+b^2}\cdot\exp(i\eta\vec{e}_b)$$

$\overset{\Rightarrow}{K}\circ\overset{\Leftarrow}{K} = a_0^2+b^2$, то, что называется нормой кватерниона. 6) 4-вектор: $\overset{\Rightarrow}{K} = ib_0 + \vec{A}$

$$a_0, b_\mu, \theta = 0,....Z = i\sqrt{b_0^2+a^2},....\cos\eta = \frac{b_0}{\sqrt{b_0^2+a^2}},...\sin\eta = \frac{-a}{\sqrt{b_0^2+a^2}}$$

$\overset{\Rightarrow}{K}\circ\overset{\Leftarrow}{K} = -(b_0^2+a^2)$

7) Интервал (надо же, как-то назвать, если специального термина в математике нет, но в теоретической физике есть инвариант, именуемый временипододобный интервал и равный $a_0^2 - a^2 = Z^2 = \det\overset{\Leftrightarrow}{K}$ >0.): $\overset{\Rightarrow}{K} = a_0 + \vec{A} = \overset{\mapsto}{S}_t$,

$$b_j = 0,.....Z = \sqrt{a_0^2-a^2},.....ch\theta = \frac{a_0}{\sqrt{a_0^2-a^2}},.....sh\theta = \frac{a}{\sqrt{a_0^2-a^2}},....\eta = 0,$$

$\overset{\mapsto}{S}_t = \sqrt{a_0^2-a^2}\cdot\exp(\theta\cdot\vec{e}_a) = \sqrt{a_0^2-a^2}(ch\theta + sh\theta\cdot\vec{e}_a)$, $\overset{\Rightarrow}{S}_t\circ\overset{\Leftarrow}{S}_t = a_0^2 - a^2$.

Пусть $d\overset{\mapsto}{S}_t = \sum_{j=0}^{3} dx_j\cdot\vec{e}_j = c\cdot dt\cdot e_0 + dx\cdot\vec{e}_1 + dy\cdot\vec{e}_2 + dz\cdot\vec{e}_3$,

$d\overleftarrow{S}_t = c\cdot dt\cdot e_0 + dx\cdot\overleftarrow{e}_1 + dy\cdot\overleftarrow{e}_2 + dz\cdot\overleftarrow{e}_3$, тогда

$d\overset{\mapsto}{S}_t\circ d\overset{\leftarrow}{S}_t = dS_t^{\,2} = c^2\cdot dt^2 - dx^2 - dy^2 - dz^2$, коротко и ясно и вовсе не надо вводить мнимое время. Если же положить скаляр $a_0 = E$, и $\vec{A} = c\vec{P}$, т.е. $\overset{\Rightarrow}{K} = E + c\vec{P}$, то $\det(\overset{\Rightarrow}{K}) = E^2 - c^2P^2 = m_0^2c^4$, другими словами инвариант – детерминант векторного комплекса есть квадрат энергии покоя. Даже вырожденный случай $(d\overset{\mapsto}{S} = 0).$, имеет физический смысл: $\det(\overset{\Leftrightarrow}{K}) = 0,.\Rightarrow .a_0 = a,$ что эквивалентно – масса покоя равна нулю и $E = h\nu.....или....E = PC.$, (это фотон или гамма квант). 8)Интервал (пространственноподобный) $\overset{\Rightarrow}{K} = ib_0 + \vec{B}^{\bullet} = \overset{\mapsto}{S}_p$,

$$a_j, \eta = 0,.....Z = \sqrt{b^2-b_0^2},.....ch\theta = \frac{b_0}{\sqrt{b^2-b_0^2}},.....sh\theta = \frac{b}{\sqrt{b^2-b_0^2}},$$

$$\overset{\mapsto}{S}_p = \sqrt{b^2-b_0^2}\cdot\exp(\theta\cdot\vec{e}_b) = \sqrt{b^2-b_0^2}\cdot(ch\theta + sh\theta\cdot\vec{e}_b),......\overset{\Rightarrow}{S}_p\circ\overset{\Leftarrow}{S}_p = b^2 - b_0^2$$

$d\vec{S}_p \circ d\overleftarrow{S}_p = dS_p^2 = dx^2 + dy^2 + dz^2 - c^2 \cdot dt^2 < 0$

Два последних случая, содержат в себе преобразование Лоренца и позволяют получить все инварианты группы Лоренца, что и было сделано Г.Минковским с использованием матриц Кэли.

9) Комплексный вектор (другие названия: бивектор, шестивектор, для электромагнитного поля это четырёхмерный ротор от четырёхмерного потенциала $F_{\mu\nu}$): $a_0, b_0 = 0$, $\theta = 0, \ldots \eta = -\dfrac{\pi}{2}$,

$$\overset{\Rightarrow}{K} = \vec{A} + i\vec{B} = \sum_{\mu=1}^{3}(a_\mu + ib_\mu)\vec{e}_\mu = \vec{C} = \sqrt{b^2 - a^2 - 2i(a_1b_1 + a_2b_2 + a_3b_3)}\exp(-i\frac{\pi}{2}\vec{e}_c)$$

Пусть $\overset{\Rightarrow}{W} = \dfrac{\vec{E} + i\vec{H}}{\sqrt{8\pi}}$, и $\overline{\overset{\Rightarrow}{W}} = \dfrac{\vec{E} - i\vec{H}}{\sqrt{8\pi}}$, тогда $\overset{\Rightarrow}{W} \circ \overline{\overset{\Rightarrow}{W}} = |W|^2 = \dfrac{E^2 + H^2}{8\pi} + \dfrac{\vec{E} \times \vec{H}}{4\pi} = W_0 + \dfrac{1}{c}\vec{S}$, - первое слагаемое есть плотность энергии электромагнитного поля, второе слагаемое – вектор Умова-Пойнтинга - плотность потока энергии э.м. поля, делённая на скорость света, всё вместе - закон сохранения энергии для э.м. поля.

10) (Без названия). Т.Калуза, используя вариант пятимерного объекта - $\overset{\Rightarrow}{K} = a_0 + ib_0 + \vec{A}$, нашёл интересное представление в ковариантном виде уравнений электродинамики Максвелла [9], подробнее развитое Ф.Клейном.

11) Можно ввести понятие «Винт», когда вектор и псевдовектор коллинеарные $a_j = \pm\lambda \cdot b_j$ или $\vec{e}_a = \pm\vec{e}_b$: $\overset{\Rightarrow}{V} = b(\lambda i - 1)\exp(-i\dfrac{\pi}{2}\vec{e}_b) = b(\lambda + i)\vec{e}_b$ в п.с.к. и $\overset{\Leftarrow}{V} = -b(\lambda + i)\vec{e}_b$ в л.с.к..

Левый винт в правой с.к. имеет вид $\overline{\overset{\Rightarrow}{V}} = b(\lambda - i)\vec{e}_b$.

Выполняются очевидные соотношения: $\overset{\Rightarrow}{V} + \overset{\Leftarrow}{V} = 0$ и $\overset{\Rightarrow}{V} + \overline{\overset{\Rightarrow}{V}} = 2a\vec{e}_a$, в последнем случае имеем действительный вектор, т.е. вращение отсутствует; $\overset{\Rightarrow}{V} - \overline{\overset{\Rightarrow}{V}} = 2bi\vec{e}_b$, осталось чистое вращение без поступательного движения.

Рассмотрим более подробно умножение двух В.К.

$$\overset{\Rightarrow}{C} = c_0e_0 + c_1\vec{e}_1 + \vec{c}_2\vec{e}_2 + \vec{c}_3\vec{e}_3 \quad \text{и} \quad \overset{\Rightarrow}{S} = s_0e_0 + s_1\vec{e}_1 + s_2\vec{e}_2 + s_3\vec{e}_3$$

в результате умножения получим новый В.К. - $\overset{\Rightarrow}{P}$.

$$\overset{\Rightarrow}{C} \circ \overset{\Rightarrow}{S} = \overset{\Rightarrow}{P} = p_0e_0 + p_1\vec{e}_1 + p_2\vec{e}_2 + p_3\vec{e}_3 = p_0 + p\vec{e}_p = \sqrt{p_0^2 - p^2}\exp(\Psi_p\vec{e}_p) \quad \ldots(14)$$

где все $c_j, s_j, p_j, \ldots . j = \overline{0,3}$ есть комплексные числа.

Всю скалярную часть p_0 этого нового В.К. назовём скалярным произведением двух начальных В.К., а всю векторную часть - $p\vec{e}_p$, векторным произведением тех же В.К.. И таким образом появляется восьмимерный аналог формулы (10):

$$\vec{\vec{C}}\circ\vec{\vec{S}} = (\vec{\vec{C}}\cdot\vec{\vec{S}}) + (\vec{\vec{C}}\times\vec{\vec{S}}), \quad ... \text{(10a)}$$

где, в полной аналогии с (10), имеем

$$\vec{\vec{C}}\cdot\vec{\vec{S}} = c_0 s_0 + c_1 s_1 + c_2 s_2 + c_3 s_3 = l_c \cdot l_s \cdot Ch\Psi = p_0 \quad ... \text{(15)}$$

$$\vec{\vec{C}}\times\vec{\vec{S}} = l_c \cdot l_s \cdot Sh\Psi \cdot \vec{e}_p = (c_0 s_1 + c_1 s_0 + ic_2 s_3 - ic_3 s_2)\cdot\vec{e}_1 +$$
$$(c_0 s_2 + c_2 s_0 + ic_3 s_1 - ic_1 s_3)\cdot\vec{e}_2 + (c_0 s_3 + c_3 s_0 + ic_1 s_2 - ic_2 s_1)\cdot\vec{e}_3 =$$
$$= p_1\vec{e}_1 + p_2\vec{e}_2 + p_3\vec{e}_3 = p\vec{e}_p \quad .. \text{(16)}$$

$l_c = \sqrt{c^2 - c_0^2}$ и $l_s = \sqrt{s^2 - s_o^2}$ длины векторных комплексов, а комплексный угол Ψ - угол между двумя В.К. $\vec{\vec{C}}$ и $\vec{\vec{S}}$.

Из формул (14-16) следует

$$Ch\Psi = \frac{p_0}{\sqrt{p_0^2 - p^2}} = \frac{\vec{\vec{C}}\cdot\vec{\vec{S}}}{l_c \cdot l_s} = \frac{\vec{\vec{C}}\cdot\vec{\vec{S}}}{\sqrt{(c^2 - c_0^2)(s^2 - s_0^2)}} \quad ... \text{(17)}$$

$$Sh\Psi = \frac{p}{\sqrt{p_0^2 - p^2}} = \frac{\left|\vec{\vec{C}}\times\vec{\vec{S}}\right|}{\sqrt{(c^2 - c_0^2)(s^2 - s_0^2)}}. \quad ... \text{(18)}$$

Приравняв знаменатели в (17) или в (18) получим:

$$p_0^2 - p_1^2 - p_2^2 - p_3^2 = (c_0^2 - c_1^2 - c_2^2 - c_3^2)(s_0^2 - s_1^2 - s_2^2 - s_3^2) \quad ... \text{(19)}$$

доказательство мультипликативности нормы или обобщённый аналог тождества Эйлера, согласно которому произведение двух сумм четырёх квадратов действительных чисел само представимо в виде суммы четырёх квадратов. Теперь это справедливо и для комплексных чисел, и, следовательно, слева может быть и восемь квадратов. Если формулу (19) переписать в виде $(\sqrt{c^2 - c_0^2})^2 \cdot (\sqrt{s^2 - s_0^2})^2 = (\vec{\vec{C}}\cdot\vec{\vec{S}})^2 - (\vec{\vec{C}}\times\vec{\vec{S}})^2$, то становиться очевидным, что это ни что иное, как восьмимерный аналог тождества Эйлера-Лагранжа, т.е. формулы (11). Если в $\vec{\vec{C}}$ и $\vec{\vec{S}}$ положить $c_0, s_0 = 0$, то формула (10а) превращается в (10), формулы (15-18) становятся стандартными определениями скалярного и векторного произведения. Более того, четырёхмерное (комплексное) векторного произведения (формула (16)), можно представить в компактном виде

$$\vec{\vec{C}}\times\vec{\vec{S}} = p_\alpha \cdot \vec{e}_\alpha = (i\cdot\varepsilon_{0\alpha kl} + \varepsilon_{\alpha+1,\alpha+2,k,l})c_k s_l \vec{e}_\alpha, \quad ... \text{(20)}$$

по дважды повторяющимися индексам происходит суммирование (по греческим от 1 до 3, по латинским - от 0 до 3). Символ ε_{0123} равен нулю, если имеются два одинаковых индекса, равен +1, если количество инверсий чётное и равен –1, при нечётном количестве инверсий. Индекс 0 коммутирующий и на знак не влияет, остальные индексы антикоммутирующие и циклические (3+1=1, 3+2=2, 2+2=1). В явном виде это выглядит так:

$$p_1 = (i\varepsilon_{01kl} + \varepsilon_{23kl})c_k \cdot s_l$$
$$p_2 = (i\varepsilon_{02kl} + \varepsilon_{31kl})c_k \cdot s_l$$
$$p_3 = (i\varepsilon_{03kl} + \varepsilon_{12kl})c_k \cdot s_l \qquad . . . (21)$$

В подтверждение утверждения сделанного в пункте 8) на стр. 14, рассмотрим объект - $\overset{\Rightarrow}{S} = z_0 e_0 + z_1\vec{e}_1 + z_2\vec{e}_2 + z_3\vec{e}_3$, где все $z_j \in C$, есть переменные величины с инвариантом $S^2 = z_0^2 - z^2$. Линейное преобразование $\Lambda = (a_{kl})$

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_0 \end{pmatrix} = \Lambda \begin{pmatrix} z_1' \\ z_2' \\ z_3' \\ z_0' \end{pmatrix} \qquad . . . (22)$$

переводящее В.К. $\overset{\Rightarrow}{Z}$ в с.к. K в В.К. $\overset{\Rightarrow}{Z'}$ в с.к. K', сохраняющее при этом инвариантную длину В.К.

$$z_0^2 - z_1^2 - z_2^2 - z_3^2 = z_0'^2 - z_1'^2 - z_2'^2 - z_3'^2$$

имеет вид:

$$\Lambda = \begin{pmatrix} 1 + \frac{Sh^2\Psi_1}{1 + Ch\Psi_9} & \frac{Sh\Psi_1 \cdot Sh\Psi_2}{1 + Ch\Psi_0} & \frac{Sh\Psi_1 \cdot Sh\Psi_3}{1 + Ch\Psi_0} & Sh\Psi_1 \\ \frac{Sh\Psi_2 \cdot Sh\Psi_1}{1 + Ch\Psi_0} & 1 + \frac{Sh^2\Psi_2}{1 + Ch\Psi_0} & \frac{Sh\Psi_2 \cdot Sh\Psi_3}{1 + Ch\Psi_0} & Sh\Psi_2 \\ \frac{Sh\Psi_3 \cdot Sh\Psi_1}{1 + Ch\Psi_0} & \frac{Sh\Psi_3 \cdot Sh\Psi_2}{1 + Ch\Psi_0} & 1 + \frac{Sh^2\Psi_3}{1 + Ch\Psi_0} & Sh\Psi_3 \\ Sh\Psi_1 & Sh\Psi_{2_2} & Sh\Psi_3 & Ch\Psi_0 \end{pmatrix} \qquad . . . (23)$$

где $\Psi_j = \theta_j + i\eta_j$, $Ch\Psi_0 = \dfrac{1}{\sqrt{1 - \left(\dfrac{dz_\mu}{dz_0}\right)^2}}$, $Sh\Psi_\mu = Ch\Psi_0 \cdot \dfrac{dz_\mu}{dz_0}$.

Преобразование (22) с матрицей (23) можно назвать обобщённым преобразованием Лоренца.

Если $\dfrac{dz_\mu}{dz_0} = \beta_\mu$ есть действительные числа, то это общее преобразование Лоренца при движении системы K' со скоростью $\vec{V}_\mu = \vec{\beta}_\mu \cdot c$ по отношению к системе K и в явном виде имеет вид:

$$x_\mu = x_\mu' + (\alpha - 1)\frac{V_\mu}{V^2}(V_1x_1' + V_2x_2' + V_3x_3') + \alpha V_\mu t'$$
$$x_0 = ct = \frac{\alpha}{c}(V_1x_1' + V_2x_2' + V_3x_3') + \alpha ct' \qquad . . . (24)$$

что соответствует матрице Λ вида, где $T = \alpha - 1$:

$$\Lambda = \begin{pmatrix} 1+T\dfrac{\beta_1^2}{\beta^2} & T\dfrac{\beta_1\beta_2}{\beta^2} & T\dfrac{\beta_1\beta_2}{\beta^2} & \alpha\beta_1\beta_4 \\ T\dfrac{\beta_2\beta_1}{\beta^2} & 1+T\dfrac{\beta_2\beta_3}{\beta^2} & T\dfrac{\beta_2\beta_3}{\beta} & \alpha\beta_2\beta_4 \\ T\dfrac{\beta_3\beta_1}{\beta^2} & T\dfrac{\beta_3\beta_2}{\beta^2} & 1+T\dfrac{\beta_3^2}{\beta^2} & \alpha\beta_3\beta_4 \\ \alpha\beta_4\beta_1 & \alpha\beta_4\beta_2 & \alpha\beta_4\beta_3 & \alpha\beta_4\beta_4 \end{pmatrix}$$

При этом угол $\Psi_j = \theta_j$ и становится действительным так как $\eta_j = 0$, $\theta_0 = arCh\alpha$, $\theta_\mu = arSh\alpha\beta_\mu$, $\alpha = \dfrac{1}{\sqrt{1-\beta^2}}$, $\beta^2 = \beta_\mu^2$, $\beta_4 = \beta_0 = 1$. И, наконец, если $\beta_2 = \beta_3 = 0$, и $\beta_1 = \beta = \dfrac{V}{c}$, то (22),(23),(24) превращаются в стандартные формулы:

$$x = \frac{x' + Vt'}{\sqrt{1-\beta^2}} ......y = y'......z = z'......t = \frac{t' + \dfrac{x'V}{c^2}}{\sqrt{1-\beta^2}}.$$

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