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Physical Interpretations of Relativity Theory

Proceedings of International Scientific Meeting PIRT-2005

(Moscow: 4 – 7 July, 2005)

Editid by M.C. Duffy, V.O. Gladyshev, A.N. Morozov, P. Rowlands

The meeting is dedicated to the 175th Anniversary of Bauman Moscow State Technical University and to honour the 100th Anniversary of Albert Einstein's epochal work of 1905

Moscow, Liverpool, Sunderland

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This volume contains papers which accepted for inclusion in the programme of lectures of meeting “Physical Interpretation of Relativity Theory” which is organized by the Bauman Moscow State Technical University, School of Computing and Technology, University of Sunderland, Liverpool University and British Society for Philosophy of Science.

The meeting is dedicated to the 175th Anniversary of Bauman University and to honour the 100th Anniversary of Albert Einstein's epochal work of 1905.

The most important single objective of the meeting in Summer 2005 is including the advantages of the various physical, geometrical and mathematical interpretations of the formal structure of Relativity Theory; and to examine the philosophical, historical and epistemological questions associated with the various interpretations of the accepted mathematical expression of the Relativity Principle and its development.

The conference is called to examine the various interpretations of the (mathematical) formal structure of Relativity Theory, and the several kinds of physical and mathematical models which accompany these interpretations.

The programme timetable, giving authors and titles of papers as presented and other details of the Moscow Meeting “Physical Interpretation of Relativity Theory” are given on the web site maintained by the Bauman Moscow State Technical University

http://fn.bmstu.ru/phys/nov/konf/pirt2005/pirt_main.html.

The meeting is intended to be of interest to physicists, mathematicians, engineers, philosophers and historians of science, post-graduate students.

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Fabri-Perot Resonator With Periodic Structures As Reflecting Mirrors Being A Basis For Gravity Waves Detection

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The task on diffraction of electromagnetic wave on periodic structures, forming Fabri-Perot interferometer, is solved. Unlike standard Fabri-Perot interferometer in the interferometer being considered, reflecting periodic structures are used as mirrors. Instrument function of such resonator is found and it is shown that under fulfillment of certain phase relations appear exponentially narrow regions of radiation transmission, position and widths of which depend on a distance between "mirrors" and periodic structures properties. It is shown that effect of radiation "leaking" through two reflecting periodic structures, forming Fabri-Perot resonator, constitutes classical counterpart of a known quantum effect - particles resonance tunneling through two potential barriers under the condition that incident particle energy coincides with energy of quantum level of potential box formed by two barriers.

It is suggested to use such Fabri-Perot resonators for measurement of small displacements of one mirror relatively to the other, and in particular, for gravity waves registration. Variation of interference pattern of Mach-Zehnder interferometer, in the legs of which such Fabri-Perot resonators are located, is analyzed, and it is shown that sensitivity of gravity waves registration by such technique substantially exceeds the known ones being used for example in LIGO and LISO installations at present.

It is indicated that use of periodic structures instead of mirrors in standard Fourier-spectrometers opens new capacities regarding the creation of compact Fourier-spectrometers with quite high spectral resolution, however spectral measurements range in such spectrometers will be narrower in comparison with standard ones.

1. Introduction

One of the most interesting problems of modern physics is direct experimental validation of gravity waves existence [1-3]. The idea of Fabri-Perot resonator in each leg of Michelson's interferometer underlies quite expensive experimental facilities being built. Mirrors, forming Fabri-Perot resonator are used as free hanging trial masses, distance between which varies under the influence of gravity wave. The idea of such laser interferometer as a technique of gravity waves detection, was first suggested in the work by M.E. Gertsenstein and the author in 1962 [4]. Minimum displacement of one mirror with respect to another, obtained by the present moment at experimental installation LIGO in the USA, is about 10^{-16} cm, however, for reliable direct experimental evidence of gravity waves existence it is necessary to increase sensitivity of such gravity antennas at six more orders.

The problem of sensitivity enhancement of such systems leads to the necessity to manufacture mirrors with great reflection factor, more than $1 - 10^{-5}$, or further enlarge interferometer legs as it is suggested to do in the space version of the system (LISO project, see [1,3]).

In the present work it is suggested to use as reflecting mirrors, forming Fabri-Perot optical interferometer, reflecting periodic structures, which as it will be shown below, possess exponentially narrow laser radiation band pass. This allows suggesting new diagrams of gravity waves detection having much higher sensitivity.

2. Fabri-Perot resonator with periodic structures as reflecting mirrors

Let Fabri-Perot resonator be formed by two flat periodic structures which are located at a distance **a** relative to each other, as it is shown in fig.1. Radiation propagates from left to right. First it falls to the first periodic structure and under fulfillment of synchronism conditions, i.e. when period of this periodic structure is close to or equal to a half of incident radiation wavelength, radiation is reflected from this structure and only some part falls to resonator. A small part of radiation falls to the second structure, then it is reflected and interferes with radiation inside resonator. If we wait for some time, the amplitude of wave steady-state electromagnetic field inside resonator will substantially exceed the amplitude of incident field, and then some part of this radiation will emerge from resonator. As it will be shown below, the amplitude of emergent field will be practically equal to the amplitude of incident radiation [6]. Our aim is to find intensity of radiation emerged from resonator, formed by two periodic structures, or rather find instrument function of such resonator.

Problem-solving technique in this case is the following. First we find task solution on light propagation in periodic structure under arbitrary boundary conditions and then we join solutions at the boundaries, found for each periodic structure. As in the case of standard Fabri-Perot interferometer the distance between periodic structures ("mirrors") in one of the interferometer's legs will vary under the influence of gravity wave, thereby varying intensity distribution in interference pattern. Substantial difference of such resonator from a standard one is that under certain phase relations (see below), instrument function acquires exponentially narrow band pass with quite sharp boundaries. The last means that sensitivity of such interferometer under phase variation or small displacement of reflective structures relatively to each other turns out to be rather high, and under rather obtainable in practice parameters can exceed the LIGO system sensitivity.

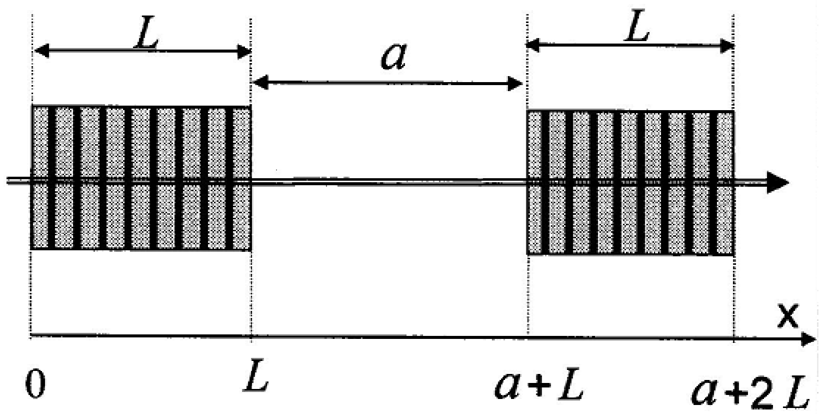

Fig. 1. *Fabri-Perot resonator with periodic reflecting structures without mirrors*

Let periodic structure be formed so that its inductivity $\varepsilon(x)$ could be written as:

$$\varepsilon(x) = \varepsilon_0 + \Delta\varepsilon\, Cos(q\, x).$$

Here $\Delta\varepsilon$ - amplitude of inductivity variation, at the same time $\Delta\varepsilon << \varepsilon_0$, *q*- periodic structure wave vector. Then, as it is known, [5-7], light propagation in such periodic structure can be described by shortened combined equations:

$$\frac{d\,\mathrm{E}_0(x)}{dx} = i\,k_0\,\Delta\varepsilon\,\mathrm{E}_\mathrm{R}(x)e^{-i\Delta k\,x},$$
$$\frac{d\,\mathrm{E}_\mathrm{R}(x)}{dx} = -i\,k_0\,\Delta\varepsilon\,\mathrm{E}_0(x)e^{i\Delta k\,x}. \tag{1}$$

Here E_0 и E_R - amplitude of incident and reflected wave correspondingly, k_0 – radiation wavevector for which accurate synchronism condition is fulfilled, and Δk defines Bragg condition mismatch. Equations (1) describe two waves propagating towards each other, synchronism conditions for which have the form: $k_{in} + k_{dif} - q = \Delta k$. For optically isotropic medium under accurate fulfillment of synchronism conditions Δk=0, and $k_{in} = k_{dif} = k_0 = q/2$. Steady-state equations (1) describe light propagation in the first (left) periodic structure $(0 \le \boldsymbol{x} \le \boldsymbol{L})$, as for equations describing light propagation in the right periodic structure in the region $(\boldsymbol{L} + \boldsymbol{a} \le \boldsymbol{x} \le \boldsymbol{a} + 2\boldsymbol{L})$, in them , unlike equations (1), $\Delta\varepsilon$ will contain phase multiplier $\boldsymbol{e}^{i\varphi}$, where φ - phase difference between periodic structures. (φ - is such phase difference which occurs between the first and the second structures, as a result of the first periodic structure continuation to the region $\boldsymbol{L} + \boldsymbol{a} \le \boldsymbol{x} \le \boldsymbol{a} + 2\boldsymbol{L}$. Such note is a sequence of use of one an the same system of coordinates for both periodic structures). It is necessary to add boundary conditions to equations (1) and similar equations for the second periodic structure:

$$\begin{gathered} E_0(x{=}0) = \mathbf{E}_0, \quad E_R(x{=}L) = \boldsymbol{e}^{-ika} E'_R(x{=}L{+}a), \\ E'_0(x{=}L{+}a) = \boldsymbol{e}^{-ika} E_0(x{=}L), \quad E'_R(x{=}2L{+}a) = 0, \end{gathered} \tag{2}$$

where light wave fields in the second periodic structure are marked with accent. Boundary conditions in the form of formulas (2) are obtained from ordinary boundary conditions after extraction of free space fields between periodic structures. Solution of the boundary problem (2) leads to the following expression for amplitude of wave, emerged from resonator:

$$\frac{E'_0(x = 2L + a)}{E_0(x = 0)} = \frac{4\left(\xi^2 - 1\right) Exp\left[ia\left(k - 2\Gamma\xi\right) + i\Psi + 2W\right]}{\left(1 - e^{2W}\right)^2 + e^{i\Psi}\left(2e^{2W} + \left(i\xi - \sqrt{1-\xi^2}\right)^2 + e^{4W}\left(i\xi + \sqrt{1-\xi^2}\right)^2\right)}, \tag{3}$$

Here $\xi \equiv \dfrac{\Delta k}{2\Gamma}$, $W \equiv \Gamma L\sqrt{1-\xi^2}$, $\Gamma \equiv \Delta\varepsilon\, k_0$, $\Psi \equiv -2ak + 2a\Gamma\xi + \varphi$. From formula (3) it is seen that under fulfillment of conditions for phase:

$$\Psi + 2\,ArcSin\left(\xi\right) = \left(2m+1\right)\pi, \qquad \left(m = 0, \pm 1, \pm 2, \ldots\right), \tag{4}$$

amplitude at resonator output reaches maximum, and near fulfillment of accurate synchronism conditions, i.e. $\xi = 0$, it will be equal to amplitude of incident radiation, $E'_0(x = 2L + a) = E_0(x = 0)$. Such resonant propagation of radiation through periodic structures has simple explanation. From quantum mechanics it is known that if free particle has energy, coinciding with energy of quantum level between two barriers, then the particle propagates through such barriers. Actually, the considered case represents classic counterpart of such resonant tunneling. It is also easy to show that field amplitude between periodic structures substantially exceeds field value outside the resonator. Solution (3) allows to find intensity of radiation emerged from such Fabri-Perot resonator, related to incident radiation intensity

$$\left|\frac{E'_0(x{=}2L{+}a)}{E_0(x{=}0)}\right|^2 = \frac{8e^{4W}(1-\xi^2)^2}{G} \equiv T\left(L, \Gamma, a, \varphi, \xi\right), \tag{5}$$

where denominator in formula (5) has the form:

$$\begin{gathered} G = 3 - 4\xi^2 + 4\xi^4 + Ch\left(4W\right) + 4\,Ch\left(2W\right)\left[-\xi^2 + \left(1 - 2\xi^2\right)Cos\left(\Psi\right)Sh^2\left(W\right)\right] + \\ + 4\,Sh^2\left(W\right)\left[Cos\left(\Psi\right) - 2\xi\sqrt{1-\xi^2}\,Sin\left(\Psi\right)Sh\left(2W\right)\right] \end{gathered} \tag{6}$$

Here $T\left(L, \Gamma, a, \varphi, \xi\right)$ - instrument function of Fabri-Perot resonator, formed by two periodic structures. From expression (5) and (6) it is seen that in the interval $-2\Gamma < \Delta k < 2\Gamma$ under $\Psi = 0$ intensity of the propagated radiation (for $\Gamma L > 1$) is exponentially small: incident radiation experiences strong reflection. Totally different situation will be under $a \ne 0, \ldots \varphi \ne 0$. In this case within interval $-2\Gamma < \Delta k < 2\Gamma$ appear quite narrow maximums of radiation transmission. The location of m-th maximum is defined from condition $\xi_m = \xi_0 + \dfrac{\pi m}{a\Gamma}$, $(m = \pm1, \pm2, \pm3, ...)$, where ξ_0 - is defined from equation $Sin(-2ak + 2a\Gamma\xi_0 + \varphi) = \xi_0$. The general form of instrument function according to formula (3) is shown in fig.2, there parameters values, under which numerical analysis was carried out, are indicated as well. If for example, a=0 and φ=0, so that the second periodic structure is the continuation of the first one, then from expressions (5), (6) follows solution, describing radiation propagation through one periodic structure with doubled interaction length (see formula (6.6.8) in book [7]).

$$T(L, \Gamma, a{=}0, \varphi = 0, \xi) = \frac{2(1-\xi^2)}{1 - 2\xi^2 + \mathrm{Cosh}\left(4\Gamma L\sqrt{1-\xi^2}\right)}$$

Under fulfillment of conditions (4) near maximum $\xi = 0$ instrument function has Lorenz form and can be represented in the form:

$$T(L, \Gamma, \xi) = \frac{1}{1 + \xi^2 \dfrac{\mathbf{e}^{4\Gamma L}(a\Gamma + 1)^2}{4}}, \quad \left(\xi \equiv \frac{\Delta k}{2\Gamma} \equiv \frac{\Delta k}{2\Delta\varepsilon\, k_0}, \Gamma L > 1\right). \qquad (7)$$

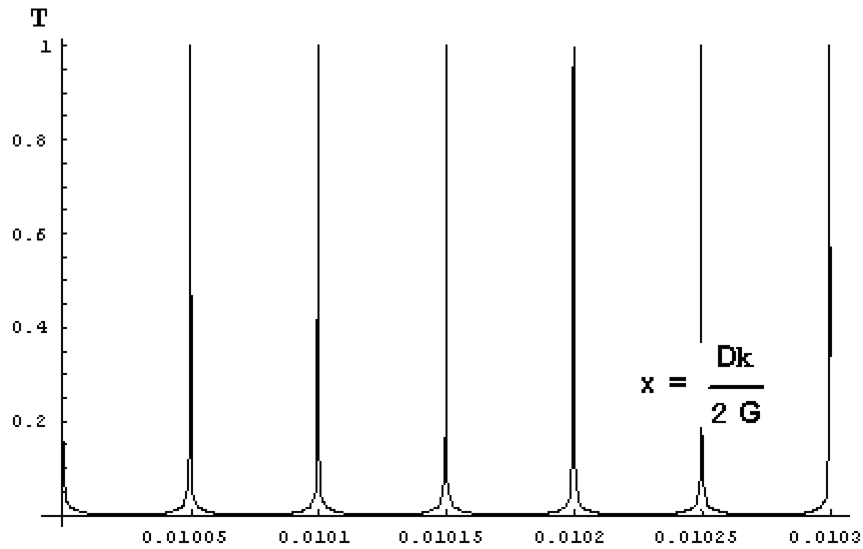

Fig.2. *Fragment of instrument function in interval of values* $0{,}01 < \xi < 0{,}0103$.
Calculation is made according to formula (3) under parameters value:
G = 10 см$^{-1}$, L = $\dfrac{p}{10}$ см; j = p ; a = 2000 p см; k = 10000 см$^{-1}$

Here $\Delta\omega \equiv c\Delta k$ - tuning-off value in frequency units, i.e. frequency deviation from value $\omega_0 \equiv c k_o$, c - light speed, $\delta \equiv 4ce^{-2\Gamma L}/a$ - maximum's width. For comparison let us give expression for instrument function of standard Fabri-Perot interferometer (see book by Malyshev [8], ch.6):

$$T_{F-P}(R, \Delta k) = \frac{1}{1 + \dfrac{4R^2}{(1-R^2)^2} Sin^2\left(\dfrac{n_e k l}{2}\right)} \cong \frac{1}{1 + \dfrac{4R^2}{(1-R^2)^2}\left(\dfrac{n_e l}{2}\right)^2 (\Delta k)^2}, \qquad (8)$$

where R – mirrors reflection factor, l – distance between mirrors, n_c - medium refraction indicator, $n_c \approx 1$, $n_c \approx 1$. Comparing the obtained formula (7) for instrument function with expression (8) we come to conclusion that under fulfillment of condition:

$$\frac{2R}{\left(1-R^2\right)} l < \frac{e^{2\Gamma L}}{2\Gamma}\left(a\Gamma+1\right) \text{ (or under } a \approx l,\ a\Gamma >> 1) \ 2\Gamma L > ln\left(\frac{4}{1-R^2}\right), \quad (9)$$

interferometer instrument function based on reflecting structures will have narrower maximum then standard interferometer.

The above obtained conclusions refer to the case of periodic media without absorption and under condition of infinitesimal divergence of incident radiation. Absorption may be accounted if in the obtained formula (3) we make formal substitution $\Delta k \rightarrow \Delta k - i\,\gamma$, where γ - light wave damping factor in periodic structure. Absorption (or enhancement) of light wave leads to destruction of condition of light wave resonant propagation and wave intensity at resonator output will be exponentially small. The analysis shows that resonant propagation will be nevertheless possible, if damping factor satisfies the following condition: $|\gamma| << e^{-\Gamma L}/2(a\Gamma+1)$. For periodic media with enhancement, when $\gamma < 0$, resonant transmission maximum width diminishes, and sensitivity on the contrary increases. That is why to increase sensitivity it is desirable to use periodic structures with electromagnetic radiation enhancement, which not only compensates damping, but also sharpens radiation transmission maximum.

Let us now consider a task when periodic structure is formed by a “step” change of medium refraction index

$$n(x)=\begin{cases} n_1, & 0<x<a, \\ n_2, & a<x<L, \end{cases} \qquad b=\Lambda-a, \quad (10)$$

at the same time $n(x)=n(x+\Lambda)$. Using matrix technique for solution of task on light wave propagation in such periodic structure (see [7], Chapter 6) we can obtain an expression for light wave amplitude at the output from symmetric Fabri-Perot resonator (i.e. under $x=m\,\Lambda$, where m - a number of layers):

$$E(k)\Big|_{x=m\Lambda} =$$

$$=\frac{Sin^2 G}{\left[A^a Sin(mG-2G)-Sin(mG-3G)\right]\left[A^b Sin(mG-2G)-Sin(mG-3G)\right]e^{ikl}-e^{ik(n_2 b-n_1 a)}\dfrac{\left(n_2^2-n_1^2\right)^2}{4n_1 n_2}Sin(kn_1 a)Sin(kn_2 b)} \quad (11)$$

Here l – distance between symmetric periodic structures, k – optical radiation wavevector and

$$A^a = e^{ikn_1 a}\left(Cos(kn_2 b)+\frac{i}{2}\left(\frac{n_1}{n_2}+\frac{n_2}{n_1}\right)Sin(kn_2 b)\right),$$

$$A^b = e^{ikn_2 b}\left(Cos(kn_1 a)+\frac{i}{2}\left(\frac{n_1}{n_2}+\frac{n_2}{n_1}\right)Sin(kn_1 a)\right), \quad (12)$$

$$G = Arc\left(Cos(kn_1 a)Cos(kn_2 b)-\frac{1}{2}\left(\frac{n_1}{n_2}+\frac{n_2}{n_1}\right)Sin(kn_1 a)Sin(kn_2 b)\right).$$

The obtained formulas describe radiation propagation in periodic structures and allow to find change of interference pattern, formed by two beams in interferometer legs, and find radiation intensity at the output from Fabri-Perot resonator as well. As it is seen from expression (11), this formula is similar to expression (3) and describes behavior of Fabri-Perot interferometer instrument function for a step periodic structure. Under small values of refraction indices difference formulas (3) and (11) are similar.

3. The technique of gravity waves detection

As it is known [1,4] under the influence of gravity wave the distance between any two free substances (points) varies

$$l = l(1+h), \tag{13}$$

where h - gravity wave amplitude. Let polarization and direction of gravity wave propagation are so that only dimensions along the direction *x*- one of interferometer's legs with Fabri-Perot resonators, vary. Optical diagram of measurements with the aid of interferometer is shown in fig.3). (This is Mach-Zehnder interferometer in two legs of which Fabri-Perot resonators are located). Then light wave amplitude in one of interferometer's legs will change, and therefore waves interference pattern will change as well. Considering interferometer's legs optical lengths as equal, we will find variation of interference pattern. For that, according to formula (10) in expression (3) we make substitution $a \to a(1+h)$. Using the known procedure we can discriminate interference term proportional to gravity wave amplitude. General formula for variation of interference term has the form:

$$Int = A(a,L,\xi)\left[a\frac{\partial A(a,L,\xi)}{\partial a}\right]h \qquad \left\{A(a,L,\xi) \equiv \frac{\mathrm{E}_0'(\mathrm{x{=}2L{+}a})}{2\,\mathrm{E}_0(\mathrm{x{=}0})} + k.c.\right\}, \tag{14}$$

where h – dimensionless gravity wave amplitude. When making conclusion (11) it was assumed that both interferometer's legs have equal legs and distance a between periodic structures in Fabri-Perot resonator. Substituting values for light wave field $A(a,L,\xi)$ from expressions (3), (4) and making necessary calculations we will obtain final expression for interference term:

$$Int = 16 e^{4W}\left(1-\xi^2\right)^2 Re\left(\frac{K}{Zn^2}\right) Re\left(\frac{e^{i\Phi}}{Zn}\right) h \tag{15}$$

Here $\Phi \equiv -ak + 2(a+L)\xi,\quad \Xi \equiv 3ak - 4a\Gamma\xi - 2\Gamma L - \varphi$ and

$$Zn \equiv \left(1-e^{2W}\right)^2 + e^{i\Psi}\left(2e^{2W} + \left(i\xi - \sqrt{1-\xi^2}\right)^2 + e^{4W}\left(i\xi + \sqrt{1-\xi^2}\right)^2\right), \tag{16}$$

$$\mathrm{K} \equiv i\,a\,e^{i\Phi}\left(1-e^{2W}\right)^2(k-2\Gamma\xi) + ake^{-i\Xi}\left(i\left(2\xi^2 - 2e^{2W} - e^{4W} + 2\xi^2 e^{4W} - 1\right) + 2\left(e^{4W}-1\right)\xi\sqrt{1-\xi^2}\right). \tag{17}$$

The obtained formulas (12) – (14) allow to estimate sensitivity of the suggested technique of measurement on interference pattern variation. The most easy way is to make it for zero maximum, i.e. near $\xi = 0$, by way of comparing line contour widths. Let us consider sensitivity of measuring technique in LIGO installation, in which standard Fabri-Perot interferometer with free hanging mirrors is used. The mirrors reflection factor R, obtained by the present moment is equal to 1- 6 10^{-6}, and distance between mirrors l = 4 10^5sm. Substituting these values to formula (8), for instrument function of standard interferometer we will obtain:

$$T = \left[1 + 1{,}11\times10^{16}\left(\Delta k\left[sm^{-1}\right]\right)^2\right]^{-1}.$$

Now we will compare it with instrument function of interferometer with periodic structures as reflecting mirrors according to formula (7). For values of parameters: a = 2000π sm, Γ =12 sm^{-1} (that corresponds to $\Delta\varepsilon \approx 1{,}2\times10^{-3}$, $k = 10^4\ sm^{-1}$) and L = 0,47 sm, numerical calculation result coincides with estimation according to formula (8) for T_{F-P}.(The number of half-wave layers will be: N=kL/π = 1400). If Γ = 20 sm^{-1} and L = 0,62 cm, under the same value of a, then formula (7) will give

$$.\ T = \left[1 + 8{,}57\times10^{27}\left(\Delta k\left[sm^{-1}\right]\right)^2\right]^{-1}.$$

It is seen that halfwidth of resonance propagation maximum is more than 6 orders smaller than corresponding maximum of standard Fabri-Perot interferometer, and that is why sensitivity of such measuring technique considerably exceeds sensitivity of standard Fabri-Perot interferometer. However region of gravity waves frequencies ω_g, to which will react such "high-Q" resonance system, is restricted from above by the value $\omega_g \le \Delta k\, c \approx 3.24\, 10^{-4}$ Hz. If the last condition is not fulfilled, spatial position of mirrors will not follow the variation of gravity field wave.

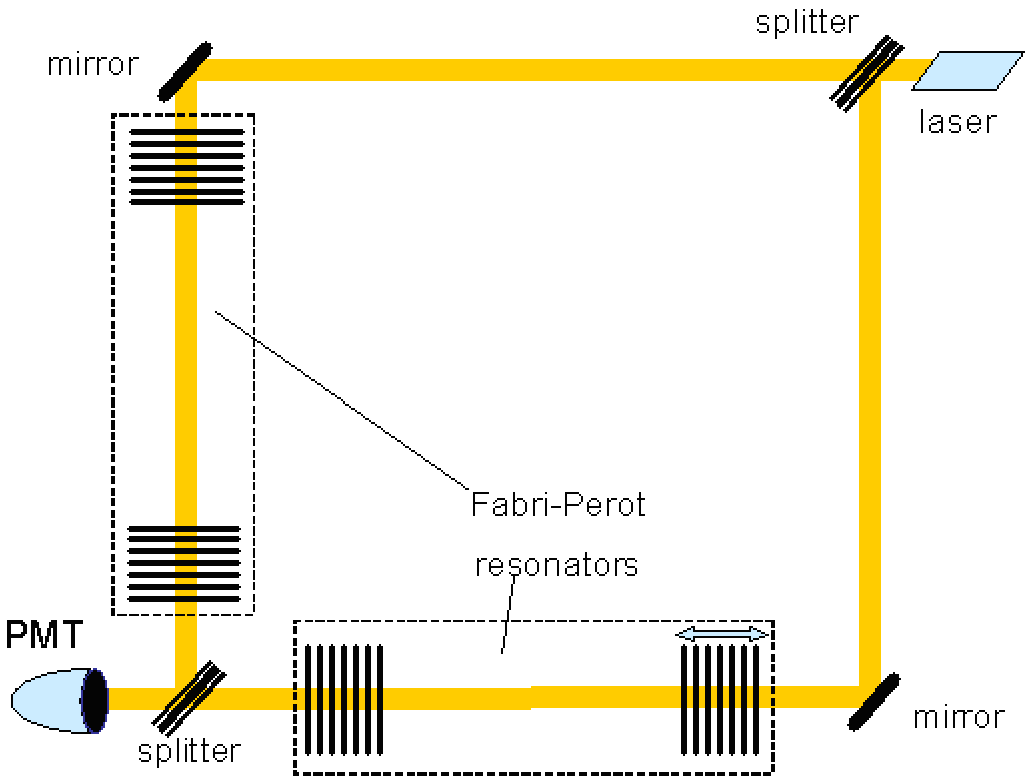

Fig.3. *Optical diagram of Mach-Zehnder interferometer with periodic reflecting structures.*

There is also one more restriction connected with fulfillment principle of uncertainty, at which paid attention V.B. Braginskij [1-3]. The principle of uncertainty requires fulfillment of condition $|\Delta x\, \Delta p| \ge \hbar$, where $\hbar$ - Plank constant, Δx - mirror coordinate variation, Δp - pulse variation. Noting that $\Delta x \approx l\, h$, where l - distance between mirrors, for minimum value of gravity wave dimensionless amplitude, which can be measured without violation of principle of uncertainty, we will obtain $h \ge \frac{1}{l}\sqrt{\frac{\hbar}{m\,\omega_g}}$, ($m$ - mirror mass). For $l = a = 2000\pi\, sm$, $\omega_g \approx 3.24\, 10^{-4}\, Hz$, $m = 10^5\, g$ we obtain for $h \cong 9{,}06 \times 10^{-20}$. Let us note that this value of h is obtained for distance between mirrors of only 628 meters and for gravity wave frequency 3,24 10^{-4} Hz.

Let us also note that system considered in this work – Fabri-Perot resonator with periodic structures as reflecting mirrors, represents an example of macroscopic quantum system with the aid of which it is possible to model quantum systems behavior, for example waves (particles) resonance tunneling, limitations in measurements, connected with principle of uncertainty and others.

Let us consider now interference term according to formula (18). Conditions under fulfillment of which factor under gravity wave amplitude in formula (11) reaches extreme value, as it is easy to see, coincide with resonance conditions, i.e. $-2\,a\,k + 2\,a\,\Gamma\xi + \varphi + ArcSin\,\xi = (2m+1)\pi$.

Exactly under fulfillment of this condition in formula (13) factor under e^{4W} vanishes and expression (12) reaches its extreme value. Value ξ, under which resonance appears, can be found from equation

$$\xi = Cos\left(m\pi + a\left(k - \Gamma\xi\right) - \varphi/2\right). \tag{18}$$

From equation (18) it is seen that if $\varphi = \pi$, $a\,k = A\pi$, where A - any integer , then $\xi = 0$ will be a solution of equation (18). That means that under accurate fulfillment of synchronism conditions appears main (with the largest amplitude) resonance. The dependence of interference term according to expression (15) near this maximum, $\xi = 0$, under a = 2000π cm, Γ = 30 cm^{-1}, L = 0,47cm, is given in fig.4. From expression (15) and fig.4 it follows that there is a certain interval of values a, ($a_1 \le a \le a_2$, see fig.4), inside which coefficient under gravity wave amplitude under the selected above values of system parameters, occurs to be rather great:

$$. \quad Int(a) \approx 1{,}2 \times 10^{24}\left(a - a_0\right)\left[sm\right] h\,. \tag{19}$$

Selecting certain value a within the indicated interval by way of interferometer tuning, (for example $a = \left(a_0 + 0.00009\right)$sm) it is possible to obtain conditions, when $Int(a) \approx 10^{20} h$. Optical diagram of small displacements measurement and values of distances between “mirrors” an be selected so that variation of interference pattern will take place in each leg in counter phase and then coefficient in formula (16) will be two times greater. Let us also note that contrast of interference pattern is rather high: intensities relation in resonance (maximum) to value outside resonance (minimum) is proportional to e^{4W}.

In the above estimations the distance between periodic structures was 628 m that is considerably smaller then distance between mirrors in LIGO installation. Besides, as it is seen from formulas (11)-(15) the use of more high-frequency optical radiation (for example, X-ray one) is preferable, and therefore dimensions of system foe gravity waves registration can be considerably smaller. As for producing periodic media, the existing techniques allow to do it, for example in optical fiber technology it is possible to roll piles of corresponding material, prepared in advance, through rolls or produce necessary medium by holographic techniques [9]. For X-ray radiation the most suitable medium could be ideal crystal with small X-ray radiation absorption.

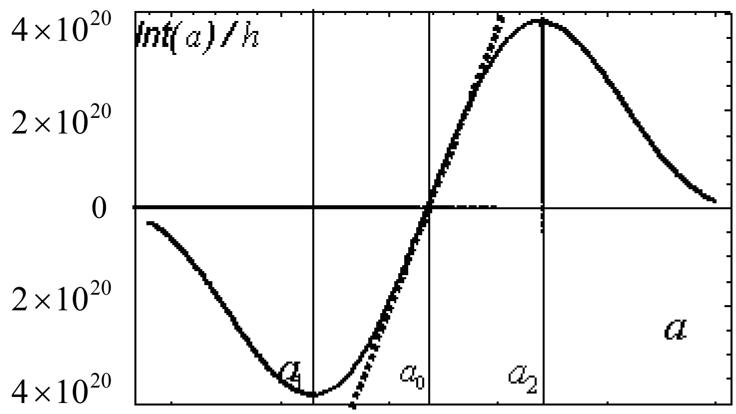

Fig.4. *The dependence of interference term on variation of distance between mirrors (periodic structures) near maximum $\xi = 0$ according to formula (12). Tangent in the point a_0 of maximum can be approximated by the right line:*

$$Int(a) \approx 1.2\,10^{24}\left(a - a_0\right)\left[cm\right]\mathbf{h}.$$

$$\left(\Gamma = 30cm^{-1}, L = 0{,}47cm,\ k = 10^4 cm,\ \ \varphi = \pi,\ \ a_0 = 62850.00013,\ \ a_1 = 62850.000075,\ \ a_2 = 62850.000185\right)$$

Thus the use of Fabri-Perot resonators with periodic structures as reflecting mirrors opens new capacities with regard to creation of small displacements measuring techniques, compact Fourier-spectrometers with rather high spectral resolution, stabilization of electromagnetic oscillations, super-narrowband filters for various types of waves, and registration of gravity waves as well.

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Riddle and problems of the standard cosmological model

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Недавние данные по анизотропии и поляризации реликтового излучения (WMAP) и крупномасштабной структуре Вселенной (каталог галактик SDSS, «лес» линий Ly_{α}) позволили независимо восстановить как космологические параметры современного Мира, так и начальные условия для его развития – спектр возмущений плотности в постинфляционной ранней Вселенной. С одной стороны, это достижение привело к прорыву в понимании физики очень ранней Вселенной (например, к запрету хаотической инфляции за исключением случая инфляции на массивном скалярном поле) и к созданию стандартной космологической модели (состав и структура темной материи, физика реионизации, теория образования первых звезд, массивных черных дыр, галактик, скоплений, крупномасштабной структуры и др.). С другой стороны, успех стандартной модели, являющийся, по сути, триумфом теории гравитации, заострил фундаментальные проблемы физики высоких энергий, поскольку такие 'краеугольные камни' космологической модели как темная материя, темная энергия, бариогенезис и инфляция современная физика описать не в состоянии. На повестке дня стоит расширение космологической модели и построение новой физики, основные контуры которой намечаются сегодня наблюдательной космологией.

Fermion interactions in the nilpotent formalism

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The algebraic nilpotent formalism, applied to the Dirac equation, reduces the basic procedure of relativistic quantum mechanics to the definition of a creation operator for the fermionic state. The definition of the fermionic state in this form leads on to explicit expressions for bosons, baryons, interaction vertices, vacuum, *C*, *P* and *T* transformations, and to nilpotent representations for weak, strong and electric interactions, and for the acquisition of mass through the Higgs mechanism.

1. The creation of the Dirac state

The Dirac state is the most efficient packaging of the 4 fundamental parameters, space, time, mass and charge.[1-3] If we define these fundamental concepts via the respective algebraic units:

Time	Space	Mass	Charge
i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$
pseudoscalar	multivariate vector	scalar	quaternion

we can compactify the algebra by defining new composite quantities, which combine the quantized and conserved nature of charge with the respective pseudoscalar, vector and scalar nature of time, space and mass:

Energy	Momentum	Rest mass
$i\boldsymbol{k}$	$\mathbf{i}\boldsymbol{i}\ \mathbf{j}\boldsymbol{i}\ \mathbf{k}\boldsymbol{i}$	$1\boldsymbol{j}$
E	$\mathbf{p}$	m

The algebra which results from either the eight primitive units, i, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, 1, $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, or the five composite ones, $i\boldsymbol{k}$, $\mathbf{i}\boldsymbol{i}$, $\mathbf{j}\boldsymbol{i}$, $\mathbf{k}\boldsymbol{i}$, $1\boldsymbol{j}$, is a group of order 64, which is isomorphic to the algebra of the Dirac γ matrices. (It can be regarded as equivalent to a particular form of twistor algebra, defining a 4-dimensional complex space, or as a geometrical algebra of form $G(4,4)$, $G(3,3)$, $G(2,3)$ or $G(3,2)$). And the combination of algebras which produces the five composite units not only affects time, space and mass. It also breaks the symmetry between the charges. So we effectively *create* weak, strong and electric charges (w, s, e), with respective pseudoscalar (timelike), vector (spacelike), and scalar (masslike), as well as quaternion,.characteristics:

Weak charge	Strong charge	Electric charge
$i\boldsymbol{k}$	$\mathbf{i}\boldsymbol{i}\ \mathbf{j}\boldsymbol{i}\ \mathbf{k}\boldsymbol{i}$	$1\boldsymbol{j}$
pseudoscalar	multivariate vector	scalar
quaternion	quaternion	quaternion

The combined Dirac or fermionic state

$$(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

is, in fact, a charge state as well as an energy state. It is a nilpotent or square root of zero:

$$(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = -E^2 + p^2 + m^2 = 0$$

which means that it is automatically Pauli exclusive. The state vector $(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ may be considered as a 4-component column vector or spinor, with components:

$$(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$
$$(i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$
$$(-i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

$$(-\,\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

The four parts include:

fermion / antifermion $\pm E$

spin up / spin down $\pm\,\mathbf{p}$ (or $\boldsymbol{\sigma}.\mathbf{p}$)

where E and $\mathbf{p}$ are either operators or eigenvalues, and for, bound states, may include field terms or covariant derivatives:

$$(\boldsymbol{ik}\partial/\partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m)$$
$$(\boldsymbol{ik}\partial/\partial t - \boldsymbol{i}\nabla + \boldsymbol{j}m)$$
$$(-\,\boldsymbol{ik}\partial/\partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m)$$
$$(-\,\boldsymbol{ik}\partial/\partial t - \boldsymbol{i}\nabla + \boldsymbol{j}m)$$

Only the first or lead term, however, provides independent information. The others follow an automatic pattern of sign changes. So, it will often be convenient to write down only this lead term, with the rest assumed. We can, therefore, as always, express the conservation principle in terms of an equivalent nonconservation principle in which the 'amplitude' ($\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$) is produced by the action of a differential operator ($\pm\,\boldsymbol{ik}\partial\,/\partial t \pm \boldsymbol{i}\nabla + \boldsymbol{j}m$) acting on a 'phase' term, and we only really need the first term of this operator ($\boldsymbol{ik}\partial\,/\partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m$). In principle, this is the only independent physical information.

For a 'free' fermion, the phase (exp ($-i(Et - \mathbf{p.r})$) provides the complete range of space and time translations and rotations, but if the E and $\mathbf{p}$ terms represent covariant derivatives or incorporate field terms, then the phase term is determined by whatever expression is needed to make the amplitude nilpotent. In other words, we don't require either the Dirac equation or a specification of 4 terms for quantum physics, only the operator:

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ .$$

Relativistic quantum mechanics is thus specified entirely by the definition of a creation operator, acting on a vacuum (the 'rest of the universe'), with which it is self-dual. In principle, the fermionic state and vacuum are the respective extremes of locality and nonlocality, each of which defines the other. The phase is the means by which they are connected. In the case of a particle bound by some interaction, the phase changes from absolute to partial variation, just as the amplitude changes from complete to partial conservation. In principle, the E and $\mathbf{p}$ terms have to incorporate all the interactions and fields to which the particle is subject; that is, the creation of a particle has to take into account all other existing particles. The fermion creation operator already defines a quantum field integral.

The ½-integral fermionic spin, and the one-handed helicity for massless fermionic states, is a routine derivation from the nilpotent formalism, as it is for the conventional one. The arbitrary direction of spin in the case of the free fermion can be described by analogy with the state of circular polarization, and we can use the nilpotent structure to define Stokes polarization parameters of the form: $I = (\boldsymbol{k}E)\,(-\boldsymbol{k}E) + (i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,(i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$; $Q = (\boldsymbol{k}E)\,(-\boldsymbol{k}E) - (i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,(i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$; $u = (\boldsymbol{k}E)\,(i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m) + (-\boldsymbol{k}E)\,(i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$; $v = i(\boldsymbol{k}E)\,(i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m) - i(-\boldsymbol{k}E)\,(i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$; so that $I^2 = Q^2 + u^2 + v^2 = v^2 = 4E^2$ before normalization.

2. Multiple meanings for *i*, *j*, *k*

The three quaternion operators $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ can be seen to have multiple meanings in the nilpotent formalism – as charges; as P, C, T transformation operators; and as vacuum generators.

(1) The primary meaning is charge, or source of the weak, strong and electric interactions. The operators do not necessarily imply the existence of nonzero units of charge. They act, rather, as sites where charge units may be generated.

(2) Premultiplying the nilpotent gives the discrete partitions of vacuum associated with the three types of charge:

$\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	weak vacuum
$\boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	strong vacuum
$\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	electric vacuum

because postmultiplication of $(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$ by any of these terms leaves the state unchanges after normalization.

(3) Pre- and postmultiplying the nilpotent transforms via *T*, *P* or *C*, the three discrete symmetries:

$\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k}$	*T* transformation
$\boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{i}$	*P* transformation
$-\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{j}$	*C* transformation

The three discrete symmetries, and the *CPT* combination can be expressed as follows:

P	$\boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{i} = (\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$
T	$\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k} = (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$
C	$-\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{j} = (-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$
CPT	$-\boldsymbol{j}\,(\boldsymbol{i}\,(\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k})\,\boldsymbol{i})\,\boldsymbol{j} = (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$

CPT connects relativity with causality, only in the nilpotent formulation, which links m via a quaternion operator with E and $\mathbf{p}$. In special relativity, we take the conjugate nilpotent $(\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau)$, where τ, the proper time, is the term required for causality. In summary, the meanings of the quaternion operators $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ include:

$\boldsymbol{i}$	strong charge	strong vacuum	*P*
$\boldsymbol{j}$	electric charge	electric vacuum	*C*
$\boldsymbol{k}$	weak charge	weak vacuum	*T*

The lead term in the Dirac nilpotent determines whether the state is fermion / antifermion, spin up / down. The three additional terms then become strong, weak and electric vacuum reflections of the state defined by the lead term. Because of the complete duality of the operator and amplitude, there is equivalent duality between fermion and vacuum, the action of the operator on vacuum (phase) producing the fermion (amplitude) as the result. So the expression $(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$ refers to either the fermion state or the continuous vacuum which represents the 'rest of the universe', and is responsible for zero-point energy. The charges act as a discrete partitioning of the continuous vacuum, a localization of the vacuum in a 3-D 'charge space' (and, by symmetry, an angular momentum space), and the three vacuum coefficients can be seen as originating in (or being responsible for) the concept of discrete (point-like) charge:

$\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	weak vacuum	fermion creation
$\boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	strong vacuum	gluon plasma
$\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	electric vacuum	*SU*(2)

The three discrete partitions are entirely orthogonal or independent of each other, and the three terms following the lead term in the 4-component Dirac spinor are effectively the 'empty' or alternative ('virtual') states to which the fermion could switch without a change in the magnitude of energy or momentum. (They could be considered as comparable with the 'holes' in condensed matter theory.) The physical manifestations of the fermion / vacuum duality include *zitterbwegung* and ½-integral spin. Taking a metaphor from biology, we could describe the motion of the fermion and vacuum combination as that of a double helix. (The same also applies to bosonic combinations of fermions and antifermions, where the directions of the helices may be either in the same or opposite senses, depending on the component spins.)

3. Interaction vertices

Where there is an interaction vertex between two fermionic / antifermionic states, the signs of E and $\mathbf{p}$ of the second term, with respect to the first, will also determine the nature of the bosonic or combined state which may be created. Because there are three operators involved – $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ – there are also three possible bosonic states. Any transformation of a fermionic state can be represented as a bosonic state in which the old state is annihilated and the new one created.

Because the state vector always represents four terms with the complete variation of signs in E and $\mathbf{p}$, the interaction vertex between any fermion / antifermion and any other will always be a real scalar (unity under normalization), with amplitude:

$$(\boldsymbol{ik}E_1 + \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)\ (\boldsymbol{ik}E_2 + \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$$

except in the case of identical fermions, where it will be 0. When the E, $\mathbf{p}$ and m values become numerically equal, the vertex can be defined as a new *combined* bosonic state, with a single phase (which depends only on the numerical values of E and $\mathbf{p}$). There are three such states, composed respectively of fermion and fermion transformed under T, C or P:

Spin 1 boson:

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \qquad T$$

Spin 0 boson:

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \qquad C$$

Bose-Einstein condensate / Berry phase, etc.:

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \qquad P$$

We can consider these bosonic types to be the respective mediators of weak, electric and strong *nonlocal* interactions, that is, interactions between fermion and vacuum. (The *local* interactions, between fermion and fermion are mediated by spin 1 bosons, $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, in which there is a change from one E state to another, through the acquisition of a scalar potential.)

Significantly, the spin 0 bosonic state cannot be massless, because, if it is nilpotent it automatically becomes zero.

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p})\ (-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p}) = 0$$

This becomes a significant factor in the Higgs mechanism. It also implies that massless fermions cannot have the same handedness as massless antifermions. The conventional derivation of spin assigns left-handedness to fermions.

4. Baryons

We have postulated an entangled system of two nilpotent states (fermion and antifermion) to describe bosons. Can we extend this idea to three nilpotent states to describe baryons? Conventionally, we consider a baryon to be made up of three fermionic components, to which we assign colour to overcome Pauli exclusion. Can we relate this concept of colour to the fundamental structure of nilpotents? Clearly we cannot have a state vector composed of three identical fermions, because

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0\ .$$

But we get nonzero solutions if only one of the three states has an active momentum component, exactly as supposed in the quantum mechanical description of spin, where there are three possible directions for the spin vector, but only one can be defined. That is,

$$(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m) \rightarrow (\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m) \rightarrow (\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m) \rightarrow (\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$

after the application of normalization. So it is possible to have a nonzero state vector if we use the vector properties of **p** (and, more specifically, **σ.p**) and the arbitrary nature of its sign (+ or –). In principle, this is nothing more than a representation of the arbitrariness of the direction of fermionic spin using the vector properties of **p** to create spatial, as well as temporal, variation (and the same applies to the combined integral spin in those bosons, such as mesons, which use the same vector properties).

Now, a state vector of the form, privileging the **p** components:

$$(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m)$$

has six independent allowed phases, i.e. when

$$\mathbf{p} = \pm\,\mathbf{i}p_x\,,\ \mathbf{p} = \pm\,\mathbf{j}p_y\,,\ \mathbf{p} = \pm\,\mathbf{k}p_z$$

but these must be *gauge invariant*, i.e. indistinguishable, or all present at once. One method of picturing this is to imagine an automatic mechanism of transfer between them. If we write the phases in the form

$$(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{i}p_x + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{j}p_y + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{k}p_z + \boldsymbol{ij}\,m)\quad +RGB$$
$$(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{i}p_x + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{j}p_y + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{k}p_z + \boldsymbol{ij}\,m)\quad -RBG$$
$$(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{i}p_x + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{j}p_y + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{k}p_z + \boldsymbol{ij}\,m)\quad +BRG$$
$$(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{i}p_x + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{j}p_y + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{k}p_z + \boldsymbol{ij}\,m)\quad -GRB$$
$$(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{i}p_x + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{j}p_y + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{k}p_z + \boldsymbol{ij}\,m)\quad +GBR$$
$$(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{i}p_x + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{j}p_y + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{k}p_z + \boldsymbol{ij}\,m)\quad -BGR$$

we see that they have exactly the same group structure as the standard 'coloured' baryon wavefunction made of R, G and B 'quarks',

$$\psi \sim (RGB - RBG + BRG - GRB + GBR - BGR)$$

That is, they have an $SU(3)$ structure, with 8 generators, and, since the E and **p** terms in the state vector really represent time and space derivatives, we can replace these with the covariant derivatives needed for invariance under a local $SU(3)$ gauge transformation.A significant aspect of this $SU(3)$ symmetry or strong interaction is that it is entirely nonlocal (though, because the mediators are massless, they become equivalent to the spin 1 bosons required for a local interaction). That is, the exchange of momentum **p** involved is entirely independent of any spatial position of the 3 components of the baryon. We can suppose that the rate of change of momentum (or 'force') is constant with respect to spatial positioning or separation. A constant force is equivalent to a potential which is linear with distance, exactly as is required for the conventional strong interaction.

The baryon representation can only exist as a unified or entangled state. It is not really a representation of a combination of 3 independent fermions. Such a representation is impossible in a conventional spinor formulation, with terms such as $p_x + ip_y$, or in any representation in which the momentum operators cannot show the full affine nature of the vector concept.

Very significantly, the full symmetry between the 3 momentum components can only apply if the momentum operators can be equally + or –. With all phases of the interaction present at the same time (perfect gauge invariance), this is equivalent to saying that left-handedness and right-handedness must be present simultaneously in the baryon state. In other words, the baryonic state must have non-zero mass via the Higgs mechanism.

The mediators of the strong force will be six bosons of the form:

$$(\boldsymbol{i}\boldsymbol{k}E - \boldsymbol{i}\mathbf{i}p_x)\ (-\boldsymbol{i}\boldsymbol{k}E - \boldsymbol{i}\mathbf{j}p_y)$$

and two combinations of the three bosons of the form:

$$(\boldsymbol{i}\boldsymbol{k}E - \boldsymbol{i}\mathbf{i}p_x)\ (-\boldsymbol{i}\boldsymbol{k}E - \boldsymbol{i}\mathbf{i}p_x)$$

These structures are, of course, identical to an equivalent set in which both brackets undergo a complete sign reversal. The important thing here is that applying any of these mediators will produce a sign change in the **p** component that leads to mass. Thic can be considered as a parity (*P*) transformation; through the quaternion operator ***i***, the one associated with the strong charge.

5. The Higgs mechanism

Imagine a virtual fermionic state with, no mass, in an ideal vacuum:

$$(\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p})$$

(Since vacuum is concerned principally with the nonconserved aspects of the fermion state (i.e. the variation in space and time, represented by the phase), we can define such a state without mass.) An *ideal* vacuum would maintain exact and absolute *C*, *P* and *T* symmetries. Under *C* transformation, $(\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p})$ would become

$$(-\boldsymbol{i}\boldsymbol{k}E - \boldsymbol{i}\mathbf{p})$$

with which it would be indistinguishable under normalization. No bosonic state would be required for the transformation.

If, however, the vacuum state is degenerate in some way under charge conjugation (as supposed in the weak interaction), then

$$(\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p})$$

will be transformable into a state with the same energy and momentum which can be distinguished from it, and the bosonic state $(\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p})\ (-\boldsymbol{i}\boldsymbol{k}E - \boldsymbol{i}\mathbf{p})$ will necessarily exist. However, this can only be true if the state has nonzero mass and becomes the spin 0 'Higgs boson':

$$(\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (-\boldsymbol{i}\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

And this will also only be possible if the mass is confined to a single sign, which we may, by convention, define to be positive.

6. $SU(2)_L \times U(1)$

The acquisition of mass in the nilpotent formalism can be related to the capacity for change of sign in the **p** term with respect to that of the *E* term. In principle, a weak isospin transition can be seen as a change of the form

$$(\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \rightarrow \underbrace{\alpha_1\ (\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)}_{\text{isospin up}} + \underbrace{\alpha_2\ (\boldsymbol{i}\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)}_{\text{isospin down}}$$

The down state introduces a degree of right-handedness which is not present in the up state, and which is not weak in origin.

Where the strong interaction is not involved, a partial **p** sign transition (involving vacuum operator ***i***) can only be accomplished by involving the electric vacuum operator (***j***) as well as the weak one (***k***). The weak interaction preserves left-handedness in fermionic states and right-handedness in antifermionic states. So, in any pure weak transition, the anti-state to the state to be annihilated and the state which is to be created must exist as a spin 1 bosonic combination.

But fermion states with mass also carry a degree of right-handedness. A transition from left- to right-handedness, involving only fermionic states (not antifermionic), requires a vacuum which we can describe as 'electric'. Only the electric vacuum carries a transition to right-handedness where the vector character (strong interaction) is absent. And, to produce a pure transition from left- to right-handedness (and *vice versa*) without a change from fermion to antifermion requires an electroweak combination ($\boldsymbol{jk}$, equivalent to $\boldsymbol{i}$):

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	left-handed fermion
$(-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	weak transition to right-handed antifermion
$(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	electric transition to right-handed fermion

Using the concept of electric 'charge' as indicating the presence of right-handedness, we may identify four possible transitions (taking the 'left-handed' / 'right-handed' transition to mean 'the acquisition of a greater degree of right-handedness'), and hence four possible intermediate bosonic states:

Left-handed to left-handed
Left-handed to right-handed
Right-handed to left-handed
Right-handed to right-handed

The left-handed / right-handed transition clearly has the nature of an $SU(2)_L$ symmetry, with the requirement of 3 generators, which are necessarily massive, to carry the right-handed unrecognised by the interaction, and 2 of which carry electric 'charge' (+ and –), in addition to one which leaves the handedness unchanged.This leaves the fourth transition state or equivalent as an extra generator with a $U(1)$ symmetry. If we assume that massive generators are necessary for a 'weak interaction', and indicate its presence, we can assign the fourth generator to the pure electric interaction. Electric charge, however, is not the sole reason for the massiveness (and hence mixed handedness) of real fermionic states. So the absence of e does not indicate that a weak generator must be massless. So, the 2 generators without e are assumed mixed, the combination producing 2 new generators, one of which becomes massless and so carries the pure electric interaction.

7. Representing interactions via symmetry groups

There are three fundamental ways of representing strong, weak and electric interactions:

(1) Through the nilpotent formalism in terms of E and $\mathbf{p}$, and their sign and component changes.

(2) Through the conversion of E and $\mathbf{p}$ in the nilpotent formalism into covariant derivatives directly derived from the symmetry groups associated with the transformation mechanisms in (1).

(3) Through the potential functions which, when added to E (and $\mathbf{p}$), produce the same effect as in (2).

Considering representation through symmetry groups, we note that the Dirac nilpotent has three terms of equal status: a pseudoscalar term ($\pm\, iE$) with a natural dipolarity connected with $SU(2)$ weak interaction; a vector term ($\pm\, \mathbf{p}$) related to strong $SU(3)$; and a scalar term (m) related to electric $U(1)$. The three symmetry groups associated with the strong, weak and electric interactions come from the spherical symmetry ($\equiv$ conservation of angular momentum) necessarily implied when we define a point source. $U(1)$ symmetry says that spherical symmetry is preserved whatever the length of the radius vector. $SU(3)$ symmetry says that spherical symmetry is preserved whatever the choice of axes. $SU(2)$ symmetry says that spherical symmetry is preserved irrespective of whether the rotation is left- or right-handed.

Applying the $SU(3)$ symmetry to the strong interaction in a baryon, we note that the covariant derivative under an $SU(3)$ local gauge transformation is:

$$\partial_\mu \to \partial_\mu + ig_s \frac{\lambda^\alpha}{2} A^{\alpha\mu}(x) \,.$$

or, in component form:

$$ip_i = \partial_i \to \partial_i + ig_s \frac{\lambda^\alpha}{2} A^{\alpha i}(x)$$

$$E = i\partial_0 \to i\partial_0 - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}(x) \,.$$

We now apply $SU(3)$ generators to the baryon state vector to obtain:

$$\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) \pm \boldsymbol{i}\left(\partial_1 + ig_s \frac{\lambda^\alpha}{2} \mathbf{A}^\alpha\right) + \boldsymbol{ij}m\right)\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) + \boldsymbol{ij}m\right)\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) + \boldsymbol{ij}m\right)$$

x active

$$\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) + \boldsymbol{ij}m\right)\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) \pm \boldsymbol{i}\left(\partial_1 + ig_s \frac{\lambda^\alpha}{2} \mathbf{A}^\alpha\right) + \boldsymbol{ij}m\right)\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) + \boldsymbol{ij}m\right)$$

y active

$$\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) + \boldsymbol{ij}m\right)\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) + \boldsymbol{ij}m\right)\left(\boldsymbol{k}\left(E - g_s \frac{\lambda^\alpha}{2} A^{\alpha 0}\right) \pm \boldsymbol{i}\left(\partial_1 + ig_s \frac{\lambda^\alpha}{2} \mathbf{A}^\alpha\right) + \boldsymbol{ij}m\right)$$

z active.

In the strong interaction, the **p** or vector term may be considered as the 'active' component, and the E term as the 'passive'

It is equally possible to represent the electroweak interaction via covariant derivatives. Deriving covariant derivatives with $\mathbf{W}^\mu$ and B^μ as the respective 4-vector generators for $SU(2)$ and $U(1)$, we have, for left-handed states:

$$\partial_\mu \to \partial_\mu + ig \frac{\boldsymbol{\tau}.\mathbf{W}^\mu}{2} - ig' \frac{B^\mu}{2} \,,$$

and, for right-handed:

$$\partial_\mu \to \partial_\mu - ig' \frac{B^\mu}{2} \,.$$

Applying these covariant derivatives to the nilpotent vertex which describes the weak interaction, we find that we can represent three components as 'active' and one as 'passive'.

$$E = i\partial_0 \to i\partial_0 + ig' \frac{B^0}{2} + ig' \frac{B^3}{2} \,,$$

and

$$ip_3 = \partial_3 \to \partial_3 + ig \frac{\boldsymbol{\tau}.\mathbf{W}^3}{2} + ig \frac{\boldsymbol{\tau}.\mathbf{W}^0}{2} \,.$$

We note here that the electroweak interaction (or the weak component of it) is defined only in terms of a 2-component vertex, such as

$$(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (-\, i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

Essentially, because of the pseudoscalar nature of the energy term, associated with $\boldsymbol{k}$, i.e. the mathematical indistinguishability of $+i$ and $-i$, the weak interaction is always defined as minimally dipolar, in the same way as the fermion always defines itself as a dipole with respect to vacuum (leading to half-integral spin). We now write a vertex for a standard electroweak transition in the form

$$(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (-\, i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) =$$

$$\left(\boldsymbol{k}\left(\partial_0 + g'\frac{B^0}{2} + g'\frac{B^3}{2} \right) + \boldsymbol{i}\left(\partial_3 + ig\frac{\boldsymbol{\tau}.\mathbf{W}^3}{2} + ig\frac{\boldsymbol{\tau}.\mathbf{W}^0}{2} \right) + \boldsymbol{ij}m \right) \times$$
$$\left(-\boldsymbol{k}\left(\partial_0 + g'\frac{B^0}{2} + g'\frac{B^3}{2} \right) + \boldsymbol{i}\left(\partial_3 + ig\frac{\boldsymbol{\tau}.\mathbf{W}^3}{2} + ig\frac{\boldsymbol{\tau}.\mathbf{W}^0}{2} \right) + \boldsymbol{ij}m \right)$$

By choice of mass term, we can write this as:

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(-\,\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) =$$
$$= \left(\boldsymbol{k}\left(\partial_0 + e\frac{\gamma^0}{2} \right) + \boldsymbol{i}\left(\partial_3 + ig\frac{\boldsymbol{\tau}.\mathbf{Z}^3}{2} \right) + \boldsymbol{ij}m \right)\left(-\boldsymbol{k}\left(\partial_0 + e\frac{\gamma^0}{2} \right) + \boldsymbol{i}\left(\partial_3 + ig\frac{\boldsymbol{\tau}.\mathbf{Z}^3}{2} \right) + \boldsymbol{ij}m \right).$$

8. Representing interactions by potentials

In the representation by potentials, using polar coordinates for $\boldsymbol{ii}\boldsymbol{\sigma}.\nabla$, we write the nilpotent potent state vector under the action of a point source in the form:

$$\left(\boldsymbol{k}\left(E + V(r)\right) + \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + ½}{\mathrm{r}} \right) + \boldsymbol{ij}m \right).$$

Here, we see the origin of the scalar ‘passive’ components for the interactions, for this operator only produces nilpotent solutions if the potential term $V(r)$ incorporates an inverse linear or Coulomb component ($-A\ /\ r$), equivalent to a $U(1)$ symmetry. If we suppose that the strong, electromagnetic and weak interactions are determined by sources with respective vector, scalar and pseudoscalar properties, the ‘passive’ or Coulomb term that each interaction requires appears to be equivalent to the scalar values associated with these. These can be equated to the coupling constants associated with these interactions, and it is these that we may expect to be unified at Grand Unification. The ‘active’ parts of the strong and weak interactions represented by the non-Coulombic potentials can then be seen to result from their vector and pseudoscalar properties.

The scalar electric term can be expected to be equivalent to a pure magnitude (a Coulomb term). The vector strong term requires an additional linear component ($-Br$). The pseudoscalar weak term requires an additional dipolar component ($-Cr^{-3}$). These three conditions give the only nilpotent solutions for the state vector, and they have the characteristics observed in the three interactions:

inverse linear	$U(1)$	scalar phase
inverse linear + linear	$SU(3)$	confinement
inverse linear + other polynomial	$SU(2)$	harmonic oscillator

9. Acquisition of mass through the Higgs boson

The coupling of a massless fermion, say $(\boldsymbol{ik}E_1 + \boldsymbol{i}\mathbf{p}_1)$, to a Higgs boson, say $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, to produce a massive fermion, say $(\boldsymbol{ik}E_2 + \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$, can be imagined as occurring at a vertex between the created fermion $(\boldsymbol{ik}E_2 + \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$ and the antistate $(-\boldsymbol{ik}E_1 - \boldsymbol{i}\mathbf{p}_1)$, to the annihlated massless fermion, with subsequent equalization of energy and momentum states. If we imagine a vertex involving a fermion superposing $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ and $(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ with an antifermion superposing $(-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ and $(-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, then there will be a minimum of two spin 1 combinations and two spin 0 combinations, meaning that the vertex will be massive (with Higgs coupling) and carry a non-weak (i.e. electric) charge. So, a process such as

$$\underset{}{(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)} \rightarrow \alpha_1\,\underset{\text{isospin up}}{(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)} + \alpha_2\,\underset{\text{isospin down}}{(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)}$$

requires an additional Higgs boson vertex (spin 0) to accommodate the right-handed part of the isospin down state, when the left-handed part interacts weakly. This is, of course, what we mean when we say that the *W* and *Z* bosons have mass. The mass balance is done through separate vertices involving the Higgs boson. In the case of baryons, the up states also acquire terms of the form $-\alpha_2$ $(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, while the more massive generations take on extra terms equivalent to α_n $(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ because of the breaking of weak charge conjugation symmetry.

10. Berry phase: a prediction

The mathematical dipolarity of the pseudoscalar weak charge appears to be the ultimate source of different phases of matter and phase transitions, when the indistinguishability of sign is allowed to effectively eliminate the weak component in fermion-fermion combinations, and so overcome aspects of Pauli exclusion. It is certainly the origin of the Berry phase and related effects (Aharonov-Bohm, Jahn-Teller, quantum Hall, Cooper pairs, etc.). In the Berry phase the spin 0 'bosonic' state

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

is such as would be required in a pure weak transition from $-\boldsymbol{ik}E$ to $+\ \boldsymbol{ik}E$, or its inverse. Because the spin 0 state is necessarily massive, time reversal symmetry (the one applicable to the transition) must be broken in the weak formation or decay of states involving the Berry phase.

11. Conclusion

Weak, strong and electric interactions and the acquisition of nonzero mass by fermions and bosons can all be related to the structure of the nilpotent state vector, and, in particular, to the relative signs associated with the *E* and **p** operators.

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Global monopoles in extra dimensions as possible brane worlds

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Multidimensional configurations with Minkowski external space-time and topological defects in extra dimensions are discussed as possible realizations of the brane world concept. We consider (i) the brane structure governed by sets of scalar fields ϕ^i with arbitrary symmetry-breaking potentials V depending on the magnitude $\phi^2 = \phi^i\phi^i$ and (ii) the brane ability to confine standard-model fields (whose self-gravity is neglected).

We first study thick $\mathbb{Z}_2$-symmetric domain walls supported by a single scalar field ϕ in 5D general relativity. Under the global regularity requirement, such configurations are shown to have always an AdS asymptotic far from the brane. Thus a thick brane with any admissible $V(\phi)$ is a regularized version of the well-known RS2 brane immersed in the AdS_5 bulk. The thin brane limit is realized in a universal manner. As regards the trapping peoperties of such branes, it is shown that the stress-energy tensor of a test scalar field inevitably diverges at the AdS horizon far from the brane, be it thin ot thick. Similar problems exist for fields of other spins due a repulsive nature of gravity in the bulk.

The situation drastically changes in higher dimensions, e.g., for static, spherically symmetric global monopoles in extra dimensions.

A monopole is formed with a hedgehog-like set of scalar fields ϕ^i such that the potential V depends on the magnitude $\phi^2 = \phi^i\phi^i$. Seven possible kinds of globally regular configurations are singled out without specifying the shape of $V(\phi)$. These variants are governed by the value of the scalar field magnitude (order parameter) ϕ_0 characterizing the energy scale of symmetry breaking. If $\phi_0 < \phi_{\rm cr}$ (where $\phi_{\rm cr}$ is a critical value of ϕ related to the multidimensional Planck scale), the monopole reaches infinite radii and has either an AdS asymptotic (if $V(\phi_0) < 0$) or a quasi-flat asymptotic with an angular defect, like global monopoles in 4 dimensions (if $V(\phi_0) = 0$). If $\phi_0 = \phi_{\rm cr}$, the monopole ends with a "flux tube" of finite constant radius. In case $\phi_0 > \phi_{\rm cr}$, there can be be either a tube with gravitational attraction towards the centre or a system with two regular centres resembling a closed cosmology. The conclusions obtained analytically are confirmed numerically for the Mexican hat potential with an additional parameter moving it up and down. Some of these models are shown to have good trapping properties.

Changing the Hilbert space structure as a consequence of gauge transformations in "extended phase space" version of quantum geometrodynamics

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In the earlier works on quantum geometrodynamics in extended phase space it has been argued that a wave function of the Universe should satisfy a Schrödinger equation. Its form, as well as a measure in Schrödinger scalar product, depends on a gauge condition (a chosen reference frame). It is known that the geometry of an appropriate Hilbert space is determined by introducing the scalar product, so the Hilbert space structure turns out to be in a large degree depending on a chosen gauge condition. In the present work we analyse this issue from the viewpoint of the path integral approach. We consider how the gauge condition changes as a result of gauge transformations. In this respect, three kinds of gauge transformations can be singled out: Firstly, there are residual gauge transformations, which do not change the gauge condition. The second kind is the transformations whose parameters can be related by homotopy. Then the change of gauge condition could be described by smoothly changing function. In particular, in this context time dependent gauges could be discussed. We also suggest that this kind of gauge transformations leads to a smooth changing of solutions to the Schrödinger equation. The third kind of the transformations includes those whose parameters belong to different homotopy classes. They are of the most interest from the viewpoint of changing the Hilbert space structure. In this case the gauge condition and the very form of the Schrödinger equation would change in discrete steps when we pass from a space-time region with one gauge condition to another region with another gauge condition. In conclusion we discuss the relation between quantum gravity and fundamental problems of ordinary quantum mechanics.

1. Introduction

One of unsolved problems of the Wheeler – DeWitt quantum geometrodynamics is that of Hilbert space structure. The Wheeler – DeWitt quantum geometrodynamics was the first attempt of constructing a quantum theory of the Universe as a whole, however, if its Hilbert space structure is not rigorously determined, one cannot consider it as full and consistent, as well as any quantum theory.

The reasons, why this problem cannot be solved in the framework of the Wheeler – DeWitt quantum geometrodynamics, are closely connected with the fact that it was thought of as a gauge invariant theory. According to the original idea of Wheeler, a wave function of the Universe, which is a basic object in quantum geometrodynamics, must depend on 3-geometry of a manifold M. In other words, if $\mathsf{Riem}(M)$ is the space of all Riemannian metrics on M, and $\mathsf{Diff}(M)$ is diffeomorphism group, the wave function must be defined on the so-called superspace of all 3-geometries, or factor space $\mathsf{Riem}(M)/\mathsf{Diff}(M)$ [1, 2]. One possible way to express this dependence would be to regard the wave function as a function of an infinite set of geometrical invariants [3]. It is not clear, however, how to put this idea into practice. Actually, the wave function depends on a 3-metric, and it was believed that, if the wave function satisfied a quantum version of gravitational constraints, it would ensure its dependence on 3-geometry only. The very requirement for the wave function to satisfy the Wheeler – DeWitt, but not a Schrödinger, equation leads to the problem of Hilbert space, in particular, it is questionable how an inner product of state vectors should be determined (for a recent review on related problems, see [4]). On the other hand, the Wheeler – DeWitt quantum geometrodynamics is based on Arnowitt – Deser – Misner (ADM) formalism, and, as some authors have emphasized [5 – 7], the latter is equivalent to some kind of gauge fixing, so there is the inconsistency between appealling to ADM formalism and the requirement for a wave function to be invariant under diffeomorphism group transformations.

In this work I shall discuss another approach to quantum geometrodynamics, namely, quantum geometrodynamics in extended phase space [8 – 10]. The main features of this approach were presented on the previous PIRT conference [11]. As was shown in [11], in the "extended phase space" approach a physical part of the wave function satisfies a Schrödinger equation, whose form, as well as a measure in Schrödinger inner product, depends on a gauge condition, or a chosen reference frame (the basic formulas will be repeated in Section 2). The situation can be illustrated by the following scheme (Fig. 1). All metrics $g_{\mu\nu}$ related by gauge transformations are unified into an equivalence class representing dynamics of some 3-geometry. Two metrics $g_{\mu\nu}$ and $g'_{\mu\nu}$, which can be obtained from each other by a coordinate transformation, correspond to the same geometry, but may answer to various gauge conditions. In this case in our approach different gauge conditions correspond to different physical Hamiltonians, say, H_1 and H_2.Every of the Hamiltonians acts in its own Hilbert space with a measure in inner product defined by a chosen gauge condition. Thus we come to the following question: How gauge transformations could change the structure of Hilbert space? To answer it, we shall consider in Section 3 three kinds of gauge transformations: residual gauge transformations, those whose parameters related by homotopy and those whose parameters belong to different homotopy classes. In Section 4 we shall point to some relation between the problems arising in quantum geometrodynamics and the problem of reduction of a wave function in ordinary quantum mechanics, which has been discussed up till now by eminent physicists (see, for example, [12 – 14]).

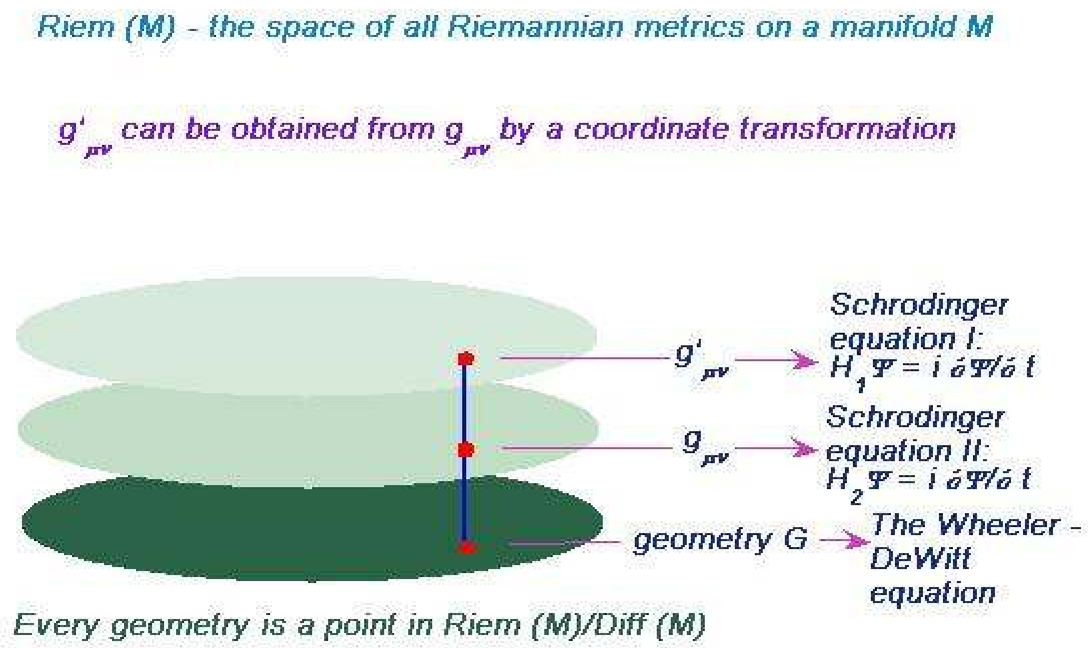

Fig. 1.

2. The Hilbert space in 'extended phase space' version of quantum geometrodynamics

In [11] we considered a simple minisuperspace model with the action

$$S = \int dt \left\{ \frac{1}{2} v(\mu, Q) \gamma_{ab} \dot{Q}^a \dot{Q}^b - \frac{1}{v(\mu, Q)} U(Q) + \pi_0 \left(\dot{\mu} - f_{,a} \dot{Q}^a \right) - i w(\mu, Q) \dot{\bar{\theta}} \dot{\theta} \right\}, \tag{2.1}$$

where $Q = \{Q^a\}$ are physical variables, $\theta, \bar{\theta}$ are the Faddeev – Popov ghosts and μ is a gauge variable, its parameterization being determined by the function $v(\mu, Q)$. In simple cases μ can be bound to the scale factor *a* and the lapse function *N* by the relation $\frac{a^3}{N} = v(\mu, Q)$.

$$w(\mu,Q)=\frac{v(\mu,Q)}{v_{,\mu}};\quad v_{,\mu}\stackrel{def}{=}\frac{\partial v}{\partial\mu}. \tag{2.2}$$

We used a differential form of gauge conditions

$$\mu=f(Q)+k;\quad k=\text{const}, \tag{2.3}$$

namely,

$$\dot\mu=f_{,a}\dot Q^a,\quad f_{,a}\stackrel{def}{=}\frac{\partial f}{\partial Q^a}. \tag{2.4}$$

The wave function is defined on extended configurational space with the coordinates $\mu,Q,\theta,\bar\theta$. In "extended phase space" version of quantum geometrodynamics we quantize ghost and gauge gravitational degrees of freedom on an equal basis with physical degrees of freedom. The motivation for it was that it is impossible to separate gauge, or "non-physical" degrees of freedom from physical ones if the system under consideration does not possess asymptotic states, and it is indeed the case for a closed universe as well as in a general case of nontrivial topology. Then, we come to the Schrödinger equation, which is derived from a path integral with the effective action (2.1) without asymptotic boundary conditions by the standard well-definite Feynman procedure, and which is *a direct mathematical consequence* of the path integral.

$$i\frac{\partial\Psi(\mu,Q,\theta,\bar\theta;t)}{\partial t}=H\Psi(\mu,Q,\theta,\bar\theta;t), \tag{2.5}$$

where

$$H=-\frac{i}{w}\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar\theta}-\frac{1}{2M}\frac{\partial}{\partial Q^\alpha}MG^{\alpha\beta}\frac{\partial}{\partial Q^\beta}+\frac{1}{v}(U-V); \tag{2.6}$$

$$M(\mu,Q)=v^{\frac{K}{2}}(\mu,Q)w^{-1}(\mu,Q); \tag{2.7}$$

$$G^{\alpha\beta}=\frac{1}{v(\mu,Q)}\begin{pmatrix} f_{,a}f^{,a} & f^{,a}\\ f^{,a} & \gamma^{ab}\end{pmatrix};\quad \alpha,\beta=(0,a);\quad Q^0=\mu, \tag{2.8}$$

M is the measure in inner product, *K* is a number of physical degrees of freedom, *V* is a quantum correction to the potential *U*, that depends on the chosen parameterization and gauge [11]: The Schrödinger equation (2.5) gives *a gauge-dependent description of the Universe*. The general solution to the equation (2.5) is

$$\Psi(\mu,Q,\theta,\bar\theta;t)=\int\Psi_k(Q,t)\delta(\mu-f(Q)-k)(\bar\theta+i\theta)dk. \tag{2.9}$$

It can be interpreted in the spirit of Everett's "relative state" formulation: Each element of the superposition (2.9) describe a state in which the only gauge degree of freedom μ is definite, so that time scale is determined by processes in the physical subsystem through functions $v(\mu,Q)$, $f(Q)$, while the function $\Psi_k(Q,t)$ describes a state of the physical subsystem for a reference frame fixed by the condition (2.3). It is a solution to the Schrödinger equation with a gauge-dependent physical Hamiltonian $H_{(phys)}$:

$$i\frac{\partial\Psi_k(Q,t)}{\partial t}=H_{(phys)}\Psi_k(Q,t), \tag{2.10}$$

$$H_{(phys)}=\left[-\frac{1}{2M}\frac{\partial}{\partial Q^a}\frac{1}{v}M\gamma^{ab}\frac{\partial}{\partial Q^b}+\frac{1}{v}(U-V)\right]\Bigg|_{\mu=f(Q)+k} \tag{2.11}$$

Solutions to Eq. (2.10) make a basis in the Hilbert space of states of the physical subsystem:

$$H_{(phys)}\Psi_{kn}(Q)=E_n\Psi_{kn}(Q); \tag{2.12}$$

$$\Psi_k(Q,t)=\sum_n c_n\Psi_{kn}(Q)\exp(-iE_nt). \tag{2.13}$$

As one can see, the spectrum and eigenfunctions of the operator $H_{(phys)}$ will depend on a chosen gauge condition. The dependence of the measure in the physical subspace on this gauge results from the normalization condition for the wave function (2.9):

$$
\begin{aligned}
&\int \Psi^*\left(\mu, Q, \theta, \bar{\theta}; t\right) \Psi\left(\mu, Q', \theta, \bar{\theta}; t\right) M(\mu, Q) d\mu\, d\theta\, d\bar{\theta} \prod_a dQ^a = \\
&= \int \Psi_k^*(Q,t) \Psi_{k'}(Q,t) \delta(\mu - f(Q) - k) \delta(\mu - f(Q) - k') M(\mu, Q)\, dk\, dk'\, d\mu \prod_a dQ^a = \\
&= \int \Psi_k^*(Q,t) \Psi_{k'}(Q,t) M(f(Q) + k, Q)\, dk \prod_a dQ^a = 1.
\end{aligned}
\tag{2.14}
$$

Therefore, the whole structure of the physical Hilbert space is formed in a large degree by the chosen gauge condition (reference frame). *One cannot give a consistent quantum description of the Universe without fixing a certain reference frame, as well as one cannot find a solution to classical Einstein equations without imposing some gauge conditions*. The attempt to give a gauge invariant description of the Universe in the limits of the Wheeler – DeWitt quantum geometrodynamics was not successful, and the problem of Hilbert space is just the fact indicating that this theory has to be modified.

On the other hand, in the "extended phase space" approach we face another problem, that for every gauge condition we have its own Hilbert space. Is there any relation between state vectors in these Hilbert spaces, or between solutions to Schrödinger equations corresponding to various reference frames? How does the structure of Hilbert space change if one varies a gauge condition? We shall try to discuss these issues in the next sections.

3. Path integral and three kinds of gauge transformations

Let us consider a spacetime manifold *M*, which consists of several regions $R_1, R_2, R_3, \ldots$, in each of them various gauge conditions $C_1, C_2, C_3, \ldots$ being imposed. It is naturally to think that such regions exist in a universe with a non-trivial topology. Just for simplicity, one can assume that boundaries $S_1, S_2, \ldots$ between the regions are spacelike and can be labeled by some time variables $t_1, t_2, \ldots$ (Fig. 2).

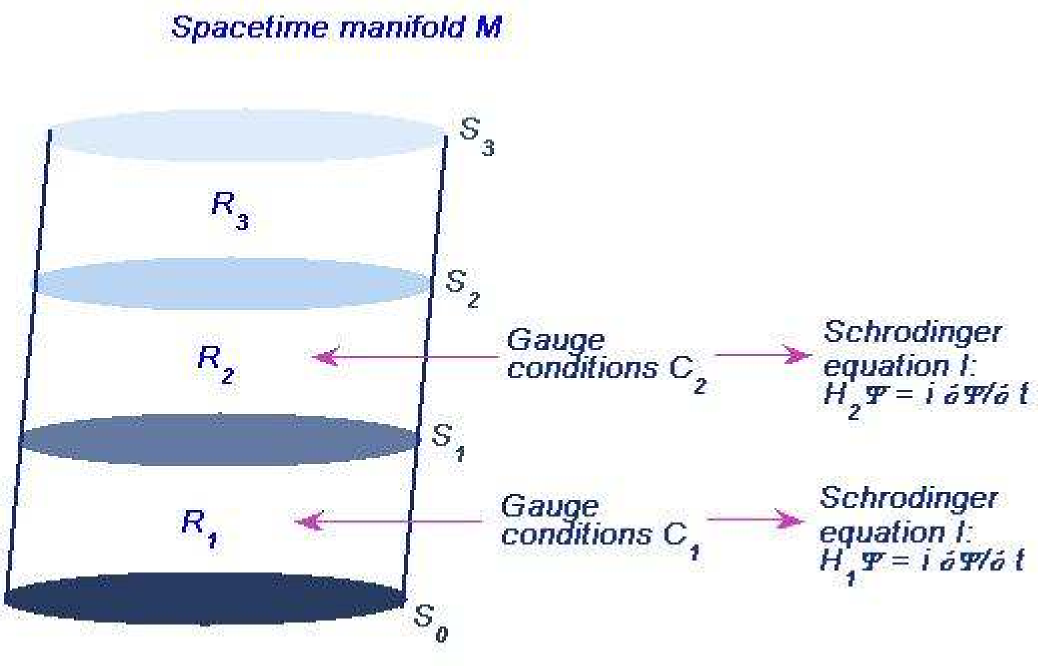

Fig. 2.

We would emphasize that the path integral approach allows us to examine this situation without any generalization of the formalism. The path integral over the manifold M is

$$
\begin{aligned}
&\int \exp(iS[g_{\mu\nu}])\prod_{x\in 3\sim} M[g_{\mu\nu}]\prod_{\mu,\nu} dg_{\mu\nu}(x) = \\
&= \int \exp(iS_{(eff)}[g_{\mu\nu}, C_1, 3\tilde{I}_1]) \prod_{x\in 3\tilde{I}_1} M[g_{\mu\nu}, 3\tilde{I}_1]\prod_{\mu,\nu} dg_{\mu\nu}(x)\times \\
&\times \exp(iS_{(eff)}[g_{\mu\nu}, C_2, 3\tilde{I}_2]) \prod_{x\in 3\tilde{I}_2} M[g_{\mu\nu}, 3\tilde{I}_2]\prod_{\mu,\nu} dg_{\mu\nu} \prod_{x\in 3|_{2}} M[g_{\mu\nu}, 3|_{1}]\prod_{\mu,\nu} dg_{\mu\nu}(x)\times\ldots
\end{aligned}
\tag{3.1}
$$

Here $S_{(eff)}[g_{\mu\nu}, C_1, 3\tilde{I}_1]$ is the effective action in the region R_1 with gauge conditions C_1, which includes gauge fixing and ghosts terms, etc.

From the viewpoint of gauge invariant approach, the path integral is not to depend on gauge conditions, in other words, we could write

$$
\begin{aligned}
&\int \exp(iS[g_{\mu\nu}])\prod_{x\in 3\sim} M[g_{\mu\nu}]\prod_{\mu,\nu} dg_{\mu\nu}(x) = \\
&= \int \langle g^{(0)}_{\mu\nu}, 3|_{0} | g^{(1)}_{\mu\nu}, 3|_{1}\rangle\langle g^{(1)}_{\mu\nu}, 3|_{1} | g^{(2)}_{\mu\nu}, 3|_{2}\rangle \prod_{x\in 3|_{2}} M[g_{\mu\nu}, 3|_{1}]\prod_{\mu,\nu} dg_{\mu\nu}(x)\times\ldots
\end{aligned}
\tag{3.2}
$$

In this case the initial state $|g^{(0)}_{\mu\nu}, 3|_{0}\rangle$, as well as intermediate states $|g^{(1)}_{\mu\nu}, 3|_{1}\rangle$, $|g^{(2)}_{\mu\nu}, 3|_{2}\rangle$, etc. are supposed to be gauge invariant (i.e. independent on ghosts and gauge conditions). This assumption would be justified only if all the states were asymptotic, but it cannot be true at least for the intermediate states. Moreover, the path integral (3.2) needs to be regularized, that implies imposing gauge condition on the surface S_1 ([10]; see also [15]). Eq. (3.2) is a generalization of the well-known quantum mechanical operation when one inserts "a full set of states" at some moment t_1. But in the present consideration we should bear in mind that the states in the regions R_1 and R_2 belong to different Hilbert spaces.

Within the region $\varnothing_1$ the evolution of the physical subsystem is determined by a unitary operator $\exp[-iH_{1(phys)}(t_1 - t_0)]$, where $H_{1(phis)}$ is a physical Hamiltonian in the region R_1 with gauge conditions C_1. Let at initial time t_0 on the surface S_0 the state of the system is given by a vector $|g^{(0)}_{\mu\nu}, 3|_{0}\rangle$. Then the state on the boundary S_1 reads

$$
|g^{(1)}_{\mu\nu}, 3|_{1}\rangle = \exp[-iH_{1(phys)}(t_1 - t_0)]\, |g^{(0)}_{\mu\nu}, 3|_{0}\rangle . \tag{3.3}
$$

However, if we gone from the region R_1 to R_2, we would find ourselves in another Hilbert space with a basis formed from eigenfunctions of the operator $H_{2(phis)}$. The transition to a new basis is not a unitary operation, as follows from the fact that a measure in the physical subspace depends on gauge conditions [11, 16] (in our minisuperspace model it is demonstrated by (2.14)). Denote the operation of the transition to a new basis in the region R_2 as $P(3|_{1}, t_1)$. Then the initial state in the region R_2 is

$$
P(3|_{1}, t_1)\exp[-iH_{1(phys)}(t_1 - t_0)]\, |g^{(0)}_{\mu\nu}, 3|_{0}\rangle \tag{3.4}
$$

and

$$
\begin{aligned}
|g^{(3)}_{\mu\nu}, 3|_{5}\rangle &= \exp[-iH_{3(phys)}(t_3 - t_2)]P(3|_{2}, t_2)\exp[-iH_{2(phys)}(t_2 - t_1)] \\
&\times P(3|_{1}, t_1)\exp[-iH_{1(phys)}(t_1 - t_0)]\, |g^{(0)}_{\mu\nu}, 3|_{0}\rangle .
\end{aligned}
\tag{3.5}
$$

So, *at any border S_i between regions with different gauge conditions unitary evolution is broken down. The operators $P(\Im_i, t_i)$ play the role of projection operators, which project states obtained by unitary evolution in a region R_i on a basis in Hilbert space in a neighbour region R_{i+1}.*

We now turn to different types of gauge transformations. It is conventionally believed that gauge conditions

$$F^{\mu}\left[g^{\lambda\rho}(x), \theta^{\nu}(x)\right] = 0 \tag{3.6}$$

should be chosen to fix completely gauge transformation parameters. Meanwhile, one knows that, in general, these conditions fix gauge parameters up to residual transformations satisfying the equations, which are consequence of (3.6):

$$\delta F^{\mu}\left[g^{\lambda\rho}(x), \theta^{\kappa}(x)\right] = 0 \quad \Rightarrow \quad A^{\mu}_{\nu}\theta^{\nu}(x) = \frac{\delta F^{\mu}}{\delta g^{\lambda\rho}}\frac{\delta g^{\lambda\rho}}{\delta\theta^{\nu}}\theta^{\nu}(x) = 0; \tag{3.7}$$

However, we should not worry about this kind of transformations since they do not change the conditions (3.6) and not affect the structure of Hilbert space.

More interesting are the transformations whose parameters can be related by homotopy. Consider two gauge conditions

$$F^{\mu}_1\left[g^{\lambda\rho}(x), \theta^{\nu}_1(x)\right] = 0, \quad F^{\mu}_2\left[g^{\lambda\rho}(x), \theta^{\nu}_2(x)\right] = 0, \tag{3.8}$$

fixing points on a gauge orbit in which a group element is parameterized by $\theta^{\nu}_1(x)$ and $\theta^{\nu}_2(x)$, correspondently. Let us assume that there exists a continuous function $L^{\nu}(r, x)$, so that

$$L^{\nu}(r, x): \quad L^{\nu}(0, x) = \theta^{\nu}_1(x), \quad L^{\nu}(1, x) = \theta^{\nu}_2(x), \tag{3.9}$$

or, more generally,

$$L^{\nu}(r, x): \quad L^{\nu}(r_1, x) = \theta^{\nu}_1(x), \quad L^{\nu}(r_2, x) = \theta^{\nu}_2(x). \tag{3.10}$$

One would say that $\theta^{\nu}_1(x)$ and $\theta^{\nu}_2(x)$ belong to the same homotopy class. Further, we could introduce a set of gauge conditions

$$F^{\mu}\left[g^{\lambda\rho}(x), \theta^{\nu}_r(x); r\right] = 0: \quad \theta^{\nu}_r(x) = L^{\nu}(r, x), \tag{3.11}$$

and identify *r* with a time variable *t*. Then, time-dependent conditions (3.11) could be interpreted as describing a smooth transition from the gauge $F^{\mu}_1\left[g^{\lambda\rho}(x), \theta^{\nu}_1(x)\right] = 0$ to $F^{\mu}_2\left[g^{\lambda\rho}(x), \theta^{\nu}_2(x)\right] = 0$. Our ability to impose the set of conditions (3.11) depends on the structure of group and related to the possibility of introducing some special coordinates in group space [2]. In our simple minisuperspace model before gauge fixing the action is invariant under one-parametric Abelian group of transformations

$$\delta t = \theta(t); \quad \delta\mu = w(\mu, Q)\dot{\theta} - \dot{\mu}\theta; \quad \delta Q^a = -\dot{Q}^a\theta, \tag{3.12}$$

so that any time-dependent gauge condition

$$\mu = f(Q, t) + k; \quad k = \text{const}, \tag{3.13}$$

would satisfy the above assumption.

Any canonical time-dependent gauge constrained physical variables and their momenta

$$\chi(Q, P, t) = 0 \tag{3.14}$$

can be reduced by Dirac-like procedure to the form similar to (3.13). In the canonical approach, choosing a simple parameterization $v(\mu, Q) = \frac{1}{\mu}$, one would find that the canonical Hamiltonian of the system *H* is proportional to the secondary constraint *T*:

$$H = \mu T = \mu\left[\frac{1}{2}P_a P^a + U(Q)\right]. \tag{3.15}$$

From the requirement of the conservation of (3.14) in time [17] one obtains

$$\frac{d\chi}{dt} = \frac{\partial\chi}{\partial t} + \mu\{\chi, T\} = 0; \tag{3.16}$$

$$\mu = -\frac{\partial \chi}{\partial t}\{\chi, T\}^{-1} = f(Q, P, t), \tag{3.17}$$

the letter can be presented in a differential form. We would like to emphasize here that, though quantization schemes using canonical time-dependent gauges (3.14) are believed to be equivalent to gauge invariant Dirac quantization [17], from the viewpoint of our approach imposing such gauge conditions implies gauge-dependent structure of physical Hilbert space.

The formalism developed in [8 – 10] can be generalized to gauge conditions explicitly depending on time. The pass integral approach includes some skeletonization procedure, which implies approximation of the gauge on each time interval $[t_i, t_{i+1}]$. In the simplest situation, we could assume that the change of gauge condition in each time interval is given by a function

$$\delta f_i(Q) = \alpha f_i(Q), \tag{3.18}$$

α is a small parameter, so that the gauge condition is a step-like function

$$\mu = f(Q) + \sum_i \alpha f_i(Q)\theta(t - t_i) + k \tag{3.19}$$

in the sense that in each interval $[t_n, t_{n+1}]$ the gauge condition does not depend on time:

$$[t_n, t_{n+1}]: \quad \mu = f(Q) + \sum_{i=0}^{n-1} \alpha f_i(Q) + \delta f_n(Q) + k\,. \tag{3.20}$$

Thus, we have come to the case of a small variation of gauge condition that was discussed in [16]. As was shown in [16], this small variation gives rise to additional terms in a physical Hamiltonian, these terms being non-Hermitian in respect to original physical subspace before variation. In our time-dependent case it means that at every moment of time we have a Hamiltonian, which acts, in its own "instantaneous" Hilbert space. The instantaneous Hamiltonian is a Hermitian operator at each moment, but one should think of it as a non-Hermitian operator in respect to a Hilbert space one had at a previous moment. The situation is different from what we have in ordinary quantum mechanics for a time-dependent Hamiltonian that acts at every moment in the same Hilbert space whose measure does not change in time. An analogy can be drawn between our situation and particle creation in nonstationary gravitational field when we also have an instantaneous Hamiltonian and instantaneous Fock basis [18].

Smooth changing of a gauge condition in time implies that solutions to the Schrödinger equation for physical part of wave function also change in a continuous and smooth manner. Another situation we face when gauge conditions in two regions fix gauge parameters which belong to different homotopy classes, and, as a rule, spacetime coordinates in these regions being related by a singular transformation. Then the gauge condition and the very form of the Schrödinger equation change in discrete steps when one passes from a space-time region with some gauge condition to a region with another gauge condition. This case is of the most interest from the viewpoint of changing the Hilbert space structure and the most difficult to treat. In any case, an initial state in a region R_i, resulting from its preceding evolution, should be written as a superposition of states in a new Hilbert space in R_i. There arise a number of questions, like: Will this superposition of states be stable? Could the breakdown of unitarity give rise to some kind of irreversibility? Could we define the change of entropy of the physical system when going to a region with different gauge conditions? Possible answers depend on a chosen model and require new non-perturbation methods.

4. Conclusion: the problem of wave function reduction and Quantum Gravity

As was pointed out by von Neumann [19], in quantum mechanics one deals with two different processes, namely, unitary evolution of a physical system in time described by the Schrödinger equation, and reduction of wave function of the physical system under observation. The whole evolution of the system can be presented by the formula

$$\begin{aligned} |\Psi(t_N)\rangle = U(t_N . t_{N-1})P(t_{N-1})U(t_{N-1}, t_{N-2})\ldots \times \\ \times \ldots U(t_3, t_2)P(t_2)U(t_2, t_1)P(t_1)U(t_1, t_0)|\Psi(t_0)\rangle \end{aligned} \tag{4.1}$$

where $P(t_i)$ are projection operators corresponding to observation at moments $t_1, t_2, t_3, \ldots, t_{N-1}$ (see, for example, [20]). There arises an analogy between the formulae (3.5) and (4.1): Indeed, we interpret any reference frame as a measuring instrument representing the observer in quantum geometrodynamics. Gauge conditions define interaction between the measuring instrument (reference frame) and the physical subsystem of the Universe. Changing the interaction with the measuring instrument makes us go to another basis in a Hilbert space and, even more, to another Hilbert space.

It enables us to hope to throw a new look to the central quantum mechanical problem of wave function reduction. Roger Penrose pointed out time and again that a solution of this problem, as well as understanding of irreversibility of physical processes, must be closely related with the progress in constructing quantum theory of gravity. In his books [12, 13] Penrose proposed a mechanism anticipating a choice among spacetime geometries, each of them corresponding to an element of quantum superposition. Details of the mechanism have not been elaborated enough, and this proposal was strongly criticized by Hawking [21]. However, the main idea that quantum gravity may help in deeper understanding of quantum mechanics seems to be fruitful. In our "extended phase space" approach we face the situation when the breakdown of unitary evolution of a physical system naturally follows from the very structure of the theory – we do not need to introduce "by hands" some special interaction, which would result in the breakdown of unitarity. In its turn, it is connected to the irreversibility of measuring processes. According to the opinion of another famous scientist, Ilya Prigogine, symmetric in time quantum dynamics described by the Schrödinger equation should be generalized to involve irreversible processes. To do it, one has to extend the class of admissible quantum operators beyond Hermitian operators and include non-unitary transformations of state vectors or density matrices ([14]; see also his Nobel prize lecture [22]). On the other side, in quantum mechanics one could examine models of interaction with a measuring instrument in which coordinates of a physical system are bound to coordinates of the instrument by means of some constraints, the latter ones are, in a sense, "gauge conditions" like those we have considered in our model with finite number degrees of freedom. Similar models of interaction with the instrument had been explored yet by von Neumann [19]. In future, some general points in quantum mechanical and quantum gravitational models of interaction may be revealed.

It may seem that there are more question than answers in this report. However, we have discussing physical interpretations of Relativity Theory already in a hundred years. So it is not surprisingly that attempts of its unification with quantum theory pose even more fundamental and intriguing questions, which have been waiting for their resolution.

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Fermion generations from a 6D brane world model

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We study the motion of higher dimensional fermions in a non-singular 6D brane background. By adjusting one parameter of the brane world potential it is possible to obtain three normalizable zero mass modes giving a higher dimensional picture of the fermion generation puzzle. The three different zero mass modes correspond to different values of the angular momentum eigenvalue with respect to the two extra dimensions. Thus the family number corresponds to the higher dimensional angular momentum. The three different normalizable spinor fields are located at different points in the brane and have different shapes. By introducing a Yukawa coupling between the fermions and a scalar field it is suggested how this difference could give rise to the mass hierarchy and mixings between different generations.

I. INTRODUCTION

In the Standard Model of particle physics [1] there are open questions which have not yet found an answer. Chief among these is the fermion family puzzle as to why the first generation of quarks and leptons (up quark, down quark, electron and electron neutrino) are replicated in two families or generations of increasing mass (the second generation consisting of charm quark, strange quark, muon and muon neutrino; the third generation consisting of top quark , bottom quark, tau and tau neutrino) In addition to explaining why there are heavier copies of the first generation fermions one would like to explain the mass hierarchy of the generations as well as the mixings between the generations which are characterized by the CKM matrix. Several ideas have been suggested such as a horizontal family symmetry [2].

Recently theories with extra dimensions have been used in a novel way to try and explain some of the open questions in particle physics and cosmology. In [3] [4] [5] the hierarchy problem (*i.e.* why the gravitational interactions are many orders of magnitude weaker than the interactions of particle physics) was addressed using large or infinite extra dimensions. Early versions of these extra dimensional models were investigated by several researchers [6] [7] [8] [9]. In contrast to the usual Kaluza-Klein picture, these recent models involve large or infinite extra dimensions with gravity acting in all the spacetime dimensions, while the other particles and fields are confined, up to some energy scale, to the $3 + 1$ dimensional brane of the observed universe. These recent extra dimensional models have also been applied to answer other questions of particle physics and cosmology. In [10] [11] [12] brane world models were constructed to explain the hierarchy of masses and/or the CKM elements of the fermions. In [13] [14] brane world models were used to explain dark energy and dark matter. General studies of such higher dimensional cosmologies can be found in [15] [16].

In this paper we attempt to address the generation problem using a brane world model in 6D. We will show how one can obtain 3 fermion families from the zero modes of a single higher dimensional spinor field. By introducing a Yukawa type interaction between the spinor field and a scalar field we will give a qualitative sketch of how a mass hierarchy and the CKM mixing elements arise.

II. 6D GRAVITATIONAL BACKGROUND

In [17] [18] [19] [20] a 6D brane world model was investigated which gave universal gravitational trapping of fields of spins $0, 1/2, 1, 2$, to the brane. The system considered was 6D gravity with a cosmological constant and some matter field energy-momentum. The action for this system was

$$S = \int d^6x \sqrt{-^6g} \left[\frac{M^4}{2} (^6R + 2\Lambda) +^6 L \right] \ , \tag{1}$$

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where $\sqrt{-^6g}$ is the determinant, M is the fundamental scale, ^6R is the scalar curvature, Λ is the cosmological constant and ^6L is the Lagrangian of the matter fields. All of these quantities are six dimensional. The ansatz for the 6D metric was taken as

$$ds^2 = \phi^2(r)\eta_{\alpha\beta}(x^\nu)dx^\alpha dx^\beta - \lambda(r)(dr^2 + r^2 d\theta^2) \ , \tag{2}$$

where the Greek indices $\alpha, \beta, ... = 0, 1, 2, 3$ refer to 4-dimensional coordinates. The metric of ordinary 4-space, $\eta_{\alpha\beta}(x^\nu)$, has the signature $(+, -, -, -)$. The functions $\phi(r)$ and $\lambda(r)$ depend only on the extra radial coordinate, r, and thus are cylindrically symmetric in the transverse polar coordinates ($0 \leq r < \infty$, $0 \leq \theta < 2\pi$). The ansatz for the energy-momentum tensor of the matter fields was taken to have the form

$$T_{\mu\nu} = -g_{\mu\nu}F(r), \qquad T_{ij} = -g_{ij}K(r), \qquad T_{i\mu} = 0 \ . \tag{3}$$

Other than satisfying energy-momentum conservation *i.e.*

$$\nabla^A T_{AB} = \frac{1}{\sqrt{-^6g}}\partial_A(\sqrt{-^6g}T^{AB}) + \Gamma^B_{CD}T^{CD} = K' + 4\frac{\phi'}{\phi}(K - F) = 0 \ , \tag{4}$$

the energy-momentum tensor was unrestricted, although it was desirable for it to satisfy physical requirements such as being everywhere finite, and being peaked near the brane.

In [18] [19] it was found that the above system had the following non-singular solution

$$\phi(r) = \frac{c^b + ar^b}{c^b + r^b} \ , \tag{5}$$

where a, b, c are constants and $a > 1$. All other ansatz functions were given in terms of $\phi(r)$.

$$\lambda(r) = \frac{\rho^2\phi'}{r} = \frac{(a-1)bc^b\rho^2 r^{b-2}}{(c^b + r^b)^2} \ , \tag{6}$$

where ρ is an integration constant with units of length, which was related to the constants a and b by

$$\frac{\rho^2\Lambda}{10M^4} = \frac{b}{a-1} \ . \tag{7}$$

The source functions also where given in terms of $\phi(r)$.

$$F(r) = \frac{f_1}{2\phi^2} + \frac{3f_2}{4\phi} \ , \qquad K(r) = \frac{f_1}{\phi^2} + \frac{f_2}{\phi} \ , \tag{8}$$

where f_1 and f_2 are constants given by

$$f_1 = -\frac{3\Lambda}{5}a \ , \qquad f_2 = \frac{4\Lambda}{5}(a+1) \ , \tag{9}$$

In [18] and [19] it was shown that for $b = 2$ this solution gave a universal, gravitational trapping for fields with spins $0, 1/2, 1, 2$ within some small distance ϵ of $r = 0$. Physically ϵ is the brane width.

For the $b \geq 2$ case one can see from (6) that the scale factor for the extra dimensions, $\lambda(r) = 0$ at $r = 0$. This raised the possibility of having a singularity on the brane, making the solutions with $b \geq 2$ unphysical. However, if one investigates invariants such as the Ricci scalar, R, one finds that they are non-singular at $r = 0$. This indicates that the zero of $\lambda(r)$ at $r = 0$ for $b > 2$ solutions is not a physical singularity. The Ricci scalar for the above solution is

$$R = \frac{2b(-5c^{2b} + 4ac^{2b} + 10c^b r^b - 22ac^b r^b + 10a^2c^b r^b + 4ar^{2b} - 5a^2r^{2b})}{(a-1)\rho^2(c^b + ar^b)^2} \tag{10}$$

It is easy to see that this is finite at $r = 0$. Other invariants such as the fully contracted Riemann tensor, $R_{AB}{}^{CD}R_{CD}{}^{AB}$, or the square of the Ricci tensor, $R_{AB}R^{AB}$ also turn out to be finite at $r = 0$. Thus we take the zero in the scale factor $\lambda(r)$ to be a coordinate rather than physical singularity. Note that $\lambda(r)$ goes to zero both at $r = 0$ and $r = \infty$ so that the metric (2) essentially becomes 4D at these locations. Thus the solution given in (5) is like the 2 brane model of [4] where two 4D branes sandwich the higher dimensional, bulk spacetime.

III. FERMIONS IN THE GRAVITATIONAL BACKGROUND

We now want to study the motion of fermion fields in the gravitational background given by (2). By making the identification that different fermion zero-mass modes (*i.e.* solutions for which the 4D part of the fermion wavefunction satisfies $\gamma_\mu \partial^\mu \psi(x_\nu) = 0$) correspond to different families, we want to see if it is possible to obtain three zero modes.

In [20] Maziashvili investigated the motion of fermions in the 6D brane solution of (2) with $b = 2$. It was found that in this case only one zero mode occurred, and thus only one family. Based on the arguments of the previous section we consider the $b > 2$ case and show that for a certain range of b it is possible to get three zero modes. The constant b controls the steepness of the scale functions $\phi(r)$ and $\lambda(r)$ thus it is reasonable that larger b should lead to a stronger confinement of the fermions to the vicinity of the brane at $r = 0$, and to a larger number of zero modes.

The 6D action and resulting equations of motion for a spinor field are

$$S_\Psi = \int d^6x \sqrt{-^6g}\bar{\Psi} i \Gamma^A D_A \Psi \quad , \qquad \Gamma^A D_A \Psi = \Gamma^\mu D_\mu \Psi + \Gamma^r D_r \Psi + \Gamma^\theta D_\theta \Psi = 0 \tag{11}$$

In the above $\Gamma^A = h^A_{\bar{B}} \gamma^B$ are the 6D curved spacetime gamma matrices, and $h^A_{\bar{B}}$ are the sechsbiens defined via $g_{AB} = h^{\bar{A}}_A h^{\bar{B}}_B \eta_{\bar{A}\bar{B}}$. In order to evaluate the 6D Dirac equation in (11) we need to calculated the spin connections

$$\omega_M^{\bar{M}\bar{N}} = \frac{1}{2} h^{N\bar{M}} (\partial_M h_N^{\bar{N}} - \partial_N h_M^{\bar{N}}) - \frac{1}{2} h^{N\bar{N}} (\partial_M h_N^{\bar{M}} - \partial_N h_M^{\bar{M}}) - \frac{1}{2} h^{P\bar{M}} h^{Q\bar{N}} (\partial_P h_{Q\bar{R}} - \partial_Q h_{P\bar{R}}) h_M^{\bar{R}} \ . \tag{12}$$

The non-zero spin connections are

$$\omega_\mu^{\bar{r}\bar{\nu}} = \delta_\mu^{\bar{\nu}} \frac{\sqrt{r\phi'}}{\rho} \ , \qquad \omega_\theta^{\bar{r}\bar{\theta}} = \sqrt{\frac{r}{\phi'}} \partial_r \left(\sqrt{r\phi'}\right) \ . \tag{13}$$

With these one can explicitly calculate the various covariant derivatives in (11)

$$D_\mu \Psi = \left(\partial_\mu + \frac{1}{2} \omega_\mu^{\bar{r}\bar{\nu}} \gamma_r \gamma_\nu \right) \Psi \ , \qquad D_r \Psi = \partial_r \Psi \ , \qquad D_\theta \Psi = \left(\partial_\theta + \frac{1}{2} \omega_\theta^{\bar{r}\bar{\theta}} \gamma_r \gamma_\theta \right) \Psi \ . \tag{14}$$

We now assume that the 6D fermion spinor can be decomposed as $\Psi(x^A) = \psi(x_\mu) \otimes \zeta(r, \theta)$ into 4D and 2D parts. We are interested in the zero-mass modes so the 4D fermion part is taken to satisfy $\gamma_\mu \partial^\mu \psi(x_\nu) = 0$. The 2D spinor can be expanded as

$$\zeta(r, \theta) = \zeta_l(r) e^{il\theta} \tag{15}$$

where $\zeta_l(r) = (f_l(r), g_l(r))$ is a two-component spinor. We take the gamma matrices of the extra space as

$$\gamma^r = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^\theta = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{16}$$

Combining equations (11) - (16) we arrive at the following equations for $f_l(r)$ and $g_l(r)$

$$\left[\partial_r + 2\frac{\phi'}{\phi} + \frac{1}{2} \frac{\partial_r \left(\sqrt{r\phi'}\right)}{\sqrt{r\phi'}} + \frac{l}{r}\right] g_l(r) = 0 \ , \qquad \left[\partial_r + 2\frac{\phi'}{\phi} + \frac{1}{2} \frac{\partial_r \left(\sqrt{r\phi'}\right)}{\sqrt{r\phi'}} - \frac{l}{r}\right] f_l(r) = 0 \tag{17}$$

The solutions for $f_l(r)$ and $g_l(r)$ are

$$\begin{aligned} f_l(r) &= a_l \phi(r)^{-2} (r\phi'(r))^{-\frac{1}{4}} r^l = a_l \frac{r^l (c^b + ar^b)^2}{((a-1)bc^b r^b)^{\frac{1}{4}} (c^b + r^b)^{\frac{3}{2}}}, \\ g_l(r) &= b_l \phi(r)^{-2} (r\phi'(r))^{-\frac{1}{4}} r^{-l} = b_l \frac{r^{-l} (c^b + ar^b)^2}{((a-1)bc^b r^b)^{\frac{1}{4}} (c^b + r^b)^{\frac{3}{2}}} \end{aligned} \tag{18}$$

where we have explicitly inserted the solution for $\phi(r)$ from (5). In order for the fermion fields to be trapped they should be normalizable with respect to the extra dimensions r, θ

$$1 = \int \sqrt{-^6g} \bar{\zeta}(r, \theta) \zeta(r, \theta) dr d\theta = 2\pi\rho^2 \int_0^\infty dr \sqrt{\frac{\phi'}{r}} (a_l^2 r^{2l} + b_l^2 r^{-2l}) \tag{19}$$

The explicit expression for $\sqrt{\phi'/r}$ can be read off from (6). From this one finds that $\sqrt{\phi'/r} \propto r^{\frac{b}{2}-1}(c^b + r^b)^{-1}$. Thus in order for (19) to be normalizable and for the particular fermion l-mode to be trapped, we want the integral

$$\int_0^\infty \frac{r^{\pm 2l + \frac{b}{2} - 1}}{c^b + r^b} dr \ , \tag{20}$$

to be finite. If (20) diverges the particular l-mode will not be trapped. This requirement that (20) be finite leads to restrictions on b for particular values of l. Using Mathematica to evaluate (20) gives

$$\frac{c^{\frac{-b}{2}+2l}\pi \sec\left(\frac{2l\pi}{b}\right)}{b} \qquad \text{if} \qquad b > 4|l|, \tag{21}$$

and (20) diverges if $b \leq 4|l|$. Thus in order to have three normalizable l-modes we require that $4 < b \leq 8$. Under these conditions the $l = 0$, and $|l| = 1$ modes are normalized and trapped, while $|l| \geq 2$ modes are not. Since the integrand in (20) is positive definite and only has possible divergences at $r = 0$ and $r = \infty$ one can come to this conclusion by investigating the $r \to 0$ and $r \to \infty$ behavior of this integrand. For $|l| = 1$ one finds that for $l = +1$ the integrand behaves as $\underset{r\to 0}{lim} \simeq r^{1+\frac{b}{2}}$, $\underset{r\to\infty}{lim} \simeq r^{1-\frac{b}{2}}$; for $l = -1$, it behaves as $\underset{r\to 0}{lim} \simeq r^{-3+\frac{b}{2}}$, $\underset{r\to\infty}{lim} \simeq r^{-3-\frac{b}{2}}$. The $r \to 0$ limit of $l = +1$ and $r \to \infty$ limit of $l = -1$ give convergent results. On the other hand the $r \to 0$ limit of $l = -1$ and $r \to \infty$ limit of $l = +1$ give convergent results only if $b > 4$. One can see the for $b > 4$ the $l = 0$ mode is normalized. For $|l| = 2$ one finds that for $l = +2$ the integrand behaves as $\underset{r\to 0}{lim} \simeq r^{3+\frac{b}{2}}$, $\underset{r\to\infty}{lim} \simeq r^{3-\frac{b}{2}}$; for $l = -2$, it behaves as $\underset{r\to 0}{lim} \simeq r^{-5+\frac{b}{2}}$, $\underset{r\to\infty}{lim} \simeq r^{-5-\frac{b}{2}}$. The $l = +2$ integral diverges at $r \to \infty$ and the $l = -2$ diverges at $r \to 0$ if $b \leq 8$. This analysis also shows that one has three normalizable modes (*i.e.* three fermion families) when the $4 < b \leq 8$.

One could also consider using the criteria that the fermion action be finite when integrated over the extra dimensions.

$$S_\Psi = \int d^6x\sqrt{-^6g}\bar{\Psi}i\Gamma^A D_A \Psi = 2\pi\rho^2 \int_0^\infty dr \frac{1}{\phi}\sqrt{\frac{\phi'}{r}}(a_l^2 r^{2l} + b_l^2 r^{-2l}) \int d^4x\sqrt{-\eta}\bar{\psi}i\gamma^\nu\partial_\nu\psi \tag{22}$$

The fermions are trapped if the integral over r in the last expression is convergent. This integral is almost the same as the last integral in (19). It differs only by a factor of $1/\phi$ which comes from the sechsbien that modifies the gamma matrices, γ^ν. The explicit expression for $\phi^{-1}\sqrt{\phi'/r}$ can be read off from (6) and (5). From this one finds that $\phi^{-1}\sqrt{\phi'/r} \propto r^{\frac{b}{2}-1}(c^b + ar^b)^{-1}$. The only change with respect to the normalization condition is that $r^b \to ar^b$ in the denominator. Thus the integral of the action over the extra coordinates will have the same convergence properties as the normalization condition, thus giving the same conclusion that three zero-mass modes will be trapped if $4 < b \leq 8$.

The existence of three zero modes depends crucially on two things: (i) allowing the exponent in (5) to satisfy $b > 2$ and (ii) taking the spacetime to be 6D. From the above analysis and also from [20] one can see that when $b = 2$ in (5) only one zero-mass mode occurs. In regard to point (ii) it was shown in [21] that the solution of (5) (6) (8) could be generalized to spacetimes of dimension greater than 6D, but for $> 6D$ the exponent in (5) was fixed as $b = 2$ leading to only one zero mass mode.

IV. MIXING ANGLES AND MASSES

By adjusting the exponent b in our gravitational background solution we have three zero mass modes which can be taken as the three generations of fermions. However there are two problems. First there is no mixing between the different generations due to the orthogonality of the angular parts of the higher dimensional wave functions. Overlap integrals like $\int_0^\infty \int_0^{2\pi} \bar{\zeta}_l \zeta_m dr d\theta$, which characterize the mixing between the different states, vanish since $\int_0^{2\pi} e^{-il\theta}e^{im\theta}d\theta = 0$ if $l \neq m$. The second short coming is that all our states are massless, whereas the fermions of the real world have masses that increase with each succeeding family. Both of these issues can be addressed by introducing a Yukawa coupling between the 6D fermions and a 6D scalar field of the form $H_p(x^A)\bar{\Psi}_l(x^B)\Psi_{l'}(x^C)$. This adds to the action a term of the form

$$S_{mix} = f\int d^4x dr d\theta \sqrt{-^6g} H_p \bar{\Psi}_l \Psi_{l'} \tag{23}$$

where f is some constant. Physically f is the Yukawa interaction coupling strength. Then taking the scalar field to have the form [11]

$$H_p(x^A) = H_p(r)e^{ip\theta} \tag{24}$$

(23) reduces to

$$S_{mix} = U_{ll'} \int d^4x \bar{\psi}_l(x^\mu)\psi_{l'}(x^\mu) \qquad \text{where} \qquad U_{ll'} = f \int dr d\theta \sqrt{-^6g} H_p(r) \bar{\zeta}_l(r,\theta)\zeta_{l'}(r,\theta) \tag{25}$$

$U_{ll'}$ will be non-zero when $p - l + l' = 0$. When $l = l'$ this is a mass term and when $l \neq l'$ this is a mixing term between the l and l' modes. One obtains a scalar field of the form in (24) if one breaks the U(1) symmetry of the extra dimensional space as in [22]. One also obtains a scalar field of the form (24) by looking at the propagation of a test scalar field in the background given by (5) (6).

To get explicit results for $U_{ll'}$ one needs an explicit form for $H_p(r)$. This can be done by solving the field equations for a test scalar field in the background given by $\phi(r)$ and $\lambda(r)$ for the different $p-$modes. A more interesting possibility is that one could replace the phenomenological matter sources given by $F(r)$ and $K(r)$ by a scalar field source which forms the brane as in [23]. This scalar field would serve the dual purpose of being the matter source that forms the brane and also provides the Yukawa coupling to the fermions that results in the masses and mixings. Here we confine ourselves to making two simple estimates by taking $H_p(r)$ to be approximately constant (*i.e.* $H_p(r) \approx$ const.) or sharply peaked at some distance from the brane (*i.e.* $H_p(r) \approx \delta(r - r_0)$). Also to simplify the analysis we take $a_l = 1$ and $b_l = 0$ so that the two-spinor takes the form $\zeta_l(r) = (f_l(r), 0)$. Looking at the case when $l = l'$ first (*i.e.* when $U_{ll'}$ represents a mass term) we find for $H_p(r) \approx \text{const.} = h$

$$U_{ll} = fh \int dr d\theta \sqrt{-^6g} \bar{\zeta}_l(r,\theta)\zeta_l(r,\theta) = K \int_0^\infty \frac{r^{2l+\frac{b}{2}-1}}{c^b + r^b} dr = K' c^{2l-\frac{b}{2}} \sec\left(\frac{2l\pi}{b}\right) \tag{26}$$

where we have successively put together constants into first K and then K'. Taking b in the range $4 < b \leq 8$ and $c > 1$ one finds that $U_{-1,-1} < U_{0,0} < U_{+1,+1}$ so that one has an increasing hierarchy of masses from lowest mass at $l = -1$ to highest mass at $l = +1$. If one takes instead $c < 1$ one gets $U_{-1,-1} > U_{0,0} > U_{+1,+1}$ so that one has an increasing hierarchy of masses from lowest at $l = +1$ to highest at $l = -1$.

If one takes instead the scalar field to have the form $H_p(r) \approx \delta(r - r_0)$ then one finds

$$U_{ll} = f \int dr d\theta \sqrt{-^6g} \bar{\zeta}_l(r,\theta)\zeta_l(r,\theta) = K \int_0^\infty \delta(r - r_0) \frac{r^{2l+\frac{b}{2}-1}}{c^b + r^b} dr = K' \frac{r_0^{2l+\frac{b}{2}-1}}{c^b + r_0^b}. \tag{27}$$

Various constants have been collected first as K and then K'. The direction of the hierarchy of the masses now depends on the value of r_0. Again taking $4 < b \leq 8$ one can see that for $r_0 > 1$ one has $U_{-1,-1} < U_{0,0} < U_{+1,+1}$, while for $r_0 < 1$ the hierarchy is reversed to the form $U_{-1,-1} > U_{0,0} > U_{+1,+1}$.

A similar analysis can be carried out with the mixings between the different "families" characterized by different l number. The mixings are delineated by $U_{0,1}, U_{1,0}, U_{1,-1}, U_{-1,1}, U_{0,-1}$ and $U_{-1,0}$. In the case of mixings the scalar field must have a non-zero angular eigenvalue (*i.e.* $H_p(r,\theta) = H_p(r)e^{ip\theta}$ with $p \neq 0$) which satisfies $p - l + l' = 0$. Thus for $U_{-1,0}$ and $U_{0,1}$ one needs $p = -1$; for $U_{1,0}$ and $U_{0,-1}$ one needs $p = 1$; for $U_{1,-1}$ one needs $p = 2$; for $U_{-1,1}$ one needs $p = -2$. We will take $H_p(r)$ to depend only on $|p|$ (*e.g.* $H_1(r) = H_{-1}(r)$) so that $U_{0,1} = U_{1,0}$, $U_{1,-1} = U_{-1,1}$ and $U_{0,-1} = U_{-1,0}$. For the assumption that $H_p(r) \approx \text{const.} = h_{|p|}$ one finds

$$\begin{aligned}
U_{0,1} &= U_{1,0} = K \int_0^\infty \sqrt{\frac{\phi'}{r}} r dr = K' h_1 c^{-\frac{b}{2}+1} \sec\left(\frac{\pi}{b}\right) \\
U_{1,-1} &= U_{-1,1} = K \int_0^\infty \sqrt{\frac{\phi'}{r}} dr = K' h_2 c^{-1} \\
U_{0,-1} &= U_{-1,0} = K \int_0^\infty \sqrt{\frac{\phi'}{r}} r^{-1} dr = K' h_1 c^{-\frac{b}{2}-1} \sec\left(\frac{\pi}{b}\right)
\end{aligned} \tag{28}$$

where various common constants such as, 2π, ρ^2, f have been combined successively as K and K'. The constant scalar field, $H_p(r) \approx \text{const.} = h_{|p|}$, has been written separately since it could in general be different for different

$|p|$. Under the assumption that the scalar field has the form $H_p(r) \approx \delta(r - r_p)$ (r_p represents the distance at which the scalar field with angular eigenvalue $|p|$ peaks) the mixing elements in (28) become

$$
\begin{aligned}
U_{0,1} &= U_{1,0} = K \int_0^\infty \sqrt{\frac{\phi'}{r}} r dr = K' \frac{(r_1)^{\frac{b}{2}}}{c^b + r_1^b} \\
U_{1,-1} &= U_{-1,1} = K \int_0^\infty \sqrt{\frac{\phi'}{r}} dr = K' \frac{(r_2)^{\frac{b}{2}-1}}{c^b + r_2^b} \\
U_{0,-1} &= U_{-1,0} = K \int_0^\infty \sqrt{\frac{\phi'}{r}} r^{-1} dr = K' \frac{(r_1)^{\frac{b}{2}-2}}{c^b + r_1^b}
\end{aligned} \tag{29}
$$

The common constants have again been combined in K and K'. By adjusting the various free constants in (28) (c, b, h_1 and h_2) or (29) (c, b, r_1 and r_2) one can make a comparison between the ratios of the $U_{i,j}$'s and ratios of the CKM matrix elements, V_{ij}.

V. SUMMARY AND CONCLUSIONS

In this paper we studied the field equations of fermions in the background of the non-singular, 6D brane solution of [17] [19]. By allowing the exponent, b, in the 4D scale function, $\phi(r)$, to take values $b > 2$ we found that we could get multiple zero mass modes which could be identified with the different fermion generations. In particular for $4 < b \leq 8$ we obtained three zero mass modes corresponding to different l eigenvalues: $l = -1, 0, 1$. When one fixes the value of $b = 2$ as in [17] one has only one zero mode [20]. Also for spacetime dimensions $> 6D$ one does not have the freedom to choose b – it is fixed as $b = 2$ [21]. For $b > 2$ the 2D scale function, λ has a zero both at $r = 0$ and $r = \infty$. However the scalar invariants such as the Ricci scalar are well behaved and non-singular over the entire range of r, indicating these points are coordinate rather than physical singularities.

By introducing a scalar field with a Yukawa coupling to the fermions one can generate both masses and mixings between the different generations. By looking at some example profiles for the scalar field it was shown that one could obtain a mass hierarchy of the fermions. The order in which the hierarchy ran (*i.e.* lowest to highest as $l = -1$ to $l = +1$ or as $l = +1$ to $l = -1$) depended on how the parameters of the 4D scale functions, $\phi(r)$, and the scalar field, $H_p(r)$, were chosen. In this brane world picture of the family puzzle the eigenvalue l played the role of the family number.

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Vacuum and Dark Energy

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A problem of vacuum energy in cosmology is reviewed. Astronomical data inidicating to a non-zero antigravitating dark energy are discussed. Possible ways to resolve 100 orders of magnitude discrepancy between theory and observation are described. Adjustment mechnism is discussed in some detail.

TWISTOR ALGEBRAIC DYNAMICS IN COMPLEX SPACE-TIME AND PHYSICAL MEANING OF HIDDEN DIMENSIONS

Vladimir V. Kassandrov [1]

Abstract. We review the algebraic field theory based completely on a nonlinear generalization of CR complex analiticity equations to the noncommutative algebra of biquaternions or, equivalently, on the structure of shear-free null congruences. Then we develop the algebrodynamical scheme on the complex extension $\mathbb{C}\mathbf{M}$ of Minkowski space-time – full vector space of biquaternion algebra. This primodial space dynamically reduces to the 6D "observable" space-time of complex null cone which naturally decomposes into the 4D physical space-time and 2D internal "spin space". A set of identical point charges ("duplicons") – focal points of the congruence – arises in the procedure. Time dynamics of duplicons is strongly correlated via fundamental twistor field of the congruence. The notions of "complex time" and of "evolution curve" necessary for the considered scheme and their probable relation to quantum uncertainty is briefly discussed.

Резюме. Представлена алгебраическая теория поля, целиком основанная на нелинейном обобщении условий комплексной аналитичности Коши-Римана на некоммутативную алгебру бикватернионов или, эквивалентно, на структуре бессдвиговых изотропных конгруенций. В работе эта алгебродинамическая схема развивается на комплексном расширении $\mathbb{C}\mathbf{M}$ пространства-времени Минковского – полном векторном пространстве алгебры бикватернионов. Исходное пространство динамически редуцируется к 6-мерному "наблюдаемому" пространству-времени комплексного изотропного конуса, которое в свою очередь разлагается на 4-мерное физическое пространство-время и 2-мерное внутреннее "спиновое пространство". При этом естественно возникает ансамбль тождественных точечных зарядов ("дубликонов") – фокальных точек конгруенции. Временная динамика дубликонов сильно коррелирована между собой через фундаментальное твисторное поле конгруенции. Кратко обсуждается с необходимостью возникающие в рассматриваемой алгебродинамической схеме понятия "комплексного времени" и "кривой эволюции", а также их возможная связь с квантовой неопределенностью.

1 Introduction

In the article we proceed to develop the *algebrodynamical* field theory presented, in particular, in [1]. We recall that in the paradigm of algebrodynamics (AD) one considers, in the spirit of Pithagorean philosophy, all fundamental physical laws and phenomena as manifestations of some basic abstract "World" structure – *the Code of the Universe* – which might be an exceptional group (e.g., the *Monster group*), algebra or geometry. The whole structure and evolution of Universe, the cathegories of time, particle, field, motion and interaction should be all predetermined and encoded in internal properties of the World structure. The known physical laws are nothing but separated *fragments* of this structure, and one could try to *forget* them all and to read *unprejudicely* the "Book of Nature" which could bring us to a grand and magestic picture of physical World quite different from that accepted in the modern theoretical physics.

In the main version of AD developed by the author (see, e.g., [2, 3, 4, 5, 6] and references therein) exceptional algebra of *quaternions* (precisely, of their complexification – *biquaternions* $\mathbb{B}$) is considered as the *World algebra*. Basic physical fields are then $\mathbb{B}$-valued functions

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of $\mathbb{B}$-variable, and field equations are nothing but the *conditions of* $\mathbb{B}$*-differentiability*, i.e. the *generalized Cauchy-Riemann equations* (GCRE). These have been introduced by the author in 1980 (see in detail [2, 3, 7, 8, 9]) and have the following invariant form:

$$dF = \Phi * dZ * \Psi, \tag{1.1}$$

where $F : \mathbb{B} \mapsto \mathbb{B}$ is a $\mathbb{B}$-differentiable function $F(Z)$ of $\mathbb{B}$-variable $Z \in \mathbb{B}$; $\Phi, \Psi : \mathbb{B} \mapsto \mathbb{B}$ are two auxiliary functions (left and right "semiderivatives") correspondent to F, and $(*)$ denotes multiplication in $\mathbb{B}$ isomorphic to the full 2×2 complex matrix algebra.

As a direct consequence of (1.1), any matrix component of $\mathbb{B}$-field $f = \{F_{AB}\},\ A, B = 1, 2$ obeys the determinant equation

$$\det \| \frac{\partial f}{\partial Z_{AB}} \| = 0 \tag{1.2}$$

which, for the case of noncommutative $\mathbb{B}$-algebra, is the *nonlinear* analogue of Laplace *harmonicity* equation in complex analysis. If one restricts the coordinates Z onto the *Minkowski* subspace $\mathbf{M}$ of the full $4\mathbb{C}$ vector space of $\mathbb{B}$-algebra represented by Hermitian matrices $X = X^+$,

$$Z \mapsto X = \begin{pmatrix} u & w \\ p & v \end{pmatrix} \tag{1.3}$$

where $u = ct + z,\ v = ct - z$ are real, $w = x - iy,\ p = x + iy$ – complex conjugated and x, y, z, t – Cartezian and time coordinates respectively, then the GCRE read as follows:

$$dF = \Phi * dX * \Psi, \tag{1.4}$$

and fundamental equation (1.2) takes the form

$$\frac{\partial f}{\partial u}\frac{\partial f}{\partial v} - \frac{\partial f}{\partial w}\frac{\partial f}{\partial p} \equiv \frac{1}{c^2}\left(\frac{\partial f}{\partial t}\right)^2 - \left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial f}{\partial z}\right)^2 = 0 \tag{1.5}$$

in which one easily recognizes the *complex eikonal equation* (CEE) [2].

On Minkowski coordinate background the theory turns to be Lorentz invariant, and one can construct an "algebraic" field theory where nonlinear complex eikonal is the basic *self-interacting* physical field.

Principles and results of (nonlinear, non-Lagrangian) algebraic $\mathbb{B}$-field theory have been presented in a number of papers (see, e.g., [3, 10, 8, 9, 11, 12, 1, 13, 6] and other papers). The GCRE and the related CEE were found to possess twistor (2-spinor) and restricted gauge structures. Moreover, *the integrability conditions for (1.4) result in identical satisfaction of (complexified) free Maxwell and Yang-Mills equations.*

In our paper [12] Lorentz invariant form of *general solution* to CEE has been found on the base of its (ambi)twistor structure. Precisely, any (almost everywhere analytical) solution of CEE can be obtained from some generating function

$$\Pi(G, \tau^1, \tau^2) \equiv \Pi(G, wG + u, vG + p) \tag{1.6}$$

of three complex variables – components of projective (null) twistor related to the points of space-time $X \in \mathbf{M}$ via the Penrose *incidence condition* [14]

$$\tau = X\xi \quad (\tau^A = X^{AA'}\xi_{A'}), \tag{1.7}$$

from which, under the choice of the gauge $\xi^T_{A'} = \{1, G\}$, one has the constraints

$$\tau^1 = wG + u, \quad \tau^2 = vG + p. \tag{1.8}$$

Any generating twistor function (1.6) gives rise to a pair of the CEE solutions. To obtain the first one, one resolves the algebraic constraint

$$\Pi(G, wG + u, vG + p) = 0 \tag{1.9}$$

with respect to G and comes to a complex (and *multivalued* in general) field $G(u, v, w, p) \equiv G(x, y, z, t)$ which *for any* Π *identically satisfies the CEE*. Secondly, one differentiates the function (1.6), resolves then again the constraint

$$P \equiv \frac{d\Pi}{dG} = 0 \tag{1.10}$$

with respect to G and substitutes the resulting field $G(u, v, w, p)$ into (1.6). In this way one comes to a function of coordinates $\Pi(G(X), \tau^1(X), \tau^2(X)) \equiv \Pi(x, y, z, t)$ which again *identically satisfies the CEE*. It was shown in [12] that the two classes above presented *exhaust the solutions to CEE*.

Geometrically, any twistor field $\mathbf{W}(X) = \{G, \tau^1, \tau^2\}$ defines a *null geodesic congruence*, i.e. a congruence of rectilinear light-like rays on $\mathbf{M}$. For twistors obtained from the constraint (1.9) this congruence is known always to be *shear-free* (SFC) and, moreover, any SFC can be constructed in this way (this is the content of so called *Kerr theorem* [14, 15]).

Conditions (1.9),(1.10) taken together specify the *caustic locus* of the related congruence so that after elimination of G the resulting equation

$$\Pi(u, v, w, p) \equiv \Pi(x, y, z, t) = 0 \tag{1.11}$$

(with function Π always obeying the CEE, see above) *determines the shape and the time evolution of caustics* which can be enormously complicated [16, 5, 13]. On the other hand, the same condition defines *singular locus* of associated Maxwell and Yang-Mills fields [10, 8] as well as of the curvature field of correspondent Kerr-Schild metric [15, 17]. Therefore, we can identify *particles as common singularities* [2] *of electromagnetic and other fields defined by any solution to GCRE (or, equivalently, by any SFC)* [16, 10, 8].

Note that these singularities possess some properties of real quantum particles due, in particular, to *overdetermined* structure of GCRE. In fact, according to the theorem of charge quantization proved in [11, 5], any bounded and isolated singularity of electromagnetic field associated with solutions of GCRE has electric charge either zero or integer multiple to some minimal, *elementary* one, i.e. to the charge of static axisymmetrical *Kerr solution* with a ring-like singularity [2, 3, 10, 9]. According to observations of Carter [18] and Newman [19], gyromagnetic ratio for Kerr-like singularities is exactly the same as for Dirac fermions. Numerous examples of SFC with complicated structure and dynamics of singular locus has been presented in our works [10, 16, 9, 8, 13].

From general viewpoint, we come in this way to a peculiar picture of physical World whose main elements are the congruence of "primodial light" rays (invisible for ordinary observer) – the flow of *Prelight* [24, 1, 13] – and the matter "born" from this "Ether-like" flow

[2] At least for the case when singular locus is finite in 3D space

at its focal points, i.e. at caustics. A consistent concept of physical Time as the *field evolution parameter* along the rays of the Prelight Flow has been also developed and discussed in our papers [24, 1, 13, 25] (see also section 3).

Unfortunately, the above presented algebraic theory for interacting fields-particles seems to be unsufficient. Indeed, generically therein particles-caustics are represented by a number of isolated 1D curves (*strings*) nontrivially evolving in time but, as a rule, unstable in shape and size and radiating to infinity. Besides, one finds no correlation in time dynamics of different strings which could model their interaction. Finally, from epistemological viewpoint, real Minkowski subspace is by no means distinguished in the structure of $\mathbb{B}$-algebra.

The last argument is of especial importance in the framework of algebrodynamics. In fact, no algebra is known whose automorphism group is isomorphic to the Lorentz group of Special Relativity. Therefore, in the algebrodynamical paradigm one is forced to accept the space-time geometry different from the Minkowski one. By this, full 4D *complex* vector space of $\mathbb{B}$-algebra seems to be the most appropriate background for algebric field-particle theory based on the $\mathbb{B}$-differentiability conditions, i.e. on the GCRE.

Complex extension $\mathbb{C}\mathbf{M}$ of Minkowski space-time indeed arises repeatedly in General Relativity, in twistor and string theories. In particular, Newman et al. [19, 20, 21] advocated the concept of complex space-time in order to obtain physically interesting solutions of electrovacuum Einstein-Maxwell system and to link together characteristics of correspondent particle-like singular sources (see the next section). In our recent paper [25] the algebrodynamics on the $\mathbb{C}\mathbf{M}$ space has been developed on the base of Newman's representation for "virtual" point charge "moving" along a complex world line in $\mathbb{C}\mathbf{M}$ and generating a congruence of complex "light-like rays". The "cut" on this null congruence by the real Minkowski slice $\mathbf{M}$, gives there rise to a SFC with interesting physical properties.

Here again, however, one deals with a rather artificial and insufficient restriction of the basic structures onto $\mathbf{M}$. Alternatively, in the framework of algebrodynamics on $\mathbb{C}\mathbf{M}$ [25], we have introduced the concept of *observable* space-time – 6D subspace of the *complex null cone* (CNC) of a point-like "observer" $\mathbf{O}$ in $\mathbb{C}\mathbf{M}$. By this, any particle-like (caustic) element $\mathbf{C}$ which could be detected by $\mathbf{O}$ lies on its CNC and has the same value of the primary twistor field as in $\mathbf{O}$ so that dynamics of $\mathbf{C}$ and $\mathbf{O}$ is strongly correlated (they "interact").

On the other hand, CNC has the topology $\mathbb{R}_+ \times \mathbf{S^3} \times \mathbf{S^2}$ (see section 3) so that it decomposes into a basic 4D space (in which time interval should be identified with 4D distance as in [26, 27]) and an orthogonal space of a 2-sphere which can be naturally treated as internal *spin space*.

Moreover, we'll see that an ensemble of identical point-like particles – *duplicons* – images of one and the same generating charge with correlated time evolution – can be naturally constructed in this framework. We'll also discuss physical sense of *quaternionic time*, *complex time* and of *evolution curve* – of notions inevitably arising in the considered algebrodynamical scheme and related probably to the origin of quantum uncertainty.

2 Newman's charge in $\mathbb{C}\mathbf{M}$ and the set of its images – duplicons

Simplest variety of a shear-free null congruence (SFC) is formed by a bundle of light rays radiated by a point particle moving along an arbitrary world line on the real $\mathbf{M}$ space [28]. Electromagnetic field $F_{\mu\nu}$ can be then associated with such SFC via integration of Maxwell equations with additional requirement on the field strength $F_{\mu\nu}k^\nu = 0$ to be orthogonal to the null 4-vector $k^\mu = \xi_A \xi_{A'}$ tangent to the congruence rays. Quite expectably, this field is just the Lienard-Wichert one. In the algebrodynamical approach, the field of exactly the same type can be obtained directly via second derivatives of the congruence field G [3, 10, 9]); however, this field necessarily corresponds to *elementary (unit) value of generating charge* [3]. Notice that in the static case the SFC generated by the charge at rest is radial and gives rise to the "quantized" Coulomb field and to the Reissner-Nördstem metric.

Newman proposed [23, 22] generalization of this construction by consideration of a point-like charge "moving" in complex extention $\mathbb{C}\mathbf{M}$ of $\mathbf{M}$ (in fact, even the case of *curved* complex space-time was studied). Then on the real slice $\mathbf{M}$ of $\mathbb{C}\mathbf{M}$ one obtains a more complicated SFC with nonzero *twist* which, in the particular case of "virtual" charge at rest, gives rise to the metric and electromagnetic field of well-known *Kerr-Newman* electrovacuum solution with a *ring-like singularity* ("Kerr ring").

Consider now Newman's construction in twistor terms [29, 30] and its generalization in the framework of algebrodynamics. Let the point singularity "moves" along a complex world line $Z_\mu = \widehat{Z}_\mu(\tau),\ \mu = 0,1,2,3$ where the parameter $\tau \in \mathbb{C}$ plays the role of "complex time" (further on we consider its own "evolution law" $\tau = \tau(s),\ s \in \mathbb{R}$). For Cartezian and "spinor" coordinates on $\mathbb{C}\mathbf{M}$ we"ll use the following representation similar to (1.3):

$$Z = \{Z^A_{\ B}\} = \begin{pmatrix} u & w \\ p & v \end{pmatrix} = \begin{pmatrix} z_0 - iz_3 & -iz_1 - z_2 \\ -iz_1 + z_2 & z_0 + iz_3 \end{pmatrix} \tag{2.1}$$

in which all z_μ (as well as u, v, w, p) are now complex-valued. Incidence relation (1.7) takes then the form

$$\tau = Z\xi \ \ (\tau^A = Z^A_{\ B}\xi^B, \quad \tau^1 = wG + u,\ \tau^2 = vG + p), \tag{2.2}$$

where the quantities $\{G, \tau^1, \tau^2\},\ G \equiv \xi^2/\xi^1$ constitute the projective twistor in $\mathbb{C}\mathbf{M}$. If now a point $Z \in \mathbb{C}\mathbf{M}$ is separated from the charge $\widehat{Z}$ by the null interval,

$$S := \det \|Z - \widehat{Z}(\sigma)\| = (u - \widehat{u}(\sigma))(v - \widehat{v}(\sigma)) - (w - \widehat{w}(\sigma))(p - \widehat{p}(\sigma)) = 0, \tag{2.3}$$

then one can find from (2.3) the field $\sigma = \sigma(Z) = \sigma(u, v, w, p)$ and, consequently, the position of charge $\widehat{Z}(Z) \equiv \widehat{Z}(\sigma(Z))$ which "influences" the point Z. This means that *twistor field* is the same both at Z and at $\widehat{Z}$. Indeed, in account of (2.3) linear system of equations

$$(Z - \widehat{Z}(\sigma))\xi = 0 \tag{2.4}$$

in a unique way fixes the ratio of spinor components $G = \xi^2/\xi^1$. System (2.4) then reads as

$$\tau \equiv Z\xi = \widehat{Z}\xi, \tag{2.5}$$

so that all twistor components are indeed equal at Z and $\widehat{Z}$. In components (2.4) takes the form

$$\left\{ \begin{array}{lcl} (u - \widehat{u}(\sigma)) + (w - \widehat{w}(\sigma))G & = & 0 \\ (p - \widehat{p}(\sigma)) + \ (v - \widehat{v}(\sigma))G & = & 0 \end{array} \right. \tag{2.6}$$

and demonstrates that the field G is *indefinite* on the world line of the charge itself (and depends on the direction in its vicinity). On the other hand, writing out (2.5) in the form

$$\left\{ \begin{array}{lcl} \tau^1 & = & \widehat{u}(\sigma) + \widehat{w}(\sigma)G \\ \tau^2 & = & \widehat{p}(\sigma) + \widehat{v}(\sigma)G \end{array} \right. \tag{2.7}$$

[3]Owing to overdetermined structure of the GCRE (or of SFC equations), see section 1

and eliminating from there the parameter σ, one concludes on the functional dependence of three twistor components, i.e.

$$\Pi(G, \tau^1, \tau^2) = 0, \tag{2.8}$$

which, as in the real case (see (1.9)), demonstrates that fundamental null congruence of "rays" generated by Newman's charge is indeed *shear-free* for any its world line (or any function Π correspondent to it).

Let us recall now fundamental property of *multivaluedness* of the GCRE solutions or, equivalently, – of the SFC defining equations (1.9) or (2.6) in particular case of Newman's congruence considered. Precisely, for any fixed point $Z = \{u, v, w, p\} \in \mathbb{C}\mathbf{M}$ one obtains a countable (finite or infinite) set of solutions $\{G, \sigma\}$ of the system (2.6) so that *generically there exists a great number of continious branches – "modes" – of twistor field functions* $\mathbf{W}(Z)$, of the eikonal field $\sigma(Z)$ and of corresponding shear-free *subcongruences*, superposing at each point Z. Notice that general concept of multivalued fields has been discussed in [13].

We see now that *any* complex ray of *any* mode of a SFC originates from a generating charge at some its position $\widehat{Z}(\sigma)$ which, therefore, is a *focal point* of the congruence. Thus, the World line of the Newman's generating charge can be considered itself as the *focal line*.

One should distinguish between focal points with indefinite value of the main spinor field G and those of caustics (envelopes of the congruence rays) at which, generically, all the field modes are well defined but two or more of them merge in one. For Newman's SFC cauistic points, in account of (2.3), satisfy the condition

$$S' = \frac{dS}{d\sigma} = \widehat{u}'(v - \widehat{v}(\sigma)) + \widehat{v}'(u - \widehat{u}(\sigma)) - \widehat{w}'(p - \widehat{p}(\sigma)) - \widehat{p}'(w - \widehat{w}(\sigma)) = 0. \tag{2.9}$$

Locus of "hupercaustic" points – *cusps* – where at least *three* modes amalgamate – are analogously defined as

$$S'' := \frac{d^2S}{d\sigma^2} = \widehat{u}''(v - \widehat{v}(\sigma)) + \widehat{v}''(u - \widehat{u}(\sigma)) - \widehat{w}''(p - \widehat{p}(\sigma)) - \widehat{p}''(w - \widehat{w}(\sigma)) - 2(\widehat{u}'\widehat{v}' - \widehat{w}'\widehat{p}') = 0 \tag{2.10}$$

and, finally, for the strongest singularities (*hypercusps*) one has $S''' = ... = 0$. More stronger singularities have degenerate codimension space and generically exist in $\mathbb{C}\mathbf{M}$ space only as isolated points ("instantons").

From (2.3) and (2.9) we easily find that at any focal point at least two modes merge together so that for any $Z = \widehat{Z}(\lambda)$, setting $\sigma = \lambda$, one gets identically $S = S' \equiv 0$. However, in the special case of a *null* World line for which

$$\det \|(\widehat{Z}'\| \equiv \widehat{u}'\widehat{v}' - \widehat{w}'\widehat{p}' \equiv 0, \tag{2.11}$$

at any point $Z = \widehat{Z}(\lambda)$ on the World line with $\sigma = \lambda$ one can easily prove that more stronger conditions for singularity are satisfied: $S = S' = S'' = S''' = 0$, so that now *four* modes merge in one and are indefinite at the focal point itself. Further on we assume that *the World line of generating charge is null.*

Note that the considered equation of null cone (2.3) in the case of real space-time $\mathbf{M}$, in particular in the framework of electrodynamics, is known as the *retardation equation.* With real Cartesian coordinates $\{x, y, z, t\}$ and natural parametrization $\widehat{t}(s) = s = \Re(\sigma)$ it takes the form

$$c^2(t - s)^2 - (x - x(s))^2 - (y - y(s))^2 - (z - z(s))^2 = 0, \tag{2.12}$$

whereas the charge moves along a real World line $\{x(s), y(s), z(s)\}$. If velocity of the charge is assumed to be smaller than the light one, $v < c$, then equation (2.12) can be proved [31] to have always two roots only one from which has $s \le t$ and satisfies thus the causality requirement. This fixes a unique position of generating charge *in the past* which forms the field (electromagnetic Lienard-Wichert field in particular) at the observation point $\{x, y, z, t\}$ propagating to it rectilinearly with the speed of light. If the latter belongs to the trajectory of the charge itself, equation (2.12) has only trivial solution $s = t$ (the retardation time equal to zero).

Situation changes drastically in complex space-time. In fact, now a whole set of solutions $\{\sigma_N\}$ of the complex null cone (CNC) equation (2.3) does exist (for any point Z) which specify a (great) number of "influence points", i.e. of *images* of one and the same charge each forming its own field mode at Z and perceiving therefore by the observer at Z as an ensemble of identical but distinguishable (via individual position and dynamics) point charges – *duplicons*.

Notice that total *number* of duplicons does not depend, as a rule, on the observation point. However, a "real" observer $\mathbf{O}$ should be *material* and, therefore, in the simplest idelized case can be identified with a focal point itself. Then one has $Z = \widehat{Z}(\lambda)$ with varying parameter $\lambda \in \mathbb{C}$ playing the role of *complex time* as evolution parameter (see below). Now four or two duplicons (the number depends on the fact is World line null (2.11) or not) merge together forming the focal point of the observer $\mathbf{O}$ himself whereas the other $N - 4$ (or $N - 2$) duplicons are perceived by $\mathbf{O}$ as an ensemble of external identical point particles which are disposed and "move" with respect to $\mathbf{O}$ in the *observable 6D space-time* defined by the CNC equation (2.3). CNC geometry and its reduction to 4D physical space-time will be studied in the next section.

3 Complex time and 6D physical geometry of complex null cone

In the algebrodynamics on real $\mathbf{M}$ where fundamental twistor field inevitably arises from the structure of $\mathbb{B}$-differentiable functions, physical time t manifests itself as a *field evolution parameter*. Indeed, the incidence relation (1.7) is invariant under a 1-parameter group of translations of coordinates $X = X^+ \in \mathbf{M}$ along the rays of correspondent null congruence:

$$x_a \to x_a + n_a t, \ (a = 1, 2, 3); \quad x_0 \to x_0 + t, \quad n_a = \xi^+ \sigma_a \xi / \xi^+ \xi, \quad \vec{n}^2 \equiv 1. \tag{3.1}$$

Presence of this symmetry demonstrates that twistor field *propagates* from any initial point with universal velocity $v = c = 1$ along 3-directions $\vec{n}$ (locally defined by the field itself) or, equivalently, is *preserved* in value along these in the 4D-sense. Just the parameter t along the rays can be treated as local physical time (see correspondent discussion in [13, 13, 25]).

When one generalizes the scheme to complex background, the same considerations lead to a more complicated and intriguing notion of time. Indeed, in $\mathbb{C}\mathbf{M}$ correspondent symmetry group of translations is $2\mathbb{C}$-parametric (the coordinates can vary across the so called *complex α-plane* [14]). We'll , however, start from an alternate representation of this field symmetry what for decompose the space $\mathbb{C}\mathbf{M}$ of biquaternion algebra into two *real quaternionic* (4D Euclidean) spaces represented by 2×2 *unitary* matrices U and V:

$$Z = U + iV, \quad U^+ = U^{-1}, \ V^+ = V^{-1}. \tag{3.2}$$

The first of these subspaces, say U, may be considered as the main coordinate space whereas the second one V will then play the role of *evolution parameter space.* Precisely, from incidence condition (2.2) and (3.2) one gets after some equivalent transformations the following translation law:

$$U = U^{(0)} + V * N, \tag{3.3}$$

where quaternion $U^{(0)}$ of initial coordinates is algebraically expressed via twistor field components, quaternion $V = v_0 + iv_a\sigma_a$ defines four real evolution parameters $\{v_0, v_a\}$ ("quaternionic time"), and *unit* quaternion $N = 1 + in_a\sigma_a$ (with vector $\vec{n}$ of the same form as in the real case (3.1)) specifies local directions of *translations preserving twistor field.*

Interestingly, the whole law (3.3) does not contain any residual of initial complex structure since all matrices therein are unitary and $(*)$ can be considered as multiplication in *real quaternion algebra* $\mathbb{H}$. In components fundamental translation law (3.3) for four coordinates $\{x_\mu\}$, $\mu = 0,1,2,3$ of the main *Euclidean* space $U \in \mathbf{E^4}$, $U = x_0 + ix_a\sigma_a$ has the following form:

$$\Delta x_a = n_a \Delta v_0 + \varepsilon_{abc} n_b \Delta v_c, \quad \Delta x_0 = n_a \Delta v_a, \tag{3.4}$$

so that one complex (two real) constraints $\Sigma(\Delta z_\mu)^2 = 0$, or in components

$$\left\{ \begin{array}{l} (\Delta x_0)^2 + (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 = (\Delta v_0)^2 + (\Delta v_1)^2 + (\Delta v_2)^2 + (\Delta v_3)^2 \equiv (\Delta t)^2, \\ (\Delta x_0)(\Delta v_0) + (\Delta x_1)(\Delta v_1) + (\Delta x_2)(\Delta v_2) + (\Delta x_3)(\Delta v_3) = 0, \end{array} \right. \tag{3.5}$$

defining the structure of *complex null cone*, are satisfied via arbitrary translation. Notice that in particular case when three of the parameters Δv_a vanish and the fourth one acquires the meaning of time $\Delta v_0 = t$, three "space" coordinates $\{x_a\}$ are translated just as in the former Minkowski case (3.1), i.e. rectilinearly and with fundamental "light" velocity whereas for additional Euclidean coordinate one has $\Delta x_0 = 0$, so that it is "freezed" and looses its dynamical sense.

Generically, the translation law (3.4) is much more complicated and rich in structure. The presence of four evolution parameters makes the dynamics of the field essentially indefinite since the *order* of continious change of the parameters is not determined by the structure. However, we can naturally assume that the role of local physical time interval is now played by the total Euclidean distance $(\Delta t)^2 = \Sigma(\Delta x_\mu)^2$ passing by the field in the coordinate space. In account of (3.5) this is equal to corresponding distance in the parameter space and twice smaller than that in the whole complex space with natural metric $\Sigma|z_\mu|^2$.

Thus, in generic complex case local physical time can be related to the translations preserving the value of basic twistor field and to the whole distance to which the field propagates via these translations. In the latter aspect such treatment of physical time is similar to that proposed in the works of H.Montanus [26] and J.Almeida [27]. They considered the model of space-time as a 4D Euclidean space $\mathbf{E^4}$ with fourth coordinate being identified with local *proper time* of a particle of matter, and with local *coordinate time interval* being the metric interval covered by the latter (in natural units). These assumptions lead to the space-time quadratic form represented by the first equation in (3.5) and, physically, are equivalent to the requirement for *all material formations to move in* $\mathbf{E^4}$ *with constant in modulus fundamental velocity* $\Sigma\left(\Delta x_\mu/\Delta t\right)^2 = 1$ $(= c^2)$. From there it follows immegiately that *3-dimensional velocity of all material formations may be only less than or equal to the fundamental one.*

Notice that in the algebrodymamics the last statement is not postulated but follows necessarily from the CNC ("observable") geometry. However, this is evident only for the velocities of propagation of "prematerial" twistor field. As for the particle-like formations represented here by focal and (hyper-)caustic points, this statement will be true for their *absolute* velocities if the World line is null (as we have already assumed). For *relative* ("observable by **O** ") velocities the situation is more complicated (see below). On the other hand, in the CNC geometry we have, in addition to the main physical space U, two more coordinates of orthogonal space V which may be parametrized by the coordinates on the sphere S^2 and interpreted (at the focal points - particles) as the coordinates of their "spin vector".

Unfortunately, the scheme based on the concept of 4D Euclidean space-time suffers from evident inconsistencies with Special Relativity. Indeed, though formally the first constraint in (3.5) can be represented in the form of Minkowski-like interval $ds^2 := dx_0^2 = dt^2 - dx_1^2 - dx_2^2 - dx_3^2$, there are no grounds for the proper time interval ds to remain invariant under the 4D rotations, as well as for the coordinate time dt to transform according to the Lorentz group. In the AD approach we have also to solve the above-mentioned problem of indefinite character of field evolution related to the presence of 3 additional free parameters v_a or, in other words, the problem of *ordering* of the 4D "quaternionic time".

Last difficulty can be partially overcomed in a rather successive way. For arbitrary field-preserving translations with four parameters $\{v_\mu\}$ particle-like formations (focal points, caustics etc.) are not preserved and, generally, dissappear from the main space $\mathbf{E^4}$. In particular, if one takes all $v_\mu = 0$ to get the field-particle distribution at *initial* moment of time $t = 0$, then, generically, one will find the space $\mathbf{E^4}$ *empty*, i.e. free from any focal points. Indeed, the latters are *holomorphically* defined as the points of a complex World line $Z = \widehat{Z}(\sigma)$, and all four equations necessary for coordinates of these points to be *unitary* can't be satisfied for any complex value of parameter σ.

Thus, the problem is to concord the structure of the main quaternionic Euclidean space U and the "real quaternionic" type of the field evolution law (3.4) with basic holomorphic structure of null congruence and its particle-like singularities. To do this, let us return back to the above introduced *observable* space-time **O**, i.e. to the 6D space-time of the CNC of an idelized observer **O** which is identified with a focal point and moves along its World line in synchro with the field evolution. On this *relative* subspace modelling physical space-time, we are now going to consider the evolution law for coordinates U preserving the structure of basic equations (2.3) and (2.9) together, i.e. *both the values of twistor field and the condition of caustic.* This extended "field-caustic preserving" automorphism has the following 1$\mathbb{C}$-parameter holomorphic form:

$$Z \to Z + \tau(Z - \widehat{Z}(\sigma)), \quad \tau \in \mathbb{C}. \tag{3.6}$$

This means that caustics reproduce themselves along complex straight lines ending/starting from a focal point, and the parameter τ along these should be considered as the only physical evolution parameter – "complex time". In the real case such situation could correspond to the "signal" propagating rectilinearly with fundamental velocity. Complex structure, however, is not ordered, so there are infinitely many "ways" connecting some two points on such a line. Moreover, one is not aware which of them is the emitter, and which – the receiver point. Note that the concept of two- (or multi- in general) dimensional time and the resulting problem of ordering of physical events has been considered in a number of works, in particular by Sakharov [32].

Uncertainty of evolution related to the complex nature of time is weaker than for the primary "quaternionic time" one but still too severe for physical applications. To overcome this difficulty, one could assume, in addition to holomorphic structure of GCRE (or of SFC equations), the existence of some "evolution curve" $\tau = \tau(s), \ s \in \mathbb{R}$ on the complex plane which establishes the order of passing of different "states" in the main "physical" space U. Such curve as well as the World line of generating charge can be enormously complicated, possess the points of self-intersection etc. Because the observer has no information about this curve, the future for him seems to be indefinite. However, a sort of *probabilities* can be introduced for different "continuations" of the evolution curve, and this procedure can open the way for explaination of quantum phenomena and the links of algebrodynamics to the Feynman's version of quantum mechanics in particular. These problems will be discussed elsewhere.

There is evidently a local correspondence between the "evolution time" τ, the parameter σ of the World line and, say, the zeroth coordinate of the observer which can be treated as the "complex proper time". In account of quaternionic form of the CNC of field evolution (3.5) we can interpret then the real part Δx_0 of "complex time increment" Δz_0 as the evolution parameter in the main coordinate space U ("translational" proper time), whereas its *imaginary part* Δv_0 – as that in the orthogonal spin space ("rotational" proper time). The existence of evolution curve implies also direct local relation between the increment of its parameter s and correspondent interval of coordinate time Δt. Explicit formulas can be easily presented.

Let us consider now in more details the above mentioned process of emission/reception of a "signal-caustic" by "charges – focal points". Let the "interacting" charges – duplicons $\widehat{Z}(\lambda), \ \widehat{Z}(\sigma), \ \sigma \neq \lambda$ are linked by the caustic. This corresponds to the solutions of the joint system of equations (2.3) and (2.9) with substitution $Z = \widehat{Z}(\lambda)$ instead of arbitrary observation point Z. This system has a *discrete* set of pairs of solutions $\{\lambda, \sigma\}$ any of which correspond to two fixed *moments of complex time* of emission/reception of the caustic signal and to two positions of the charges involved into the process. By this, if the moment of observation λ "lies in the future" with respect to the "moment of influence" σ, i.e. corresponds to greater values of monotonically increasing parameter s of the evolution curve, than the observer $\widehat{Z}(\lambda)$ would seem to *receive* the signal from the charge at $\widehat{Z}(\sigma)$, and the latter would *emit* the signal towards $\widehat{Z}(\lambda)$. The converse situation is evident. However, one should not forget that in essence this is *one and the same* charge *interacting with itself in its own past or future.*

Thus, all the *events* in complex space-time are well-defined and predetermined by the only structure of the World line of generating charge (focal line of the fundamental congruence). These are: the total number of charges – duplicons ("observed" by anyone of them and *permanently* correlated by twistor field), their relative coordinates and the points on the World line at which this correlation between each pair is stronly *amplified* (the charges are involved into the emission/reception process). However, the precise picture of evolution remains indefinite and should be completed by additional information about (extremely complicated and exceptional from mathematical viewpoint) form of evolution curve.

A microscopic object (focal point) is detectable for an idealized micro-observer only at some discrete moments of time when the caustic line links them together. Near these, *relative* instantaneous velocity of the object is easily proved to be equal to its *absolute* one and, for the considered case of a *null* World line, is always less or equal to the fundamental velocity (speed of light). These conclusion seems to be true also for a "classical" macroscopic object which is nearly permanently linked with similar "classical" observer by *a net* of caustic lines. A detailed construction (in two utmost cases – quantum (microscopic) and classical (macroscopic)) of correlated pairs "object - observer" exchanging by caustic "signals" will be presented elsewhere.

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Dark matter and search capability for its particles

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On dark matter we understand the matter which may be detected only due to its gravitational interaction on environment bodies and particles. The main evidences of dark matter existence are the plotting of rotational curves for galaxies (fig. 1) and microlensing of the electromagnetic galactic radiation. An alternative to the dark matter existence is the presupposition that the gravitational forces are getting stronger with distance. However more than the successive explanation observed anomalies is based on availability of dark matter in galaxies. At the same time let us mention that the assessed density of Universe (about 1 hydrogen atom in 1 м^3) is too small, therefore "the dark energy" for the dynamics description of Universe is still introduced [1].

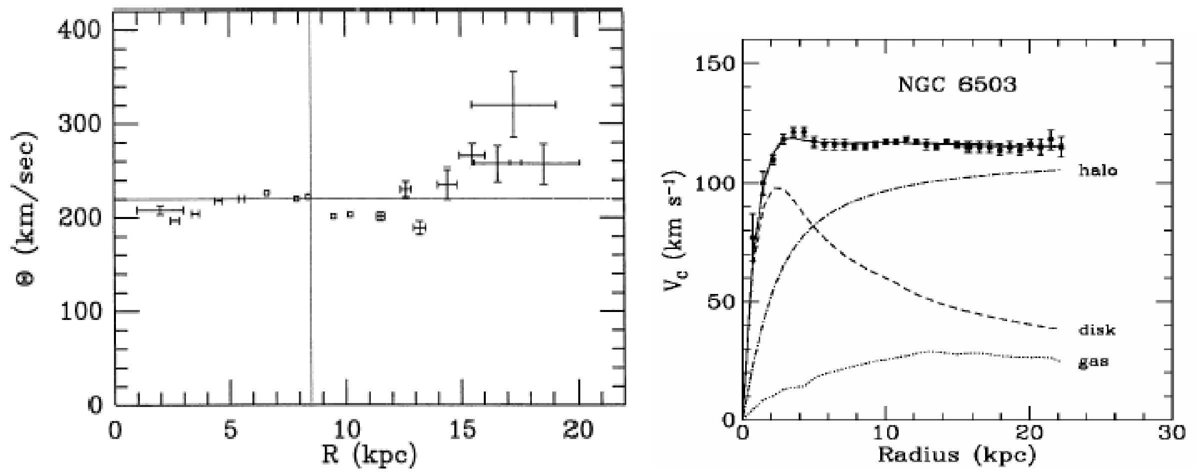


Fig. 1 The rotational curves for the galaxies "Milky Way" and NGC 6503

Observational data denote existence about 90% of a galaxies mass, which is being detected only by gravitational effects. It is common practice to discriminate three types of dark matter:

- Baryonic dark matter
- Cold dark matter
- Hot dark matter.

The last two species are nonbarionic nature. It is necessary to use the cold dark matter for the explanation of star beginning and/or stars restraining in galaxies. Particles of nonbarionic cold dark matter may be the presumable particles of elementary particle physics. For example there are axions, WIMPs, SIMPs, stranglets, technibaryons, ChaMPs. Therefore the registration of dark matter particles is actual both for astrophysics and for elementary-particle physics.
Let us consider the parameters of cosmic WIMPs

MASS	M_W	**10 – 5000 GeV/c^2 (~ 10 – 5000 m$_P$)**
VELOSITY	V_W	**10^5 --10^6 m/s**
DENSITY	ρ_W	**0,3 (GeV/c^2)/cm^3**
CROSSECTION	σ_W	**<10^{-10} pbarn (~10^{-44} cm^2)**
FLUX	Φ_W	**~ 5 $*$ 10^4 1/(cm^2 s)**

For comparison we will lead to masses of other particles

MASSES OF OTHER PARTICLES

NEUTRINO	<30eV/c^2
AXION	10 eV/c^2
NEUTRALINO	>20 GeV/c^2
MAJORANA FERMION	>20 GeV/c^2
FAST NEUTRONS	>1 MeV/c^2

The nonbarionic nature of WIMPs and the absence of an electric charge permits the registration of these strange particles only by mass presence in case of the frontal particle collision with a nuclei of ordinary matter.

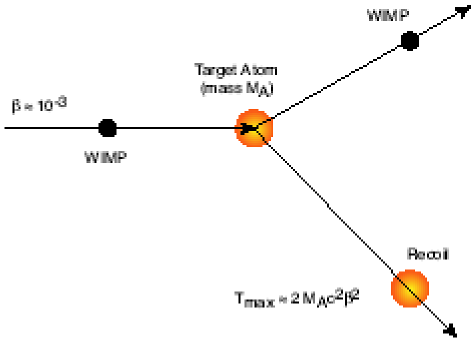


Fig.2 Model of interaction between WIMP - atom

As the result of a collision the recoiled atom receives an energy. It can be registered by various methods:
A) in gases, recoiled atoms produce ionization, that may be detected and measured by electronic devices. In some gases there may be the scintillation (the emergence of bursts of radiation) in case of the deceleration of an ionized atom motion;
B) in a condensed media (some fluids and crystals) the scintillation also there may be. The radiation intensity depends on the energy of recoil atoms;
C) if a recoil energy is large, then in a condensed media the excitation of acoustic waves produced along the path of a registered particle is possible (the crackles and the splits in media along a track like a bolt);
D) in crystals a recoil energy can be transformed in lattice oscillations (phonons). These oscillations are to be recorded at cryogenic temperatures by the bolometric technique;
E) in semiconductors (for example the silicon or germanium) the electric charge freed by a recoiled atom can be registered as in the case A);
F) it is possible to measure the change of an atomic magnetic moment of conventional substance, due to its collision with WIMP.
Data of similar measurements may be used for:

- particle detection;
- determinations of their nature (masses, speeds, flux densities, spin);
- possible correlation with models of higher-dimensional, i.e. string theory and theory of dark energy;
- revealing of facts for the mutual influence of gravitation and strong interaction;

For the purpose of conducting of such measurements within the last years by the various scientific groups a large number of experimental facilities was created. Their classification can be realized in a number of ways. Let us lead to one of them [1]:

ANAIS	**CASPAR**	**CDMS**	**CRESST**	**CUORE**
DAMA	**Drift**	**Edelweiss**	**Genius**	**HDMS**
IGEX	**LIBRA**	**MIMAC-He3**	**Majorana**	**NAIAD**
ORPHEUS	**Picasso**	**ROSEBUD**	**UKDMC**	**XENON**

The results received at facilities CDMS (USA) and DAMA (Italy) are considered as the most significant. Let us mention that apart from experiments, in which it is possible to register WIMP directly, in a number of other experiments it is able to obtain oblique evidences of the existence of dark matter particles. In its turn, one can break the direct experiments on two classes. In the first class(CDMS, Edelweiss, Zeplin - 1) the total energy of nuclear recoil of a detector working medium is registered, and the signal is extracted from background, created by the nuclei collisions with the other particles. In the second (DAMA) the modulation of a count rate is registered. The modulation of a flow of dark matter particles can be conditioned by Earth motion around the Sun, and consequently the detector movement through a halo of dark matter of our galaxy. In standard experiments the modulation is low (< 2%).

Other possibility of an emergence of modulation of flow is the orientation change of a detector concerning vector of the solar velocity relative to the centre of our galaxy.

Underneath the results of two experimental groups are given.

The Sierra Grande curve is plotted from a long exposure germanium experiment in which a search for both daily and annual modulation has been performed, and the results from the daily modulation search are shown. No significant signals are seen.

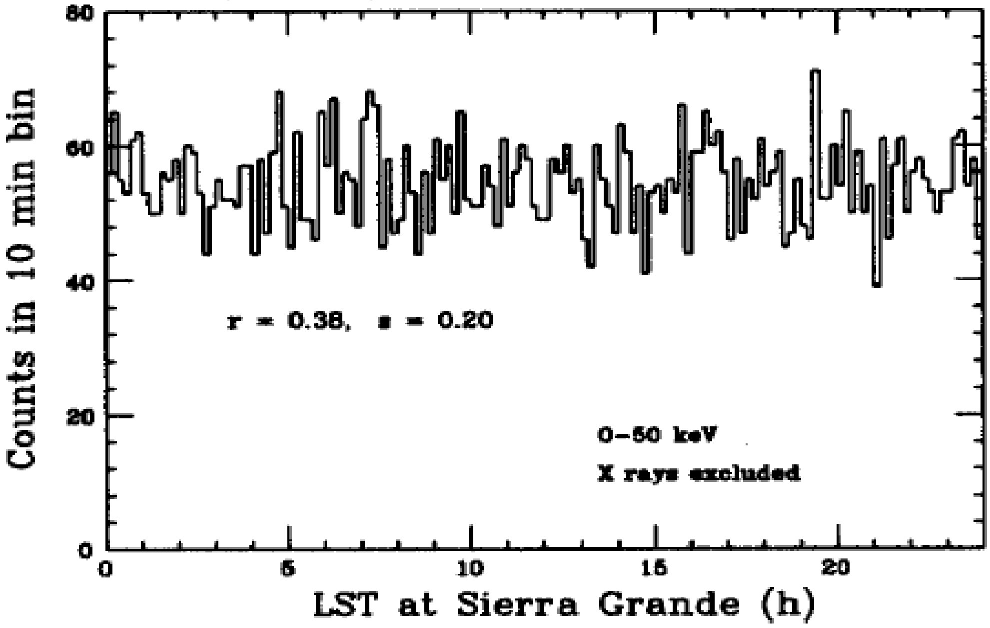


Fig. 3 Background rate from 428.1 days of data binned in 10-minute intervals and folded to look for daily modulation

An example of an annual modulation search in the DAMA program is shown below.

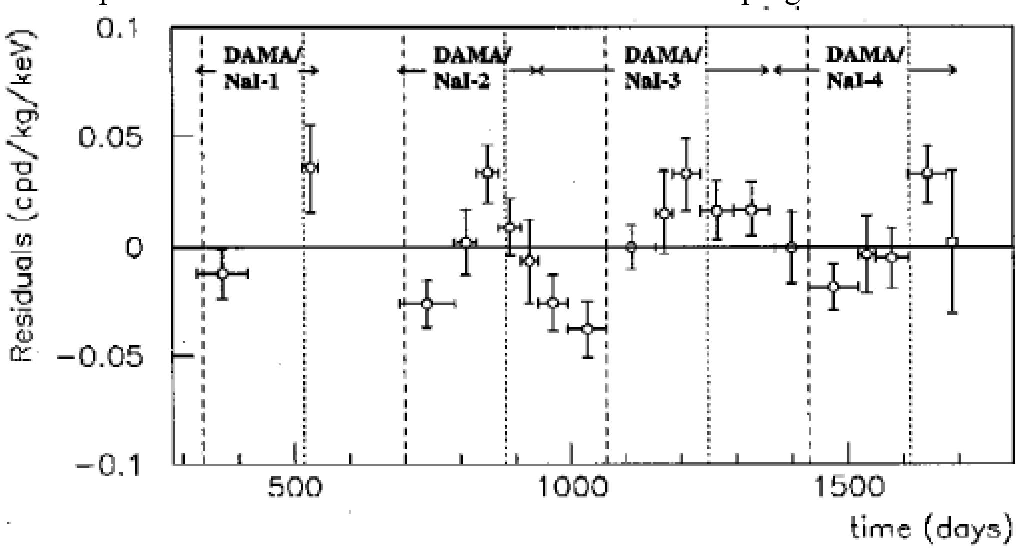


Fig.4 Results of an annual modulation search using ~ 4 years of data from the DAMA experiment

Coherent principle of registration. Estimates of the registration probability by liquid He-II using

The experimental devices used for the registration of WIMP, as evidenced by the foregoing, must satisfy to contradictory requirements:

- to posses high sensitivity to rare events;
- to have the significant noise protection level, i.e. insensibility to low background, created by other particles and/or effects.

Therefore the only narrow class of precise recorders will do for the purpose of a feebly interacting particles registration. The success of a laser application in precision measurements is defined in large degree by coherence of their radiation. The considerable share of measurable progress of low magnetic fields is determined by the use of coherent states in superconductors. Other manifestations of quantum substance properties in macroscopic scale appear in Bose-Einstein condensate [5]. Some manifestations have been much studied in a liquid helium.

The liquid helium is the subject often used in experiments with elementary particles. The reactions have been examined and the interaction cross sections during exposure of the liquid helium by photons with energies to GeVs, high-speed neutrons and protons are measured.

Let us make an estimation of the possibilities of the WIMP registration with the help of devices, using the liquid helium. Already since 40th of past century it was known that in the liquid He-II the excited states can be two types (fig. 5).

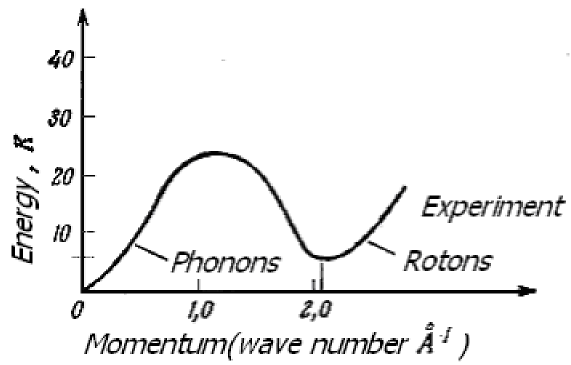


Fig.5 Types of excitations in liquid He-II

Really, this experimental curve is explained on the assumption that in a liquid He-II there are not only phonons but there is also the other type of excitation. Phonon excitations correspond to the rearrangements by large distances of Bose particles (the long-wave density fluctuations), and rearrangements at short distances d (d is about the interatomic distance) correspond to the other elementary excitations - the rotons.

In case when a particle hits in the liquid helium it is possible to excite phonons and rotons. Let us consider the registration possibility of WIMPs with the help of phonon excitation in a liquid He-II. It is well known that for production of elementary excitation in a liquid He-II (for example of phonon) it is necessary that [2]

$$\varepsilon_{ph} < v_W \cdot p_{ph} \qquad (1)$$

From given work [3]

$$\varepsilon_{ph} \sim 1{,}3 \cdot 10^{-30}\ \mathrm{J},$$

$$p_{ph} > 6{,}6 \cdot 10^{-30}\ \frac{\mathrm{kg\,m}}{\mathrm{s}}.$$

If to accept the speed of WIMP is

$$v_W \sim 10^5\ \frac{\mathrm{m}}{\mathrm{s}},$$

then the condition (1) is being done.

The estimations of a phonon energy show that the very small energy for their excitation is needed. Indeed

$$\varepsilon_{ph} \approx \frac{hc}{\lambda}, \tag{2}$$

where c is a sound velocity in the liquid He-II ($c \approx 0{,}2 m/s$), λ is a wavelength. Assuming the length $\lambda \approx 0{,}1$ mm, will receive

$$\varepsilon_{ph\ min} \approx 10^{-11} \mathrm{eV}\ .$$

This value agrees well with experimental data. So there are reasons to believe that in case of a WIMP entry into the device with a superfluid He-II the excitation is born.

Estimating the probability magnitude of a particle detection by the coherent state in the liquid helium, let us use the following argument. Assume that a healing length ξ determines the atom location in the liquid helium, then the total transversal interaction cross section of WIMP-coherent atomic state becomes equal to $N^* \sigma_{a-W}$ (where N* is a number of atoms in a coherent state, σ_{a-W} is a cross section of atom-WIMP interaction), instead of σ_{a-W}, as in the case of an interaction with atoms in normal fluid. If $d \sim 4{,}1\ \overset{o}{A}$ and $\xi \sim 60$ nm the increase of the detector cross-section is equal $\mu = (\frac{\xi}{d}) = (\frac{6}{4{,}1})\ (\frac{10^{-8}}{10^{-10}}) \approx 150$. In other words, a registration probability of the particle incoming in a device by coherent states of the liquid helium about hundred times higher than by individual atoms of a normal fluid.

Principle of WIMP registration by devices with a weak link

Let us suppose now, that the liquid He - II occupies two volumes, joined by a weak link (for example by a pipe with a small cross-section) (ref. to fig. 6).

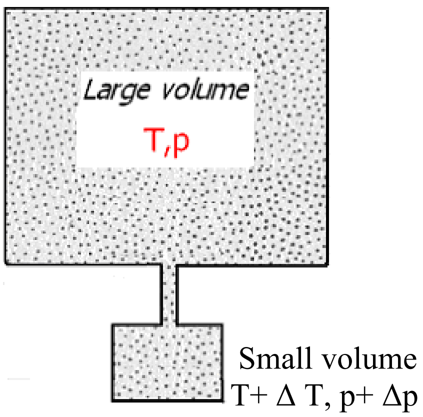


Fig. 6 Volumes with Bose-condensate joined by the weak link

If the various pressures act in volumes, then the superfluid component of the liquid helium will be bypass from one volume into the other (the thermomechanical effect).

The creation of an additional difference of chemical potentials Δμ between two reservoirs will lead to emergence of flow pulsations with Josephson frequency $\mathbf{f_j} = \frac{\Delta\mu}{h}$. Indeed, let the wave function ψ_k describing the state of an He atom, is presented by dependence

$$\psi_k(\vec{r},t) = \mathrm{A}(\vec{r},t)\, \mathrm{e}^{i\phi(\vec{r},t)} \tag{3}$$

and satisfies to time-dependent Schroedinger equation

$$\mathrm{i}\hbar \frac{\partial \psi_k}{\partial t} = \hat{\mathrm{H}} \psi_k . \tag{4}$$

By use of equation (3), we rewrite (4) in form

$$-\mathrm{i}\hbar \frac{\partial \phi_k}{\partial t} = \varepsilon_k , \tag{5}$$

where ε_k is an energy of 'k' state. Because in the liquid He-II the coherent and the incoherent states exist, then we can carry out the averaging in phases of states. It is evident that after the averaging the incoherent states give the zero contribution to a phase variation, and the coherent states posses the uniform energy, i.e. a wave-function phase. So it is possible let to go the index 'k' in case of description for the liquid He-II superfluid component. If the volume with the liquid He-II is divided into some parts, then in every part will arise its proper coherent state (we shall characterize its by the new index 'i' and for simplicity will put i = 1, 2). Under change of an internal energy in one of parts of volume and availability of a weak relationship between parts the interference of states arises, resulting in pulsations of any hydrodynamic magnitudes: pressure P, temperature T, flow rate v_S, etc. The phase difference of coherent states obeys to equation

$$\frac{\mathrm{d}(\phi_1 - \phi_2)}{\mathrm{dt}} = \frac{\mathrm{d}(\delta)}{\mathrm{dt}} = \frac{\Delta\varepsilon}{\hbar} \tag{6}$$

where $\Delta\varepsilon$ is a difference of internal energies in various parts of a volume.

Let WIMP fall into a device (a detector), containing two volumes of liquid He-II, joined by the week link (the pipe with a diameter in some nanometers). The new particle exciting the phonon alters its chemical potential

$$\mu = \left(\frac{\partial \mathrm{U}}{\partial \mathrm{N}}\right)_{S,V} , \tag{7}$$

where U is the internal energy, N is the particles number, S is the entropy, V is the substance volume. The change of a chemical potential μ will transfer the liquid He-II (as it will be shown below) in one of halves in an excited state. Therefore we rewrite now equation (6) in form

$$\frac{\mathrm{d}\delta}{\mathrm{dt}} = 2\pi \frac{\Delta\mu}{\mathrm{h}} . \tag{8}$$

In accordance with equation (8) the oscillations with an linearly varying phase have to arise in the medium. The flow magnitude through the weak link has to range under the law

$$\Phi = \Phi_0 \sin\delta , \tag{9}$$

where $\delta = 2\pi\ f_j\ t$ и $f_j = \dfrac{\Delta\mu}{h}$ is Josephson oscillation frequency. The device, in which was demonstrated the oscillations effect of sound oscillations when establishing the difference of chemical potentials Δ μ in the liquid He-II is described in paper [3].

Thus, it is possible to register still weaker (for approximately four order) excitation energies, because the difference of chemical potentials $\Delta\mu \sim 10^{-15}$ eV corresponds to registered Josephson frequency.

In device [3] the chemical potential difference Δ μ has been produced by the application of an electric field. Another possibility is the creation of Δ μ by the particles injection into one of device cavities. The calculation of the Δ μ value is made on the basis of Gross - Pitaevskii equation.

Quantity determination of the chemical potential variation in Bose-condensate

It should be borne in mind that already from the most general considerations [4] a collision probability in Bose systems grows in comparison with classic, and because of Heisenberg ratio even the weak potential may result in significant momentum transfer, whereby in case of an extended structure - on the creating of an angular momentum.

Imagining superfluid component of the liquid helium as Bose-Einstein condensate, the full number of particles N and the condensate energy E is expressible in terms of its temperature T and the chemical potential μ

$$\mathrm{N} = \sum_{i=0}^{i=\infty} N_i = \sum_{i=0}^{i=\infty} \frac{1}{e^{(\varepsilon_i-\mu)/kT}-1} \quad , \tag{10}$$

$$\mathrm{E} = \sum_{i=0}^{i=\infty} \varepsilon_i N_i = \sum_{i=0}^{i=\infty} \frac{\varepsilon_i}{e^{(\varepsilon_i-\mu)/kT}-1} \; , \tag{11}$$

here ε_i is the energy of a state, N_i is the particles number in this state, T is the condensate temperature. At that $\varepsilon_i > \mu$, otherwise the negative values N_i are occurred .

On the other hand, considering the closed quantum system with a fixed temperature T, it is possible to believe that equations (10) and (11) determine the chemical potential μ

$$\mu = \mathrm{f(N,T)}\,. \tag{12}$$

So the addition of particles to a system, which was in the stationary equilibrium state with a fixed temperature, will cause to increase the chemical potential.

It is interesting to note that the Bose particles condensation in the ground system state is not a direct consequence of the provision of a particle interaction, as this shall be allowed by the classic thermodynamics or classical statistics, and it is exclusively conditioned by the quantum statistics (the induced radiation draws Bose particles into a ground state, in which already are so much particles).

After condensation the absorbtion of new particles in Bose condensate will cause the change of μ. The appearing of an additional Δμ, causing the occupation of excited states, may manifest the Bose particles excitation (in particular, the phonon excitation), the energy quantity of which is well under a recoil atom energy in a nonquantized liquid, but it is sufficient for an experimental registration [3].

Mention also that the knockout of particles from Bose-Einstein condensate or the excitation of an atom chain, being in a coherent state, also could lead to a phonons production.

In principle, the Δμ determination can be derived from modified Gross-Pitaevskii equation. As it is known [5], the equation itself describes the physics of weakly interacting systems at low temperatures. In time-dependent Schroedinger equation

$$\mathrm{i}\hbar \cdot \frac{\partial \psi}{\partial t} = \hat{\mathrm{H}}\, \psi(\vec{r},t), \tag{13}$$

Hamiltonian operator has the specific kind

$$\hat{\mathrm{H}} = -\frac{\hbar^2}{2m}\nabla^2 + \mathrm{U_{ext}}(\vec{r}) + \mathrm{g}\left|\psi(\vec{r},t)\right|^2 , \tag{14}$$

here ∇^2 is Laplacian, U_{ext} is the external field potential, and g is the constant, corresponding to the interaction between particles. In the stationary case it may be assumed

$$\psi(\vec{\mathrm{r}},\mathrm{t}):=\phi(\vec{\mathrm{r}})\exp(-\mathrm{i}\mu\mathrm{t}/\hbar),$$

resulting in the solution of equation for μ

$$(-\frac{\hbar^2}{2m}\nabla^2 + \mathrm{U_{ext}}(\vec{r}) + \mathrm{g}|\phi(\vec{r})|^2)\ \phi(\vec{r}) = \mu\ \phi(\vec{r}) \ . \tag{15}$$

At that
$$\mathrm{g} = \frac{4\pi\ \hbar^2 \mathrm{a}}{\mathrm{m}}, \tag{16}$$

here a is the scattering length of so-called S-wave. In case of low temperatures $\phi^2(\vec{\mathrm{r}}) = \mathrm{n}(\vec{r})$, where $\mathrm{n}(\vec{r})$ is number density of particles. One of consequences of equation is the determination of the healing length ξ in Bose-Einstein condensate

$$\xi = \frac{1}{\sqrt{8\pi \cdot \mathrm{n} \cdot \mathrm{a}}}. \tag{17}$$

The use of formula (16) and (17) allows to define the sound velocity in Bose-Einstein condensate. The description of collective excitations with a low energy is accomplished by Gross-Pitaevskii equation if Bogoliubov approximation accounted for [6]. The Bogoliubov dispersion law for sonic waves serves as one of equation solutions

$$\hbar\omega = \sqrt{\frac{\hbar^2 q^2}{2m}\left(\frac{\hbar^2 q^2}{2m} + 2gn\right)}, \tag{18}$$

here $\vec{q}$ is the propagation vector of an excitation. One can obtain the linear dispersion law from law (18) within the limit of small excitation pulses

$$\omega = \mathrm{cq}, \tag{19}$$

where c is a sound velocity in Bose-Einstein condensate

$$\mathrm{c} = \sqrt{\frac{\mathrm{gn}}{\mathrm{m}}} = \frac{\hbar}{\sqrt{2}\mathrm{m}}\frac{1}{\xi}. \tag{20}$$

When $T \approx 2{,}176$ K, the healing length $\xi \approx 60$ нм, the sound velocity $c \approx 0{,}2$ м/с, that on order of magnitude coincides with the experimental data [3].

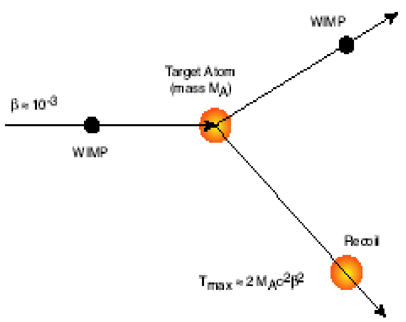
In case of WIMPs hit into Bose-Einstein condensate the system becomes by the open quantum system, and its evolution and a variation of Δμ are derived from a solution of stochastic Schroedinger equation. In the proposed model of WIMP - Bose-particle interaction the stochastic Schroedinger equation is written as

$$\mathrm{d}\psi = -\frac{i}{\hbar}\hat{\mathrm{H}}\,\psi \cdot \mathrm{dt} - \frac{\lambda}{4}(\mu - \overline{\mu})^2 \cdot \psi \cdot \mathrm{dt} + \sqrt{\lambda}(\mu - \overline{\mu})\psi \cdot \mathrm{dq(t)}. \tag{21}$$

The last component on the right side of equation describes Wiener random process. At that λ is the constant of a WIMP – Bose-particle interaction.

Estimates of the registration probability by a classic liquid

On the basis of classical concepts, will lead to an evaluation of a detector efficiency, using the liquid argon. This estimate it is possible then to compare with an evaluation of efficiency of the detector thick with the liquid superfluid He-II. Believing that in case of the hit of WIMP in a detector results in the collision of WIMP with a particle of a working medium, the energy of which will be further specified, let us define the effective cross section of a detector by following ratio. Let



$$\mathrm{E_{abs}} = \int_{\mathrm{V}} \Phi(\mathrm{v}) \cdot \mathrm{mv} \cdot \sigma_{\mathrm{D}}(\mathrm{v}) \cdot \mathrm{dv}, \tag{22}$$

is the energy absorbed in a detector medium in a unit time, $\Phi(\mathrm{v})$ is the particle flux, falling on a unit square of a detector working medium, mv is the particle (WIMP) momentum. Because the WIMP velocity distribution is unknown, and the transversal cross section of a WIMP-atom reaction has the large range of uncertainty, then we restrict ourselves by quantity estimations of (22)

$$\mathrm{E_{abs}} = \Phi_{\mathrm{W}} \cdot \mathrm{T_W} \cdot \tau \cdot \sigma_{\mathrm{D}} = \Phi_{\mathrm{W}} \cdot \mathrm{E_W} \cdot \beta^2 \cdot \tau \cdot \sigma_D, \tag{23}$$

here τ is a measurement duration. Assume that $E_{abs} = 40$ keV, $\Phi_W = 4 \cdot 10^5$ 1/sm^2 s, $E_W = 10^{11}$ eV, $\beta = 10^{-3}$, then in case of a registration time τ = 1 day the evaluation of a needed detector cross-section is equal to

$$\sigma_{\mathrm{D}} = \frac{4 \cdot 10^4}{4 \cdot 10^5 \cdot 10^{11} \cdot 10^{-6} \cdot 0.86 \cdot 10^5} = 1.16 \cdot 10^{-11} sm^2.$$

On the other hand, take

$$\sigma_{\mathrm{D}} = \frac{\rho \cdot N_A \cdot \sigma_{a-W} \cdot V}{\mu}, \tag{24}$$

where μ is an atomic mass, N_A is Avogadro constant, $\sigma_{a\text{-}W}$ is a transversal cross-section atom–WIMP reaction, ρ is a working medium density, we will have the necessary volume of a detector medium is determined from

$$V = \frac{\sigma_{Д}}{\rho / \mu \cdot N_A \cdot \sigma_{a-W}}. \tag{25}$$

If the operating fluid is argon with ρ = 1,42 g/sm^3, μ = 40 g/mol, $\sigma_{a\text{-}W} = 10^{-44}$ см$^2 = 10^{-8}$ pbarn,

$$V = \frac{1.16 \cdot 10^{-11}}{1.42/40 \cdot 6.02 \cdot 10^{23} \cdot 10^{-44}} = 0.54 \cdot 10^{11} sm^3 = 5.4 \cdot 10^4 m^3, \tag{26}$$

that coincides with the evaluation of a count rate 10^{-4} -- 10 event /kg/day.

So, the particle detection with the help of He-II as Bose-Einstein condensate has at least two priorities:

1. The availability of the registration threshold (light neutral particles can not excite the phonon).
2. Drastically improvement of the registration probability (the significantly greater interaction cross section) due to coherence of Bose-Einstein condensate.

In comparison with cryogenic solid-state sensors (the phonons registrations) the WIMP registration with the help of coherent states in Bose-Einstein condensate has the advantages:

- The greater interaction cross section (at least in N=$\xi/a \cdot \mu_{Ge}/\mu_{\mathrm{He}}$ times), since in a coherent state atoms are bounded, and in a crystal lattice the atoms oscillate independently.
- Lower threshold of a sensitivity on energy (the phonon frequency in helium $\nu_{\mathrm{He}} \sim 10^1 - 10^3$ Hz, whereas in Ge $\nu_{\mathrm{Ge}} \sim 10^5 - 10^8$ Hz).
- The reaction cross section of WIMP-He is easier theoretically to calculate, than for WIMP-heavy atom (Ar, Si, Ge).
- Not too low temperatures are needed for a registration (for He-II ~2.7 K, and for Ge ~ 0.01 K).

Concerning the problems of a WIMP registration, we would like to make a remark about conditions of its registration. It remains open the question of an advantage of an experiment in an underground laboratory in relation to a space or a stratospheric experiment in connection with difficulties of finding of a particle flux direction. It is necessary for a reliable registration the networking of underground laboratories with its geodesic reference, development of data exchange protocols, synchronous connection with radio telescopes networks and a gravitational antenna network. On the other hand, perhaps the experiments on a particle collision on accelerators (for example in Fermilab and of course, on putting in LHC (CERN)) can play the constitutive role in a registration of weakly interacting massive particles, because the WIMP flux is hardly fixed in space, and the particle flux density may artificially alter.

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On the Formalism of Physical Theories

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The role of the formalism used to describe the observed phenomena is discussed. It is shown that the mathematical apparatus generated by the consciousness takes part in the creation of the world picture. The choice of the base of this formalism is arbitrary. The example is given that shows the appearance of the "indefinites relation", that has the same quantitative form with the traditional one but does not refer to any physical reasons. The possibility to construct a theory based on the postulate of the stochastic Universe and to provide the possibility to observe the given properties is discussed and the corresponding algorithm is given. The discussed approach is shown to have close connection with the anthropic principle.

1. Introduction

The aspiration for obtaining not only qualitative, but quantitative description of the world applicable for the forecast of the phenomena and their calculation demands the use of mathematics which is the universal and non-contradictive language. On the one hand, the mathematics is generated not by Nature, but by human consciousness, on the other hand, when describing a phenomenon we limit ourselves with some sides of it and regard only a model of it. That is why the appearing structures known as laws of Nature possess two particular features.

1. Due to the limitations when modeling the laws of Nature are true only for some region of parameters characteristic for a model. (E.g., the ideal gas).
2. The artificial structures brought into the model by the consciousness can a) assist and b) prevent such understanding of the world machinery that essentially affects the human life and world view. (E.g., the tunnel transition predicted by the quantum mechanics).

Much depends on the foundations of the mathematical structures used to describe the laws of Nature. If we chose the complex numbers algebra as such a foundation and demanding the mathematical condition of analyticity of the functions defined over it, we get the Cauchie-Riemann conditions – the basis for the physics of hydro dynamical processes. If we chose the biquaternions algebra, then the mathematical condition of analyticity of the functions defined over it will be the equations that correspond to the free electromagnetic field in physics. Therefore, the mathematical conditions are on the one hand formal, but on the other have deep physical meaning, and one can't but think about the reasons of these links of the formalism with our notions of physical laws.

Some other possibilities should also be mentioned. In the recent paper by L.Kauffman (Kauffman 2004) there is an arbitrary Lie algebra supplemented by the elements that appear to suffice the demand of the fulfillment of Leibnitz rule for the derivative of the product and Jacoby identity. The formal corollaries of the introduced mathematical operations ("discrete derivative") result in the equations that have the same forms as the diffusion equation, Schroedinger equation, Hamilton equations, gauge theories equations with their metric tensors and connections. No physical assumption is made, no physical sense is discussed. But if we attribute the sense of, say, time and coordinates to the elements of the algebra, we immediately get the well-known equations of the theoretical physics.

This makes one think that having chosen the right formalism, one can not only get some known equations of physical theories "for free", but also get some other equations. These other equations could either describe the patterns that had not been noticed by the researchers, or lead to the paradoxes in the results that seem still desirable. (The above mentioned situation with the prediction of the tunnel transition was realized in experiment and then – very effectively – in technology). As a matter of fact, both leading modern concepts – quantum mechanics and relativistic physics – demonstrate this deductive approach. Their formalisms play such great roles that their "philosophical backgrounds" are disputed even nowadays. From the point of view of the understandable to all classical physics, these backgrounds are exotic.

That is why the known two-gaps interference experiment is treated in so many ways: regular quantum mechanics and its particle-wave dualism; Feynman-Hibbs approach and path integrals; Bohm's "quantum potential"; not very popular in physical community but still existing theories of plural Universes starting from Everett's paper (Everett 1957). Thanks to the instrumentalist tendencies in science and to the tremendous lot of results achieved by quantum mechanics these disputes are not so evident. This can not be said about the relativity theory which is the established theory with not so evident results.

A question arises: is it possible to make such a choice of the formalism while designing a theory that, on the one hand, it will describe at least part of the known effects, and on the other hand, allow the existence of the effect that is considered desirable? Surely, this "volunteering" approach is very vulnerable for criticism and full of risk. Successful "trials" in experimental physics remained in history; no one remembers unsuccessful "trials". The experimental check of the presumably challenging results would have been difficult due to the natural inertia of thinking. Nevertheless, it should be noted that the theoretician can not suggest the experimental check of what is contradicting the theory he uses or constructs. That is why the theoretician supporting the theory common today won't suggest the perpetual mobile project not because the impossibility of this machine is the law of Nature or because its existence is rejected by the French Academy, but because the existence of this machine contradicts the logically closed formalism of the modern theory.

J.Wheeler (Wheeler 1982) justly pointed at the role of questions that we pose in the process of cognition. Actually, the situation in science characteristic for this or that period of time is defined by the sequence of questions we try to answer. In this sense the question of the formalism choice should be analyzed though public excitement should be avoided.

2. On the subjective uncertainty in non-perturbing measurements

Since there is no universal and independent language of experiment and any experiment is treated in frames of some theory, we conclude that the role of the theory is decisive. The footing of the theory is the logical structure, and everything that can not be determined strictly is presented by the statistical averages that can be also processed with the help of logics. The known problem of the scattering of the particle on the two gaps led to the objectification of the probability theory notions and to the introduction of the principle of complementarities. The participation of the experimenter's consciousness in the creation of the physical reality was discussed, but the discussion was not finished. As far as I can judge an important aspect of the measurement procedure was never touched.

The Heisenberg's uncertainty principle

$$\Delta X \Delta P = \hbar \tag{1}$$

is usually treated as a manifestation of the particle-wave dualism – physical phenomenon characterizing the micro world. The simplest way to obtain eq. (1) uses such notions as the de Brogle wave of the micro particle and the diffraction of waves on the gap. It is always underlined that it is impossible to measure the parameters of micro particle in such a way that its further behavior is not disturbed.

Let us describe the experiment in such a way that the physical complementarities principle is not involved. This approach is based on the ideas of (Kauffman 2004).

Take a particle moving along the axis of the reference system. The goal is to define the product of its coordinate by the velocity in the given point. Let us use the "non-perturbing" method of measurements. We have to perform four measurements with the help of a clock and a ruler: coordinates and times in the initial and final points. The coordinates difference divided by the time interval gives the value of velocity. The smaller is the time interval the more precise is the velocity value.

The question is: which of the two coordinates should be taken as the coordinate of the position of the particle? This question is connected with the question of the sequence of the measurements used. If we take an intermediate point as the particle location, then the instrumental error causes the irreducible error. But the question has the principal aspect too.

Write down the results for the product of coordinate by the velocity for both sequences of measurements with regard to the instrumental errors, $\Delta x, \Delta t$:

$$(X \pm \Delta x) \cdot \frac{(X' \pm \Delta x) - (X \pm \Delta x)}{(T' \pm \Delta t) - (T \pm \Delta t)} = (X \pm \Delta x) \cdot \frac{(X' - X) \pm 2\Delta x}{(T' - T) \pm 2\Delta t}$$
$$\frac{(X' \pm \Delta x) - (X \pm \Delta x)}{(T' \pm \Delta t) - (T \pm \Delta t)} \cdot (X' \pm \Delta x) = \frac{(X' - X) \pm 2\Delta x}{(T' - T) \pm 2\Delta t} \cdot (X' \pm \Delta x) \quad (2)$$

Here X and X' are the coordinates for the initial and final positions, T and T' are the corresponding times. (Here the coordinates are not the operators). We see that the results differ. This is principal because there is no reason to prefer one order of measurements to the other. Here appears the uncertainty caused solely by the observer's will but not by the properties of the physical world. Let us evaluate this uncertainty.

Find the difference

$$VX' - XV = \frac{(X' - X) \pm 2\Delta x}{(T' - T) \pm 2\Delta t} \cdot ((X' - X) \pm 2\Delta x). \quad (3)$$

The increase of accuracy in the measurements of the coordinate corresponds to the decrease of the instrumental error, Δx, while the increase of accuracy in the measurements of the velocity corresponds to the decrease of both (X'-X) and (T'-T). There is a limit of the accuracy of measurements equal to the instrumental accuracy. In this case let us take $X' - X = \Delta x; T' - T = \Delta t$. Then the last expression gives

$$VX' - XV \sim \frac{(\Delta x)^2}{\Delta t} \quad (4)$$

Thus, the product calculation uncertainty due to the arbitrary choice of the order of measurements is characterized by the expression well-known from the probability theory. If the ratio in the r.h.s. of eq. (4) is constant, then it describes the square of the displacement of the particle performing one-dimensional random walk with step Δx every Δt seconds.

To measure length and time (and mass) let us choose the so called Planck units combined out of physical constants.

$$L = \sqrt{\frac{\hbar G}{c^3}}; T = \sqrt{\frac{\hbar G}{c^5}}$$
$$L \approx 10^{-35} m; T \approx 10^{-43} s \quad (5)$$

$$M = \sqrt{\frac{\hbar c}{G}}$$
$$M \approx 10^{-7} kg \quad (6)$$

If the accuracies of the instruments correspond to these values, i.e. $\Delta x = L$, $\Delta t = T$, then

$$\frac{(\Delta x)^2}{\Delta t} = \frac{\hbar G}{c^3}\sqrt{\frac{c^5}{\hbar G}} = \sqrt{\frac{\hbar G}{c}} = \frac{\hbar}{M} \quad (7)$$

It follows that measuring the product of the coordinate of the particle with the mass equal to one Planck's unit of mass by its momentum, and using the instruments that do not perturb the motion of the particle and have the accuracies of one Planck's unit of length for measuring length and one Planck's unit of time for measuring time, we get the expression for the uncertainty due to the observer's will

$$PX' - XP = \widetilde{\Delta}(XP) \sim \hbar \quad (8)$$

When we use the mentioned system of objects and units, this uncertainty corresponds to that defined by eq. (1).

Obviously, the particle with mass equal to one Planck's unit of mass can not be considered a micro particle. And the accuracies of length and time measurements are far beyond any known achievements. Nevertheless, the result seems interesting since it illustrates the probabilistic character of measurements (not only in quantum mechanics) from the unusual side. The role of the observer which was so intensively discussed by the founders of the quantum mechanics is presented explicitly.

The appearance of $\hbar$ in the r.h.s. of eq. (8) looks very spectacular, but it has no connection with eq. (1). If we take more reasonable values for the accuracies of length and time measurements, e.g., $\Delta x = 10^{-10}$m , $\Delta t = 10^{-15}$s, and take a proton with mass $m = 10^{-29}$kg, then we get again

$$m(VX'-XV) = m\frac{(\Delta x)^2}{\Delta t} = 10^{-34} \sim \hbar \tag{9}$$

Contrary to (1) this uncertainty has a purely subjective character. Even if we manage to overcome this problem in the numerical calculations in the quantum mechanical problems, it stays in principle.

The preliminary conclusion is: the known formalism is not defined sufficiently well. Particularly, the procedure of indirect measurement is not strictly defined and this can affect the interpretation of the results. In the regular quantum mechanics, when we pass to operators the commutators appear and we can follow their link to the Poisson brackets, the latter being such objects of the theory that have the physical sense. But on the other hand, the same commutators can be linked with the measurement procedure. This can affect the algorithms of the corresponding numerical calculations. It is unclear how to overcome this problem since no reason for this or that choice of the order of measurements can be given.

The function of the observer's consciousness

Let us regard a cloud of points on the experimental plot, for example, the dependence of the coordinate of a particle on time. If there is no additional information about the nature and character of the process, then the only thing to do is to find the approximating function that fits the experimental data best. Analyzing this function, one can get some information on the situation observed. But to do this one would inevitably use some concept that has definitions and axioms as background. For example, if we decide that the cloud of points is distributed around a straight line, then we conclude that the particle moves in a uniform straight way, hence, the sum of the forces acting on it is equal to zero. This conclusion corresponds to the definition of force as a value proportional to acceleration, and to the axiom of the inertial frames' existence (1-st law of dynamics). All the points that are not on this line are treated as resulting from errors or as small corrections due to the phenomena not taken into consideration (dissipation). If the allocation of these points is far enough from the straight line and the density of the cloud is large, then other approximating functions can be used their choice being performed by the experimenter. For example, the choice of a parabolic function presumes the existence of the force field the source of which must be found. Thus, we see that the density and the allocation play an important role. If they are large, the observer's consciousness starts to play an important role in the interpretation of data. If they are little, there is a risk not to find an important feature of the process. In this case the criterion for the optimal density and the allowable allocation of points is produced by the observer's consciousness which operates with some chosen concept.

Let us regard the situation (Siparov 1997) when we observe the particle performing the (one dimensional) random walk with the step equal to δx every δt seconds. Let us follow the change of location of this particle by the periodically switching on the measuring device for Δt seconds every $A\Delta t$ seconds ($A > 1$) ("stroboscope"). With this the random character of the (original) motion would not be presented explicitly. Let us introduce the following scales hierarchy. The time of a single step δt is less than the time of the observation Δt which is less than the time t of the process as a whole. That is

$$\delta t < \Delta t << t \tag{10}$$

Let us regard only the following particular case. Let δt depend on time but rather weakly, that is eq. (10) remains true during all the experiment. Actually, it means that the motion resembles Levi flights. If the parameter α characterizing Levi flights (Shlesinger et al 1995) is larger than two, $\alpha > 2$, then the average square displacement $< x^2 >^{1/2}$ obtained in experiment will fit the approximate plot corresponding to the regular random walk with constant δx and δt

$$\langle x^2 \rangle^{1/2} = \sqrt{b\frac{(\delta x)^2 t}{\delta t}} \tag{11}$$

where b is the dimensionless constant.

Let us consider the last equation as the solution of a dynamical equation in which $< x^2 >^{1/2}$ plays the role of displacement. Let us take $X(t)$ and $T(t)$ instead of δx and δt in eq. (11)

$$\langle x^2 \rangle^{1/2} = \sqrt{b\frac{X^2(t)t}{T(t)}} \tag{12}$$

This expression (12) presumes the use oh the ergodic hypothesis according to which the average of realizations (l.h.s) coincides with the average at $t \rightarrow \infty$ (r.h.s). Now we find the cloud of experimental points with the help of the "stroboscope" and try to approximate it by a straight line. Let us find such values of steps $X(t)$ that could provide the needed points. The following condition must be sufficed

$$b\frac{X^2(t)}{T(t)} = v^2 t \tag{13}$$

where $v = const.$ In this case observing the motion of the particle "with the help of the stroboscope", we can regard the possibility of the straight homogeneous motion. Introduce the function

$$C_s \equiv b\frac{X^2(t)}{T(t)} - v^2 t = 0 \tag{14}$$

If the set of points visited by the particle is dense enough (δt is small enough), then during one flash of the "stroboscope" lasting for Δt seconds ($\Delta t >> \delta t$), there would be the points for which the step length $X(t)$ suffices eq. (14). This makes it possible to observe the uniform straight motion, while the scattering of the other points on the plot (originating in our case from the random character of the process) could be interpreted as the experimental error.

In the same way the conditions corresponding to other types of motion (accelerated, etc.) could be found. The choice of the linear approximating function and the $C_s = 0$ condition in the form of eq. (14) corresponds to the fact that they suffice the postulate of the inertial frames existence. Nevertheless, it is clear that one can observe the other motions in the same set of points if he uses other approximating functions. Since in real experiment we don't know how the particle does *really* move and we can judge upon this only interpreting the results of observations, the suggestion we have made can not be rejected at once however strange it could seem.

The results of the experimental data processing (e.g., least square method) depend both on the characteristics of the process and on the process organization (δt and Δt ratio, A value). In case of the micro particles observations the main role is played by the choice of the approximating function. The corresponding historical example is the interpretation of the results of the heated body radiation measurements given by Wien, Rayleigh and Jeans and Planck.

Thus, C_s function (C - – Consciousness, s – stochastic) is the function of the observer's consciousness. The need for the introduction of a function connected with the observer's consciousness was discussed in (Catania 1990, Stapp 1994, Capra 1994). Notice also that the need for the unified approach to the description of micro and macro phenomena is realized long ago (Feynman and Hibbs 1968, Siparov 1994), but this problem is not solved up to now. To exclude the effect of the $T(t)$ choice on the investigation of the macroscopic objects of classical mechanics one should only demand that for large t the $T(t)$ value is essentially smaller than t, and this is usually so for any measurements.

The above said provides the possibility, for example, to postulate that the Universe possesses the stochastic properties – in the same sense that we have in mind while postulating that the space-time is uniform. The principal law of such “twinkling” Universe is that very variability that J.Wheeler mentioned in (Wheeler 1982). The approach philosophically close to this was developed by A.Sakharov (Sakharov 1984).

The recognition of the arbitrariness in the choice of the foundation of a theory means that while constructing the mechanics based on the Hamilton-Lagrange formalism that uses the least action principle one should act in a different manner than usually. One should pass from the variation problem with fixed ends to the variation problem with free ends, while the role of the transversality condition will be played by the condition $C_s = 0$. From the formal point of view it means the following. The traditional approach presumes that there exists the determinate Universe and we are able to observe it and describe it by fixing certain coordinates and times, fitting the approximating function and proclaiming it a law. Considering the Universe to be “twinkling” we realize that indicating a (arbitrary) law we are principally able to observe its performance in the world around if we choose the appropriate conditions of observations.

One should not think that this is easy. Since the main tendency of science is the coordination of its concepts in various fields, the reduction of the results to the form acceptable by the scientific community and the interpretation of these results would demand significant efforts both in experiment and philosophical world view. These efforts might well appear to be incommensurable with the supposed advantages of the new approach, and such situation took place a lots of times. That is why the “arbitrary” law that can in principle be observed must be chosen with regard to the far off consequences. The experimental check up could be performed only in case when the importance of these consequences is sufficient.

The base of the new formalism providing the possibility to describe <u>the observable</u> Universe with the postulated stochastic properties is the following system of (Lagrange) equations

$$\begin{cases} (L - v\dfrac{\partial L}{\partial v})\Big|_{t=t} T(t) - (L - v\dfrac{\partial L}{\partial v})\Big|_{t=T_1} T_1 + \dfrac{\partial L}{\partial v}\Big|_{t=t} X(t) - \dfrac{\partial L}{\partial v}\Big|_{t=T_1} X(T_1) = 0 \\ C_s = 0 \end{cases} \tag{15}$$

If there are no fields produced by their sources and the particle moves freely (in the twinkling Universe) and we want it in this case to move in a uniform straight way, then we should take C_s according to eq. (14) and solve system (15) to find the Lagrange function L_0 for the free particle. Taking into account the possible fields and complicating the system of equations one could find the equations of motion with regard to forces. In frames of this approach one can speak of the uniform and isotropic space-time only with regard to a certain averaging over some region.

One could also regard a reference frame in which the <u>free</u> particle moves according to eq. (11), i.e. it has an acceleration decreasing with time as $t^{-3/2}$. This system will become inertial at large t (in appropriate units). This is analogous to the situation in the general relativity in which the space-time region far from material masses has the vanishingly small curvature.

When constructing the electrodynamics in the stochastic twinkling space-time one can use the corresponding results for the discrete space, e.g. (Plohotnikov 1988).

In quantum mechanics the transportation of the stochastic properties from a micro object to the space-time as a whole (Catania 1990) provides the possibility to describe the interaction of a quantum particle with a macroscopic instrument in a unified way. Besides, the finite character of the first interval T_1 is in accord with the ideas and results given in (Kobe & Aquilera-Navarro 1994) where the uncertainty relation for energy and time was discussed.

If the particle moves according to eq. (12), then it also has the limiting value of observable velocity. Really, in this case the $x(t)$ dependence is the distribution of points in the vicinity of the parabola branch. Calculating the dispersions $D(T)$ and $D(T')$ with regard to $T' - T >> \Delta t$, we get the limiting velocity $v = (1/2)dD(T)/dT$. This means that though the “superlimiting” velocity is possible, one can observe it only if the time of a single observation Δt is less than δt. This is due to the choice of $\langle x^2 \rangle^{1/2}$ as a displacement.

Therefore, we see that the two independent postulates of the existing theory – inertial frames existence and limiting value of velocity existence – can be replace by one: the Universe possesses the stochastic properties. Those two postulates are corollary of this one. In the traditional theory the result of the observation is the state of the concrete (quantum) object. In this theory the observer takes part in the creation of the observable world as a whole. The physical laws equations are only some structures superimposed on the world and originating from the observer's consciousness. Out of all the set of random points-events we observe only those that suffice best to the chosen structure. This means that the consciousness that has a) formed the arbitrary world picture, b) has provided it with some properties and c) has constructed the non-contradictive structure for its description will perform its observation observing its own self. The results of such observation presented to another consciousness would help to form the same picture with the same properties in that consciousness, and this would finally lead to the objectification of the world. The huge amount of information collected by various observers conceals the stochastic character of the initial picture, everyone sees (and observes in experiments of any kind) the picture the convention of which has been composed during the history. One can easily see that the anthropic principle naturally follows from this approach.

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Dynamics of lattice models of media and physical vacuum

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1.Introduction.

Modern field theory is constructed on the base of continuous model of physical vacuum. However, if physical vacuum is some media, we should propose, that such media should have distinct microscopic structure. We know that limit equilibrium state of every media at low temperature is crystalline lattice state. So, from this point of view, we can propose, that physical vacuum resembles the space-ordered structure such as crystalline lattice.

In this paper we would like to discuss some properties of physical vacuum as ***crystalline lattice***.

2. Structure and dynamics of physical vacuum as crystalline lattice.

We propose that physical vacuum is constructed as a result of very small particles (praparticles) packing as a result of attraction according to Newton gravitational law at large distance and repulsive law at small distance. The vacuum lattice constant “a” and mass value “M” of praparticles can be estimated from dimension considerations, using fundamental constants: velocity of light ($c=3x10^{8}$m/s), gravitational constant ($G=6.67x10^{=11}m^{3}/kgs^{2}$) and Max Plank constant ($\hbar=1.05x10^{-34}$Js). As a result of such estimation [1] we have the order of mentioned value: vacuum lattice constant (so-called Plank length) $a{\sim}10^{-35}$ m (closed to the dimension of praparticles) and mass of praparticles $M{\sim}10^{-8}$kg.

If we take into account attraction between praparticles only, we have gravitational crystalline lattice, which is unstable and should be compressed into point state. Maybe state of strongly compressed vacuum really have existed in past, but now we have large enough extent of physical vacuum. According to proposition of compression and next expansion of physical vacuum, the crystalline lattice of physical vacuum should be inhomogeneous and does not coincide with known crystalline structures. However, at the first step we shall consider physical vacuum crystalline structure as homogeneous, at last at small enough scale. For example we shall consider structure of physical vacuum as face centered cubic lattice with lattice constant $a=10^{-35}$ m, packed from particles with mass equal to $M=10^{-8}$kg.

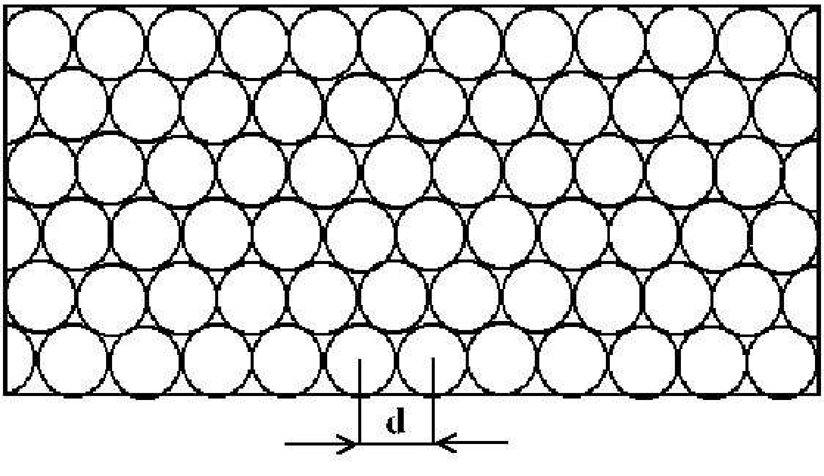

Fig.1. Structure of globular photonic crystal with lattice constant d»a.

Recently new type of solid state objects has been synthesized – so-called globular photonic crystals. In these crystals crystalline structure are forming as a result of packing of globules, size d of which is essential larger, than known in nature crystalline lattice constant a~10^{-10}**m**, i.e. d»a (see Fig.1).

There are the resembling structures in nature, known as natural opals. Artificial opals consist from amorphous silica globules. Size of such globules is equal to 200-400 nm. Amorphous silica globules form cubic face centered lattice. Corresponding lattice constant d is comparable to wavelength of visible or ultraviolet electromagnetic waves.

The simplest example of crystalline chains, used in lattice dynamics, is so-called monoparticle chain with additional bonds [1].

For such type chain (see Fig.2) we have following law of motion, taking into account only nearest neighbor:

$$m\ddot{u}(l) = -\gamma_0 u(l) - \gamma\left[2u(l) - u(l-1) - u(l+1)\right], \tag{1}$$

where $u(l)$- is the displacement of particle with number l $(l = 0,1,\ldots)$.

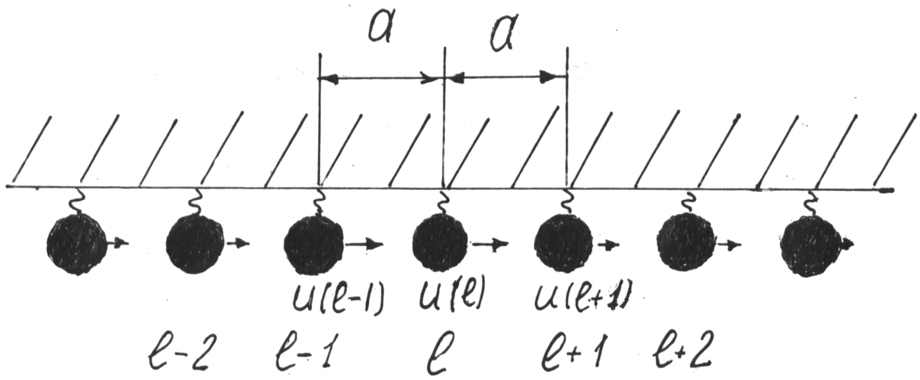

Fig.2. One-dimensional crystalline lattice with additional bonds.

Dispersion law for such type chain is

$$\omega^2(k) = \frac{\gamma_0}{m} + 4\frac{\gamma}{m}\sin^2\frac{ka}{2}, \tag{2}$$

where a - is the lattice constant, γ_0 and γ - corresponding force constants.

If $\gamma_0 = 0$ and $\gamma > 0$ from (2) we found acoustical branch dispersion law:

$$\omega = 2\frac{S}{a}\sin\frac{ka}{2}, \tag{3}$$

where $S^2 = \frac{\gamma}{m}a^2$, and k - wave vector of the corresponding flat wave. The simplest one-dimensional crystalline model of the physical vacuum is the monoparticle chain with the dispersion law (3). In this case value S has a sense of velocity of light ($S\approx$c) and fundamental constant "a" is the elemental translation. As we pointed before and this constant is close to 10^{-35} m. So at small wave vector k in (3) we can use linear approximation: $\omega = ck$.

Such relation is the first approximation for photonic dispersion law of the physical vacuum.

If $\gamma_0 > 0$ and $\gamma > 0$ we have dispersion law of crystalline chain with positive mass of corresponding quasiparticles:

$$\omega^2 = \omega_0^2 + 4\frac{S^2}{a^2}\sin^2\frac{ka}{2}, \tag{4}$$

where $\omega_0^2 = \frac{\gamma_0}{m}$ and $S^2 = \frac{\gamma}{m}a^2$.

Accordingly, when $\gamma_0 > 0$ and $\gamma < 0$ we have optical branch with negative mass of corresponding quasiparticles:

$$\omega^2 = \omega_0^2 - 4\frac{S^2}{a^2}\sin^2\frac{ka}{2}. \qquad (5)$$

At last when $\gamma_0 < 0$ and $\gamma > 0$ we have:

$$\omega^2 = -\omega_0^2 + 4\frac{S^2}{a^2}\sin^2\frac{ka}{2}. \qquad (6)$$

where $\omega_0^2 = \frac{|\gamma_0|}{m}$ and $S^2 = \frac{\gamma}{m}a^2$.

We shall use linear approximation for photonic dispersion law ($\omega = ck$) and propose that that the new lattice constant d (d » a; a – is lattice constant of the initial vacuum lattice) of physical vacuum takes place due to structural phase transition in vacuum. For three-dimensional case physical sense of the period d is the diameter of the globules, forming new crystalline phase state of the physical vacuum as a result of the structural phase transition from the initial cubic lattice, lattice constant of which is equal to "a". Thus we describe physical vacuum as globular photonic crystal, constructed from compact packed globules, with new lattice constant, equal to d. In this case we have at the edge of the new first Brillouine zone the following photonic frequency value:

$$\omega_1 = c\frac{\pi}{d}, \qquad (7)$$

and accordingly for $k = 0$ (Γ-point):

$$\omega_0 = 2c\frac{\pi}{d} \qquad (8)$$

Approximately we may use (4-6) relations for description of photonic band laws in photonic crystal if we use d - constant instead of a. The relation $\omega = 2\frac{c}{d}\sin\frac{kd}{2}$ corresponds to the lowest photonic band; (5) - to the second and (4) - to the third one (S=c d=a). At small wave-vector value we may use quasirelativistic approximations as :

$$\omega_1 = ck \qquad (9a)$$

$$\omega_2^2 = \omega_0^2 - c^2k^2 \qquad (9b)$$

$$\omega_3^2 = \omega_0^2 + c^2k^2 \qquad (9c)$$

$$\omega^2 = -\omega_0^2 + c^2k^2 \qquad (9d)$$

In these relations c-constant has a sense of velocity of light. So we can conclude that photon-like dispersion law (9a) correspond to photons, (9c) - to relativistic particles with positive rest mass, (9b) - to relativistic particles with negative rest mass, (9d) - to relativistic particles with imagine rest mass. Thus in photonic crystals we have the unusual situation, when photons become heavy particles with negative or positive mass. For dispersion law, described (5) and (7b) equations, we have come to conclusion that the sign of $\left(\frac{d\omega}{dk}\right)$ - value is negative. Consider the situation, when light ray falls onto the surface of photon crystal at zero angle opposite to unit vector ***i*** with light velocity equal to ***c* = -** c***i*.** So in this case group velocity of light vector $\left(\frac{d\omega}{dk}\right)\boldsymbol{i}$ is opposite to wave vector ***k*** = = ***ki*** (***i*** - is the unit vector, opposite to the direction of incident ray). Group velocity of light vector $\left(\frac{d\omega}{dk}\right)\boldsymbol{i}$ has the same direction as **c = -** c***i*** – vector and phase velocity of light vector $\frac{\omega}{k}\boldsymbol{i}$ has the same direction as ***k*** wave vector**.** For normal incidence of light ray onto the surface of photonic crystal we have:

$$\frac{\omega}{k}\boldsymbol{i} = \mathbf{c}/\mathrm{n}, \tag{10}$$

where **c**= **-** c$\boldsymbol{i}$ **-** is the velocity of light vector out of the crystal and n is refraction index. So in this case sign of n is negative. Accordingly for the third branch the sign of $\left(\frac{d\omega}{dk}\right)$ - value is positive and sign of n also is positive.

For any case we have common relation for index of refraction as:

$$|n| = \frac{ck}{\omega(k)}, \tag{11}$$

where $\omega(k)$ - is the dispersion laws according to (3-5) or (7a-7c). The sign of n is negative if the sign of $\left(\frac{d\omega}{dk}\right)$ - value is negative and the sign of n is positive if the sign of $\left(\frac{d\omega}{dk}\right)$ - value is positive.

We have proposed that physical vacuum is some type of globular photonic crystal with lattice constant L=d, corresponding to weak interaction length. According to the known theory [2] heavy boson energy is close to $E_w = 90\Gamma eV$. So we can estimate weak interaction lattice constant L from (8)-relation:

$$L = \frac{2\hbar c\pi}{E_w} = \frac{2\cdot 1{,}05\cdot 10^{-34}\cdot 3\cdot 10^{8}\cdot 3{,}14}{90\cdot 10^{9}\cdot 1{,}6\cdot 10^{-19}} = 2{,}75\cdot 10^{-17}\,m\,. \tag{12}$$

Such value is in correspondence with that, known from weak interaction field theory [2].

Globular photonic crystals model for physical vacuum, developed by us, predicts the existence not only heavy photon with positive mass, but also the heavy photon with negative mass according to (7b) relation. Such type excitations so far have not been observed.

Dispersion laws (6) and (7d) predict also the existence of so-called tachyon-like particles in physical vacuum. We suggest that such type particles might be elemental excitations, corresponding, for instance, to longitudinal electromagnetic waves, which are unstable at small wave vector value.

Each elemental globule of real photonic crystal is a spherical resonator. Total symmetric mode of such resonator corresponds to scalar waves, propagating along the photonic crystal. Main resonance frequency of such resonator is close to value: $\omega_r = S\frac{\pi}{L}$, where L - is the size of the globule, S – sound velocity. In common case the global resonance modes with frequency ω_r have deformational nature and behave as tensor-type excitations. Dispersion law of the corresponding tensor-type waves may be written as:

$$\omega^2 = \omega_r^2 + S^2k^2. \tag{13}$$

Elemental excitations of such kind waves are even type. So such type excitations we shall call “evennons”. If S-constant relates to the electromagnetic wave velocity: S=c, we have a new type of electromagnetic waves – tensor or even type electromagnetic waves and corresponding elemental excitations - “eventons”. Note, that gravitational waves have also tensor type symmetry properties. So we can suppose that gravitational waves are some kind of deformational perturbations of physical vacuum as real media and are one type of even electromagnetic waves. Thus elemental excitations of gravitational waves -gravitons- are indeed eventons.

3. Opportunity of generation and observation of even-type waves in media and physical vacuum

Now there is the problem of experimental observation of even-type sound and also electromagnetic waves. We have proposed that using two-photon scattering of light, excited in real condensed matter (crystal or liquids) and also in globular photonic crystal might solve this task. Nanoresonator inelastic light scattering of light in globular photonic crystal has been recently recorded in experimental work [3]. The results of this work are illustrated by Fig.3. Continuous laser with monochromatic line of generation used for exciting of the spontaneous secondary emission in this case. We can see several Stokes(S) and antiStokes(A) satellites, corresponding to excitations of different nanoresonator modes of opal globules. The intensity of S and A satellites are essentially less, then exciting line intensity. Such property is typical for spontaneous scattering of light. If high-power laser pulses are used for excitation of inelastic light scattering, so-called stimulated light scattering may be observed. In this case the intensities of exciting and satellites lines become comparable. To now such experiments have been realized by author of this paper with N.V. Tchernega and A.D. Kudryavtzeva. Stimulated nanoresonator light scattering in globular photonic crystals (artificial opals) have been observed by using of giant (Q-switched) ruby (694.3 nm) laser pulses with intensity close to 10^8-10^{10}W/cm^2.

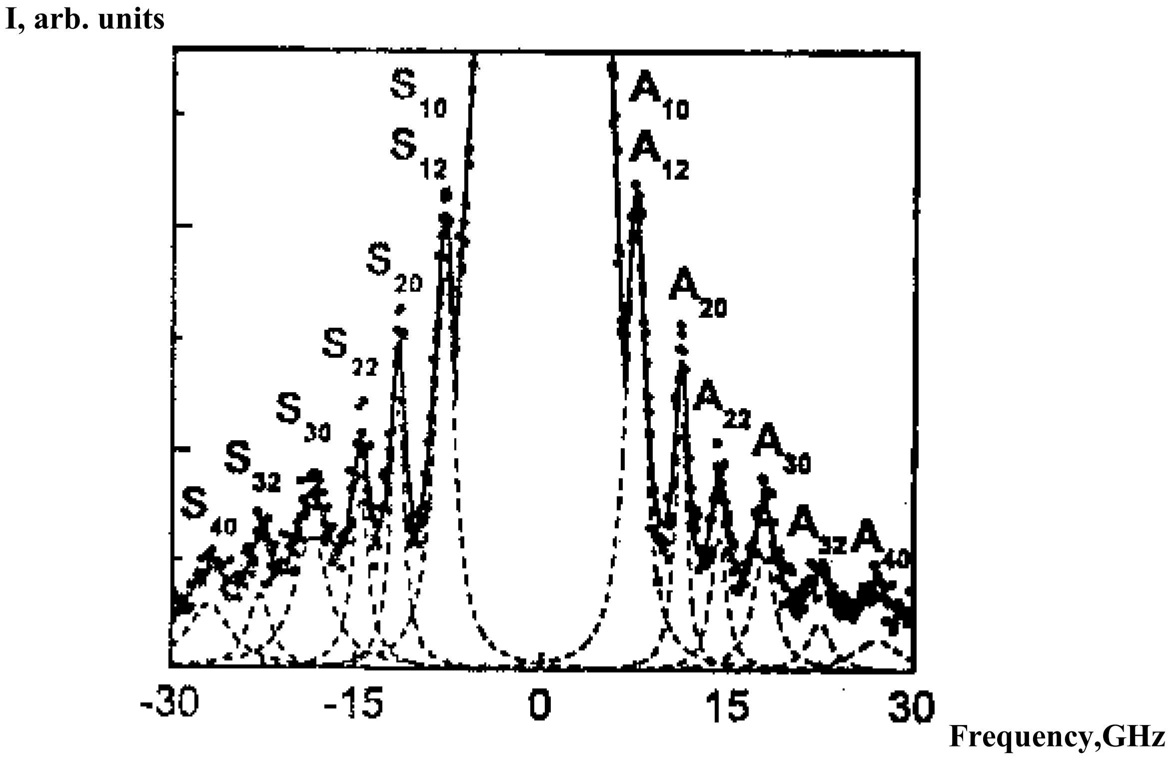

Fig.3. Spectra of nanoresonator scattering of light in globular photonic crystal, corresponding to excitation of even-type modes, obtained in work [3].

In this case only one S-satellite was observed. The intensity of this S-satellite is comparable with intensity of exciting line (694.3 nm). As a result of stimulated nanoresonator light scattering even-type sound waves should be generated in globular crystal. Such waves are propagating in media with velocity, close to known value of sound velocity in the condensed media. The recording of even-type sound waves may be realized with the help of piezoelectric detectors. According to symmetry properties of such waves, at the "vacuum-media" boundary such waves should be converted into electromagnetic even type (gravitational) waves and vice versa.

Another type of scattering has been observed for fused silica (see Fig. 3). In this case we can see distinct low-frequency (60 cm^{-1}) maximum, known as Boson peak. Boson peak emerges as a result of scattering on nanoparticles, existing in fused silica; the size of these nanoparticles is near 2 nm.

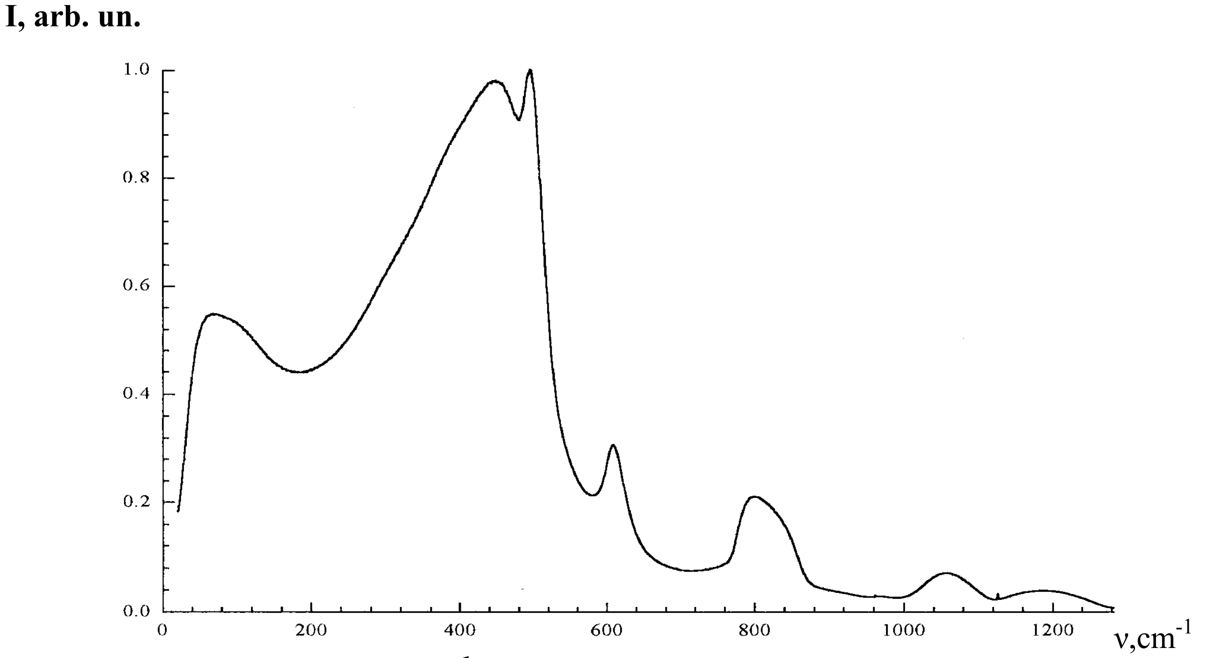

Fig.4. Low-frequency (60 cm^{-1}) maximum (Boson peak), according to nanoclasters of amorphous quartz in Raman scattering spectra (at left), corresponding to tensor-type spontaneous process.

Even-type waves can be also excited as a result of Raman scattering on total symmetrical molecular modes in liquids (C_6H_6, CS_2 and others) and crystals. In these cases the mode frequencies correspond to 500-1000 cm^{-1} value, i.e. to 10^{13} Hz –range. Accordingly as a result of stimulated Raman scatterings of light the generation of corresponding eventon-type waves should be take place. Experimental setup for stimulated Raman scattering observation is illustrated by Fig. 5.

Recently infrared emission simultaneously with Raman scattering of light has been predicted [4] and observed. So we have the opportunity to record even type waves, generated as a result of stimulated Raman scattering, by means of infrared emission recording.

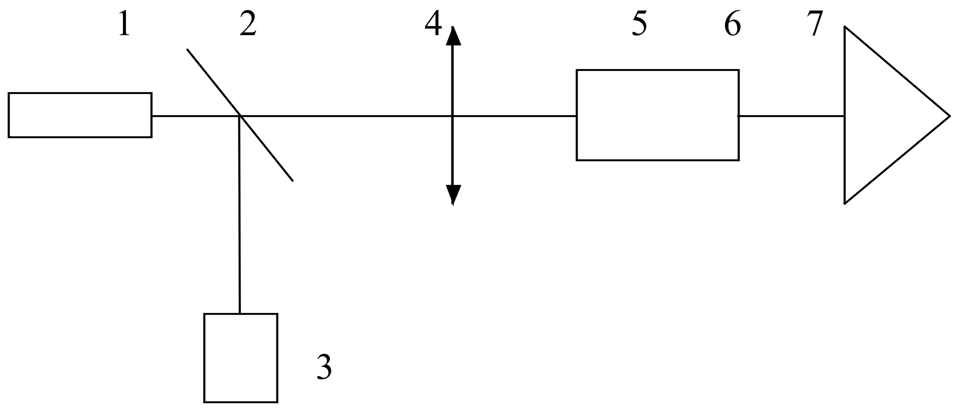

Fig.5. Experimental setup for stimulated Raman scattering observation.
1 - laser, 2- glass plate, 3 – system of laser emission recording, 4- lens, 5 – sample, 6- exit window (philter), 7- detector.

In our recent work [5] we have shown that the efficiency of inelastic light scattering essentially increased for ultra dispersive substances, placed into resonator cuvette. In these conditions the threshold for stimulated Raman scattering observation should be essentially decreased.

4. Conclusion.

Thus we have come to the next results.

1. Dispersion law of acoustical branch of real crystalline lattices resembles photon dispersion law of physical vacuum. Accordingly we should expect dispersion of light velocity for large enough wave vector value.
2. Physical vacuum may be described as globular photonic crystal with super-lattice constant equal to $\sim 10^{-17}$m (distance of weak interactions).
3. Relativistic law of dispersion of elemental particles may be obtained from more common relations by using of lattice models of physical vacuum. The particles with negative and imaginary mass should also exist.
4. Even-type emission may be generated in real photonic crystals, molecular media and in physical vacuum as a result of stimulated light scattering.

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The Ether Concept in Modern Physics

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The ether in modern physics interprets the formal structure of Relativity (Special and General), and suggests ways of unifying Relativity with Quantum Mechanics. The modern ether is a relativistic medium, compatible with geometrical, non-classical formulations of physics. It serves as a disclosing model, indicating the relationship between quasi-classical Poincare-Lorentz Relativity – couched in terms of a Lorentzian ether – and Relativity expressed in the geometrized tradition established by Einstein and Minkowski. The vortex-sponge mechanical analogue, which removes long-standing methodological objections to Poincare-Lorentz Relativity, is the most promising analogue. When geometrized it provides an equivalent to Einstein's space-time of General Relativity. The vortex-sponge resembles a Dirac ether. It can be regarded as a chaotic medium, generating phenomena interpreted by dynamic algebras, or nilpotent theories. Use of the concept no longer implies an adverse attitude to the Einstein-Minkowski tradition, and the long-standing ether–relativity polemic is a sterile misconception.

The "Ether Question" 1905-2005.

The ether in 21st C physics is a continuum theory, generally non-classical, which interprets fundamental activity in terms of space-time geometry or action in a medium. For full development, it requires a mechanical analogue to supplement the geometrical interpretation.. There are too many particular ether theories to review. This paper concentrates on the comprehensive ether theories which cover SR & GR, and which promise to unify relativity with QT, ED and cosmology. The modern ether is the consequence of two programmes. One is the Poincare-Lorentz programme; the other is the Einstein-Minkowski programme. Both interpret the accepted formal structure of relativity, and are practically indistinguishable. Once regarded as mutually exclusive ways of interpreting relativity, they are now regarded as alternative ways of interpreting the formal structure which can be "mapped" or translated into each other Philosophy and history of science are needed to correct misconceptions about ether and relativity which originated between 1910-1920 and which still confuse the "ether question". A small number of ether theorists link the term to an anti-Einstein, anti-relativity polemic and thereby discredit its use, but modern ether theory does not deny the positive achievements of Einstein's relativity, Minkowski's geometry, non-classical physics, multi-dimensional geometrization, GMD, and relativistic cosmology. The ether-versus-relativity polemic has delayed acceptance of the ether concept in present-day physics. The rise of Einstein's Special Relativity, with its abandonment of the then-prevailing concept of ether, caused it to be set aside as a redundant idea. The rise of Einstein's General Relativity, with its transformation of long-standing concepts of space and time, suggested that the classical Newtonian ether was incapable of interpreting 20th C physics. The legitimacy of the ether concept has been an enduring question in physics since 1920. Many misconceptions abound concerning "absolute" and "relative" definitions of ether, and its many roles. The concept has a complex history, as have other fundamental terms like mass, space, time, energy. Too many ether theorists fail to clarify which are the essential features of an ether, and whether the several kinds of ether, found in contemporary physics, are radically different from each other, or whether they are all aspects of one fundamental medium. In the early 19th C, ether was a subtle medium made up of fine-scale matter, but this gave way to ether as a non-ponderable medium in which matter was a configuration. The evolving concept of modern ether, and the new concept of the electron played major roles in the evolution of early relativity between 1890-1910. The main early expositions of special relativity were Poincare-Lorentz relativity using a classical ether, and Einstein-Minkowski relativity expressed in terms of geometrized Space-Time. The Poincare-Lorentz exposition of relativity has continued to this day as a minority programme, and much modern ether theory has come out of it. Because of this there is an unfortunate tendency to associate the term ether with a narrow definition (absolutist, classical, Newtonian) linked to the Poincare-Larmor-Lorentz programme, which was developed by Ives, Builder, Prokhovnik, et.al Even more unfortunate was the association of the ether with a polemicists' campaign against Einstein's Relativity, the geometrization of physics, and the use of non-Euclidean geometry. This polemicists' campaign has, at various times, been linked to anti-semitism, and pseudo-science made to serve political, metaphysical and theological interests. This polemic, continued by an active minority, brought the concept into disrepute, and keeps alive misconceptions about ether which are completely unjustified. The other great source of modern ether theory is quantum theory (see below).

Ether, Geometrized Physics & Non-Euclidean Space-Time.

The geometrizing of physics, associated with the Einstein- Minkowski exposition of special relativity took place for good reasons. Unresolved difficulties faced the Lorentz theory of electrons, the electromagnetic world view, and the undetected ether. Failure to provide a mechanical interpretation of the ether robbed matter (and thereby instruments) of any satisfying material underpinning at a time when matter was increasingly thought of as an ethereal state. The conceptual and methodological impasse was escaped by Minkowski's geometrizing of the Lorentz theory of electrons, and fusing it with Einstein's early special relativity. Einstein later developed the "Einstein-Minkowski" geometrized exposition to interpet gravitation, using non-Euclidean geometry. Non-classical general relativity, and relativistic cosmology became exemplary physical theories. Geometrized physics became normative after the success of General Relativity in the 1920s, and led to the later development of Geometrodynamics. Later, interpretations of GR and cosmology were devized in classical terms, using the evolved P-L programme, but these followed Einstein, and reinterpreted what he did. They could not replace him, let alone establish that his methods were in error. In fact, the ether was recast in relativistic terms, though the P-L classical ether remained as an optional element in a quasi-classical subgroup within a larger body of relativistic ethers.

The major concepts of ether developed within two main classes, though there were many other types. One main class contains ethers defined as "Space with Physical Properties" (Einstein, Eddington, Whittaker), or "Ether as Field" (Steinmetz, Dirac). These were compatible with general relativity and non-classical theories. They enjoyed a long history, and in the 19^{th} C were discussed by Clifford, Riemann and Pearson. A second main class includes the classical ethers, serving to provide a background Euclidean space in which Newtonian absolute time prevailed, and energy and momentum were conserved. Because they interpret relativistic effects from a classical base, they are termed "quasi-classical" or "pseudo-classical" theories. This is the ether of Poincare-Lorentz Relativity, as presented by Ives, Builder and Prokhovnik. Generally, this ether lacked a mechanical analogue, which was essential for a stronger conceptual and methodological foundation. Failure to detect this ether by universally repeated experiment remains a major conceptual flaw in these Lorentzian theories because their most vital feature remains undetected by science. Between 1920 and 1950 there emerged a transformed ether theory out of which came the ether of present day physics. The Lorentz Theory of Electrons gave birth to the exposition of special relativity in terms of observations conducted with rods and clocks, subjected to specific synchronization techniques and slow instrument transport velocities. This Rod-Contraction; Clock-Retardation Ether Theory (Erlichson) is associated with Lorentz, Larmor, Broad, Ives, Builder, Prokhovnik, Mansouri and Sexl, Levy and many present day advocates of a Poincare-Lorentz interpretation of the agreed relativistic formal structure. A comprehensive, non-ad hoc derivation of Special Relativity was established and extended to cover General Relativity and Cosmology. These quasi-classical interpretations of GR and cosmology were openly recognized as valid, consistent alternative presentations of relativistic physics by eminent relativists like Eddington, but there seemed to be no pressing reason why the geometrized, non-classical Einstein formulation should not remain the norm. Einstein's theory was better understood and was conceptually and methodologically superior.

Hartley, Kelly & the Vortex Sponge: The Kelvin-Larmor Dynamic Ether Analogue

In the 1950s, Hartley, a colleague of Ives at Bell :Laboratories, developed the Kelvin-Larmor vortex-sponge mechanical ether analogue to provide a new dynamic interpretation of matter, ether, space and time. This was developed in later years, to the present time, by E M Kelly, Dmitriyev, Winterberg et al. to interpret quantum mechanical and other fundamental phenomena. The vortex-sponge provides a mechanical analogue which underpins the Poincare-Lorentz theories and removes conceptual and methodological weaknesses by remechanizing the Lorentzian world-view. If this is not done, geometrization remains the only obvious way of expressing relativity. The mechanism was imagined as a classical array of gyrostats held in a frictionless framework, which could take different equivalent forms, including hydrodynamical analogues. The vortex-sponge put a classical rod and clock at each point in a classical, Euclidean space, with Newtonian clock time regulated throughout. The passive, featureless Lorentzian ether gave way to a dynamical modified Kelvin-Larmor ether which filled or defined space and time. Material particles were represented as spherical standing waves, in a random atmosphere of vortex rings. These are equivalent to miniature Langevin clocks, idealized interferometers, and the combined rods-and-clocks used by Builder, Prokhovnik, Jennison and Clube to extend the Poincare-Lorentz ether-based theories to cosmology, and to set up a space-time metric. A relatively "large-scale; long-time" view of phenomena, in which the wave-particles retained their form, provides a complete interpretation in Lorentzian terms of SR, GR and much of cosmology. The imaginary surveying operations and measurements with rods and clocks could be referred to an ultimate set of standards motionless in the urether. However, a non-Lorentzian interpretation was equally available (see below) following Einstein and Minkowski.

If a view is taken of phenomena over much smaller length scales, and shorter time intervals, the instruments become disordered by the turbulence of the vortex-sponge and wave-particles used as ultimate measuring instruments encounter intrinsic lower limits to meaningful measurement of spatial intervals and clock-time ordering. There are random dislocations of measuring rods, and clock action. Instrument activity becomes discontinuous. The dynamic pulsations of the vortex-sponge require higher-order geometries and the classical scheme is no longer adequate. Energy exchange between wave-particle and ether is in fixed units or quanta, and a way is suggested of incorporating quantum mechanics and relativity into the same analogue. At the present time, E M Kelly, V Dimitiyev and F Winterberg continue to extend the range of the vortex sponge. Donnelly (University of Oregon) has produced laboratory-scale vortex-sponges using supercooled liquids, which exhibit quantum mechanical behaviour, and this research has provided a growing body of vortex-sponge theory applied in aeronautics, hydrodynamics, meteorology, and cryogenics. It is a quantum-mechanical medium, able to bear wave-particles which obey the principles of general relativity. It discloses relationships between the P-L classical exposition of relativity, and the normative Einstein geometrical exposition which shows them to be aspects of the same thing, rather than mutually exclusive rivals.

Einstein's Ether & the Vortex-Sponge.

Space and time measurements using a wave-particle as a combined rod-and-clock, in a vortex-sponge ether, in a gravitational field, as viewed from different platforms, gives the familiar "Einstein-GR" relation for the space-time interval. Ives developed an equivalent relation (the "chronotopic interval") within the context of Poincare-Lorentz Relativity, without using a mechanical analogue. This relation can be given a classical, or a non-classical interpretation. Definitions concerning basic rods and clocks (rigid or distortable?) determine whether or not the chronotopic interval is given a classical interpretation (referred to a background Lorentzian frame) or a non-classical formulation, in terms of non-Euclidean space-time geometry in the gravitational case. Both space-time geometries are practically equivalent. One can map back and forth between them. They are the space-time counterpart of two maps, drawn to different projections, used for navigating between two places on earth. Each map projection (e.g. Mercator's, or "equal-area" projection) has its own convention for calculating direction and distance at each latitude, because the metric scales differ. A navigator uses the simplest projection or geometry adequate for the task. The appropriate geometry might be decided by the coarse or fine scale nature of the job. No projection or geometry is more natural, or more real than any other, because the different metrics are not possessed by the earth – they are conventions for enabling direction and distance to be computed. This is exactly the same with the quasi-classical and the non-classical geometries, projections and metrics by which the Poincare-Lorentz and the Einstein-Minkowski programmes survey and compute space-time intervals associated with the same physical problem.

The vortex-sponge, originally presented as a classical mechanism of gyrostats and linkages, or a hydrodynamical equivalent, can be geometrized and interpreted in non-classical terms. Kron's geometrization of any mechanical and electromagnetic ensemble shows a general technique for achieving this. The vortex-sponge geometrized is a world-ether, equivalent to the space-time of Einstein's geometrized general relativity, and possessing the characteristics of the fundamental plenum of geometrodynamics depending on whether a coarse or fine scale view is taken. The "static" or geometric presentation of space-time, can be split into the "Frame-Space" perspectives in which laboratory measurements take place. Measurements in these laboratory frames can be correlated and interpreted using the non-Classical Einstein-Minkowski tradition, or the Poincare-Lorentz pseudo-classical tradition. It is a matter of choice and convention in defining ultimate measuring units as Ives realized, though he preferred the Lorentzian interpretation.

Kostro's recent research into Einstein's later writings identifies a concept equivalent to ether, which Kostro terms "Einstein's Ether" This is an example of "Ether as Field", or "Ether as Space-Time", proposed by engineers (Steinmetz) and physicists (Whittaker, Dirac) after 1920. Kostro refers to a dynamic and a static image. In the dynamic image, the motion of reference spaces is studied in the (clock) time of the laboratory reference frames. In this frame space perspective, the position of the reference spaces changes in time. The vortex-sponge, represented by a classical array of gyrostats or the hydrodynamical equivalents, can model frame-space phenomena and it provides an underpinning for the Poincare-Lorentz ether theory. The static or geometric image presents space and time fused together into the space-time continuum, and the reference space-times are composed of world-lines and instantaneous spaces. Kostro argues that this relativistic ether, or world ether, is not composed of world-lines or instantaneous spaces, but is a four-dimensional continuum made up of events. Geometrizing the wave-particle of the vortex-sponge into an event-particle and surveying space-time with it, leads to the same result. Einstein's Ether, and the geometrized vortex-sponge "World-Ether" are the same. Modern ether theory is not incompatible with Einstein's Relativity, or developments such as Geometrodynamics (which deal with a fine-scale perspective). The misconceived "ether versus relativity" and "Lorentz versus Einstein" polemics, have delayed a fruitful unification of two main programmes in Relativity, that of Einstein and Minkowski, and that of Poincare and Lorentz. They have hindered the subsequent development of a unified relativistic ether theory, and associated the term "ether" with archaic ideas.

At present there has been no unequivocal experimental distinction drawn between these two programmes, which have practically the same formal structure. Undisputed detection of ether drift would favour the Poincare-Lorentz programme in the context of relativity, but would by no means destroy the validity of geometrized, non-classical relativity. Present day ether drift experiments have not yet amassed sufficient evidence to favour the claims of the Poincare-Lorentz exposition, despite careful and persistent work carried out with modified MM apparatus, Fizeau double toothed wheels, and the Sagnac apparatus. Individual experiments may raise a question mark, but these tests need to be repeated many times, by disinterested experimenters, to separate misinterpreted results (given wide publicity by over-enthusiastic polemicists) from the genuine observations of colleagues whose work deserves consideration. Sometimes unsubstantiated claims that ether drift has been detected are acclaimed uncritically by ether theorists looking for supportive experimental evidence. Over-eagerness to accept the results of single experimenters, working unseen, based on relatively few tests, is a betrayal of scientific caution and has done much to bring the ether hypothesis into still deeper disrepute. It will require hundreds of undisputed detections of ether drift, carried out by impartial investigators in first class laboratories, all over the world, with impartial witnesses, and publication of meticulous records, before the normative status of Einstein's relativity is called in question. The question of drift is vital. It must be addressed – but it is up to ether theorists to beware the unjustified claims of anti-relativity polemicists if they are to win a fair hearing for their ideas.

Ether, Cosmology & Gravitation.

Einstein's relativity became the norm when it was extended from the Special Theory to cover gravitation and relativistic cosmology. Any comprehensive ether theory must cover the same ground, and a considerable body of present-day ether theory does so. The most important group is expressed in terms of Poincare-Lorentz theory, using the techniques of the rod-contraction; clock-retardation exposition. S. J. Prokhovnik interprets cosmology using Poincare-Lorentz relativity, with an ether as reference frame, and taking rod-contraction and clock-retardation as real phenomena. Prokhovnik does not advocate ether as a hidden mechanism, but he recommended the absolute reference frame of Poincare and Lorentz, and he uses the "instrument transport" techniques of Langevin, Ives and Builder for synchronizing clocks in inertial frames. Prokhovnik stresses the causal significance of absolute velocities, and defines absolute as "relative to the universe". He treats the galaxies as the fundamental particles of the universe at large, which define an expanding frame filled with background radiation. Distribution of "particles" is seen as homogeneous by an observer on any one galaxy-particle. The universe expands according to Hubble's law, and the cosmological principle holds. Prokhovnik shows that the Robertson-Walker metric applies and defines a unique, observable, cosmological reference frame in which light is propagated in all directions with a speed always measured as "c". In thought experiments one can refer to non-isotropy in the speed of light with respect to a moving body in the frame, but this is not observable because rod contractions and clock retardations make the as-measured speed "c". This is a common feature of the entire group of Poincare-Lorentz ether theories, and is the source of much criticism. Prokhovnik claims that astronomy has revealed a unique, fundamental frame, within which moving bodies are effected by motion. Velocity with respect to this expanding reference frame can be estimated from the 2.7K microwave background radiation. The expansion of the frame provides a measure of cosmic time, which enables a clear, paradox-free exposition of relativity to be presented. The work of Prokhovnik and Builder has been developed further by Paparadopoulos. Wegener argues that the "spray substratum" of Milne meets the requirements set out by Builder and Prokhovnik for a universal reference frame.

Clube presents theories of gravity and cosmology, along Poincare-Lorentz lines, using an ether described as a superfluid or material vacuum. Clube develops the de Sitter-Atkinson gravitational theory as an approximation to a more fundamental Lorentz-Dicke gravitational theory. It models the production of particle pairs in the physical vacuum (ether) and relates a wide range of astrophysical and cosmological phenomena within a Lorentzian theory of gravity, couched in terms of a static model of the universe. Its chief features include redshift arising from vacuum processes; baryonic and non-baryonic matter formation in the ether; and the suggestion that phase-locking of fundamental particles may involve a principle more important than relativity. It should be compared with the work of Surdin.

The theories of Arminjon, Broekaert, Podlaha, and Sjodin form a consistent group, which can be related to work by Clube, Roscoe, Surdin, Prokhovnik, Builder, Rongved, d'Atkinson, Cornish, and Ives. The vortex-sponge analogue, though not forming an essential part of these theories, can provide a mechanical underpinning. Arminjon's work typifies this school of thought. Gravitation is modelled after Euler as an Archimidean thrust in a fluid ether. Particles of ponderable matter are localised flows in the ether, and creation and annihilation is represented in fluid terms – an old idea, developed to a high order by Karl Pearson and Schuster in the 1880s. Gravitation is a "smoothed-out" macro-force, in which ether pressure plays a crucial role. This is compatible with Hartley's vortex-sponge interpretation. The ether fills the homogenous space of SR, and the effects are interpreted by rod contractions and "Larmor clock slowing". Gravitation is due to an apparent variation in ether density, or heterogenity of space, with gravitational rod-contraction and clock slowing. These theories constitute a major development of the Poincare-

Fitzgerald-Larmor-Lorentz-Ives programme. Interpretations of space-time employ two metrics. There is a flat background metric (against which rod-contractions and clock slowings are defined), and a physical metric, which is curved in the gravitational case. This is the fundamental characteristic of the Poincare-Lorentz programme, which has the "Ives Group of theories" as perhaps its most representative member. The ether is identified with the background metric. The "uncorrected" readings of rods and clocks defines the "physical metric". The quasi-classical expositions of relativity (d'Atkinson, Cornish, Clube, Rongved) refer all instrument readings back to the ether .This is a legitimate way of interpreting the relativistic formal structure which a minority prefer to use. The majority argue that until the background ether is detected by universally repeated experiment in a clear and undisputed manner, the instrument readings should be taken as "read-off", and not reduced into an undetected ether-state. This is the orthodox relativistic position, which is superior on methodological grounds. They are equivalent. One can map from one to the other. They are by no means mutually exclusive. Ives' "chronotopic interval" paper is worth consulting in this respect.

Arminjon and his colleagues explore various options for developing these theories. Rod contraction can be treated as anisotropic, lying along the ether pressure gradient and line of gravitational acceleration (Arminjon), or as being isotropic (Podlaha and Sjodin). The anisotropic assumption gives the Schwarzschild exterior metric in the static case with spherical symmetry, giving the same observable results (light ray behaviour) as does general relativity starting with the Schwarzschild metric. Problems are encountered with the weak equivalence principle, but Arminjon argues that these will occur with general relativity also, with anisotropic metrics. These problems are not encountered with the isotropic case.

Broekaert offers an alternative scalar interpretation of general relativity, following geometrical conventions introduced by Poincare, and presenting gravitationally modifed Lorentz Transforms. Depending on which isotropic scaling functions are applied, one set of LT distinguish between a "natural geometry" which is affected by gravitation, and a co-ordinate geometry which is not affected by gravitation. A spatially varied speed of light is suggested. Different scaling functions give a different set of transforms giving the invariant (locally observed) speed of light and the local Minkowski metric. This group of theories contains work by Sjodin on gravitation and determination of one-way velocity of light; and by Podlaha, which presents a comprehensive ether model of the physical vacuum and wave-particle which is similar in many ways to the vortex-sponge and wave-particles of Hartley.

Grand Comprehensive Theories.

Several comprehensive ether theories cover a wide range of phenomena from the quantum to the cosmic scale. Examples include the theories of Cavalleri, Dimitriyev, and Winterberg. In a brief review all that can be done is to summarize a few major features and direct the reader to the references.

G. Cavalleri has developed a comprehensive ether theory based on a stochastic medium made up of vortex elements for interpreting QED and a wide range of electonic and electromagnetic phenomena. Cavalleri proposes that the zero point field of QED is caused by the classical EM radiation of all the particles in the universe, emitted since the "big bang", and that it may be regarded as a real ether. Motion of rods and clocks through this ether produces the length contraction and clock retardation of the Poincare-Lorentz or Ives group of theories. Cavalleri shows that the power spectral density is Lorentz invariant, and that experiments of the Michelson-Morley type, and their equivalent, cannot detect ether drift. However, if the power spectral density is limited to prevent infinite energy density in a zero point field regarded as real, the relativistic invariance is lost, and there should be a privileged observer for which the zero point field ether is isotropic. Cavalleri suggests that this ether might be detected by accelerating a charged hydrogen atom in a synchrotron. An energy of 20Tev would be needed to detect an effect interpreted as "friction in vacuo". Cavalleri, like Winterberg, rejects the metaphysics of the Copenhagen school, and aims for realism. He regards the zitterbewegung of Schroedinger as a real phenomenon located in the physical vacuum and not an illusory effect due to uncertainty. He regards the zero point field as the producer of fluctuations in velocity direction which amplify fluctuations in electron position, and generate quantum mechanical effects. His ether is a space filling stochastic medium reacting with matter within the framework of Poincare-Lorentz relativity.

V.P.Dmitriyev proposes a comprehensive theory covering a wide range of fundamental physical phenomena from QM to GR., based on mechanical ether analogues developed from the models of Kelvin, Hicks, McCullogh and Larmor. He shows that solid, liquid and mechanical ether analogues are equivalent, and unusually presents many of his findings in terms of a solid elastic continuum analogue. Much of his work is also expressed using the vortex-sponge. Dmitriyev remarks that the Yang-Mills theory of physical fields can be used to create a complete theory in terms of a solid body with singularities, but he reverses this procedure to devize a theory of physical fields and particles using an elastic solid ether. There is no empty space. All space is occupied by physical vacuum or ether through which all interactions are transmitted, including EM waves and gravity waves. Beginning with a linear-elastic substratum, Dmitriyev shows that this is Lorentz-invariant because it is practically incompressible, and that the universe is practically static. Material particles are approximately modelled by localised energetic excitations (solitons) in the ether, which ideally should be modelled in non-linear terms. Models of quantum particles, gravitation and GR are provided. GR is treated as a sub-algebra in the non-linear theory of elasticity. The solid continuum model, provided with internal rotation is equivalent to the vortex-sponge, which Dmitriyev uses, along with dipole models, to interpet microphenomena and asymmetry in the macroscopic world. As with Winterberg's model, Dmitiyev's system is hierarchical, having six "levels" of phenomena, which are classical mechanics; GR and QM; the solid substratum; the solid substratum with internal rotation; the vortex-sponge regarded as chaotic turbulence in the primary medium; and the primary medium. It would be better if the whole picture could be expressed in terms of the vortex-sponge alone.

F. Winterberg , like most modern ether theorists, accepts the formal structure of relativity but is critical of attempts to develop the geometrical theories by adding dimensions of space-time, starting with the adding of a fifth dimension by Kaluza and Klein to unify gravity and EM, and ending with the multidimensional space-times of superstring theory. He believes (as do many supporters of P-L ether theory) that physical reality is 4-d space-time which must be made the foundation of any theory which avoids "physical impossibilities or absurdities" like infinite stresses in zero diameter strings. Winterberg works within the Poincare-Lorentz framework, with rod-contraction and clock retardation being real phenomena caused by motion through a substratum. Insisting that all modern ether theory must be within a quantum mechanical framework, Winterberg models the substratum as a superfluid ether full of quantized vortices. The superfluid ether is made up of an equal number of positive and negative masses called Planckions, densely packed together which preserve the zero-point energy fluctuations of the physical vacuum, but make the average vanish. The analogue interprets vector gauge bosons, charge and charge quantization; special relativity as a dynamic symmetry; gauge invariance; Dirac spinors; and elementary particle mass as a function of Planck mass. For many of its functions, Winterberg's ether resembles the vortex-sponge. Indeed in considering its energy spectrum, Winterberg used results from liquid helium theory (phonon-roton structure) presented by Feynman in 1954, and supported by recent laboratory investigations on vortex-sponges using low temperature helium.

Winterberg's model is hierarchical. Elementary particles are bound states in the superfluid quantum mechanical ether. The model provides a classical description of Schroedinger's zitterbewegung derived from Dirac's equation. A QM interpretation is obtained compatible with relativity. The model analyses wave function collapse in a "realistic, objective" manner, rejecting the Copenhagen interpretation. Like Prokhovnik, Winterberg regards very large systems of galaxies as defining a privileged frame of reference at rest in the ether. This ether is Heisenberg's fundamental field, admitting wave modes of superluminal velocity. Winterberg argues that an absolute-space-time structure allows for a realistic interpretation of wave-function collapse. He regards the Minkowski space-time continuum, and the Riemannian manifold, as illusions caused by true physical distortions which should be interpreted in terms of Poincare-Lorentz theory using a real, absolute ether. This reflects the attitude of many ether theorists..

Dirac Ether & the Physical Vacuum

An ether was proposed by Dirac to unify electrodynamics and quantum mechanics in a manner different from that found in the QED programme. E.M. Kelly used the vortex-sponge to effect this unification, but others have tried to develop Dirac's ether, to achieve the same result, without reference to the dynamical analogue. De Haas's recent efforts are important contributions. De Haas defines Dirac's ether as a revived Maxwell's ether, which requires that the non-gauge invariant stress-energy tensor has non-zero, non-symmetric magnitude in space-time. If so, the Lorentz force and Poincare force can be obtained, with Poincare force represented by a translation pressure, or "something rotating in the Maxwellian operational ether". (In the vortex-sponge, these are forces resulting from ether pressure due to the atmosphere of fine-scale vortex rings between the wave-particles). Dirac regarded electric potential and velocity field as "physically real". Maxwell's operational ether required that its electromagnetic stress-tensor was zero, and it was overtaken as a concept by Lorentz's ether, in the early days of relativity, which was defined in terms of motion (SR – not the accelerations of the GT) and not by the "real" existence of Maxwellian ether-stresses. In the Maxwellian ether, magentism was due to inner rotations of space; in Einstein's space-time (GR) rotations were replaced by geodisic movements in curved space, with the gravitational part of the stress-energy tensor being zero. A minority of physicists continued to develop the Maxwellian approach. The vortex-sponge might effect a reconciliation of these several approaches (Maxwell, Lorentz, Einstein). De Haas accepts that GR allows for an operational ether, though Einstein never incorporated ED or QM into GR The operational ether of Dirac, though a "real, physical ether" with charge flow velocity being an ether velocity, is not a substantial ether fixed to absolute space. It must be distinguished from an "Aristotelian substance connected to Euclidean, absolute space" – which is the obsolete ether concept of so many anti-relativity, anti-Einstein "dissidents" who advocate it to the detriment of ether theory in general. De Haas identifies the problem central to modern Dirac ether as being formulation of the stress-energy tensor. Maxwell interpreted magnetic stress as arising from rotation in the ether, but Einstein-Minkwoski flat space time (SR) and the curved space-time of GR contain no rotations by definition, and their spaces and time are orthogonal. Dirac's ether can contain non-orthogonalities in space and time, as required by Maxwellian EM, by using an antisymmetric tensor or 6-vector following a path indicated by Minkowski. Developing a 6-vector Lorentz Transformation matrix yields a 6-d electromagnetic space-time additional to 4-d Einstein-Minkowski space-time. De Haas points out that Dirac explored a Poincare-Lorentz programme (spherical electron held together with ether pressure), and an Einstein-Minkowski programme (point charge without extension; no ether pressure; an ether free metric) before cultivating the neo-Maxwellian ether of his later years. Referring to von Laue' work unifying Newtonian mechanics with SR (1911) and Minkowski's work of fusing Maxwell's ED and SR (1908), De Haas presents his own development of a Dirac ether which all contemporary ether theorists should consider.

Another promising line of development is indicated by S Bell, who quantizes general relativity using QED in an analysis which (like that of Carroll) draws on techniques from information science, computing, systems processing and signal analysis. She shows how inverse square law of force, and Bohr's quantization of angular momentum can be derived from vortex theory and SR. Though she does not use the ether concept, this work, together with that of Carroll and Rowlands, shows how theories of the physical vacuum (or ether) might be greatly extended and unification achieved.

Cavalleri, Puthoff, Winterberg and Surdin have developed comprehensive theories of the physical vacuum, based on ther zero point field, and related them to large and small scale phenomena. Cavalleri's and Winterberg's work has been summarized above. Surdin assumes that all the laws of physics originate in the ZPF and he sought the cosmology which best suited stochastic electrodynamics. Early SED used steady-state cosmology and was criticised because it could not explain 2.7K background radiation. Surdin's ambitious programme sought to use a classical model of the physical world, to show that quantum phenomena are a consequence of SED, to obtain a cosmology compatible with the large numbers hypothesis, to relate gravitational and EM forces, and to explain cosmic background radiation and other cosmological effects. Surdin employs Newtonian Mechanics to derive the well-known GR effects of gravitational redshift, perihelion advance, and bending of light by massive bodies. He takes the same quasi-classical, Lorentzian approach found in Ives, Builder, Prokhovnik and Clube. In his later work, Surdin attempts to unify GR with SED and develops a steady-state cosmology with an expanding closed universe, with matter density constant. He proposes a mechanism for particle creation. He argues that the existence of a real zero point field or ether overcomes objections levelled against standard steady-state theory because it accounts for cosmic background radiation and continuous creation of matter. Surdin provides an example of a comprehensive theory, encompassing cosmology, GR, particle creation, electrodynamics and microphenomena. Puthoff is another example of a theorist who treats the zero point field as a dynamic ether in which gravity is an induced effect caused by the field being loaded by large scale ponderable matter. These theories based on the ether as zero point field or physical vacuum take the ether concept beyond the point reached by Lorentz, Ives, Builder and Prokhovnik by giving it a dynamical character. There are considerable differences between the various comprehensive theories and more effort should be directed to synthesising a unified theory from them. The Poincare-Lorentz programme is the foundation of most of these theories.

Ether & Matter

The role of material particles, their structure, and the mechanism of particle creation are vital elements in comprehensive ether theories, granted that fundamental particles provide the ultimate combined-rod-and-clock systems for surveying space-time. The wave-particle in Hartley's vortex-sponge has already been mentioned. Similar wave particles are suggested by Podlaha who models ponderable matter as being made up of spherical standing waves, sent out from a centre, reflected at an envelope, and returned simultaneously to centre. This can be treated as an "idealized interferometer" and used to set up a Poincare-Lorentz interpretation of SR and GR. Podlaha postulates the existence of matter waves of another kind, which spread with the velocity of light and are essential for particle stability, possibly playing the same role as the vortex-ring atmosphere in the vortex-sponge which holds the wave-particle together by ether pressure. Jennison's theories of material particles is based on theoretical and experimental work at Canterbury, where he created a fundamental particle serving as a combined rod-and-clock system. Jennison does not specifically mention the ether, and he accepts the geometrical formulations of relativity which offend some classically minded supporters of the Poincare-Lorentz programme. Jennison studied perfectly lossless entrapment of monochromatic radiation in confined spaces, termed "phase-locked cavities" which obey Newton's first and second laws; exhibit quantized momentum at microscopic level; and are fit to serve as proper rods and clocks. Jennison stresses the need to identify the rods and clocks which are the best instruments (and methods) for calibrating the space-time metric. Jennison and others argue that phase-locked cavities fulfill this role. Mackinnon applied phase-locking modelling to de Broglie matter-waves and devised the 'soliton' with waves phase-locked at the centre. Jennison devised a rotating matter wave particle, and derived a stable three dimensional system to serve as a combined proper clock and relativistically rigid measuring rod (in which velocity of sound is the speed of light). These instruments are used to calibrate the space-time metric. Clock time is "very real" and "cannot be assumed to exist where matter itself cannot exist" – such as in an environment where there is entirely free radiation without rest mass. Electrons and protons can be used as clocks. Pair production can produce proper clocks whose proper time starts from the moment of formation. Fundamental questions are raised concerning the meaningfulness of time and space measure beyond the province of the best available rod and clock – such as within the minimum measurable intervals of the vortex sponge. Jennison explains how to construct macroscopic measuring rods and clocks using laser light and microwave radiation, for practical experimental investigations. Other important discussions of the nature of fundamental roda and clocks are given by Kostro and Prokhovnik. Kostro's papers on the three-wave hypothesis, and the soliton, are important discussions on the inter-relation between ultimate particles, particle creation, and the nature of fundamental "best" rods and clocks. Simon has considered the role of the electron in this role, within the context of Eddington's analysis of GR and QM. Eddington regarded a particle as a conceptual carrier of measurable properties subject to probability in space-time. He considered replacing the physical reference system of objects by an ideal standardized reference object which was "a fluid, permeating all space like an ether" This was defined mathematically, in non-classical terms. These are a few examples of studies of how reference particles are defined, and how postulated mechanisms of particle creation are related to particular theories of ether, ZPF or physical vacuum.

Present Situation for Modern Ether Theory

Modern ether theory is relativistic in that the measuring operations which it defines are described by the accepted formal structure of Relativity. Any departure from Relativity theory is beyond experimental detection by current techniques, though the Sagnac experiment continues to raise questions which require answers. The supposed detections of ether drift will require confirmation by multiple, independent findings by disinterested parties before they are accepted. Most ether theories are practically identical to Einstein's Relativity. Most modern ether theory is couched in non-classical terms, and multi-dimensional spaces and times, though much of it can be given a "quasi-classical" interpretation as has been acknowledged by leading relativists (Eddington) since 1920. The ether is an abstract construct, a "disclosing model", for facilitating the classification and analysis of observations. Its use is justified by its ability to solve problems efficiently, checked by experiment. It is ontologically non-Realistic: it is a provisional instrument which must evolve and undergoe be transformations to take into account new discoveries which require a complete revolution in what constitutes an ether. Ether theory is not anti-relativity, nor anti-Einstein's relativity. This must be clearly established or the concept will be rejected. Destructive misconceptions are spread by naïve Realists who argue that a classical ether, within the Poincare-Lorentz programme, is more rooted in objective reality, because it employs Newtonian 3-d space and absolute time. This is a totally unjustified assumption. A classical, Newtonian ether is as much an imaginary construct of the mathematicians as is a superstring or brane. Polemicists have used the classical ether concept to support an absolutist metaphysic for religious reasons (Hazelett and Turner), whilst condemning Einstein's relativity as the source of subversive irrationalism. This unjustified misuse of the ether concept is the result of bad history, bad philosophy and bad science. It delays acceptance of the ether concept by the community of Science, and causes many modern ether theorists to use alternative expressions, such as "vacuum field", "physical vacuum" or "cosmological plenum" rather than the obvious term. Redundant concepts of ether must be set aside, and the futile ether-versus-relativity polemic ended. Ether theorists should concentrate much more on bringing order to the range of present day theories, with a view to integrating them into a comprehensive whole. Leaving a disarray of scattered, isolated fragmented theories will not serve. Re-inventing Ives and Lorentz in different guises will not do. Modern ether theory must do more than show that it can interpret General Relativity and Cosmology, as geometrized physics has done – this is merely the basic qualification for being taken seriously. This had to be done because it was perceived that relativity made the ether concept untenable. It has been done, and must be followed by an integrated ether interpretation of the full range of major physical theories, from which new and creative insights can be obtained.

Future Developments in Ether Theory?

Modern ether theory has given rise to several promising development programmes which break new ground. The following indicate trends which promise to take ether theories into a new phase.. Ether can be treated as a seat of symmetry breaking mechanisms. Vortex theory is finding a key role in symmetry breaking analysis, and in creating new analogues of general realtivity, some of them derived from the vortex-sponge, or from studies of superfluid helium and turbulent superfluids (Pismen, Voklovik).

Mathematical formulations of ether have always been important (Clifford, Eddington, Maclaren) and today one finds interesting concepts of ether as a "generator" of mathematical systems: hypercomplex numbers, dynamic algebras, and tesselated spaces.(Trell, Kassendrov, Santilli). These mathematical ethers are often the result of techniques borrowed from communications theory, information science, fractals, or chaos theory.

Ether as the physical vacuum is a vital area of growth. The earlier ether theories postulating mechanisms for "creation out of nothing" or out of the vacuum or ZPF, have been joined (for example) by neo-quaternion theory, and nilpotent theory (P Rowlands) Dr Rowlands (Liverpool) has developed an interpretation of the physical vacuum which emerges as a mathematical property of the Dirac nilpotent operator, and for any individual fermion, the vacuum represents the rest of the universe. Three vacuum operators leave this original operator unchanged. These correspond to the vacuum mediated by weak, strong and electric charges. The fermion state vector expressed as a four-component spinor specifies the fermion and its three vacuum "reflections". These partitions of the vacuum are discrete, but the combined "gravitational" or "total vacuum" is not. His vacuum is the carrier of nonlocality because it directly expresses Pauli exclusion. The continuous vacuum is connected with irreversible time, the Higgs mechanism, renormalization, zero-point energy, the Casimir effect and thermodynamics. The crossover between discrete nilpotent and continuous vacuum emerges as inertia, and if treated as a "gravity plus inertia (nilpotent) theory", GTR avoids singularities, nonlinearity and non-renormalizability. It can be quantized and yields accelerating cosmological redshift and background radiation. The nilpotent operator incorporates proper time, and hence causality, whereas Einstein's theory excludes it as a separate parameter and defines the space-time scalar product as an invariant. In the restricted cases Einstein considered, with causality introduced *ad hoc,* quantum and the vacuum can be avoided and left undiscussed. This is a limited instance, which requires an artificial concept of simultaneity, unknown in a quantum context, which leads to the many difficulties (conceptual and methodological) which adverse critics associate with STR. Rowlands argues that these disappear in all physically realistic cases. An examination of the proper times shows that the "twin paradox" involves an asymmetry which makes the apparent simultaneity the result of first order approximation. Dr Rowlands argues that the exclusion of proper time from STR allows the absolute frame and absolute time to be ignored in the theory's fields of application, though these are required outside the immediate confines of the theory to preserve Einstein's concept of causality. This suggests that there must therefore be some form of absolute frame of reference, as well as "absolute birthordering of all quantum events". Advocates of ether theory should heed Rowland's warning that it is an entirely different question whether or not this absolute frame and vacuum, which is quantum rather than classical in origin, can be derived from the Poincare-Lorentz version of the ether. He concludes that though Poincare, Lorentz, Lodge and Larmor define the boundaries of STR, the full quantum theory of the Dirac state, which goes far beyond the supposed 4-dimensionality of space and time, takes physics to an order of understanding which lies beyond the confines of the Poincare-Lorentz-Einstein-Minkowski dispute.

The information theoretical aspects of the physical vacuum (ether) and matter is receiving much attention. Information science, computer science, systems processing, and signal analysis are being used to interpret the physical vacuum. Non-classical, geometrized ether theories, generating new mathematical interpretations are developing out of the dynamical interpretations (vortex sponge) and the GMD equivalents. Communications signal theory was always fundamental to dynamical ether theory, and played a central role in the mechanism of wave-particle creation postulated by Hartley and Jennison. Recent work by S Bell, and J Carroll, show how vortex theory, and communications signal theory reveal the hidden mechanisms which determine particle structures, particle stability, and the appropriate space-time metric. This "correlation technique" provides a method for developing the vortex-sponge from a "large-scale; long-time period" model, to one which interprets the short-period, small-scale phenomena which become manifest at the "minimum measurable intervals" scale. Wave particle dynamics is then unstable and three spatial dimensions are no longer adequate for interpretations. The ether is then a correllating mechanism, linking Relativity, and Quantum Mechanics Its activity finds expression in Clifford algebra, dynamic algebras, nilpotent theories, and space-time vortices. The work of J E Carroll represents current work in this field, though he does not use the word "ether" to identify the physical vacuum. He develops correlation theory to relate special relativity, Clifford algebra and quantum theory, splitting scalar signals into even and odd to give sense of direction in space and time co-ordinates. Scalar signals "hide" structures which find expression in multivectors, geometries of different orders, and Clifford algebra. The correlation technique (hailed by some as heralding a revolution in the way physics is defined and understood) gives rise to models of space-time which include orthogonal vectors, spinors, differences in chirality. Three dimensions at least are required for an observable isotropic space. The fourth dimension cannot be equivalent to one of these 3 dimensions of isotropic space and the natural metric for a 4-d space is that of special relativity in space-time, following Einstein and Minkowski. Note how the analysis establishes the Lorentzian programme of ether, particles as configurations in ether, and operations with rods and clocks, as the other face of geometrized special relativity. The route from signals in ether to the Eisntein-Minkowski metric is direct, and of course, one can reverse the derivation. Once more the invalid claims of the anti-Einstein polemical ether theorists stand revealed. The correlation technique can be developed by embedding (3+1) space time in 6-d space time to gain insight into the issues governing observations where signals are randomly fluctuating. It is interesting that correlation technique, and Clifford algebra, applied in the technology of image processing, now finds application in interpreting gravitation, relativity and quantum theory. Correlation analysis of 4-d space-time shows that a +ve metric leads to contradictions, but the relativity metric gives sensible results. Mermin shows that reliable, meaningful observations are the outcome of correlations, and that correlations in space-time cannot exist between two objects moving at speeds greater than that of light. Any modern ether theorist should pay great attention to correlation theory which relates the physical theories (signals in physical vacuum, observations, wave-particle structure) to geometrized formulations and mathematical expositions.

Conclusion

The above developments favour the vortex-sponge ether analogue, which is a quantum-mechanical entity, and which requires orders of space-time in excess of 4-d on the microscale and for fullest exposition. It is only for certain interpretations that a 4-d expression will suffice. Several promising lines of development for modern ether theory are evident. The sterile ether-versus-relativity polemic must be set aside before anything positive can be accomplished. Next, a greater degree of unification must be sought within the comprehensive theories which interpet relativity (Special and General), effects of physical vacuum, cosmology, space-time metric and Quantum Mechanics. Providing an equivalent and alternative "second interpetation" to the formal structure of General Relativity, already provided by the "Einstein school" isn't enough to justify ether theory. Ether theorists must show that the concept points towards creative theories in the future. A start can be made by accepting both the Einstein-Minkowski and the Poincare-Lorentz programmes as valid. The geometrized vortex-sponge is equivalent to the space-time continuum of GR. When a very small scale perspective is taken, the vortex-sponge becomes the foam-like "punctured, fluctuating" continuum of GMD, requiring multi-dimensional geometrical interpretations. Treating the ether as a generator of mathematical decriptions (or first interpretations) seems particularly promising (Rowlands, Santilli, Trell). Much could be done by analysing the ether in terms of information science and signal theory, following the example of Carroll's correlation technique. The rapidly growing studies of chaotic media, fractals, and symmetry-breaking mechanisms suggest what the ether is in the 21st C. It is an increasingly important class of unifying disclosing models, and associated theories, usually presented in non-Euclidean, multi-dimensional terms, which contains a quasi-classical "Newtonian sub-group" which can be identified with the Poincare-Lorentz programme, the Ives group of theories and the Lorentzian ether. It is a "scale-dependent" model. Large-scale, long-(clock)-time observations can be correlated with a simpler geometry than much smaller-scale measurements. The purely mathematical formalisms (first interpretations) generated by the ether require more study. They may closely resemble the dynamic algebras, and the nilpotent expositions of contemporary theories (Kassandrov; Rowlands).

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Octonions and vacuum stability

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The paper addresses one of nontrivial octonion related facts. According to paper gr-qc/0409095, the most stable space-time state is the one described by real Dirac matrices in 11-dimensional space of signature $1(-)\,\&\,10(+)$. The internal subspace is 7-dimensional, and its stability is due to a high "zero" energy packing density when using an oblique-angled basis from fundamental vectors of lattice E_8 for the spinor degrees of freedom. The nontrivial fact consists in the following: Dirac symbols with octonion matrix elements can be used to describe states of the space of internal degrees of freedom if and only if the space corresponds either to stable vacuum states or states close to the just mentioned ones. The coincidence of the internal space dimension and signature for absolutely different and independent approaches to the consideration of this issue seems to predetermine the internal space vacuum properties and the apparatus, which is able to constitute the basis of the unified interaction theory.

1. Introduction

4-dimensional Riemannian space with metric tensor $g_{\alpha\beta}$ is considered. The Greek letters take on values $\alpha,\beta,\ldots=0,1,2,3$. The metric tensor is assumed nonsingular, so Dirac symbols (DS) γ_α according to

$$\gamma_\alpha\gamma_\beta+\gamma_\beta\gamma_\alpha=2g_{\alpha\beta} \tag{1}$$

can be introduced at every point. This paper considers algebraic properties of DS at some spatial point. The coordinate system is assumed to be locally Cartesian and tensor $g_{\alpha\beta}$ equal to

$$g_{\alpha\beta}=\mathrm{diag}\left(-1,1,1,1\right). \tag{2}$$

In the DS theory there are a number of problems, the solution to which directly affects the physical interpretation of DS involving constructions, however, such that there is no complete understanding in regard to their solution method. Mention two of them.

It is well known that symbols $\{\gamma_\alpha\}$ can be realized as Dirac matrices (DM) above any number field (real, complex, quaternion numbers) as well as above the octonion body. The realization in the form of square matrices 4×4 is meant. On the other hand, DS can be realized in the form of real square matrices $N\times N$, $N\geq4$. Of interest is the question: What is the relation between these two realization types? In particular, what are the characteristics of the subspaces of internal degrees of freedom that are introduced additionally in each complication of the number body used?

For physics, the octonion realization is of a special interest, in particular, for the reason that using any number body except for the octonion one does not allow us even to pose the question of explanation of the irreversibility of actual processes on the basis of time-reversible fundamental laws. The irreversibility phenomenon may be explained only in transition to the formulation of the physical laws in terms of octonions. But in this most interesting case some of the theorems do not hold, on the basis of which the polarization density matrix is introduced and conclusions on the correspondence between tensors and bispinors are reached. The problem is to give an answer to the question: To what extent are those results for the correspondence between tensors and bispinors, which have been found for real matrix realizations of DM, valid for the octonion DM?

This paper makes an attempt to give answers to the two above-formulated problems.

2. DM realization above a real field in Riemannian spaces of a dimension higher than four

The Riemannian spaces of dimension $n\geq4$ have been studied in connection with construction of matrix spaces (MS), that is the Riemannian spaces, in which the internal degrees of freedom properties are introduced through Dirac matrices γ_A. Subscripts $A, B, \ldots$ take on values $A, B = 1, 2, \ldots, n$, while the γ_A's themselves are realized as square matrices $N \times N$ and satisfy relations

$$\gamma_A \gamma_B + \gamma_B \gamma_A = 2 g_{AB} \cdot E \tag{3}$$

Here E is the unit matrix in the space of internal degrees of freedom.

The internal degrees of freedom are related, first, with transformations

$$\gamma_A \to \gamma'_A = S(x) \gamma_A S^{-1}(x), \tag{4}$$

and, second, with the transition to Riemannian spaces of larger dimensions and different signatures.

The MS theory in multidimensional Riemannian spaces with real realizations of DM is discussed in detail in refs. [1]-[3]. These papers also prove the following:

The realization of DM above a real field frequently entails the notion of so-called maximum MS, in which the set of the quantities, the generatrices for which are DM, coincides with the set of all matrices of a given dimension. The maximum MS have odd dimension n.

$$n = 2k + 1,$$

where k is a positive integer. Their signature is therewith of form $(k+1)(+) \,\&\, k(-)$ or differs from that by a number of "minuses", which is a multiple of four.

The DM, which can be introduced in the Riemannian space possessing the above properties are square matrices $N \times N$, with N relating to k as

$$N = 2^k.$$

In any MS, either anti-Hermitizing matrix D or Hermitizing matrix C can be introduced. The D or C are determined as

$$D \gamma_a D^{-1} = -\gamma_a^+; \quad C \gamma_a C^{-1} = \gamma_a^+; \quad a = 0, 1, 2, \ldots, N-1.$$

The matrix D or matrix C can be used to introduce a Hermitean matrix set, whose existence, in its turn, is needed to introduce the concept of polarization density matrix.

3. Complex numbers and quaternions

The results relating to determination of properties of those multidimensional Riemannian spaces, in which the real DM algebra is mapped isomorphously to the DM algebra in 4-dimensional space of signature $(-+++)$ in realization of the latter above the real, complex and quaternionic fields are summarized in Table 1.

Table 1. Parameters characterizing the isomorphism between DM realized above different number fields and DM realized above the real number field

A method for satisfaction of determining relation $\gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha = 2 g_{\alpha\beta}$ with $\alpha = 0,1,2,3$ and signature $(-+++)$	
With the help of real matrices	**With the help of matrices 4×4, but using different number bodies**
Matrices 4×4. 16 parameters.	Real number field
Subset of matrices $(4 \times 4) \otimes (2 \times 2)$. 32 parameters. In this realization method, an additional internal subspace of dimension 1 is actually introduced. Gauge group $U(1)$	Complex field. The transition to the matrix notation is performed using isomorphism $\mathrm{I} \Leftrightarrow E_{4\times4} \otimes i\sigma_2$

Subset of matrices $(4\times 4)\otimes(4\times 4)$. 64 parameters. In this realization method, an additional internal subspace of dimension 3 is actually introduced. Gauge group $SU(2)$	Quaternion field. The transition to the matrix notation is performed using isomorphism $\left.\begin{matrix} \mathrm{I}_1 \Leftrightarrow E_{4\times 4}\otimes i\rho_2\sigma_1 \\ \mathrm{I}_2 \Leftrightarrow E_{4\times 4}\otimes i\sigma_2 \\ \mathrm{I}_3 \Leftrightarrow E_{4\times 4}\otimes i\rho_2\sigma_3 \end{matrix}\right\}$

As it follows from Table 1, the dimension of the internal space that appears in the transition from one number field to another is the same as the number of imaginary units in the number field. The Riemannian spaces are therewith subspaces of maximum MS.

4. Octonion DS

Irrespective of the fact that octonions are discussed extensively in the literature (see, e.g., [4], [5]), nevertheless, here we present some information about these unusual numbers. Naturally, we will do this briefly and only to the extent, which is needed for the consistency of the discussion.

The algebra of octonion imaginary units $\{e_N\}$ is determined as

$$e_M e_N = -\delta_{MN} e_0 + C_{MNK} e_K \tag{5}$$

Here: $M, N, K = 1, 2, ..., 7$; C_{MNK} are quantities completely antisymmetric in their indices; nonzero components are:

$$C_{123} = C_{145} = C_{246} = C_{347} = C_{176} = C_{257} = C_{365} = 1\,. \tag{6}$$

Quantity $\Delta[A,B,C]$ is called the associator of three octonions A,B,C:

$$\Delta[A,B,C] = \frac{1}{2}\{(AB)C - A(BC)\}. \tag{7}$$

The whole specificity of the octonion algebra against the matrix algebra is that the associators (7) are nonzero.

Perform the linear real transformation of symbols e_M of the following form:

$$\left.\begin{matrix} e_M \to e'_M = G_{MN}\cdot e_N \\ e_0 \to e'_0 = e_0 \end{matrix}\right\}. \tag{8}$$

Consider properties of tensor G_{MN} in the 7-dimensional Euclidean space, in which the base vectors are symbols e_M. The substitution of e'_M into

$$e'_M e'_N = -\delta_{MN} e_0 + C_{MNK} e'_K\,, \tag{9}$$

which symbols e'_M should satisfy, leads to the following two relations:

$$\left.\begin{matrix} G_{MK} G_{NK} = \delta_{MN} \\ G_{MA} G_{NB} C_{ABC} = C_{MNS} G_{SC} \end{matrix}\right\}. \tag{10}$$

Quantities G_{MN} produce 14-parametric group G_2 of rank 2. According to the universally adopted classification, group G_2 is attributed to the exceptional Lie group category. Detailed information about the group G_2 can be found, e.g., in ref. [6].

It is known in advance that in the case of DS realization above the octonion body any isomorphous mapping of the appearing DS apparatus to the matrix apparatus cannot exist in principle. So the question is quite appropriate: Do the matrix realizations of DS have any bearing on the octonion DS whatsoever?

To answer this question, make it our aim to construct the DS realization in the form of real DM in a multidimensional Riemannian space, which would satisfy the following requirement:

When in algebraic operations with octonion DS C_{ABC} play actually no role, the algebraic operations should map to the algebra of real DM of an appropriate dimension. This is true for the algebraic operations with DS $\{\gamma_\alpha\}$ near real DM $\{\overline{\gamma}_\alpha\}$.

To meet this requirement, suppose that in the scheme under discussion there is the smallness parameter $0<\lambda<<1$, such that all matrix elements $(\gamma_\alpha-\overline{\gamma}_\alpha)$ modulo are of the order of λ. Write the matrices γ_α as

$$\gamma_\alpha=\overline{\gamma}_\alpha+f_{\alpha;0}\cdot e_0+f_{\alpha;N}\cdot e_N; \quad (N=1,2,...,7) \tag{11}$$

Matrices (11) will satisfy relation (1), if small matrices $\{f_{\alpha;0},f_{\alpha;N}\}$ are of the form

$$f_{\alpha;0}=[s_0,\overline{\gamma}_\alpha]_-; \quad f_{\alpha;N}=[s_N,\overline{\gamma}_\alpha]_-, \tag{12}$$

where $\{s_0,s_N\}$ are arbitrary small real matrices 4×4. Upon substitution of (12) into (11) it turns out that octonion DS are written in the form

$$\gamma_\alpha=\overline{\gamma}_\alpha+[s_0,\overline{\gamma}_\alpha]_-\cdot e_0+[s_N,\overline{\gamma}_\alpha]_-\cdot e_N. \tag{13}$$

The substitution of (13) into (1) shows that in the first smallness order the C_{ABC} drop out and play no role. This means that the algebra with generatrices satisfying relation

$$[e_M,e_N]_+=-2\delta_{MN}e_0, \tag{14}$$

$$[e_M,e_N]_-=2C_{MNK}e_K, \tag{15}$$

can be mapped in the first order of smallness to the algebra of real DM, in which instead of seven imaginary units $\{e_N\}$, seven matrix imaginary units $\{\mathrm{I}_N\}$ are used. The specific form of the real DM satisfying either above-formulated requirement can be as follows:

$$\left.\begin{array}{lll} e_1\Leftrightarrow \mathrm{I}_1=E_{4\times4}\otimes i\rho_2\sigma_1\otimes\sigma_1 & e_2\Leftrightarrow \mathrm{I}_2=E_{4\times4}\otimes i\sigma_2\otimes\sigma_1 & e_3\Leftrightarrow \mathrm{I}_3=E_{4\times4}\otimes i\rho_2\sigma_3\otimes\sigma_1 \\ e_4\Leftrightarrow \mathrm{I}_4=E_{4\times4}\otimes i\rho_1\sigma_2\otimes\sigma_3 & e_5\Leftrightarrow \mathrm{I}_5=E_{4\times4}\otimes i\rho_2\otimes\sigma_3 & e_6\Leftrightarrow \mathrm{I}_6=E_{4\times4}\otimes i\rho_3\sigma_2\otimes\sigma_3 \\ e_7\Leftrightarrow \mathrm{I}_7=E_{4\times4}\otimes E_{4\times4}\otimes i\sigma_2. & & \end{array}\right\} \tag{16}$$

The resultant multidimensional Riemannian space has dimension 11 and signature $1(-)\&10(+)$. The DM in the space is written as:

$$\left.\begin{array}{l} \overline{\gamma}_0=-i\rho_2\sigma_1\otimes E_{4\times4}\otimes E_{2\times2}; \\ \overline{\gamma}_1=\rho_1\otimes E_{4\times4}\otimes E_{2\times2}; \quad \overline{\gamma}_2=\rho_2\sigma_2\otimes E_{4\times4}\otimes E_{2\times2}; \quad \overline{\gamma}_3=\rho_3\otimes E_{4\times4}\otimes E_{2\times2}; \\ \overline{\gamma}_4=i\rho_2\sigma_3\otimes i\rho_2\sigma_1\otimes\sigma_1; \quad \overline{\gamma}_5=i\rho_2\sigma_3\otimes i\sigma_2\otimes\sigma_1; \quad \overline{\gamma}_6=i\rho_2\sigma_3\otimes i\rho_2\sigma_3\otimes\sigma_1; \\ \overline{\gamma}_7=i\rho_2\sigma_3\otimes i\rho_1\sigma_2\otimes\sigma_3; \quad \overline{\gamma}_8=i\rho_2\sigma_3\otimes i\rho_2\otimes\sigma_3; \quad \overline{\gamma}_9=i\rho_2\sigma_3\otimes i\rho_3\sigma_2\otimes\sigma_3; \\ \overline{\gamma}_{10}=i\rho_2\sigma_3\otimes E_{4\times4}\otimes i\sigma_2. \end{array}\right\} \tag{17}$$

Expressions (13) have the meaning of the ones in the first order of smallness for the first four DM among eleven DM. The expressions for all the eleven DM in the first order of smallness are derived from

$$\gamma_\alpha=\overline{\gamma}_\alpha+[s,\overline{\gamma}_\alpha]_-; \quad \gamma_{N+3}=\overline{\gamma}_{N+3}+[s,\overline{\gamma}_{N+3}]_-; \quad N=1,2,...,7, \tag{18}$$

where

$$s=s_0\cdot E+s_N\cdot \mathrm{I}_N. \tag{19}$$

Thus, in the linear approximation the octonion DM γ_α can be treated as ordinary matrices, if for the basic matrices, in the vicinity of which the expansion proceeds, real DM are used in 11-dimensional Riemannian space of signature $1(-)\&10(+)$. Pay attention to the fact that except for the reality no other properties of DM in 11-dimensional Riemannian space have been used in this consideration. This means that instead of system (17) that DM system can be used in the consideration, which has been derived in [7] from system (17) through transition to the oblique-angled basis produced by simple root vectors of Lie algebra E_8.

In the general case the following rule remains valid: If it was possible to realize DS with the help of octonion DM 4×4, then after that one can transfer from one realization to another using transformations G_2.

5. Discussion

Although the octonion DS can be written in the form of matrices 4×4 in the general case, but the algebra of the matrices possesses no associativity and, hence, cannot be mapped to the algebra of ordinary real matrices in a multidimensional space. In the linear approximation, however, the algebra of octonion DM is mapped to that of real DM in 11-dimensional Riemannian space of signature $1(-)\,\&\,10(+)$. One of possible DM systems in this space is of form (17).

The result obtained is of interest for several reasons.

Reason 1 is that the correspondence found by us between octonion DS and real DM in a multidimensional Riemannian space leads to the Riemannian space, in which the most stable vacuum state appears. Ref. [7] shows that the most stable vacuum state both among the internal subspaces of dimensions other than 7 and among DM of different spinor basis structure is the DM realization in the form of real matrices, in which the oblique-angled basis from the set of fundamental vectors of lattice E_8 is used. In this realization, the internal space dimension is 7; the specific form of the lattice DM is presented in ref. [7] and the matrix of transition from the orthonormal basis to the lattice one is given, e.g., in [7], [8].

Reason 2 is that using any number body, except for the octonion body, in physical theories does not allow us even to pose the question of explanation of the irreversibility of actual processes on the basis of time-reversible fundamental laws with writing the latter in terms of any number field. The irreversibility phenomenon may be explained only in transition to the formulation of the physical laws in terms of octonions.

In this connection note that it has been long since the physicists have paid attention to the existence of an evident contradiction: on the one hand, the dynamic equations describing fundamental interactions possess time reversibility; on the other hand, actual processes that occur in the Nature are irreversible. R. Penrose in [9] writes: "...It is hard to understand how our immense Universe could "sink" into one or another of the states with being unable to even imagine in what time direction to start! ...the only explanation ... remains: not all accurate physics laws are symmetric in time!...".

If DM are realized above the octonion body, then the transition amplitudes automatically cease to be associative. While this just means that the reversibility in time does disappear at the level of fundamental processes in the microworld. In fact, if $A_1, A_2, A_3, \ldots$ are amplitudes of the transitions from initial state t_0 to states arising at times $t_1, t_2, t_{3,\ldots}$, then the amplitude for one of the paths of the process proceeding in the time-forward direction should be found according to rule

$$\left(\left(A_1\cdot A_2\right)\cdot A_3\right)\ldots, \tag{20}$$

while the conjugate amplitude for the process running in the time-backward direction should be found according to rule

$$\left(A_1\cdot\left(A_2\cdot A_3\ldots\right)\right). \tag{21}$$

At the level of real, complex and quaternionic numbers expressions (20), (21) lead to the same probabilities of transitions. But as soon as octonions come into use, the equality between expressions (20), (21), generally speaking, disappears. Moreover, the body of octonion numbers is the only one possessing this property. This means that we may necessarily resort to the octonion quantities for explanation of the irreversibility of processes.

The above considerations and results justify the multiple attempts to consider the octonion wave functions for half-integer spin particles. We only point out to refs. [5], [10], [11] as typical papers from the standpoint of the method for consideration of octonion Dirac matrices. The method of these papers is valid only to the quadratic approximation, as in these papers there is either explicit or implicit transition to so-called split octonions (introduction of the outer imaginary unit commutating with all octonions) or the octonion composition rule is replaced by the open product. Similar (or equivalent) techniques restore the associativity of the modified number body and allow the standard matrix apparatus to be employed. However, in so doing a most interesting part of the octonion specificity is lost.

A method for description of the half-integer spin particle dynamics is the method of mapping of tensors to bispinors developed in a number of papers (see, e.g., [3]). In the method, one of principal objects is bispinor matrix Z. For the octonion implementation of DS, the matrix Z exists in the linear approximation and, as it follows from (13), coincides with S^{-1}. Through multiplication on the right by the projectors, states with different quantum numbers can be separated from the bispinor matrix. For example, one of the subgroups of group G_2 is $SU(3)$. In the general case there is no bispinor matrix, however, the results obtained using the methods for consideration of the transformations of DM 4×4, which are suggested in ref. [12], remain valid.

Thus, the vacuum stability requirement can be made consistent with using the most general number body. In so doing any violation of the bounds of the 7-dimensional internal Euclidean space will result in vacuum instability (and appearance of tachyons as a consequence).

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Number and the geometry of space-time

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Nowadays the well-known saying of Pythagoras: Everything is number is usually understood as a metaphor, having only a very indirect correlation with the structure of the real world. Pythagoras program of reducing that structure to ratios of whole numbers seems naive. Our modern knowledge of the real and complex generalizations of the number concept does not seem to affect this conclusion. Nor does Hamilton s great discovery of the generalization of complex numbers, the quaternions, appear to offer any prospect of the reduction of the physical to the numerical (incidentally Hamilton regarded this discovery as his greatest achievement).

The reason this beautiful idea cannot be true is often said to be a consequence of Frobenius Theorem. This states that those developments of the number concept which preserve the properties of the rationals come to an end with the real and complex numbers. Since the quaternions do not commute, they are no longer numbers properly speaking. The octonions, discovered shortly after the quaternions, seem to be even less convincing candidates for the status of number in the full sense, because of the loss of associativity. Special relativity at first sight seemed to renew the hope of finding a connection between the geometry of physical space and this most fundamental of mathematical concepts. However, neither Minkowski Space nor any of its multidimensional generalizations has disclosed any immediate connection with the study of number fields. It is probably due to this fact that the basic language of modern relativity theory is tensor analysis, which as mathematical machinery is not directly numerical, although the structure of the real and complex numbers provides its basis.

For most physicists the absence of a simple and natural relationship between the objects they study (and notably the fundamental geometric structures of space-time) and the structure of number fields goes unremarked. Many find it strange that a few of their colleagues would still like to realize the dream of Pythagoras and Hamilton. However, when contemplating the desired characteristics of the unified Theory about which they sometimes speculate, physicists often claim that it should be based on a minimal number of the simplest and most elegant principles, although they are usually rather reluctant to say which principles they have in mind.. From a philosophical standpoint, the existence of close connections between physical geometry and algebraic or even more strikingly, arithmetic structures seems a very natural idea, and one providing a clue to the simplest and most beautiful principles lying at the basis of the physical world. At what point is it possible to break into this circle of geometric and arithmetic concepts and gain a clearer understanding of the way in which they intertwine? Could such an understanding help overcome the obstacles which have obstructed the realization of Pythagoras and Hamilton's vision?

One such approach has been largely ignored by both physicists and mathematicians. We refer to the commutative and associative hypercomplex numbers and the corresponding linear Finsler Spaces. Before developing this approach, we take note of the following: besides the very familiar examples of correspondences between geometric and arithmetic notions, such as the real line and the complex plane, there exists a less obvious example., namely the association of a pseudo-Euclidean plane and the corresponding dual numbers, sometimes termed the hyperbolically complex plane (1). The main difference between these numbers and their complex analogues is that the square of the imaginary unit is equal not to 1 but to +1, which implies that the absolute value of a dual number is expressed not by the sum but by the difference of squares, which in its turn coincides with the interval, the main invariant of the pseudo-Euclidean plane.

This pairing involves properties every bit as rich as those seen in the case of the pairing between the Euclidean plane and the complex numbers.. Addition for dual numbers corresponds to parallel transfer and multiplication corresponds to extensions and boosts, the analogues of rotation in space-time. We can also consider analytical functions of dual numbers as conformal transformations of the pseudo-Euclidean plane, each having a distinct physical meaning. The lack of attention to this fact on the part of physicists can be explained by the lack of the correspondence in question in the case of 3 and 4 dimensional pseudo-Euclidean spaces and also by the lack of any physical interpretation for the absolute value of a gradient of the scalar potential in the framework of Special Relativity. The closest analogue for this geometric object is the 4 velocity (on a plane it will be a 2-velocity); but its absolute value should always be equal to unity, whereas in the conjectured theory of a hyperbolic complex potential the corresponding value can vary from zero to infinity. It seems that Hermann Weyl first came closest to finding a solution to this problem. He suggested that each point in space-time should be characterized not only by a metric tensor but also by a scalar. However he never proposed any simple or natural physical meaning for this scalar.

The correspondence between the dual numbers and the pseudo-Euclidean plane should be taken just as seriously as the correspondence between the complex numbers and the Euclidean plane or between the real numbers and the line. It gives a further reason to ask why it is that there appear to be only one 1-dimensional space and two 2 dimensional spaces that have such a natural pairing with algebras (actually there is a third 2 dimensional case: the dual numbers which correspond to the Galilean plane. Why does no such beautiful correspondence exist for 3 and 4 dimensional spaces? Notice the Frobenius theorem no longer blocks such a correspondence, for it says nothing about the dual numbers and the corresponding pseudo-Euclidean plane, although the latter is in some respects closer to real physical geometry than the 1 dimensional and 2-dimensional Euclidean spaces.

A property specific to the algebra of dual numbers is that it can be represented as a direct sum of 1 dimensional algebras. What about considering a direct sum of 3 and 4 dimensional real algebras as a generalization to the multidimensional case? It is easy to see that such algebras exist and that they satisfy desirable basic conditions such as commutativity and associativity of multiplication. But from the viewpoint of the Frobenius Theorem, such algebras are defective, because they have s-called zero divisors or non-invertible elements. The outcome of division by such elements, like division by zero, is unspecified. Algebras having more than one element without an inverse are usually viewed as defective. Attention is not paid to the fact that division by non-zero elements is well specified in such algebras. Besides, in the Special Theory of relativity and on the pseudo-Euclidean plane the existence of zero divisors is actually a reflection of basic features of the corresponding geometrical space, namely the existence of isotropic directions, or in other words, of a light-cone structure.

There is also another and significantly more important problem. The spaces corresponding to the 3 and 4 component generalization of dual numbers are neither Euclidean nor pseudo-Euclidean. In fact they are quite unlike any of the spaces normally considered in connection with the generalization of algebraic or arithmetic concepts and operations. The chief difference marking them out is that the main invariants of such spaces, namely the generalized concepts of distance and interval which they admit, appear not to be connected with forms of the power 2, but with forms of higher power.

The field of modern abstract mathematics which deals with such structures is called Finsler Geometry. The basic subject matter of Finsler Geometry is the study of curved manifolds and metric functions of the most general form. At the basis of the investigation of these structures, which turn out to possess a rich meaning in terms of numerical concepts, lie the axioms of linear algebra and the metric functions on such spaces are polynomials. These features make these geometries and the associated numerical structures quite akin to the Euclidean and pseudo-Euclidean constructions, but with the distinction that the metric polynomials in the case of the Finsler spaces are taken to be nonquadratic. There is no agreed term in the mathematical community for the corresponding spaces. We will christen them polylinear spaces. Today the theory of polylinear spaces is just at the beginning of its development (2), although the close connection between some of the representative spaces under study and the well-developed theory of hypercomplex numbers should be of definite help in understanding at least some of their geometrical features.

So we have two striking facts: on the one hand, because of the no go consequence of the Frobenius Theorem, we cannot expect to find a neat correspondence between the structure of numbers and that of the multidimensional spaces studied in physics of a kind that would have satisfied Pythagoras. On the other hand, we see that mathematicians and physicists have not yet really developed the study of geometries that might turn out to correspond to more or less appropriate generalizations of the concept of number. One conclusion could be that we just have to live with these facts, and pursue the investigation of physical geometry and numerical structures as separate intellectual enterprises. But there is an alternative point of view. In particular there is the possible program of trying to construct appropriate mathematical models of physical space not only on the basis of Minkowski geometry, but also on the basis of the polylinear Finsler spaces, which we have seen ARE associated with numerical structures. At first sight such a mathematical framework might seem too abstract to serve as the basis for a physical model. But if we look back a hundred years, we see that the classical space-time of Galileo and Newton was considered to be the only candidate for the geometry of the physical Universe. So the idea of using some metric different from either the pseudo-Euclidean or Riemannian form to describe that geometry seems quite an appealing further step, or at any rate not a wholly unnatural one

There are infinitely many polylinear Finsler spaces. However, if we restrict to

a) the 4 dimensional spaces
b) the metric form of minimal power
c) those spaces connected with commutative and associative number fields
d) those which allow us to recover Minkowski space as a limiting case.

Then we find these requirements constrain the construction so effectively that there is only one candidate. The space which is thus picked out is a linear Finsler space associated with numbers generated by a direct sum of four real algebras and having a metric function named the Berwald-Moor metric after the two mathematicians who first investigated it. This space is quite unlike the usual quadratic spaces in that it has a designated basis consisting of special isotropic vectors. In this basis the metric function has the following beautiful and laconic form

$$|H| = |\Pi^{n}{}_{i=1} h'_i|^{1/n}, \tag{1}$$

where $|H|$ generalizes the interval of a vector; h_j are its isotropic components and n is the space dimension. As was demonstrated in (2) it is more natural to think of the manifolds associated with this metric as Multi-dimensional Times rather than Multi-dimensional Spaces, since any of their non isotropic straight lines can be interpreted as the proper time of some inertial reference system. The metric function of this 4 dimensional time relates the value of each vector to the fourth root of the product of its components. At first glance, it seems there is no parameter with respect to which we can perform the transition in the limit either to the Euclidean or the Minkowki space metric. However, let us take a closer look at this problem.

For the sake of simplicity, Minkowski space is usually studied in the 3 or even 2 dimensional case. Adopting a similar tactic, let us study the Berwald-Moor metric for $n = 3$. There is no point in considering the case of $n = 2$, because then the Finsler space is just isomorphic to the pseudo-Euclidean plane. For the 3 dimenssional time, the Berwald-Moor metric has the form

$$|H|^3 = h_1'h_2'h_3'. \tag{2}$$

Its pseudo-Euclidean analogue is known to be quite different

$$|X|^2 = x_1^2 - x_2^2 - x_3^2\ . \tag{3}$$

Thus it seems improbable that the geometries specified by these metrics could have anything in common. However, let us pass from the orthonormal basis in which expression (3) is written to one of the isotropic bases () associated with the orthonormal basis by the relations

$$e_1' = \frac{1}{\sqrt{3}}(e_1 + \cos\varphi \cdot e_2 + \sin\varphi \cdot e_3);$$

$$e_2' = \frac{1}{\sqrt{3}}(e_1 + \cos(\varphi+120^o)e_2 + \sin(\varphi+120^o)\cdot e_3); \qquad (4)$$

$$e_3' = \frac{1}{\sqrt{3}}(e_1 + \cos(\varphi+240^o)\cdot e_2 + \sin(\varphi+240^o)\cdot e_3),$$

where φ is an arbitrary parameter. Then the quadratic form (3) becomes absolutely symmetrical:

$$|X|^2 = x_1'x_2' + x_1'x_3' + x_2'x_3'. \qquad (5)$$

In this representation, the pseudo-Euclidean metric takes on a resemblance to the Berwald-Moor metric function (2) and this clarifies the assertion about the resemblance of the corresponding geometries.

Let us now look at this same assertion from the opposite side and perform a transformation of the Berwald-Moor metric (2) from the isotropic basis f_1', f_2', f_3' to the basis which is the Finslerian analogue of the orthonormal basis f_1, f_2, f_3. In this case, the new basis can be associated with the isotropic one by the following expressions:

$$f_1 = f_1' + f_2' + f_3';$$

$$f_2 = \frac{1}{\sqrt{2}}(\sin\psi\cdot f_1' + \sin(\psi+120^o)f_2' + \sin(\psi+240^o)\, f_3'); \qquad (6)$$

$$f_3 = \frac{1}{\sqrt{2}}(\cos\psi\cdot f_1' + \cos(\psi+120^o)\, f_2' + \cos(\psi+240^o)\cdot f_3').$$

Here ψ is another arbitrary parameter. Taking the vectors in this basis, the metric function (2) becomes

$$|H|^3 = h_1^3 - \frac{3}{2}h_1(h_2^2+h_3^2) - \frac{\sqrt{2}}{2}(\sin3\psi\cdot h_2^3 - \cos3\psi\cdot h_3^3) + \frac{3\sqrt{2}}{2}(\sin3\psi\cdot h_2h_3^2 - \cos3\psi\cdot h_2^2h_3), \quad (7)$$

In the limit

$$h_1 >> h_2, h_3 \qquad (8)$$

which is usual in Special relativity, the expression thus obtained gives

$$|H|^3 \approx h_1^3 - \frac{3}{2}h_1(h_2^2+h_3^2), \quad (9)$$

to the third infinitesimal order. In its turn, this last expression is equivalent to the ordinary quadratic form of the pseudo-Euclidean space to the power of 3/2:

$$(|X|^2)^{3/2} = (x_1^2 - x_2^2 - x_3^2)^{3/2} = (x_1^2(1 - \frac{x_2^2 - x_3^2}{x_1^2}))^{3/2} \approx x_1^3 - \frac{3}{2}x_1(x_2^2+x_3^2). \quad (10)$$

Therefore, for $h_1=x_1 >> h_2=x_2$, $h_3=x_3$: $|H| \approx |X|$.

Thus, two completely different metric forms (2) and (3) demonstrate a close affinity, at least in some types of basis.

However, the observed similarity between the fundamental metric forms of both geometries does not give rise to the correspondence principle. To recover this principle, some nontrivial inner properties of the symmetry groups of each space have also to be similar. A pseudo-Euclidean space of any given dimension possesses a six-parameter group of continuous linear transformations which is the analogue of the Poincare Group of ordinary Minkowski spacetime. This six-parameter group contains the three-parameter group of translations and the rotation group of the same dimension. In the case of the latter group, one parameter is responsible for the spatial rotations, and the other two for the boosts. The corresponding group of a three-dimensional time has not six, but five parameters and consists of the three-parameter subgroup of translations and the И-parameter subgroup of transformations. These transformations are very similar to boosts, the only difference being that, unlike boosts, they commute. It might seem however, that the difference in the number of independent parameters is crucial: that it deprives us of any hope of recovering the Poincare group (or its modified analogue) as the fundamental group of continuous symmetries of the multidimensional spacetime — and that without the Poincare group (and its subgroup, the Lorentz group) one cannot envisage recovering the essential structure of current physics.

Nonetheless, a solution to this problem seems in prospect. Since Finsler spaces, apart from the ordinary transformations that leave the interval between points unaffected, also possess other types of symmetries (3) which turn out to be fixed. Conformal mappings, which preserve the angles between directions, could be regarded as forming a distant analogy for such symmetries in the case of spaces with the ordinary quadratic form. However, the analogy in question is incomplete, because in the case of Finsler spaces such symmetries are much more varied than in the conformal case. In particular, all the nonlinear transformations that alter the ψ parameter in the previous formulas to any other real value ψ' are fixed for this three-dimensional time. Specifically, these transformations are related to the rotations of the three-dimensional pseudo-Euclidean Space around the time-like axis. In spite of the fact that the three-dimensional time intervals are not invariant under these transformations, the transformations conserve distances in the two-dimensional subspace.

Figure 1 shows surfaces with their points equidistant from two fixed points (labeled T and –T) in the cases of a) pseudo-Euclidean space and b) Finsler space in affine coordinates. In the pseudo-Euclidean case, the points of such a surface are the simultaneous events which take place in an inertial reference system represented by the straight line passing through T and –T. In principle, the same physical meaning can be assigned to the corresponding surface in the case of three-dimensional time, only now, instead of a plane, we have a set of points satisfying the relation

$$h_1'h_2'h_3'+ (h_1'+h_2'+h_3')\, T^2 = 0, \tag{11}$$

which follows from the requirement that the lengths should be equal. Now the events represented by this plane cannot be considered simultaneous for every point of the straight line (–T, T) as in the pseudo-Euclidean case of Special Relativity. The only point for which they are all simultaneous is T. This is the point which must be occupied by an observer if he is to call all the events on the surface (11) simultaneous. From the philosophical standpoint, this feature represents the most radical difference between the physical consequences of Minkowski and Finsler geometry. Notice that the key difference between Special Relativity and classical physics remains the same the absolute character of simultaneity is rejected. Instead, in the framework of Special Relativity the notion of simultaneity is applicable only to isolated reference systems whose world lines are represented by parallel straight lines. In multi-dimensional time, this relativisation of the notion of simultaneity of events reaches its logical limit, since now the notion of simultaneity holds not for the whole set of parallel straight lines of the space, but only for a single point.

There are grids on both figures designed to show the planes of relative simultaneity. For the observer at point T, the lines of these grids play the role of circular and radial co-ordinates. What is important to, note here is that in the near neighbourhood of the centres of the two co-ordinate grids (i. e. in the neighbourhood of the point associated with the location of the observer) these co-ordinate systems almost completely overlap. In practice, this means that, if confined to local experiments, an observer will be unable to detect the difference between the metric properties of the three-dimensional pseudo-Euclidean space and the multi-dimensional time of the same dimension. As for the more essential distinctions, they become apparent closer to the periphery of the plane of simultaneity and are maximized on its boundary, which has a circumference 2T in diameter (see Fig 1 a)) in the pseudo-Euclidean case; while I the case of the three-dimensional time it is a broken hexagon ABCDEF (the hexagon consists of fragments of the edges of cubes, and the main diagonal of the cube is again equal to 2T (see Fig 1 b)).

What is unusual here is that in the three-dimensional time all the radial lines of the plane of relative simultaneity come together at the three symmetrical opposed vertices of the hexagon enclosing the plane. The three remaining vertices behave as centres of repulsion. Therefore, on the one dimensional celestial sphere of the two-dimensional physical space of an observer located in a Finslerian manifold, there turn out to be six fixed points. In the subjective awareness of the observer, three of these six points (namely the ones where the radial lines converge) extend are smeared over one third of the horizon each; while all the remaining points of the hexagon ABCDEF are focused at the remaining three points. This is certainly one of the major differences in the physical consequences of the two geometries. Notice, however, that it is a difference realized only at the periphery of the space visible to the observer and is virtually imperceptible in her close vicinity.

The nonlinear transformations now play the same role as rotations in the three-dimensional pseudo-Euclidean space. The plane of relative simultaneity is invariant under such transformations, which act so as to transform circular and radial lines into themselves. The linear transformations, which use the proper time, also look like boosts of the pseudo-Euclidean space, but it is the nonlinear transformations which generate the two-parameter commutative group and transform the special cone-like surfaces into themselves. The generators of theses surfaces in this case are the radial lines in the planes of relative simultaneity.

When the Four-dimensional Minkowski space is compared to the space with the Berwald-Moor metric having the same dimension, the resulting picture remains much the same. Thus, in the symmetric isotropic basis the usual quadratic form

$$|X|^2 = x_1^2 - x_2^2 - x_3^2 - x_4^2 \tag{12}$$

becomes

$$|X|^2 = x_1'x_2' + x_1'x_3' + x_1'x_4' + x_2'x_3' + x_2'x_4' + x_3'x_4', \tag{13}$$

which to some extent resembles the basic metric form in the case of four-dimensional time:

$$|H|^4 = h_1'h_2'h_3'h_4'. \tag{14}$$

In its turn, this last expression reduces, in the basis analogous to the orthonormal basis, to the expression

$$|H|^4 = h_1^4 + h_2^4 + h_3^4 + h_4^4 - 2\,(h_1^2h_2^2 + h_1^2h_3^2 + h_1^2h_4^2 + h_2^2h_3^2 + h_2^2h_4^2 + h_3^2h_4^2) + 8h_1h_2h_3h_4, \tag{15}$$

which after simple transformations, takes the form

$$|H|^4 = h_1^4 - 2\,(h_2^2 + h_3^2 + h_4^2)\,h_1^2 + 8\,(h_2h_3h_4)\,h_1 - 2\,(h_2^2h_3^2 + h_2^2h_4^2 + h_3^2h_4^2) + h_2^4 + h_3^4 + h_4^4. \tag{16}$$

Provided the condition

$$h_1 >> h_2, h_3, h_4 \tag{17}$$

is satisfied, this last expression gives

$$|H|^4 = h_1^4 - 2\,(h_2^2 + h_3^2 + h_4^2)\,h_1^2 + (h_2^2 + h_3^2 + h_4^2)^2, \tag{18}$$

within the accuracy of the infinitesimal terms of third and fourth order. Note that this final expression is the perfect square of the classical quadratic form in Minkowki space.

The same holds of the continuous symmetry groups. Unlike the Poincare group that is so well-known amongst physicists, the group of transformations of the Finsler space under examination here has seven rather than ten parameters. Four of these parameters are responsible for parallel transfer, the other three are responsible for boosts. If we then add to these transformations the three-parameter group which leaves invariant the symmetrized t wo-vector form

$$((A, B)) = 1/2\,((A, A, A, B)\,/|A|^2 + (A, B, B, B)\,/|B|^2) \tag{19}$$

then it becomes possible, as in the three-dimensional case, to simulate the spatial rotations in addition to the translations and boosts. This is so because although the form (19) is not linear in each of its vectors, it is nevertheless very close to an ordinary scalar product for all its other properties. When these transformations are added, the designate symmetries of the four-dimensional time becomes equivalent in practice to the Poincare group of Minkowski space, both qualitatively and quantitatively. It is true there is still a certain mathematical difference in the properties of these groups but what is important is that both have features which connect them closely with the observed symmetries of our physical universe. The question which is more adequate to the task of providing a precise and testable description of that physical reality demands further serious investigation.

Notice that the Finsler space under consideration here can be claimed to have one significant advantage. Its group of nonlinear symmetries is essentially more extensive than the corresponding group of conformal transformations in Minkowski space. This wider group contains the transformations that preserve the form created by the four vectors

$$(A, B, C, D) / (|A||B||C||D|). \tag{20}$$

and this expression provides a generalization of the notion of the scalar product of ordinary quadratic geometries.

In addition to the mentioned limiting cases and the similarity of the subgroups of the group of symmetries, there is a further important feature that might incline us to consider the Berwald-Moor metric rather than that of Mikowski as a candidate for the metric of the space we inhabit. This is the essential difference between forward motion and rotary motion in that space, a difference already remarked on by Newton. In the case of rectilinear uniform motion, it is impossible to conclude unambiguously what really moves the purported moving body or the surrounding Universe. This makes the notion of forward speed a relative one. It is much less clear whether the same applies to uniform rotary motion. The issue has long been the subject of intensive investigation and debate, particularly that focused on the meaning and justification of Mach s principle. No consensus has so far been reached.

The question is seen in a new light if we try to describe physical phenomena using a geometry with the Berwald-Moor rather than the Minkowski metric function. The shift in perspective comes from the fact that in a multi-dimensional time, the rotations and translations belong to completely different types of continuous symmetries.

Unfortunately there is no simple way to draw a four-dimensional affine space as in Figure 1 b) Where we could show the radial lines and equidistant points on the three-dimensional hyperplane of simultaneity. One can see that the features of the resultant picture would be the same in principle. The radial lines would extend uniformly from the centre, filling the three-dimensional flat space, passing gradually beyond the hyperplane into the fourth dimension, reaching the specific dodecahedron (which would form the analogue of the hexagon considered in the three-dimensional case) and ultimately converge at its four vertices. As for the limiting dodecahedron, it is formed by the two intersecting light cones. The vertex of one of these is the observers standpoint, while the other is its mirror image with its origin at a point symmetrical to the vertex of the first light cone. As a result, the radial lines fill the three-dimensional region, but this is no longer flat, but essentially four dimensional. As a result, its boundary has 14 vertices, 24 edges and 12 faces. Figure 2 shows the projection of such a dodecahedron on the three-dimensional internal firmament — the observed celestial sphere. Different points of this firmament and different directions within it are not of equal status. But the difference in their status only becomes apparent at huge astrophysical distances, so the problem of detecting such features in the real world is a hard one.

From the point of view of an observer who resides in such a four-dimensional time, this dodecahedron is just the most distant end of the visible universe. Therefore to reveal a discrepancy between predicted observations in the two models the one based on the Finsler space and the other on Minkowski geometry data from very remote sources has to be compared. To make such comparison possible, the data has to arrive from sufficiently separated points. Unfortunately the solar system which forms the current limit is far too small in proportion to the visible universe to permit such comparison. Therefore, at least for the time being, the direct measurement of angles and distances, like Gauss similarly inspired efforts of two centuries ago, is not going to help us decide which of the two geometries, Riemannian or Finslerian, better fits the structure of physical space. In saying this, however, we do not preclude that other, subtler distinctions between the two geometries might provide a basis for choosing between them.

Such indirect evidence might in particular be supplied by observations of quasars, since these are the most remote known objects. It is known that the behaviour of quasars exhibits serious anomalies which were not at all anticipated from within the framework of current relativistic astrophysics. In attempting to calculate the distances at which quasars lie from the frequency at which their luminosity varies, to account for the observed oscillations in luminosity we have either to assign incredibly small transversal dimensions to quasars or else conclude that the waves inside quasars are propagating at superluminal speeds. Either claim conflicts sharply with our current cosmological understanding. But both can comfortably be accommodated by the hypothesis of the Finslerian distortion of the signals being emitted by quasars.

More direct evidence that the metric of real-world geometry is that of Berwald-Moor is provided by investigations of the angular distribution of the temperature fluctuations in the cosmological microwave background radiation. It has been claimed that in these, a dodecahedral shape for the boundaries of the space of our universe was registered. See Reference (5). In the four dimensional time, Finsler space based model discussed above, the boundaries of the space seen by the observer are also dodecahedral, only this time the sides of the dodecahedrons are ordinary squares rather than pentagons. In addition, because our figure 1 b) is closed in two dimensions, it is now embedded in four dimensional rather than three-dimensional space. Still, since the observational probes of the cosmological background radiation so far carried out offer no evidence as to the shapes of the sides of the dodecahedron, or any other details, it remains disputable which of the two kinds of figures described above should be regarded as the preferred candidate.

When a multi-dimensional time is substituted for Minkowski space, the degrees of freedom of the system are essentially increased that is the effect of using the Finslerian generalization of the metric tensor rather than the standard Riemannian tensor. In Einstein s theory, the independent components of the metric tensor are identified with the gravitational field potentials. However, whereas in the case of four-dimensional Riemannian space the metric tensor has 10 independent components, in the case of a Finsler Space of the same dimensions, the corresponding tensor has no fewer than 35.

A further argument for considering the Berwald-Moor, as against the quadratic, metric is to be found in the images of the fractal Julia sets. These sets are today familiar from the corresponding (and strikingly beautiful) computer-generated images in the case of the Euclidean plane, corresponding to the complex numbers. But they also have a primitive structure in the higher-dimensional case, in particular that constructed on the basis of the quaternions. By replacing Euclidean space by a higher dimensional time associated with the commutative and associative hypercomplex numbers, we likewise obtain three dimensional images notice that these can naturally be regarded as snapshots of four-dimensional objects evolving in time.

So in conclusion: the geometry of four-dimensional time allows interpretations of its structures such that these turn out to correspond in a natural way with structures familiar from both classical and relativistic physics. Moreover, the adoption of curved Finsler space as a geometrical framework for the description of real world structure may make possible a reduction of not only the gravitational field, but also the other fundamental fields of physics to pure geometry whilst staying within four dimensions, thus, realizing the vision of Einstein and Wheeler. And to the extent that this pure geometry can be conceived as itself having a purely numerical basis, we should also have fulfilled an even older and compelling vision that of Pythagoras and Hamilton

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Classical scalar vacuum: the modern treatment.

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> When I talk about reintroducing the ether, I do not mean to go back to the picture of the ether that one had in the 19th century, but I do mean to introduce a new picture of the ether that will conform to our present ideas of quantum theory.
>
> P.A.M. Dirac (1963)

At the dawn of quantum physics it was definitely shown that physical vacuum provides for so called 'null oscillations' being unremovable even at zero temperature limit and related as usually to the virtual fluctuations of electromagnetic field. In general quantum field theory it is accepted to associate vacuum(s) with variety of lowest energetic states for all kinds of the quantized fields. As inevitable consequence of introduction *ab initio* into Standard Model of hypothetical scalar field we get a fundamental concept of the 'Higgs vacuum' as a basic vacuum state which is as far as necessary for justification of the theory but so much problematical due to permanent lack of the Higgs bosons in the current experiment. There are yet more questions to the different exotic vacuum states (such as planckian vacuum, grand unification vacuum, inflatons, string vacuum, etc.) to be capable to produce 'spontaneously' the usual matter.

In General Relativity (GR) and Astrophysics the realistic vacuum is considered as an ubiquitous, weakly gravitating medium which appears to be responsible for the phenomena of 'dark matter and energy' in the Universe and also for its accelerated expansion discovered in 1998.

As a whole we get the distinction in energy density between the classical and quantum vacuums in more than 50 orders, both (!) in conformity with the corresponding experiments. No comments, darlings.

In context of the *staton* treatment of gravitation one can see the way out of situation in development of adequate approach to realistic, well defined, scalar vacuum [1]. This implies that quantum vacuum represents the highly exited bound states of scalar condensate localizing just in the domain of interactions of particles. At the same time particles and their composites ('quarks' of three generations, etc.) should themselves be considered as some special soliton (vortex) states of scalar background being in dynamical equilibrium (to be stable for the stable particles) both with surrounded quantum and classical vacuums.

However, properties of quantum vacuum can never be spread to the macroscopic scales (in a sense, no interaction – no vacuum to be defined). The only exceptions are the collective anisotropic Casimir effects which however are also localized well enough. As for the classical low-energetic vacuum, it is always free and presents always and everywhere. In this respect so-called 'hierarchies problems' for vacuums should be considered not so far as 'great problems' but rather as not a well enough posed (in terms of traditional quantum field theory – QFT) question.

What is really a serious challenge to QFT that is the long-term unrevealing of the Higgs bosons (HB). But from our point the problem is not how to find the HB but just how to remove them without violation of basic features of SM. Well, if there is some gauge condition providing a 'grace exit' of HB from SM.

The point is that the staton approach does not provide for any other free carriers of scalar field besides 'statons' (i.e. effective pairs of '+' and '-' statons) in the laminar component or 'psions' (effective neutrino-antineutrino pairs) for the vortex component of scalar 'condensate'.

In order to avoid the ambiguities, it is worth to notify that we are talking about the *free* scalar field, both laminar and vortex, as about the *'condensate'* (in the quotation marks) as well, just in a sense (and due to) that ‘statons’ and ‘psions’ are assigned to be some *‘quasiparticles’*, but of course only effectively composed from two dual components that is without any real coupling similar to the Cooper couples.

In the free ‘condensate’ (classical vacuum) the propagation of any wave perturbations proves to be possible only in a form of electromagnetic waves, so that interaction is transferring by photons. In the bound condensate (quantum vacuum) such a role passes on to intermediate bosons (and hypothetical gluons). In that picture there is no place for HB, to be an artifact of phenomenological Higgs mechanism providing the masses for particles and fields. So, the Moor has done his duty, let him go away.

Note that in staton treatment of Gravity there is no place for the gravitational waves as well, they are to be as far elusive phenomenon in GR as so the discussed Higgs bosons in Particle Physics.

[1] See another presented reports of our as necessary.

Binary geometrophysics and the twistor program of R.Penrose

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В докладе изложены основы бинарной геометрофизики, представляющей собой новый подход к построению объединенной теории пространства-времени и физических взаимодействий [1]. В отличие от всех известных физических теорий, в этом подходе с самого начала не постулируется существование классического пространства-времени, а предлагается его построение, исходя из понятий алгебраической теории бинарных систем комплексных отношений, вводимых между элементами двух множеств (своеобразной бинарной геометрии), трактуемых как начальные и конечные состояния микросистем.

Предложенная теория опирается на идеи квантовой теории S-матриц, многомерных геометрических моделей физических взаимодействий типа теории Калуцы-Клейна и теории прямого межчастичного взаимодействия Фоккера-Фейнмана. В этом подходе существенно используется идея о макроскопической природе пространства-времени, согласно которой классические пространственно-временные представления справедливы лишь при описании достаточно сложных макросистем и теряют силу в микромире, где им на смену приходят иные закономерности.

В докладе был представлен ряд результатов в рамках данной теории, касающихся, главным образом, способа объединения известных видов физических взаимодействий: сильных и электрослабых. В этом подходе гравитационные взаимодействия не являются первичными, а определяются другими взаимодействиями и возникают вместе с понятиями классического пространства-времени.

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The speed of gravitation

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A regular approach looks the speed of gravitation as a speed of weak waves of the metrics. This study realizes a new approach, defining the speed as a speed of travelling waves in the field of gravitational inertial force. D'Alembert's equations of the field show that this speed is equal to the light velocity corrected with gravitational potential. The approach leads to the new experiment to measure the speed of gravitation, which, having "detectors" like as planets and their satellites a base, is not linked to deviation of geodesic lines and quadrupole mass-detectors with its specific technical problems.

1. Introduction

We take a pseudo-Riemannian space with the signature $(+---)$, where time is real and spatial coordinates are imaginary, because the projection of four-dimensional impulse on the spatial section of any given observer is positive in this case. We also sign space-time indices Greek, while spatial indices Roman. So, the time term in d'Alembert's operator $\Box = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$ will be positive, while the spatial part (Laplace's operator) will be negative $\Delta = -g^{ik}\nabla_i\nabla_k$.

Applying the d'Alembert operator to a tensor field, we obtain the *d'Alembert equations* of the field. The non-zeroes are the d'Alembert equations containing the field-inducing sources. The zeroes are the equations without the sources. If no the sources, the field is free. This is a free wave. There is the time term $\frac{1}{a^2}\frac{\partial^2}{\partial t^2}$ including the linear velocity a of the wave. So, being applied to gravitational fields, the d'Alembert equations give a possibility to calculate the speed of propagation of gravitational attraction (the *speed of gravitation*). In the same time the result may be different depending from a way we define the speed as the velocity of waves of the metric or something else.

A regular approach set forth the speed of gravitation as follows [1, 2]. One considers the space-time metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$, composed of a Galilean metric $g^{(0)}_{\alpha\beta}$ (wherein $g^{(0)}_{00} = 1$, $g^{(0)}_{0i} = 0$, $g^{(0)}_{ik} = -\delta_{ik}$) plus tiny corrections $\zeta_{\alpha\beta}$ defining a weak gravitational field. Because $\zeta_{\alpha\beta}$ are tiny, we can lift and lower indices with the Galilean metric tensor $g^{(0)}_{\alpha\beta}$. The quantities $\zeta^{\alpha\beta}$ are defined with the main property of the fundamental metric tensor $g_{\alpha\sigma}g^{\sigma\beta} = \delta^\beta_\alpha$ as $(g^{(0)}_{\alpha\sigma} + \zeta_{\alpha\sigma})g^{\sigma\beta} = \delta^\beta_\alpha$. Besides the approach defines $g^{\alpha\beta}$ and $g = \det\|g_{\alpha\beta}\|$ to within higher order terms as $g^{\alpha\beta} = g^{(0)\alpha\beta} - \zeta^{\alpha\beta}$ and $g = g^{(0)} + \zeta$, where $\zeta = \zeta^\sigma_\sigma$. Because $\zeta_{\alpha\beta}$ are tiny we can take Ricci's tensor $R_{\alpha\beta} = R^{\cdots\sigma}_{\alpha\sigma\beta}$ (the Riemann-Christoffel curvature tensor $R_{\alpha\beta\gamma\delta}$ contracted by two indices) to within higher order terms withheld. Then the Ricci tensor for the metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$ is

$$R_{\alpha\beta} = \frac{1}{2} g^{(0)}_{\mu\nu} \frac{\partial^2 \zeta_{\alpha\beta}}{\partial x^\mu \partial x^\nu} = \frac{1}{2}\Box\,\zeta_{\alpha\beta},$$

that simplifies Einstein's equations $R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = -\kappa T_{\alpha\beta} + \lambda g_{\alpha\beta}$, wherein this case means $R = g^{(0)\mu\nu}R_{\mu\nu}$. In the absence of substance and λ-fields ($T_{\alpha\beta} = 0$, $\lambda = 0$), that is in emptiness, the Einstein equations for the metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$ become

$$\Box \zeta_{\alpha}^{\beta} = 0.$$

Actually, these are the d'Alembert equations of the corrections $\zeta_{\alpha\beta}$ to the metric $g_{\alpha\beta} = g_{\alpha\beta}^{(0)} + \zeta_{\alpha\beta}$ (*weak waves of the metric*). Taking the flat wave along $x^1 = x$, we see

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} \right) \zeta_{\alpha}^{\beta} = 0,$$

so weak waves of metric travel at the light velocity in an empty space.

This approach leads to an experiment, based on that geodesic lines of two infinitely close test-particles deviate in a field of waves of the metric. A system of two real particles connected through a spring (a quadrupole mass-detector) shall react to the waves. The most of such experiments were linked with Weber's detector since 1968. The experiments have not arrived to a result until now, because of problems with measuring precision and other technical problems [3].

Is the approach that above the best? Really, the resulting d'Alembert equations are derived from the formula of the Ricci tensor, which was obtained under the substantial simplifications of higher order terms withheld. Eddington [1] wrote that a source of this approximation is a specific reference frame which differences from Galilean reference frames with the tiny corrections $\zeta_{\alpha\beta}$, an origin of which could be very different, not only gravitation. This is a "vicious circle", Eddington wrote. So, the regular approach has got its own drawbacks as follows:

1. The approach gives the Ricci tensor and the d'Alembert equations of the metric to within higher order terms withheld, so the velocity of waves of the metric calculated from the equations is not finally exact theoretical result;
2. A source of this approximation are the tiny corrections $\zeta_{\alpha\beta}$ to a Galilean metric, an origin of which may be very different, not only gravitation;
3. Two bodies attract one another, because of the transfer of gravitational force. A wave travelling in the field of gravitational force is not the same that a wave of the metric — these are different tensor fields. When a quadrupole mass-detector registers a signal, then the detector reacts a wave of the metric in accordance with this experiment theory. Therefore it is possible that quadrupole mass-detectors would be good to discover waves of the metric, however the experiment is only oblique to measure the speed of gravitation.

The reasons lead us to consider gravitational waves as waves travelling in the field of gravitational force, that provides two important advantages:

1. The mathematical apparatus of chronometric invariants (physical observable quantities in the General Theory of Relativity) define gravitational inertial force F_i without the Riemann-Christoffel curvature tensor [4, 5]. Using the methods, we can deduce the exact d'Alembert equations for the force field, that give an exact formula for the velocity of waves of the force;
2. Experiments to register waves of the force field, having "detectors" like as planets or their satellites a base, does not link to the quadrupole mass-detector and its specific technical problems.

2. The new approach

A base here is the mathematical apparatus of chronometric invariants, developed by Zelmanov in 1940's [4, 5]. Its essence is that, if an observer accompanies his reference body, his observable quantities (chronometric invariants) are projections of four-dimensional quantities on his time line and the spatial section, made by projecting operators $b^{\alpha}=dx^{\alpha}/ds$ and $h_{\alpha\beta}=-g_{\alpha\beta}+b_{\alpha}b_{\beta}$ which fully define his real reference space. For instance, chr.inv.-projections of any world-vector Q^{α} are $b^{\alpha}Q_{\alpha}=Q_0/\sqrt{g_{00}}$ and $h^i_{\alpha}Q^{\alpha}=Q^i$, while chr.inv.-projections of any world-tensor of the 2nd rank $Q^{\alpha\beta}$ are $b^{\alpha}b^{\beta}Q_{\alpha\beta}=Q_{00}/g_{00}$, $h^{i\alpha}b^{\beta}Q_{\alpha\beta}=Q^i_0/\sqrt{g_{00}}$, $h^i_{\alpha}h^k_{\beta}Q^{\alpha\beta}=Q^{ik}$. Observable properties of the space are derived from the non-commutativity of chr.inv.-derivating operators $\frac{{}^*\partial}{\partial t}=\frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i}=\frac{\partial}{\partial x^i}+\frac{1}{c^2}v_i\frac{{}^*\partial}{\partial t}$. Those are the chr.inv.-vector of gravitational inertial force F_i, the chr.inv.-tensor of angular velocities of the space rotation A_{ik}, and the chr.inv.-tensor of deformation rate of the space D_{ik}, namely

$$F_i=\frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i}-\frac{\partial v_i}{\partial t}\right), \qquad \sqrt{g_{00}}=1-\frac{\mathrm{w}}{c^2},$$

$$A_{ik}=\frac{1}{2}\left(\frac{\partial v_k}{\partial x^i}-\frac{\partial v_i}{\partial x^k}\right)+\frac{1}{2c^2}\left(F_iv_k-F_kv_i\right), \qquad v_i=-c\frac{g_{0i}}{\sqrt{g_{00}}},$$

$$D_{ik}=\frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}, \qquad D^{ik}=-\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}, \qquad D=D^k_k=\frac{{}^*\partial \ln\sqrt{h}}{\partial t},$$

where $h_{ik}=-g_{ik}+\frac{1}{c^2}v_iv_k$ is the metric chr.inv.-tensor, $h=\det\|h_{ik}\|$, w is gravitational potential, v_i is the linear velocity of the space rotation. Observable non-uniformity of the space is set up by the Christoffel chr.inv.-symbols $\Delta^i_{jk}=h^{im}\Delta_{jk,m}$, which are built just like as the Christoffel regular symbols $\Gamma^{\alpha}_{\mu\nu}=g^{\alpha\sigma}\Gamma_{\mu\nu,\sigma}$ using h_{ik} instead of $g_{\alpha\beta}$.

Four-dimensional generalization of the chr.inv.-quantities F_i, A_{ik}, and D_{ik} had been obtained by Zelmanov in 1960's [6] as follows $F_{\alpha}=-2c^2b^{\beta}a_{\beta\alpha}$, $A_{\alpha\beta}=ch^{\mu}_{\alpha}h^{\nu}_{\beta}a_{\mu\nu}$, $D_{\alpha\beta}=ch^{\mu}_{\alpha}h^{\nu}_{\beta}d_{\mu\nu}$, where $a_{\alpha\beta}=\frac{1}{2}(\nabla_{\alpha}b_{\beta}-\nabla_{\beta}b_{\alpha})$ and $d_{\alpha\beta}=\frac{1}{2}(\nabla_{\alpha}b_{\beta}+\nabla_{\beta}b_{\alpha})$.

So forth, following the study [7], we consider a field of the gravitational inertial force $F_{\alpha}=-2c^2b^{\beta}a_{\beta\alpha}$, which chr.inv.-projections are F^i and the mentioned $F_i=h_{ik}F^k$. The d'Alembert equations of the vector field $F^{\alpha}=-2c^2b^{\beta}a_{\beta\cdot}^{\cdot\alpha}$ in the absence of its sources are $\Box F^{\alpha}=0$. Their chr.inv.-projections (the *d'Alembert chr.inv.-equations*), generally speaking, can be deduced as follows

$$b_{\sigma}g^{\alpha\beta}\nabla_{\alpha}\nabla_{\beta}F^{\sigma}=0, \qquad h^i_{\sigma}g^{\alpha\beta}\nabla_{\alpha}\nabla_{\beta}F^{\sigma}=0.$$

After some algebra we obtain the d'Alembert chr.inv.-equations for the field of the gravitational inertial force $F^{\alpha}=-2c^2b^{\beta}a_{\beta\cdot}^{\cdot\alpha}$ in their final form. They are

$$\frac{1}{c^2}\frac{{}^*\partial}{\partial t}(F_kF^k)+\frac{1}{c^2}F_i\frac{{}^*\partial F^i}{\partial t}+D^k_m\frac{{}^*\partial F^m}{\partial x^k}+h^{ik}\frac{{}^*\partial}{\partial x^i}[(D_{kn}+A_{kn})F^n]-$$
$$-\frac{2}{c^2}A_{ik}F^iF^k+\frac{1}{c^2}F_mF^mD+\Delta^m_{kn}D^k_mF^n-h^{ik}\Delta^m_{ik}(D_{mn}+A_{mn})F^n=0,$$

$$\frac{1}{c^2}\frac{{}^*\partial^2 F^i}{\partial t^2}-h^{km}\frac{{}^*\partial^2 F^i}{\partial x^k \partial x^m}+\frac{1}{c^2}(D_k^i+A_{k\cdot}^{\cdot i})\frac{{}^*\partial F^k}{\partial t}+\frac{1}{c^2}D\frac{{}^*\partial F^i}{\partial t}+$$
$$+\frac{1}{c^2}F^k\frac{{}^*\partial F^i}{\partial x^k}+\frac{1}{c^2}\frac{{}^*\partial}{\partial t}[(D_k^i+A_{k\cdot}^{\cdot i})F^k]+\frac{1}{c^2}(D_n^i+A_{n\cdot}^{\cdot i})F^n D+$$
$$+\frac{1}{c^2}\Delta_{km}^i F^k F^m+\frac{1}{c^2}F_k F^k F^i-h^{km}\left\{\frac{{}^*\partial}{\partial x^k}(\Delta_{mn}^i F^n)+\right.$$
$$\left.+(\Delta_{kn}^i\Delta_{mp}^n-\Delta_{km}^n\Delta_{np}^i)F^p+\Delta_{kn}^i\frac{{}^*\partial F^n}{\partial x^m}-\Delta_{km}^n\frac{{}^*\partial F^i}{\partial x^n}\right\}=0.$$

Called on the formulas for chr.inv.-derivatives, we transform the first term in the vector d'Alembert chr.inv.-equations to the form

$$\frac{1}{c^2}\frac{{}^*\partial^2 F^i}{\partial t^2}=\frac{1}{c^2 g_{00}}\frac{\partial^2 F^i}{\partial t^2}+\frac{1}{c^4\sqrt{g_{00}}}\frac{{}^*\partial \mathrm{w}}{\partial t}\frac{{}^*\partial F^i}{\partial t},$$

so waves of gravitational inertial force travel with a velocity, a modulus of which is

$$u=\sqrt{u_k u^k}=c\left(1-\frac{\mathrm{w}}{c^2}\right).$$

Because waves of the field of gravitational inertial force transfer gravitational interaction, this waves speed is the *speed of gravitation* as well. The speed depends on the potential w of the field itself, that lead us to the next conclusions:

1. In a weak gravitational field, a potential w of which is negligible but its gradient F_i is non-zero, the speed of gravitation equals the light velocity;
2. According to this formula, the speed of gravitation shall be lesser than the light velocity near bulk bodies like stars or planets, where gravitational potential is perceptible. On the Earth surface slowing gravitation down shall be 21 cm/sec. Gravitation near the Sun shall be 6.3×10^4 cm/sec slow than light.
3. Under gravitational collapse ($\mathrm{w}=c^2$) the speed of gravitation becomes zero.

Let us turn out from theory to experiment. An idea to measure the speed of gravitation as a speed to transfer the attracting force between space bodies had been proposed by mathematician Dombrowski in his conversation with me a decade before. But in the absence of theory the idea had not arrived to experiment in that time. Now we have an exact formula for the speed of waves travelling in the field of gravitational inertial force, so we can propose an experiment to measure the speed (a Weber detector reacts to weak waves of the metric, so it is inapplicable to put this idea into experiment).

So, the Moon attracts the Earth surface looking to her stronger than the opposite. As a result, the flow "hump" in the ocean surface follows the moving Moon producing ebbs and flows. Analogous "hump" follows the Sun, its height is more lesser. A satellite in an Earth orbit has the same ebb and flow oscillations, its orbit lowers and lifts for a little following the Moon and the Sun as well. A satellite in airless space does not any friction to the contrary of viscous water in the ocean. A satellite is a perfect system, which reacts to the flow carrying instantly. If the speed of gravitation is limited, in this case the moment of the satellite's maximal flow rise should be late from the lunar/solar upper transit with the time that waves of gravitational force field travelled from the Moon/Sun to the satellite.

The Earth gravitational field is not absolute symmetric, because of the imperfect terrestrial globe. A real satellite reacts to the field defects during its orbital flight around the Earth — the height of its orbit oscillates about decimetres that would be substantial noise in the experiment. From this reason a geostationary satellite would be the best. Such satellite, having an equatorial orbit, requires an angular velocity the same as that of the Earth. As a result, the height of a geostationary satellite above the Earth does not depend on defects of the Earth gravitational field. The height could be measured by a laser range-finder with high precision almost without interruption, providing a possibility to register the moment of the maximal flow rise of the satellite perfectly.

In accordance with our formula the speed of gravitation near the Earth is minus 21 cm/sec of the light velocity. In this case the maximum of the lunar flow wave in a satellite orbit shall be about 1 sec late from the lunar upper culmination. The lateness of the flow wave of the Sun shall be about 500 sec after the upper transit of the Sun. A question is how much precisely could be registered the moment of the maximal flow rise of a satellite in its orbit, because the real maximum can be "fuzzy" in time.

3. Effect of the curvature

If a space is homogeneous ($\Delta^i_{km}=0$) and it is free of rotation and deformation ($A_{ik}=0$, $D_{ik}=0$), then the d'Alembert chr.inv.-equations for the field of gravitational inertial force take the form

$$\frac{1}{c^2}\frac{{}^*\partial}{\partial t}(F_k F^k)+\frac{1}{c^2}F_i\frac{{}^*\partial F^i}{\partial t}=0,$$
$$\frac{1}{c^2}\frac{{}^*\partial^2 F^i}{\partial t^2}-h^{km}\frac{{}^*\partial^2 F^i}{\partial x^k\partial x^m}+\frac{1}{c^2}F^k\frac{{}^*\partial F^i}{\partial x^k}+\frac{1}{c^2}F_k F^k F^i=0,$$

so waves of gravitational inertial force can be even in this case.

Waves of metric are linked with the space-time curvature deriving from the Riemann-Christoffel curvature tensor. If the first derivatives of the metric (the space deformations) are zeroes, then its second derivatives (the curvature) are zeroes too. Therefore waves of metric have not a place in a non-deforming space, while waves of gravitational inertial force are possible therein.

In connection with this fact, following the study [7], the next question rises. How much affects the curvature on waves of gravitational inertial force?

To answer this question, let us remember that Zelmanov [4], following the way that the Riemann-Ciristoffel tensor was introduced, after taking the non-commutativity of the second chr.inv.-derivatives of a vector ${}^*\nabla_i{}^*\nabla_k Q_l-{}^*\nabla_k{}^*\nabla_i Q_l=\frac{2A_{ik}}{c^2}\frac{{}^*\partial Q_l}{\partial t}+H_{lki}^{\cdot\cdot\cdot j}Q_j$ had arrived to the chr.inv.-tensor $H_{lki}^{\cdot\cdot\cdot j}$ like Schouten's tensor [8]. Its generalization gives the curvature chr.inv.-tensor $C_{lkij}=\frac{1}{4}(H_{lkij}-H_{jkil}+H_{klji}-H_{iljk})$, which has all properties of the Riemann-Christoffel tensor in the observer's spatial section. So the spatial chr.inv.-projection $Z^{iklj}=-c^2R^{iklj}$ of the Riemann-Christoffel tensor $R_{\alpha\beta\gamma\delta}$, after contraction by h_{ik} twice, is $Z=D_{ik}D^{ik}-D^2-A_{ik}A^{ik}-c^2C$, where $C=C_j^j=h^{lj}C_{lj}$ and $C_{kj}=C_{kij}^{\cdot\cdot\cdot i}=h^{im}C_{kimj}$.

In the same time, as it was shown in Synge's book [9], in a space of a constant four-dimensional curvature $K=\text{const}$ we have $R_{\alpha\beta\gamma\delta}=K(g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma})$, $R_{\alpha\beta}=-3Kg_{\alpha\beta}$, $R=-12K$. So forth, having the formulas a base, after calculation the spatial chr.inv.-projection of the Riemann-Christoffel tensor we arrive to that in a constant curvature space $Z=6c^2K$. Equalizing it to the same quantity in an arbitrary curvature space, we obtain a correlation between the four-dimensional curvature K and the observable spatial curvature in the constant curvature space

$$6c^2K = D_{ik}D^{ik} - D^2 - A_{ik}A^{ik} - c^2C.$$

If the four-dimensional curvature is zero $K = 0$, and also the space does not deformations $D_{ik} = 0$ (its metric is stationary $h_{ik} = \mathrm{const}$), then no waves of the metric there exactly. In such space the observable three-dimensional curvature is

$$C = -\frac{1}{c^2}A_{ik}A^{ik},$$

it is non-zero $C \neq 0$, if only the space rotates $A_{ik} \neq 0$. If, aside for these, the space does not rotate, then its observable curvature also becomes zero $C = 0$. Even in this case the obtained d'Alembert chr.inv.-equations show the presence of waves of gravitational inertial force.

What this implies? As a matter of fact that gravitational attraction is an everyday reality of our world, so waves of gravitational inertial force transferring the attraction shall be incontrovertible. Therefore we arrive to the alternative:

1. Waves of gravitational inertial force depend on a curvature of space, then the real space-time is not a space of constant curvature;
2. Either waves of gravitational inertial force do not depend on the curvature.

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The Casimir effect and the gravitation

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The detection and the research of the neutrinos background of Universe are the attractive problems. These problems do not seem the unpromising one in the case of the high neutrinos density of Universe. It was offered before to use the low energy neutrinos background of Universe for the explanation of the gravitational phenomena with the quantum position attracting the Casimir effect for this. As a result it was connected the gravitational constant with the parameters characterizing the electroweak interactions. If now we shall be based on the results of the experiments fixing the equality of the gravitation mass and the inert one then it can consider that the spectrum of the particle masses is defined by their interaction with the neutrinos background of Universe. This statement is confirmed what the rest mass of the photon is equal to zero in contradistinction to the masses of the vector bosons W^+, W^-, Z° which's interact with the neutrinos immediately.

1. Introduction

The making of a physical theory embracing all an energy spectrum of interactions is a fairly difficult task. In consequence a construction of asymptotical theories both in high-energy and low-energy ends of this spectrum was justified historically. The most considerable success attended the work in the high-energy approximation, in a result of which was made quantum electrodynamics (QED) giving the prediction confirming experimentally with the remarkable precision. Naturally, that this theory became the imitation specimen by the construction of the analogous theories of the strong interaction (the quantum chromodynamics (QCD)) and the weak one (Salam-Weinberg model). The every possible theories describing continuum with the large number of particles such as the theories of solid bodies, liquids, gases, plasma, an electromagnetic radiation, shells, nuclei at low energies and also General Relativity (GR) is related to the opposite end of the energy spectrum. GR look the exclusion against this background, what gave the reason to consider the gravitation is only the effect of the existence of the space-time curvature.

Admittedly the theories wrecking the present idea are appearing in the second half of XX century such as the two-tensor gravitation theory [1], in which was made the attempt to rewrite the theory of nuclear interactions into the geometrical language. It can attribute to like works also and the gauge theory of the dislocations and disclinations [2]. In consequence of this the transfer to the geometrization description as the most comfortable one in the long-wave length range for any interactions is the logical one. Thereby it is wrecked fully the exceptionality of the gravitation and the forces corresponding to it are not to be distinguished between others, such as Yukawa forces and the Van der Waals forces. So it is necessary to show that the gravitation interaction is not a fundamental one, but the one is induced by others interactions as possible hypothetical ones. The more so, that the gravitational constant $G_N \approx 6.7 \cdot 10^{-39}\ GeV^{-2}$ (it is used the system of units h/2π = c = 1, where h is the Planck constant and c is the light speed) is a suspiciously small value and a dimensional one furthermore (as is known the latter prevent to the construction of the renormalizable quantum theory).

Before building the theory of the induced gravitation on the base of the hypothetical interactions and the hypothetical particles it was necessary to verify the possibility of the use of the known particles and the known interactions for this purpose. Naturally that the neutrinos are the most suitable particles for this taking into account their penetrating ability, which allow them to interact with all the substance of the macroscopic body - not with the surface layer only. As is known [3], already in 30th Gamow and Teller offered to use the neutrinos for the explanation of the gravitation, but their mechanism provided the direct exchange of the pairs consisting of a neutrino and an antineutrino and therefore the one does not correspond to the modern conceptions of the theory of interactions.

Bashkin's works appearing in 80th on a propagation of the spin waves in the polarized gases [4] allowed to make the supposition [5], that the analogous collective oscillations are possible under certain conditions as well as in the neutrinos medium. Since the collective oscillations can induce an interaction between particles, Bashkin's works make us to pay attention to the background neutrinos [6] (under which we shall imply antineutrinos, too) filling our Universe. If the effective temperature of the Universe neutrinos is the fairly low one then it is fulfilled one of conditions ($\lambda \gg r_w$, where λ is the de Broglie's wave-length of a neutrino and r_w is the weak interaction radius of an one [4]) of the propagation of the spin wave in the polarized gases. As a result the quantum effects become the determinative ones in such medium and the interference of the neutrinos fields (being the consequence of the known identity of elementary particles) must induce the quantum beats, which will be interpreted as zero oscillations of a vacuum. In consequence of this the mathematical apparatus [7] applied by the description of the Casimir effect [8] can be used.

2. The Casimir effect

We shall be interesting in quantum beats arising by the interference of the falling polarized flow of the relict neutrinos on the macroscopic body with the scattered one at this body. Let's suppose for this the neutrinos have the zero rest mass (the other version [6] will not be considered), so that the direction of their spin is connected hardly with the direction of their 3-velocity. In consequence of this only those neutrinos can be considered as ones forming the polarized flow, which propagate along straight line connecting specifically two particles of different macroscopic bodies. It explains the anisotropy of the zero quantum oscillations, which is necessary to obtain the right dependence ($1/R$) of the energy of the two-particle interaction on the distance R between particles in the Casimir effect.

Let's consider two macroscopic bodies with masses m_1 and m_2 and with the fairly long distance R one from another. We shall regard, that the bodies contain $2m_1l$ and $2m_2l$ particles correspondingly (where the normalizing factor l is connected with cross-section σ of the neutrino upon the particle), implying thereby the statistics averaging of the properties of the elementary particles constituting the bodies. If the particles of the macroscopic bodies had interacted with all neutrinos being incident on them then these particles might have been considered as the opaque boundaries, which induce Casimir effect on the straight line. By this the energy of the interaction of the particles would have been equal to [7]

$$\varepsilon_{AB} = \sum_{n=1}^{\infty} \frac{\pi n}{2R_{AB}} - \int_0^{\infty} \frac{\pi x}{2R_{AB}} dx = \frac{i}{2} \int_0^{\infty} \frac{\pi(it)/R_{AB} - \pi(-it)/R_{AB}}{\exp(2\pi t) - 1} dt = -\frac{\pi}{24R_{AB}}, \quad (2.1)$$

(A is a number of a particle of the first macroscopic body and B is a number of a particle of the second body). On account of the weakness of the interaction of neutrinos with particles we are confined to a first approximation, so that the energy E of the interaction of two macroscopic bodies is equal to

$$E \approx \sum_{A=1}^{2m_1 l} \sum_{B=1}^{2m_2 l} \varepsilon_{AB} \,. \quad (2.2)$$

Neglecting the dimensions of the bodies in comparison with interval R between them ($R_{AB} \approx R$), we shall have finally

$$E \approx -\frac{2m_1 l \cdot 2m_2 l \cdot \pi}{24R} = -\frac{G_\nu m_1 m_2}{R}, \quad (2.3)$$

where $G_\nu = \pi l^2/6$.

3. The estimate

Consider the scattering of the neutrino upon the charge lepton, induced by the exchange of the neutral Z° boson (taking account of the low energy of the relict neutrinos) only. The amplitude of the process in the lower approximation can be written down as

$$M = 4\frac{G_F}{\sqrt{2}}\left(\overline{\nu_L}\gamma^j \nu_L\right)\left[\left(-\frac{1}{2}+\xi\right)\overline{e_L}\gamma_j e_L + \xi \overline{e_R}\gamma_j e_R\right] =$$
$$= \frac{G_F}{\sqrt{2}}\left[\left(-\frac{1}{2}+\xi\right)\bar{e}\gamma_j (I-\gamma_5)e + \xi\bar{e}\gamma_j (I+\gamma_5)e\right]\left[\bar{\nu}\gamma^j (I-\gamma_5)\nu\right], \tag{3.1}$$

in consequence of this the square of the amplitude (spin-average) will take the form

$$\left\langle M^2\right\rangle = 64G_F^2\left[\left(-\frac{1}{2}+\xi\right)^2 (p'\cdot k')(p\cdot k) + \xi^2 (p'\cdot k)(p\cdot k') - \left(-\frac{1}{2}+\xi\right)\xi m^2 (k\cdot k')\right], \tag{3.2}$$

where γ_i, γ^i are the Dirac matrices, e is a bispinor describing of a charge lepton, p is its original of 4-momentum and p' is the finite 4-momentum of it (m is the rest mass of a charge lepton); ν is bispinor, describing of the neutrino, k is its original 4-momentum and k' is the finite 4-momentun of it; + is the symbol of the Hermitian conjugation; $G_F \approx 1.166 \cdot 10^{-5}$ GeV^{-2} is Fermi's constant. Here and further

$$\xi = \sin^2\Theta_W, \quad \gamma_5 = -i\gamma_1\gamma_2\gamma_3\gamma_4, \quad \psi_L = \frac{1}{2}(I-\gamma_5)\psi, \quad \psi_R = \frac{1}{2}(I+\gamma_5)\psi \tag{3.3}$$

(I is the unit matrix and Θ_W is the weak angle). By analogy we can get the square of the scattering amplitude of the antineutrino upon the charge lepton as

$$\left\langle M^2\right\rangle = 64G_F^2\left[\left(-\frac{1}{2}+\xi\right)^2 (p'\cdot k)(p\cdot k') + \xi^2 (p'\cdot k')(p\cdot k) - \left(-\frac{1}{2}+\xi\right)\xi m^2 (k\cdot k')\right] \tag{3.4}$$

As a result the cross-section of the scattering for the neutrino proves to be equals to the cross-section of the scattering for the antineutrino in the low-energy approximation (the energy of the neutrino $\omega \ll m$) and they are written down as

$$\sigma^Z = \frac{4G_F^2\omega^2}{\pi}\left[\left(-\frac{1}{2}+\xi\right)^2 + \xi^2 - \left(-\frac{1}{2}+\xi\right)\xi\right]. \tag{3.5}$$

Note that σ^Z proves to be minimal for $\xi = 1/4$. As the low energy neutrinos scarcely are able to change the spin direction of the particles of a macroscopic body, their scattering must be accompanied the collision radiation. In consequence of this the cross-section has the form $\sigma_\nu = k\sigma^Z$, where for the charge leptons the factor $k = k_e$ depends on the fine structure constant $\alpha \approx 1/137$ only, while for the quarks the factor $k = k_q$ must depends on the running coupling constant α_s too, which define the collision radiation by gluons.

For the crude estimate of the constant G_ν let us consider the scattering the relict neutrino upon the electron only, supposing that

$$\sigma_\nu = \pi l^2, \quad k_e = \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right), \tag{3.6}$$

Besides substituting the middling

$$\langle\omega\rangle = \frac{\int_0^\infty \frac{\omega^3 d\omega}{\exp(\omega/T)+1}}{\int_0^\infty \frac{\omega^2 d\omega}{\exp(\omega/T)+1}}, \tag{3.7}$$

instead of ω we receive the following value of the constant

$$G_\nu = \sigma_\nu / 6 \approx 10^{-38}\, GeV^{-2}, \tag{3.8}$$

($T \approx 1.9$ $K \approx 1.64 \cdot 10^{-13}$ GeV, $\langle\omega\rangle \approx 3.15$ $T \approx 5.166 \cdot 10^{-13}$ GeV, $\xi \approx 0.23$), which is near to the known value of the gravitational constant G_N [12].

4. Conclusion

So the gravitational phenomena can be explained by the presence of the collective oscillations in the neutrinos medium. In consequence it might be worthwhile to return to the potential

$$V(R) = \frac{A}{R} e^{-BR}$$

of which Seeliger [9] suggested to substitute the Newton potential and to note the gravitational potential (it is possible in an any approximation) as

$$V(R) = \frac{1}{R} \sum_{i=1}^{n} A_i \, e^{-B_i R}$$

in the general case where the constants A_i and B_i characterize the different media. By this we can be based on the theory of the strong gravity (see, for example the work [10]). Moreover, having the neutrinos Universe and taking account of the Fermi-Dirac statistics we can recollect about the Sakharov hypothesis [11] using the idea of the metrical vacuum elasticity for the explanation of the gravitational interactions. But the main idea is it now for us what the normal matter (not neutrinos) acts as the Brownians by the help of which it can make the attempt to estimate the statistics characterization of the Universe neutrinos background. In the capacity of one from such indicator we offer to use the particles masses, which are connecting with the scattering cross-section of the neutrinos. Note in tie with it, what we can ignore the photon collision radiation by the neutrinos scattering on the hadrons which's the quark resonator because of the existence of the additional degree of freedom in comparison with the electron. Exactly the resonance scattering causes to a gain in the hadrons masses by a factor of 10^3 in comparison with the electron mass.

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Lightlike limits of massive particles in general relativity as catastrophes

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Exterior gravitational fields of massive Schwarzschild's, Kerr's and NUT's particles have an algebraic type ***D.*** When a velocity of fast moving particles tend towards the light velocity along z axis and the total energy of each particle is constant (i.e. a rest mass of the particle tends towards zero) together with Kerr's angular momentum along z axis and with NUT's parameter which also tend to constants then the gravitational fields of these fast moving particles tend towards the wave's fields of the ***N*** and ***III*** algebraic types as their limits. The lightlike limit of massive particle can be described as cusp catastrophe on the level of Weyl's matrix with the change of gravitational field symmetry of such source. In considered cases this is the phase transition of the gravitational field from ***D*** type into ***N*** type or ***III*** type (transition of one "phase" to another). Petrov's algebraic types are different "phases" of gravitational field. As result of the lightlike limit procedure we can obtain the lightlike limit's particle as a scalar particle or as a spinning particle with a helicity. It is shown that the lightlike sources in General Relativity "no have hairs"

1. Introduction

In the classical electrodynamics a task of finding the field of electric charge when its velocity tends to the light velocity is well known [1]. The limiting field of such fast moving charge particle approaches to the field of monochromatic electromagnetic plane wave. The similar problem in General Relativity (GR) for Petrov's symmetric 6x6 curvature matrix of the gravitational field of a fast moving particle is considered in [2].

However a correct solution of two last problems is connected with use of generalized functions (δ-functions of Dirac) [3]. In this case the limit velocity V tends to the light's velocity c (here we have $V \to \tilde{n} = 1$), and the rest mass tends to zero ($m_0 \to 0$) so that a total relativistic energy of the particle is a constant, $E = const$. We shall call this procedure as ***a lightlike limit.*** We introduce Weyl's matrix as the Petrov symmetric 3×3 traceless complex matrix in 3D Euclidean space.

The similar limit on the level of Weyl's matrix eigenvalues we will call this procedure as ***a lightlike limit on the level of Weyl's matrices.*** Such lightlike limit can be described as the cusp catastrophe.

The aim of this paper is to summarize results of lightlike limits of the Schwarzschild, Kerr and NUT particles on the Weyl's matrices level and investigations connected with the theory of catastrophe (phase transitions of algebraic types of gravitational fields.

2. Lightlike limit of a massive particle as catastrophe

An exterior gravitational field of a rest massive particle is described by the Schwarzschild solution [4], which belongs to an algebraic type ***D***

$$\widehat{W}_D = \frac{m_0}{r_0^3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{2.1}$$

Here $\widehat{W}_D$ is a canonical traceless symmetric Weyl matrix of ***D*** type of Petrov's algebraic classification [5]; m_0 is a rest mass of the particle; r_0 is a radial variable. If the particle velocity tends to the light velocity along *z*-axis when a total relativistic energy of the particle is constant (i.e.

$m_0 \to 0$), then a limiting field will be described as the gravitational plane wave of Petrov's ***N*** type with a singular source. Such procedure is ***the lightlike limit on the Weyl matrix level.***

The lightlike limit on the metric level for the Schwarzschild particle in the isotropic coordinates was considered in [6] and in the Kerr-Schild coordinates in [3,7] We will investigate here the lightlike limit on the Weyl matrix level for the Schwarzschild-like particle [3,8]. In this case the orthogonal transformation with a matrix

$$\widehat{T} = \begin{pmatrix} \cosh\psi & -i\sinh\psi & 0 \\ i\sinh\psi & \cosh\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.2}$$

must be used under condition $V \to 1$. The matrix $\widehat{T}$ has property $\widehat{T}^{-1}\widehat{T} = \widetilde{\widehat{T}}\widehat{T} = \widehat{T}\widehat{T}^{-1} = \widehat{T}\widetilde{\widehat{T}} = 1$ with a transpose matrix $\widetilde{\widehat{T}} = \widehat{T}^{-1}$; $i^2 = -1$; $\cosh\psi = (1-V^2)^{-1/2}$ and $\sinh\psi = V(1-V^2)^{-1/2}$. We take lightlike limit under condition $V \to 1$ and correspondingly $\psi \to \infty$.

At the resting frame of reference for the gravitational field of massive particle on the level of Weyl's matrix we can write

$$\widehat{W} = \widehat{T}\widehat{W}_D\widehat{T}^{-1} = \frac{E\varepsilon^2}{R^3}\widehat{W}(\varepsilon), \tag{2.3}$$

where $\varepsilon = (1-V^2)^{-1/2}$; $E = \dfrac{m_0}{\varepsilon}$ is a total particle energy, $E = const$; $R^2 = \rho^2\varepsilon^2 + (z+Vt)^2$; $\rho^2 = x^2 + y^2$. Weyl's matrix $\widehat{W}(\varepsilon)$ is

$$\widehat{W}(\varepsilon) = \varepsilon^2\widehat{C}_D + 3(\widehat{W}_N^{(E)} + iV\cdot\widehat{W}_N^{(B)}), \tag{2.4}$$

$\widehat{C}_D$ is Weyl's matrix of ***D*** type,

$$\widehat{C}_D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \tag{2.5}$$

"an electric type matrix" $\widehat{W}_N^{(E)}$ and *"an magnetic type matrix"* $\widehat{W}_N^{(B)}$ are two parts of ***N*** type Weyl's matrix

$$\widehat{W}_N = \widehat{W}_N^{(E)} + i\widehat{W}_N^{(B)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + i\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2.6}$$

In the electrodynamics there are an similar situation in the transition from laboratory inertial frame of reference with an electric field to any other inertial frame of reference, $\vec{F} = \vec{E} + i\vec{B}$, where $\vec{E}$ is a strength of the electric field and $\vec{B}$ is a strength of the magnetic field.

The Weyl's matrix of ***N*** type (the gravitational wave type) can be rewritten as the skeleton's representation

$$\widehat{W}_N = l\,\widetilde{l}, \tag{2.7}$$

where $\widetilde{l} = (1, \quad i, \quad 0)$ is a lightlike vector-row, $\widetilde{l}\,l = 0$.

In [3] it is shown that under condition $V \to 1$ the factor

$$\frac{E\varepsilon^2}{R^3} \quad \to \quad \frac{2E}{\rho^2}\delta(z+t), \tag{2.8}$$

where $\delta(z+t)$ is the δ-function of Dirac.

Then we find for lightlike limit of Weyl's matrix

$$\widehat{W}(\varepsilon) \;\; \rightarrow \;\; 3\widehat{W}_N , \tag{2.9}$$

i.e. this limit gives Weyl's matrix of the gravitational plane wave.

From the point of view of a catastrophe theory [9] such lightlike limit is the catastrophe. To discover this fact we put an eigenvalue problem for Weyl's matrix

$$\widehat{W}\, X = \lambda\, X , \tag{2.10}$$

where X is a vector- column and λ is a eigenvalue.

The characteristic equation is written as

$$\det(\widehat{W} - \lambda\,\widehat{I}) \propto \lambda^3 + p\lambda + q = 0 , \tag{2.11}$$

where $\widehat{I} = diag(1,1,1)$ is an identity matrix; $p = -3\varepsilon^4$ and $q = -2\varepsilon^6$.

The equation (2.11) can be considered as the extremum equation of a "potential function"

$$U(\lambda, p, q) = \frac{\lambda^4}{4} + \frac{p\lambda^2}{2} + q\lambda , \tag{2.12}$$

which describes the cusp catastrophe.

On Fig.1 a surface of the cusp catastrophe and its projection onto the plane of control parameters p and q can be seen. The cusp point ($p = q = 0$) is point of the second kind's phase transition. In our case this is the phase transition of the gravitational field from ***D*** type into ***N*** type (transition of one "phase" to another). Petrov's algebraic types are different "phases" of gravitational field. The parameter p plays a role of the temperature here, the derivative $\partial U / \partial p$ plays the role of an entropy and $\partial^2 U / \partial p^2$ corresponds to a thermal capacity [7,8].

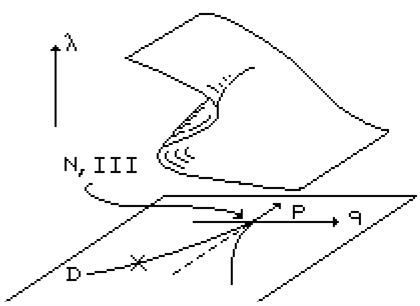

Fig.1 The cusp catastrophe's surface and its projection onto the plane of control parameters *p* and *q*.

The discriminant of the equation (2.11) is $Q = (p/3)^3 + (q/2)^2$ When $Q = 0$ we have semicubical parabola $p = -3(q/2)^{2/3}$ respective to Weyl's matrix of ***D*** type (see Fig.1). Our case is marked by a cross ($q < 0$)

The equation (2.11) has three roots: $\lambda_1 = \lambda_3 = -\lambda_2 / 2 = -\varepsilon^2$. Then the potential function (2.12) has following values: $U(\lambda_1) = U(\lambda_3) = p^2/12$ and $U(\lambda_2) = -2p^2/3$. In the point of cusp can be seen jumps of second derivatives $\Delta(\partial^2 U / \partial p^2) = 1/6$ and $\Delta(\partial^2 U / \partial p^2) = -4/3$ which are accompanied by the jump of the Weyl matrix's rank from $r = 3$ (***D*** type) to $r = 1$ (***N*** type), where Δ is the jump of the second derivative's value of the potential function U.

The Weyl matrix (2.3) has two eigenvectors. The first eigenvector X_1 corresponds to eigenvalue λ_2 and under the lightlike limit operation $(\varepsilon \rightarrow 0, V \rightarrow 1)$ tends to lightlike eigenvector $L = il$: $\widetilde{X}_1 = (-i/(1-\varepsilon^2)^{-1/2},\ 1,\ 0) \rightarrow \widetilde{L}$, where

$$\widehat{W}_N L = 0 . \tag{2.13}$$

The second eigenvector $\widetilde{X}_2 = (0,\ 0,\ 1)$ is also the eigenvector of Weyl's matrix of ***N*** type, $\widehat{W}_N X_2 = 0$. So the lightlike limit gives the Weyl matrix of ***N*** type and this limit corresponds to the second phase transition (Fig.1).

We have a change of symmetry of gravitational equations' solutions under such lightlike limit. The Schwarzschild-like's limiting metric as result of the lightlike limit can be written [3,7-8]

$$g_{\mu\nu} = \delta_{\mu\nu} - 8Hl_\mu l_\nu, \tag{2.14}$$

where $\mu, \nu = 0,1,2,3$; $\delta_{\mu\nu} = diag(1,-1,-1,-1)$; $H = -2E\delta(z+t)\ln(x^2+y^2)$; $l_\mu = \delta_\mu^0 + \delta_\mu^3$; $l_\mu l^\mu = 0$.

The metric (2.14) describes a singular source and is an exact solution of Einstein's exact equations

$$\Box g_{\mu\nu} = -8\pi T_{\mu\nu}, \tag{2.15}$$

where $\Box$ is D'Alamber's operator in Minkowski space-time, $T_{\mu\nu} = 2E\delta(z+t)\delta(x)\delta(y)l_\mu l_\nu$ is a lightlike radiation's singular energy-momentum tensor.

The metric (2.15) has two Killing's vectors: *lightlike vector* $\xi_L = (\partial/\partial t + \partial/\partial z)$ and *spacelike vector* $\xi_Z = (x\,\partial/\partial y - y\,\partial/\partial x)$, which defines the axial symmetry and in polar coordinates it equals to $\partial/\partial\varphi$.

The Schwarschild-like solution has four Killing's vectors: *timelike vector* $\xi_T = \partial/\partial t$ and three *spacelike vectors*: $\xi_X = (y\,\partial/\partial z - z\,\partial/\partial y)$; $\xi_Y = (z\,\partial/\partial x - x\,\partial/\partial z)$; $\xi_Z = (x\,\partial/\partial y - y\,\partial/\partial x)$. Further the Lorentz boost is applied to Killing's vectors of Schwarzschild's solution and when it is tended towards velocity of light then vector ξ_Z will be invariable and vectors $\widehat{L}\xi_T$, $\widehat{L}\xi_X$, $\widehat{L}\xi_Y$ will be degenerated into lightlike vector.

Therefore the lightlike limit of massive particle can be described as cusp catastrophe on the level of Weyl's matrix with the change of gravitational field symmetry of such source. In this case we have a scalar lightlike particle.

3. Lightlike limit of a massive NUT particle as catastrophe

An exterior gravitational field of a massive particle describing by the NUT solution [10-11] has an algebraic type ***D***. Under the lightlike limit when the particle velocity tends to the light velocity along z axis, the total energy of the particle is constant (i.e. $m_0 \to 0$) and NUT's parameter $b \to B = const$ then the limiting field will be the gravitational wave of ***N*** type without the NUT parameter [12,13].

The NUT metric can be written in both forms NUT and Misner [10-11]. If we will take the NUT solution in these forms for NUT's any parameter we find nonzero components of curvature tensor: $R_{11} = -2R_{22} = -2R_{33} = -R_{44} = 2R_{55} = 2R_{66} = -2\alpha(r)$; $R_{14} = -2R_{25} = -2R_{36} = -2\beta(r)$, where Petrov's map for indexes is used and

$$\alpha(r) = \frac{b^4 + 3b^2 m_0 r - 3b^2 r^2 - m_0 r^3}{(b^2+r^2)^3}; \tag{3.1}$$

$$\beta(r) = \frac{b(b^2 m_0 - 3b^2 r - 3m_0 r^2 + r^3)}{(b^2+r^2)^3}. \tag{3.2}$$

Weyl's matrix in this case is

$$\widehat{W}_{NUT} = (\alpha + i\beta)\,\widehat{W}_D = -\frac{m_0 - ib}{(r-ib)^3}\,\widehat{W}_D, \tag{3.3}$$

where the canonical the Weyl matrix of ***D*** type is

$$\widehat{W}_D = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.4}$$

At the resting frame of reference for the gravitational field of NUT particle on the level of Weyl's matrix we can write

$$\widehat{W} = \widehat{T}\,\widehat{W}_{NUT}\,\widehat{T}^{-1}. \tag{3.5}$$

An applying of the lightlike limit's procedure to Weyl's matrix of NUT solution leads to the limit of matrix (3.5)

$$\widehat{W} \to \frac{6(E-iB)}{\rho^2}\delta(z+t)\begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{6(E-iB)}{\rho^2}\delta(z+t)\,\widehat{W}_N. \tag{3.6}$$

Thus the limiting Weyl's matrix for NUT solution is the matrix of ***N*** type. Now we can eliminate the NUT parameter B by means of rotation in complex plane with $\tan\varphi = -B/E$ and with a new choice of parameter E as $\sqrt{E^2+B^2} \to E$.

As in the case with Schwarzschild's particle here we also have the lightlike limit of Weyl's matrix is a catastrophe from the point of view of the eigenvalue problem. The cusp point ($p = q = 0$) is the point the phase transition of NUT's gravitational field from ***D*** type into ***N*** (Fig.1). Also under such limiting procedure can be seen the change of the symmetry of gravitational fields as degeneration of Killing's vectors into the Killing lightlike vectors.

Therefore the lightlike limit of massive the NUT particle can be described as cusp catastrophe on Weyl's matrix level with the change of gravitational field symmetry of such source and loss of the NUT parameter. So and in this case we have a scalar lightlike particle.

4. Lightlike limit of a massive Kerr particle as catastrophe

An exterior gravitational field of a spinning massive particle is described by the Kerr solution [14], which has an algebraic type ***D***. If the particle velocity tends to the light velocity along z axis when the total energy *E* of the particle is const (i.e. the rest mass $m_0 \to 0$) and Kerr's angular momentum along *z* axis $L_Z = a\,m_0 \to J\cdot E$ then the limiting gravitational field will be by the gravitational wave of ***III*** type with the spinning singular source [3,7,15]. Here $a = L_Z/m_0$ is Kerr's relative angular momentum, J is the limit of Kerr's relative angular momentum under the lightlike limit and $J = const$.

If we will write the Kerr solution in Boyer-Lindquist coordinates we can find all nonzero components of a curvature tensor in a slow spinning approximation (i.e. $a = L_Z/m_0$ is a small) with the using of Petrov's map for indexes of the curvature tensor : $R_{11} = -2R_{22} = -2R_{33} = 2m_0/r^3$; $R_{15} = R_{24} = 2R_{26} = 2R_{35} = (6\,a\,m_0/r^4)\sin\theta$; $R_{14} = -2R_{25} = -2R_{36} = (6\,a\,m_0/r^4)\cos\theta$. The Weyl matrix in this case is

$$\widehat{W}_{Kerr} = \begin{pmatrix} R_{11}+iR_{14} & iR & 0 \\ iR_{15} & -\frac{1}{2}(R_{11}+iR_{14}) & \frac{1}{2}iR_{15} \\ 0 & \frac{1}{2}iR_{15} & -\frac{1}{2}(R_{11}+iR_{14}) \end{pmatrix}. \tag{4.1}$$

For the gravitational field of Kerr's fast moving particle at the resting frame of reference the Weyl matrix can be written as

$$\widehat{W} = \widehat{T}\,\widehat{W}_{Kerr}\,\widehat{T}^{-1}, \tag{4.2}$$

where the matrix $\widehat{T}$ is defined by equation (2.2).

The lightlike limit's procedure is applied to Weyl's matrix of Kerr's solution and leads to the limit of matrix (4.2)

$$\widehat{W} \to \frac{3}{\rho^2}\delta(z+t)\left(2E\begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{\pi J}{2\rho}\sin\theta\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & i \\ -1 & i & 0 \end{pmatrix}\right). \tag{4.3}$$

We can transform this matrix by elementary transformations into Weyl's matrix of the ***III*** algebraic type (a wave type). On the other hand the matrix (4.3) is a superposition of two matrices: ***N*** and ***III*** algebraic types and we immediately have resulting ***III*** algebraic type (see [17]).

As in the case with Schwarzschild's and NUT's particles we have the lightlike limit of Weyl's matrix of Kerr's solution is the catastrophe from the point of view of the eigenvalue problem. The cusp point ($p = q = 0$) is the point the phase transition of Kerr's gravitational field from ***D*** type into ***III*** type (Fig.1).

A lightlike limiting metric which corresponds to the limiting matrix (4.3) has two Killing's vectors only: *lightlike vector* $\xi_L = (\partial/\partial t + \partial/\partial z)$ and *an axial spacelike vector* $\xi_Z = \partial/\partial\varphi$ in polar coordinates. The Kerr solution has two Killing vectors: *timelike vector* $\xi_T = \partial/\partial t$ and *an axial spacelike vector* $\xi_Z = \partial/\partial\varphi$.

The Lorentz boost is applied to Killing's vectors of Kerr's solution together with the lightlike procedure and leads to $\xi_Z \to \xi_Z$; $\xi_Z \to \xi_L$.

Therefore the lightlike limit of massive Kerr's particle can be described as cusp catastrophe on Weyl's matrix level with the change of gravitational field symmetry of such source.

In the case of Kerr's particle with the angular momentum along z axis we have a spinning lightlike particle with a helicity $L_Z = \pm JE$.

When Kerr's relative angular momentum $a = L_Z / m_0$ is perpendicular to z axis then the lightlike limit procedure leads to loss of limiting value of the Kerr's relative angular momentum J . In this case under the lightlike limit we have the same result as for the Schwarzshcil's solution and the lightlike limit's particle is the scalar particle.

Thus the describing lightlike procedure for Schwarzschild's, NUT's and Kerr's solutions leads to new lightlike particle with only two freedom parameters: the total energy and the helicity. The other physical parameters are lost under such limiting process. So we can say that the lightlike sources in General Relativity "no have hairs" [7,16].

5. On lightlike pencil

As well known a focusing effect of lightlike geodesic lines is absent if all lightlike particles run in the same direction. In this case we can construct the lightlike pencil as a superposition such singular sources. This superposition is an exact solution of the linear gravitational equations [3,7,16]. Thus for monochromatic lightlike particles with the same total energy E and helicity L_Z we will have metric in cylindrical coordinates for an infinite lightlike pencil as

$$ds^2 = dudv - d\rho^2 - \rho^2 d\varphi^2 - 2Hdv^2 - 8L_Z dvd\varphi, \tag{5.1}$$

where $u = t - z,\ v = t + z,\vdots\ H = -4E\ln(\alpha\rho),\quad L_Z = JE$.

This metric is reduced to

$$ds^2 = dudv - d\rho^2 - \rho^2 d\varphi^2 - 2Hdv^2 \tag{5.2}$$

by the coordinate transformation $u \to u - 8L_Z\varphi$. Therefore, the infinite monochromatic lightlike pencil has not of a constant spin (helicity).

However, if we have the monochromatic lightlike ray with an endpoint which not equal the infinity then such ray may be have the helicity for

$$H = -4E \cdot \Theta(v)\ln(\beta\rho) \tag{5.3}$$

with a step function

$$\Theta(v) = \Theta(t+z) = \int_{-\infty}^{t+z} \delta(\tau)d\tau \text{ ,(5.4)}$$

where $\delta(v)$ is Dirac's singular function.

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Einstein equations for a remarkable generalized Lagrange space $GL^{2(n)}(M, g_{ij}(x, y^{(1)}, y^{(2)})$

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The generalized Lagrange space $GL^{2(n)}$ provides a convenient relativistic model. The purpose of this paper is to study the Einstein equations in the case of $g_{ij}(x, y^{(1)}, y^{(2)}) = e^{2\sigma(x, y^{(1)}, y^{(2)})}\gamma_{ij}(x)$, giving the complete calculation for a remarkable metric tensor; the connection with the classical Einstein equations of the Riemannian space is also inferred.

On the base manifold M the metric tensor $g_{ij}(x) = e^{2\sigma(x)}\gamma_{ij}(x)$ was introduced by Watanabe S., Ikeda S., and Ikeda F. **[WII]**. The Einstein and the Maxwell equations for the first order generalized Lagrange space $GL^{(n)}(M, g_{ij}(x, y^{(1)}) = e^{2\sigma(x, y^{(1)})}\gamma_{ij}(x))$ were studied by R.Miron and R.Tavakol **[MAN]**. Generalized Einstein-Yang Mills equations for the same space were studied by V. Balan **[B].**

1. The coefficients of the canonical metrical N-linear connection

Let *M* be a *n* dimensional C^∞ differentiable manifold endowed with the metric tensor

$$g_{ij}(x, y^{(1)}, y^{(2)}) = e^{2\sigma(x, y^{(1)}, y^{(2)})}\gamma_{ij}(x) \tag{1.1}$$

defined on Osc^2M where $\sigma \in F(Osc^2 M)$ is a given function and $\gamma_{ij}(x)$ is a Riemannian metric tensor field. Consider also N the canonical nonlinear connection **[M]** with the coefficients:

$$\begin{cases} \underset{(1)}{N}{}^i{}_j = \gamma^i_{jk} y^{(1)k} \\ \underset{(2)}{N}{}^i{}_j = \dfrac{1}{2}\left(\dfrac{\partial \gamma^i_{kj}}{\partial x} - \gamma^i_{rh}\gamma^h_{kj}\right) y^{(1)k} y^{(1)r} + \gamma^i_{kj} y^{(2)k} \end{cases} \tag{1.2}$$

We know that on the total space *E* exists an unique *N*-linear connection depending only on the Lagrangian *L* which satisfies Matsumoto's axioms

$$g_{ij|m} = 0; \quad g_{ij} \overset{(A)}{|}_m = 0; \quad T^i{}_{jk} = 0; \quad \underset{(A)}{S}{}^m{}_{ij} = 0 \quad ; \ A = 1,2 \tag{1.3}$$

This is the *N*-linear canonical metrical connection *CΓ(N)* and it has the coefficients given by:

$$L^i{}_{jk} = \frac{1}{2} g^{is}\left(\frac{\delta g_{ks}}{\delta x^j} + \frac{\delta g_{sj}}{\delta x^k} + \frac{\delta g_{jk}}{\delta x^s}\right) \tag{1.4}$$

$$\underset{(A)}{C}{}^i{}_{jk} = \frac{1}{2} g^{is}\left(\frac{\delta g_{ks}}{\delta y^{(A)j}} + \frac{\delta g_{sj}}{\delta y^{(A)k}} + \frac{\delta g_{jk}}{\delta y^{(A)s}}\right) \qquad A = 1,2$$

Proposition 1.1 *For the considered metric the coefficients of CΓ(N) are:*

$$L^i{}_{jk} = \gamma^i_{jk} + \underset{0}{\Lambda}{}^i{}_{jk} \ ; \quad \underset{(A)}{C}{}^i{}_{jk} = \underset{A}{\Lambda}{}^i{}_{jk} \quad ; \quad A = 1,2 \tag{1.5}$$

where $\quad \underset{B}{\Lambda}{}^i{}_{jk} = \delta^i_k \overset{B}{\sigma}_j + \delta^i_j \overset{B}{\sigma}_k - \gamma_{jk} \overset{B}{\sigma}{}^i \qquad B = 0,1,2$

and $\quad \overset{B}{\sigma}_j = \dfrac{\delta\sigma}{\delta y^{(B)j}} \quad ; \quad \overset{B}{\sigma}{}^i = \gamma^{is} \overset{B}{\sigma}_s \quad ; \quad y^{(0)} = x \quad ; \quad \delta^i_j$ *is the Kronecker symbol.*

Corollary 1.1 $\underset{B}{\Lambda}{}^i{}_{jk} = 0$ **,** $B = 0,1,2$ *if and only if σ is constant.*

2. Torsions and curvatures

Let **T** be the tensor of torsion of an *N*-linear connection *D*. For any vector fields $X, Y \in \chi(E)$ we have:

$$\mathbf{T}(X,Y)=D_X Y-D_Y X-[X,Y] \tag{2.1}$$

This tensor can be evaluated for the pairs of tensor fields $\left(X^H, Y^H\right)$, $\left(X^H, Y^{V_A}\right)$, $\left(Y^{V_A}, Y^{V_B}\right)$, A,B=1,2 and for the canonical metrical *N*-linear connection *CΓ(N)*. By direct calculation we obtain:

Theorem 2.1 *The d-tensor fields of torsion for CΓ(N) has the local components given by:*

$$T^i{}_{jk}=0\ ;\quad \underset{(0)}{\overset{1}{T}}{}^i{}_{jk}=r_m{}^i{}_{jk}y^{(1)m}\ ;\quad \underset{(0)}{\overset{2}{T}}{}^i{}_{jk}=\frac{1}{2}\left(r_q{}^i{}_{mj}\gamma^m_{kp}-r_q{}^i{}_{mk}\gamma^m_{jp}\right)y^{(1)q}y^{(1)p}+r_m{}^i{}_{jk}y^{(1)m}$$

$$\underset{(A)}{\overset{0}{P}}{}^i{}_{jk}=\underset{A}{\Lambda}{}^i{}_{jk}\quad (A=1,2)\ ;\quad \underset{(1)}{\overset{1}{P}}{}^i{}_{jk}=-\underset{0}{\Lambda}{}^i{}_{jk}\ ;\quad \underset{(1)}{\overset{2}{P}}{}^m{}_{ij}=\gamma^m_{kr}\gamma^r_{ij}y^{(1)k}-\gamma^m_{ij} \tag{2.2}$$

$$\underset{(2)}{\overset{1}{P}}{}^i{}_{jk}=0\ ;\quad \underset{(2)}{\overset{2}{P}}{}^i{}_{jk}=-\underset{0}{\Lambda}{}^i{}_{jk}\ ;\quad \underset{(12)}{\overset{0}{P}}{}^i{}_{jk}=0\ ;\quad \underset{(12)}{\overset{1}{P}}{}^i{}_{jk}=\underset{1}{\Lambda}{}^i{}_{jk}\ ;\quad \underset{(12)}{\overset{2}{P}}{}^i{}_{jk}=\underset{2}{\Lambda}{}^i{}_{jk}$$

$$\underset{A}{\overset{B}{S}}{}^i{}_{jk}=0\quad A=1,2\,;B=0,1,2$$

Theorem 2.2 *The d-tensor fields of curvature of CΓ(N) are locally expressed in the shape:*

$$R_b{}^a{}_{pq}=r_b{}^a{}_{pq}+\left(\delta^a_p\overset{0}{\sigma}{}_{bq}-\delta^a_q\overset{0}{\sigma}{}_{bp}\right)+\left(\gamma^{as}\gamma_{bq}\overset{0}{\sigma}{}_{sp}-\gamma^{as}\gamma_{bp}\overset{0}{\sigma}{}_{sq}\right)+\delta^a_b\left(\overset{0}{\sigma}{}_{pq}-\overset{0}{\sigma}{}_{qp}\right)+$$
$$+\gamma_{bt}\left(r_s{}^a{}_{pq}\overset{1}{\sigma}{}^t-r_s{}^t{}_{pq}\overset{1}{\sigma}{}^a\right)y^{(1)s}++\gamma_{bt}\left(r_s{}^a{}_{pq}\overset{2}{\sigma}{}^t-r_s{}^t{}_{pq}\overset{2}{\sigma}{}^a\right)y^{(2)s}\,; \tag{2.3}$$

$$\underset{(1)}{P}{}_b{}^a{}_{pq}=\left(\delta^a_p\overset{01}{\sigma}{}_{bq}-\delta^a_q\overset{10}{\sigma}{}_{bp}\right)-\gamma^{as}\left(\gamma_{bp}\overset{01}{\sigma}{}_{sq}-\gamma_{bq}\overset{10}{\sigma}{}_{sp}\right)+\gamma^{as}\left(\gamma_{bp}\overset{0}{\sigma}{}_q\overset{1}{\sigma}{}_s-\gamma_{bq}\overset{0}{\sigma}{}_s\overset{1}{\sigma}{}_p\right)+$$
$$+\delta^a_b\left(\overset{01}{\sigma}{}_{pq}-\overset{10}{\sigma}{}_{qp}\right)-2\gamma^{as}\gamma_{pq}\left(\overset{0}{\sigma}{}_b\overset{1}{\sigma}{}_s-\overset{0}{\sigma}{}_s\overset{1}{\sigma}{}_b\right)-\delta^a_p\overset{0}{\sigma}{}_q\overset{1}{\sigma}{}_b-\delta^a_q\overset{0}{\sigma}{}_p\overset{1}{\sigma}{}_b\,; \tag{2.4}$$

$$\underset{(2)}{P}{}_b{}^a{}_{pq}=\left(\delta^a_p\overset{02}{\sigma}{}_{bq}-\delta^a_q\overset{20}{\sigma}{}_{bp}\right)-\gamma^{as}\left(\gamma_{bp}\overset{02}{\sigma}{}_{sq}-\gamma_{bq}\overset{20}{\sigma}{}_{sp}\right)+\gamma^{as}\left(\gamma_{bp}\overset{0}{\sigma}{}_q\overset{2}{\sigma}{}_s-\gamma_{bq}\overset{0}{\sigma}{}_s\overset{2}{\sigma}{}_p\right)+$$
$$+\delta^a_b\left(\overset{02}{\sigma}{}_{pq}-\overset{20}{\sigma}{}_{qp}\right)-2\gamma^{as}\gamma_{pq}\left(\overset{0}{\sigma}{}_b\overset{2}{\sigma}{}_s-\overset{0}{\sigma}{}_s\overset{2}{\sigma}{}_b\right)-\delta^a_p\overset{0}{\sigma}{}_q\overset{2}{\sigma}{}_b-\delta^a_q\overset{0}{\sigma}{}_p\overset{2}{\sigma}{}_b\,; \tag{2.5}$$

$$\underset{(12)}{P}{}_b{}^a{}_{pq}=\left(\delta^a_p\overset{12}{\sigma}{}_{bq}-\delta^a_q\overset{21}{\sigma}{}_{bp}\right)-\gamma^{as}\left(\gamma_{bp}\overset{12}{\sigma}{}_{bq}-\gamma_{bq}\overset{21}{\sigma}{}_{sp}\right)+\gamma^{as}\left(\gamma_{bp}\overset{1}{\sigma}{}_q\overset{2}{\sigma}{}_s-\gamma_{bq}\overset{1}{\sigma}{}_s\overset{2}{\sigma}{}_p\right)+$$
$$+\delta^a_b\left(\overset{12}{\sigma}{}_{pq}-\overset{21}{\sigma}{}_{qp}\right); \tag{2.6}$$

$$\underset{(1)}{S}{}_b{}^a{}_{pq}=\delta^a_p\overset{1}{\sigma}{}_{bq}-\delta^a_q\overset{1}{\sigma}{}_{bp}-\gamma^{as}\left(\gamma_{bp}\overset{1}{\sigma}{}_{sq}-\gamma_{bq}\overset{1}{\sigma}{}_{sp}\right)+\delta^a_b\left(\overset{12}{\sigma}{}_{pt}\gamma^t_{lq}-\overset{12}{\sigma}{}_{qt}\gamma^t_{lp}\right)y^{(1)l}\,; \tag{2.7}$$

$$\underset{(2)}{S}{}_b{}^a{}_{pq}=\delta^a_p\overset{2}{\sigma}{}_{bq}-\delta^a_q\overset{2}{\sigma}{}_{bp}-\gamma^{as}\left(\gamma_{bp}\overset{2}{\sigma}{}_{sq}-\gamma_{bq}\overset{2}{\sigma}{}_{sp}\right); \tag{2.8}$$

where

$$\overset{A}{\sigma}{}_{cd}=\left(\frac{\delta\overset{A}{\sigma}{}_c}{\delta y^{(A)d}}-\overset{A}{\sigma}{}_c\overset{A}{\sigma}{}_d+\frac{1}{2}\gamma_{cd}\overset{A}{\sigma}{}^t\overset{A}{\sigma}{}_t\right)\quad (A=1,2)$$

and

$$\overset{AB}{\sigma}{}_{cd}=\left(\frac{\delta\overset{A}{\sigma}{}_c}{\delta y^{(B)d}}-\overset{B}{\sigma}{}_c\overset{A}{\sigma}{}_d+\frac{1}{2}\gamma_{cd}\overset{B}{\sigma}{}^t\overset{A}{\sigma}{}_t\right)\quad (A,B=1,2\ ;\ A\neq B).$$

By a straightforward computation we obtain the Ricci tensors and the scalars of curvature and, by consequence, we can write the Einstein equations.

Theorem 2.3 *With respect to the canonical metrical N-linear connection the Einstein equations for the space $GL^{2(n)}$ endowed with the metric tensor $g_{ij}(x, y^{(1)}, y^{(2)}) = e^{2\sigma(x,y^{(1)},y^{(2)})}\gamma_{ij}(x)$ are given by:*

$$R_{bp} - \frac{1}{2}\gamma_{bp}R = \chi \overset{0}{\underset{0}{T}}{}_{bp} \ ; \ \underset{(1)}{S}{}_{bp} - \frac{1}{2}\gamma_{bp}R = \chi \overset{1}{\underset{0}{T}}{}_{bp} \ ; \ \underset{(2)}{S}{}_{bp} - \frac{1}{2}\gamma_{bp}R = \chi \overset{2}{\underset{0}{T}}{}_{bp} \ ;$$

$$\overset{1}{\underset{(A)}{P}}{}_{bp} = \chi \overset{01}{\underset{(A)}{T}}{}_{bp} \ ; \ \overset{2}{\underset{(A)}{P}}{}_{bp} = \chi \overset{02}{\underset{(A)}{T}}{}_{bp} \ ; \ (A=1,2) \ ; \ \overset{1}{\underset{(12)}{P}}{}_{bp} = \chi \overset{12}{\underset{(1)}{T}}{}_{bp} \ ; \ \overset{2}{\underset{(12)}{P}}{}_{bp} = \chi \overset{12}{\underset{(2)}{T}}{}_{bp} \qquad (2.9)$$

Theorem 2.4 *We have the following conservation law:*

$$\left(R^b_p - \frac{1}{2}R\delta^b_p\right)_{|b} + \overset{1}{\underset{1}{P}}{}^b_p \overset{1}{|}_b + \overset{1}{\underset{2}{P}}{}^b_p \overset{2}{|}_b + \overset{2}{\underset{1}{P}}{}^b_p \overset{1}{|}_b + \overset{2}{\underset{2}{P}}{}^b_p \overset{2}{|}_b = 0 \ ;$$

$$\left(\underset{(1)}{S}{}^b_p - \frac{1}{2}R\delta^b_p\right)\overset{(1)}{|}_b - \overset{2}{\underset{(1)}{P}}{}^b_{p\,|b} + \overset{2}{\underset{(12)}{P}}{}^b_p \overset{(2)}{|}_b = 0 \ ; \qquad (2.10)$$

$$\left(\underset{(2)}{S}{}^b_p - \frac{1}{2}R\delta^b_p\right)\overset{(2)}{|}_b - \overset{2}{\underset{(2)}{P}}{}^b_{p\,|b} + \overset{2}{\underset{(12)}{P}}{}^b_p \overset{(1)}{|}_b = 0 \ .$$

Corollary 2.1 *If the nonlinear connection N satisfy the integrability conditions then the conservation law is:*

$$R^b_{p|b} - \frac{1}{2}R_{|p} = 0 \ ; \quad \underset{(A)}{S}{}^b_p \overset{(A)}{|}_b - \frac{1}{2}R \overset{(A)}{|}_p = 0 \quad ; \ (A=1,2). \qquad (2.11)$$

3 Applications

Let us consider the Liouville vector fields $z^{(1)}, z^{(2)}$ given by:

$$z^{(1)i} = y^{(1)i}, \ z^{(2)i} = y^{(2)i} + \frac{1}{2}\underset{(1)}{M}{}^i{}_j y^{(1)j} . \qquad (3.1)$$

With respect to the canonical nonlinear connection *N*, these fields depend only on the Riemannian metric $\gamma_{ij}(x)$ and are defined by:

$$z^{(1)i} = y^{(1)i}, \ z^{(2)i} = y^{(2)i} + \frac{1}{2}\gamma^i_{jk}y^{(1)j}y^{(1)k} . \qquad (3.2)$$

We present the complete calculation for the case of :

$$\sigma\left(x, y^{(1)}, y^{(2)}\right) = \frac{1}{2}\left\|y^{(1)}\right\|^2 = \frac{1}{2}\gamma_{ij}z^{(1)i}z^{(1)j} . \qquad (3.3)$$

Proposition 3.1 *The following results hold:*

$$\overset{0}{\sigma}_k = 0 \ ; \quad \overset{1}{\sigma}_k = z^{(1)}_k \ ; \quad \overset{2}{\sigma}_k = 0 \quad ;$$

$$L^i{}_{jk} = \gamma^i_{jk} \ ; \quad \underset{(1)}{C}{}^i{}_{jk} = \delta^i_k z^{(1)}_j + \delta^i_j z^{(1)}_k - \gamma_{jk}z^{(1)i} \ ; \quad \underset{(2)}{C}{}^i{}_{jk} = 0 \quad ;$$

$$\overset{0}{\sigma}_{ij} = 0 \ ; \quad \overset{01}{\sigma}_{ij} = 0 \ ; \quad \overset{02}{\sigma}_{ij} = 0 \ ; \quad \overset{0}{\sigma}{}^t \overset{A}{\sigma}_t = 0 \ ; \quad \overset{0}{\sigma}_t \overset{A}{\sigma}{}^t = 0 \qquad (A\text{-}0,1,2); \qquad (3.4)$$

$$\overset{1}{\sigma}_{ij} = \delta_{ij} + \frac{1}{2}\gamma_{ij}\left\|z^{(1)}\right\|^2 - z^{(1)}_i z^{(1)}_j \ ; \quad \overset{1}{\sigma}{}^t \overset{1}{\sigma}_t = \left\|z^{(1)}\right\|^2 \ ; \quad \overset{1}{\sigma}_t \overset{A}{\sigma}{}^t = 0 \quad (A=0,1,2) \ ;$$

$$\overset{1A}{\sigma}_{ij} = 0 \ ; \quad (A = 1,2) \qquad \overset{2}{\sigma}_{ij} = 0 \quad .$$

Proposition 3.2 *The torsion d-tensor fields are:*

$$\overset{A}{\underset{(0)}{T}}{}^i{}_{jk} = \overset{A}{\underset{(0)}{T}}{}^i{}_{jk} \ ; \quad \overset{0}{\underset{(1)}{P}}{}^i{}_{jk} = \underset{(1)}{C}{}^i{}_{jk} \ ; \quad \overset{0}{\underset{(2)}{P}}{}^i{}_{jk} = 0 \ ;$$

$$\overset{1}{\underset{(1)}{P}}{}^i{}_{jk} = 0 \ ; \quad \overset{2}{\underset{(1)}{P}}{}^i{}_{jk} = \overset{2}{\underset{1}{P}}{}^i{}_{jk} \ ; \quad \overset{1}{\underset{(21)}{P}}{}^i{}_{jk} = \underset{(1)}{C}{}^i{}_{jk} \ ; \quad \overset{1}{\underset{(2)}{P}}{}^i{}_{jk} = \overset{2}{\underset{(2)}{P}}{}^i{}_{jk} = \overset{0}{\underset{(21)}{P}}{}^i{}_{jk} = \overset{2}{\underset{(21)}{P}}{}^i{}_{jk} = 0 \ ; \qquad (3.5)$$

$$\overset{B}{\underset{A}{S}}{}^{i}{}_{jk} = 0 \quad (A, B = 0,1,2).$$

Proposition 3.3 *The curvature d-tensor fields are:*

$$\overline{R}_{b}{}^{a}{}_{pq} = r_{b}{}^{a}{}_{pq} + g_{bt}\left(r_{s}{}^{a}{}_{pq} z^{(1)t} z^{(1)s} - r_{s}{}^{t}{}_{pq} z^{(1)a} z^{(1)s}\right);$$

$$\underset{(A)}{\overline{P}}{}_{b}{}^{a}{}_{pq} = 0 \quad (A = 1,2); \quad \underset{(12)}{\overline{P}}{}_{b}{}^{a}{}_{pq} = 0; \tag{3.6}$$

$$\underset{(1)}{\overline{S}}{}_{b}{}^{a}{}_{pq} = \delta^{a}_{p} \overset{1}{\sigma}_{bq} - \delta^{a}_{q} \overset{1}{\sigma}_{bp} + g^{as}\left(g_{bq} \overset{1}{\sigma}_{sp} - g_{bp} \overset{1}{\sigma}_{sq}\right);$$

$$\underset{(2)}{\overline{S}}{}_{b}{}^{a}{}_{pq} = 0.$$

Proposition 3.4 *The Ricci tensors of curvature and the curvature scalars have the following expressions:*

$$\overline{R}_{bp} = r_{bp} + \gamma_{bt}\left(r_{0}{}^{a}{}_{pa} z^{(1)t} - r_{0}{}^{t}{}_{pa} z^{(1)a}\right);$$

$$\underset{(A)}{\overset{1}{\overline{P}}}{}_{bp} = \underset{(A)}{\overset{2}{\overline{P}}}{}_{bp} = \underset{(12)}{\overset{1}{\overline{P}}}{}_{bp} = \underset{(12)}{\overset{2}{\overline{P}}}{}_{bp} = 0 \quad \text{(A=1,2)};$$

$$\underset{(1)}{\overline{S}}{}_{bp} = (1-n)\overset{1}{\sigma}_{bp} + \gamma^{as}\left(\gamma_{bp} \overset{1}{\sigma}_{sa} - \gamma_{ba} \overset{1}{\sigma}_{sp}\right); \qquad \underset{(2)}{\overline{S}}{}_{bp} = 0; \tag{3.7}$$

$$\overline{R} = \left(r + 2r_{st} z^{(1)s} z^{(1)t} + 2(1-n)\gamma^{bp} \overset{1}{\sigma}_{bp}\right) e^{-2\sigma};$$

$$\underset{(A)}{\overset{1}{\overline{P}}} = \underset{(A)}{\overset{2}{\overline{P}}} = \underset{(12)}{\overset{1}{\overline{P}}} = \underset{(12)}{\overset{2}{\overline{P}}} = 0 \quad \text{(A=1,2)};$$

$$\underset{(1)}{\overline{S}} = 2(1-n)\gamma^{bp} \overset{1}{\sigma}_{bp} e^{-2\sigma}; \quad \underset{(2)}{\overline{S}} = 0. \tag{3.8}$$

Proposition 3.5 *The Einstein equations for the space endowed with this metric are:*

$$\frac{1}{2} r\gamma_{bp} + \gamma_{bp}\left[(n-1)\gamma^{ij} \overset{1}{\sigma}_{ij} - r_{st} z^{(1)s} z^{(1)t}\right] = \chi \underset{(0)}{\overset{0}{T}}{}_{bp} - \gamma_{bt}\left(r_{0}{}^{a}{}_{pa} z^{(1)t} - r_{0}{}^{t}{}_{pa} z^{(1)a}\right);$$

$$\underset{(1)}{\overline{S}}{}_{bp} - \frac{1}{2} g_{bp} R = \chi \underset{(1)}{\overset{1}{T}}{}_{bp}; \tag{3.9}$$

$$\frac{1}{2} g_{bp} R = -\chi \underset{(0)}{\overset{2}{T}}.$$

Observation We write the first equation in this shape in order to emphasize the relation between the Einstein equations of the space and the Einstein equations of the Riemannian space $V^{n} = \left(M, \gamma_{oj}(x)\right)$.

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A COSMOLOGICAL MODEL WITH ROTATION OF A BIANCHI TYPE IX.

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The paper [1] is about cosmological inflation with rotation for the metric of a Bianchi type IX. We obtained a nonstationar model with rotation for metric

$$ds^2 = g_{\alpha\beta}\theta^\alpha\theta^\beta$$

,where $g_{\alpha\beta} = diag(1, -1, -1, -1)$,
$\theta^{\hat{0}} = dt - R\nu_A e^A$, $\theta^{\hat{1}} = Rk_1 e^1$, $\theta^{\hat{2}} = Rk_2 e^2$, $\theta^{\hat{3}} = Rk_3 e^3$,
k_1, k_2, k_3 -constants,¿0,
$\alpha, \beta = 0, 1, 2, 3;$ $A = 1, 2, 3;$

$$e^1 = \cos x^2 \cos x^3 dx^1 - \sin x^3 dx^2,$$

$$e^2 = \cos x^2 \sin x^3 dx^1 + \cos x^3 dx^2,$$

$$e^3 = -\sin x^2 dx^1 + dx^3,$$

$R = R(t)$, ν_1, ν_2, ν_3 -constants.

We suppose $\nu_1 \neq 0$, $\nu_2 = \nu_3 = 0$, $k_2 = k_3$, $k_2^2 = k_1^2 - \nu_1^2$. We use the metric in tetradic form. The sources of this model gravity field are a concomitance anisotropic liquid with tensor of energy-momentum in tetradic form, which is given by

$$T^{(1)}_{\alpha\beta} = (\tilde{\varepsilon} + \pi)\tilde{u}_\alpha\tilde{u}_\beta + (\sigma - \pi)\chi_\alpha\chi_\beta - \pi\eta_{\alpha\beta}$$

and a nonconcomitance ideal liquid with tensor of energy-momentum, which is given by

$$T^{(2)}_{\alpha\beta} = (p + \varepsilon)u_\alpha u_\beta - p\eta_{\alpha\beta}.$$

We have $\tilde{u}_A = (1, 0, 0, 0)$, $\chi_A = (0, 1, 0, 0)$, $u_2 = u_3 = 0$, $p = 0$.

We found the cosmological solution with $R(t) = C_2 e^{\sqrt{C_1}t}$, where C_2 and C_1 -constants, ¿0.For this metric we have the kinematic parameters of model, such as the expansion of model, the rotation of model, the acceleration of model; the shear of model is ziro.

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On the physical content of the principle of general covariance

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Abstract

Ever since the inception of General Relativity the Principle of General Covariance (GCP) has been matter of debate and confusion. In our view the physical meaning of coordinates is related to the question of the possible physical significance of that principle. We believe that the latter may be taken as an appropriate generalized principle of relativity with physical content. With the purpose of throwing light over these issues, after presenting our version of the GCP, we define and construct quasi-Minkowskian coordinates associated to the word-line of an observer who transports an orthonormal tetrad (QMCC). We view the QMCC as the coordinates that would be obtained by that observer by applying operational protocols valid in flat space-time to get the Lorentzian coordinates of an event. The set of all the QMCC is in general an infinite family all of whose members collapse to the usual Lorentzian coordinates when the observer is in free fall, his or her space triad does not rotate and the curvature of space-time vanishes. This implements the idea that the set of all the operational protocols which are equivalent -in the sense of assigning the same numerical values- to obtain the Lorentzian coordinates of events in flat space-time split into inequivalent subsets of operational prescriptions under the presence of a gravitational field or when the observer is not inertial. Something similar must happen with all the physical quantities. Other considerations will be presented.

1 Introduction

Since first formulated nine decades ago the question of the possible physical content of the GCP has been a subject of polemic and confusion. Thus Kretschmann [1] in 1917 claimed the GCP to be devoid of physical content and that given enough mathematical ingenuity any theory could be set in a general covariant form. Einstein [2] begrudgingly accepted the objection stating however the heuristic value the GCP had in searching for a good theory and that that was a reason to prefer General Relativity to Newtonian gravitation which -in his opinion- would only be awkwardly casted into generally covariant form. Einstein was soon proved wrong as Cartan [3] in 1923 and Friedrichs [4] in 1927 found serviceable generally covariant formulations of Newtonian gravitation theory. See also Misner *et al*(1973, ch 12) [5]. In his excellent book Fock [6]makes interesting and critical remarks about the term "general relativity" adopted by Einstein to name his theory of gravitation and the connection of the term with general covariance that, in his view, is merely a logical requirement that is always satisfiable. Fock rightly points out that though Einstein had agreed with Kretschman objection as to the physical vacuity of the GCP his agreement was rather formal, because actually to the end of his life Einstein related the requirement of general covariance to the idea of some kind of "general relativity" and with the equivalence of all frames of reference. The subject has subsequently been addressed in several ways, for example, by Anderson [7](1967), Stachel [8](1986, 2002), Norton [9](1993) and Ellis and Matravers [10](1995). All these works while attemting to clarify the formulation and meaning of the GCP in our opinion fail to give it a specific expression susceptible of physical verification or contradiction. And certainly whatever the claim about the physical content of the GCP might be that should be subject to experimental test to be confirmed or refuted. Ellis and Matravers [10] point out how physicists and astrophysicists in fact almost always use preferred coordinate systems not merely to simplify the calculations but also to help define quantities of physical interest, and that this suggests that we should reconsider and perhaps refine the dogma od general covariance. In that spirit we present in this contribution a proposal for the GCP that may be proved wrong should that be the case, that is, that in principle may be falsifiable in the Popperian sense. The plan of the paper is as follows: In Section 2 we define a principle of general relativity and take it as the GCP. In Section 3 we construct a family of coordinates which are useful to endow with physical meaning the GCP. Some consequences of our formulation of the GCP and further work to be done to explore its potential will be indicated in Section 4.

2 The GCP as a principle of general relativity

We will first formulate a principle of general relativity (GRP) as a physical principle subject to experimental test. To do that we need a few preliminay definitions: Let F be a physical quantity. Let $Q[F]$ be the set of all the different operational protocols -but equivalent in the sense that they yield the same values- that may be used to measure F in flat space-time when working in Lorentzian coordinates: $\eta_{\mu\nu} = diag(+1,+1,+1,-1)$. The physical meaning of F is given by $Q[F]$. Let S be an inertial frame and L the Lorentz group of transformations. Then if $\Lambda \in L$ one has that the action of Λ on S and F implies the following:

$$\Lambda : S \longrightarrow S',\ \Lambda : F \longrightarrow F' \Longrightarrow Q[F] = Q[F'] \ \forall\ \Lambda \in L \text{ and } \forall\ F.$$

Let O be an observer and C his world-line given by its equations $x^\lambda = f^\lambda(\tau)$, where the x^λ's, $\lambda = 0, 1, 2, 3$, are any particular smooth but otherwise generic local coordinates and τ is O's proper time. O's four-velocity is $u^\lambda = \dfrac{dx^\lambda}{d\tau} = \dot{f}^\lambda(\tau)$,
$u^\lambda u_\lambda = -c^2$. O transports an orthonormal tetrad $e_{(\nu)}$ along his world-line whose components verify

$$e^\lambda_{(\nu)} e_{(\mu)\lambda} = \eta_{\nu\mu} \quad , \quad e^\mu_{(0)} = \frac{u^\mu}{c} \ . \tag{1}$$

The transportation law of the tetrad is given by the covariant derivatives of their components with respect to τ according to

$$\frac{De^\mu_{(\sigma)}}{d\tau} = \frac{1}{c^2}(u^\mu a^\nu - u^\nu a^\mu) e_{(\sigma)\nu} + \frac{1}{c}\omega_\alpha u_\beta \epsilon^{\alpha\beta\mu\nu} e_{(\sigma)\nu} \ , \tag{2}$$

where

$$a^\nu = \frac{Du^\nu}{d\tau} \ , \quad \epsilon_{\alpha\beta\gamma\delta} = (-g)^{\frac{1}{2}}[\alpha\beta\gamma\delta] \ , \quad \epsilon^{\alpha\beta\gamma\delta} = -(-g)^{-\frac{1}{2}}[\alpha\beta\gamma\delta] \ ,$$

$$[\alpha\beta\gamma\delta] = \begin{cases} +1 & \text{if } \alpha\beta\gamma\delta \text{ is an even permutation of } 0123 \\ -1 & \text{if } \alpha\beta\gamma\delta \text{ is an odd permutation of } 0123 \\ 0 & \text{if } \alpha\beta\gamma\delta \text{ are not all different} \end{cases}$$

$$g = \det ||g_{\alpha\beta}|| \quad ,$$

and the $\omega_\alpha' s$ are the covariant components of a rotation pseudovector such that $u^\alpha \omega_\alpha = 0$.

Let us now introduce new coordinates, $\tilde{x}^\lambda = (c\tau, \tilde{x}^i) = (c\tilde{t}, \tilde{x}^i), \quad i = 1, 2, 3$, that verify the following conditions:

1. C is described in the $\tilde{x}^\lambda$ coordinates by: $\tilde{x}^i = 0 \;, \quad \tilde{t} = \tau$.
2. The restriction of the metric in the $\tilde{x}^\lambda$ coordinates on C is: $\tilde{g}_{\mu\nu} \mid_C = \eta_{\mu\nu}$, and $\dfrac{\partial \tilde{g}_{\mu\nu}}{\partial \tilde{x}^\lambda} \mid_C = 0$, when the four-acceleration of O and the four-rotation of the tetrad vanish: $\mathbf{a} = \omega = 0$.
3. $\mathbf{e}_{(\alpha)} = \dfrac{\partial}{\partial \tilde{x}^\alpha} \mid_C \iff e^\lambda_{(\alpha)}(\tau) = \dfrac{\partial x^\lambda}{\partial \tilde{x}^\alpha} \mid_C$.
4. The $\tilde{x}^\lambda$'s become the usual Lorentzian coordinates in a neighborhood of C when $\mathbf{a} = \omega = 0$ and the curvature tensor vanishes, $R_{\alpha\beta\mu\nu} = 0$, whithin that neighborhood.

The $\tilde{x}^\lambda$'s will be denoted henceforth by QMC$C\omega$ (cuasi-Minkowskian coordinates relative to the world-line C and to the tetrad $\mathbf{e}_{(\alpha)}$ subject to the rotation ω).

Principle of General Relativity (GRP)

Let us have a generic smooth enough space-time $(\mathbf{M}, \mathbf{g})$, $\mathbf{M}$ and $\mathbf{g}$ respectively denoting the manifold and the metric. Let O be an observer in that space-time of world-line C that uses any type of QMC$C\omega$. Let F be any -generally multicomponent- physical quantity. And let us denote by $\tilde{F}$ the same quantity -or its components- in the QMC$C\omega$, $\tilde{x}^\lambda$'s.

We shall say that the Principle of General relativity (GRP) is verified when the following two conditions hold:

- The equations describing the behaviour of the physical quantities are covariant and they become their corresponding ones in Lorentzian coordinates in flat space-time $(\mathbf{M}, \eta)$ on C when they are expressed in terms of the $\tilde{x}^\lambda$'s and $\mathbf{a} = \omega = 0$. (This implies the Equivalence Principle in general, but curvature dependent terms may still appear if there is no minimal coupling or higher order derivatives are involved in the equations.)
- One has $Q[\tilde{F}] \subseteq Q[F]$, $\forall\ F$ and $\forall$ QMC$C\omega$.

Principle of General Covariance (GCP)

The GCP is the above GRP.

- It implies the Equivalence Principle.

- It is a principle of general relativity with physical content as previously defined.

What is called for now is to verify that this interpretation of the GCP is coherent with the theory and agrees with experience. To that end we proceed to construct the QMC$C\omega$ in the next Section.

3 Construction of the QMC$C\omega$'s

The $\tilde{x}^\lambda$'s will be constructed by expressing the x^λ's as power series of the $\tilde{x}^i$'s with coefficients depending on τ about C.

We get

$$x^\lambda = f^\lambda(\tau) + e^\lambda_{(k)}\tilde{x}^k + \frac{1}{2}(e^\lambda_{(\alpha)}\tilde{\Gamma}^\alpha_{ik}(\tau) - e^\rho_{(i)}e^\sigma_{(k)}\Gamma^\lambda_{\rho\sigma}(\tau))\tilde{x}^i\tilde{x}^k + \Phi^\lambda(\tilde{x}) \ , \tag{3}$$

with

$$\tilde{\Gamma}^\alpha_{ik}(\tau) = A^\alpha_{ik\nu}(\tau)a^\nu + B^\alpha_{ik\nu}(\tau)\omega^\nu + O^\alpha_{ik}(2) \ , \tag{4}$$

where the $A^\alpha_{ik\nu}$ and $B^\alpha_{ik\nu}$ are arbitrary save by being sufficiently smooth and the constraints

$$A^\alpha_{ik\nu} = A^\alpha_{ki\nu} \ , \ B^\alpha_{ik\nu} = B^\alpha_{ki\nu} \ , \ A^\alpha_{ik\nu}e^\nu_{(0)} = B^\alpha_{ik\nu}e^\nu_{(0)} = 0 \ ; \tag{5}$$

$O^\alpha_{ik}(2) = O^\alpha_{ki}(2)$, is an arbitrary smooth enough fuction of second order in the $\tilde{a}^i$'s and $\tilde{\omega}^i$'s, that is, of second order in the a^ν's and ω^ν's, otherwise $O^\alpha_{ik}(2)$ is a function of τ as so is $\Gamma^\lambda_{\rho\sigma}(\tau)$, as both are defined on C;

$$\Phi^\lambda(\tilde{x}) = \Psi^\lambda(\tilde{x}) + \Phi^\lambda_{(0)}(\tilde{x}) \ , \tag{6}$$

with $\Psi^\lambda(\tilde{x})$ verifying

$$\Psi^\lambda \mid_C = \frac{\partial \Psi^\lambda}{\partial \tilde{x}^i} \mid_C = \frac{\partial^2 \Psi^\lambda}{\partial \tilde{x}^i \partial \tilde{x}^j} \mid_C = 0 \ , \tag{7}$$

and vanishing whenever $\mathbf{a}, \omega$ and the curvature tensor, $\mathbf{R}$, in a finite neighborhood of C, all vanish; the latter is equivalent to the vanishing of the $\tilde{\Gamma}^\lambda_{\mu\nu}$'s in that neighborhood; otherwise the Ψ^λ's apart from being sufficiently smooth are arbitrary;

$$\Phi^\lambda_{(0)}(\tilde{x}) = \sum_{l,m,n} \frac{1}{l!m!n!} C^\lambda_{lmn}(\tau)(\tilde{x}^1)^l(\tilde{x}^2)^m(\tilde{x}^3)^n \ , \tag{8}$$

with

$$l \geq 0, \ m \geq 0, \ n \geq 0, \ l+m+n \geq 3, \text{ and } l, m, n \text{ all being integers.}$$

The $C^\lambda_{lmn}(\tau)$'s may be systematically calculated by the following algorithm:
Consider the equation

$$\frac{\partial^2 \Phi^\lambda_{(0)}}{\partial \tilde{x}^i \partial \tilde{x}^k} = -\frac{\partial x^\rho}{\partial \tilde{x}^i}\frac{\partial x^\sigma}{\partial \tilde{x}^k}\Gamma^\lambda_{\rho\sigma}(\tilde{x}^\mu) + e^\beta_{(i)}e^\gamma_{(k)}\Gamma^\lambda_{\beta\gamma}(\tau) \ , \tag{9}$$

where the first term on the rhs is taken as dependent, in general, on the $\tilde{x}^\mu$'s, while the second term only depends on τ as is evaluated on C. Eq. (9) is a consequence of considering the equation for the transformation of the Christoffel symbols on C and using eqs. (3), (6), and (7). The $C^\lambda_{lmn}(\tau)$'s are found by using the power series for $\Phi^\lambda_{(0)}$ given in eq. (8) and taking successive derivatives of eq. (9) with respect to the $\tilde{x}^j$'s, taking the result on C, and doing it all along as if $\tilde{\Gamma}^\alpha_{\mu\nu} = \Psi^\lambda = 0$ at all points.

That way any $C^\lambda_{lmn}(\tau)$ may be expressed in terms of the $C^\lambda_{l'm'n'}$'s of lower order: $l'+m'+n' < l+m+n,\ l' \leq l,\ m' \leq m,\ n' \leq n$; the $e^\lambda_{(k)}(\tau)$, the $\Gamma^\alpha_{\mu\nu}(\tau)$'s, and the partial derivatives of the $\Gamma^\alpha_{\mu\nu}$'s with respect to the x^λ's up to order $\leq l+m+n-2$.

Fixing the Ψ^λ's, $A^\alpha_{ik\nu}$'s, $B^\alpha_{ik\nu}$'s and the $O^\alpha_{ik}(2)$'s uniquely determines a set of corresponding QMC$C\omega$. If space-time is flat and also $\mathbf{a} = \omega = 0$, it follows that $\Psi^\lambda = A^\alpha_{ik\nu} = B^\alpha_{ik\nu} = O^\alpha_{ik}(2) = 0$, and the entire family of the QMC$C\omega$'s collapses to the unique usual Lorentzian coordinates corresponding to the choiced tetrad $e^\lambda_{(\nu)}$. This does not mean that if space-time is not flat and/or if $\mathbf{a} \neq 0$, or $\omega \neq 0$, by taking $\Psi^\lambda = A^\alpha_{ik\nu} = B^\alpha_{ik\nu} = O^\alpha_{ik}(2) = 0$, the resulting QMC$C\omega$ would be Lorentzian, as these do not symply exist for non-flat space-times or in non-inertial reference frames.

4 Some consequences

The relationships of the generic given coordinates x^λ and two different sets of QMC$C\omega$, $\tilde{x}^\lambda$ and $\tilde{\tilde{x}}^\lambda$, may differ at most by terms of second order in the $\tilde{x}^i$'s and $\tilde{\tilde{x}}^i$'s if the observer is not in free fall (C is not a geodesic) and/or his/her choiced transported tetrad rotates, which corresponds to the freedom allowed to choose the $\tilde{\Gamma}^\alpha_{ik}(\tau)$'s via eq. (4), or by terms of third order in the same variables if the observer is in free fall and its reference tetrad is paralell transported, corresponding to the freedom allowed to choose the function Φ^λ when the space-time is not flat.

If the observer O is in free fall and his reference tetrad does not rotate the Equivalence Principle tells us that when he is at a point P at proper time τ_P, he should measure for the square, dl^2, of the spatial distance between P and any other very close point Q of QMC$C\omega$, $(c(\tau_P + \delta\tau_P), \delta\tilde{x}^i)$, corresponding to the given coordinates $x^\lambda = f^\lambda(\tau_P) + \delta x^\lambda$,

$$dl^2 = \delta_{ik}\delta\tilde{x}^i\delta\tilde{x}^k = e^{(k)}_\alpha e_{(k)\beta}\delta x^\alpha \delta x^\beta \tag{10}$$

that results from inverting the coordinates transformation in eq. (3). It is easy to see that one has at P

$$e^{(k)}_\alpha e_{(k)\beta} = g_{\alpha\beta} - \frac{g_{0\alpha}g_{0\beta}}{g_{00}} \ , \tag{11}$$

that yields the usually accepted result for dl^2 in eq. (10) [11].

It follows from eq. (3) that the values of tensor quantities measured on the world-line C corresponding to two different sets of QMC$C\omega$'s -but with the same choiced reference tetrad- should be identical. This is clearly not so for their ordinary partial derivatives with respect to the spatial QMC$C\omega$ coordinates. In future work this question will be considered in more detail and the general theory here presented will be applieed to some concrete examples.

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Conservation laws and causality

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1. Introduction

In different branches of science (in thermodynamics, physics and mechanics) one assigns a different physical meaning to the concept of ``conservation laws".

n areas of physics related to the field theory and in the theoretical mechanics ``the conservation laws" are connected with a conservative physical quantity or with a conservative object. (These are conservation laws that below will called ``exact".)

In mechanics and physics of continuous media the concept of ``conservation laws" is related to the conservation laws for energy, linear momentum, angular momentum, and mass that establish the balance between the change of physical quantities and external action. These are balance conservation laws.

In thermodynamics the conservation laws are associated with the principles of thermodynamics.

It turns out that there exists a connection between exact and balance conservation laws. It is just a connection that enables one to explain a causality of physical phenomena. (In thermodynamics such a connection is described by the principles of thermodynamics.)

These results have been obtained owing to the skew-symmetric differential forms that possess the evolutionary properties (and hence they were given the name evolutionary ones).

In present paper it will be shown that the closed exterior skew-symmetric differential forms, which describe exact conservation laws for physical fields (exact conservation laws), are obtained from the evolutionary forms that are assigned to the conservation laws for material media (to the balance conservation laws). From this it follows that material media generate physical fields, and this disclose the causality of physical phenomena.

The evolutionary forms arise under describing physical processes in material media. Unlike to the exterior forms, the basis of which are differentiable manifolds and manifolds with structures of any type (i.e. manifolds with closed metric forms), the basis of evolutionary forms is made up by deforming manifolds (with unclosed metric forms). The manifolds constructed of trajectories of material system elements are examples of deforming manifolds.

2. The exact conservation laws

Exact conservation laws are those that state an existence of conservative physical quantities or objects. Closed exterior differential forms correspond to exact conservation laws.

It is known that the exterior differential form of degree p (p - form) can be written as [1]

$$\theta^p = \sum_{i_1 \cdots i_p} a_{i_1 \cdots i_p} dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_p} \quad 0 \le p \le n \tag{1}$$

Here $a_{i_1 \cdots i_p}$ are functions of variables $x^{i_1}, x^{i_2}, \ldots, x^{i_p}$, n is the dimension of space, $\wedge$ is the operator of exterior multiplication, $1, dx^i, dx^i \wedge dx^j, dx^i \wedge dx^j \wedge dx^k, \cdots$ is the local basis which satisfies the condition of exterior multiplication:

$$\begin{aligned} &dx^i \wedge dx^i = 0 \\ &dx^i \wedge dx^j = -dx^j \wedge dx^i \quad i \ne j \end{aligned} \tag{2}$$

The differential of exterior form θ^p is expressed as

$$d\theta^p = \sum_{i_1 \cdots i_p} da_{i_1 \cdots i_p} dx^{i_1} dx^{i_2} \ldots dx^{i_p} \tag{3}$$

[From here on the symbol $\sum$ will be omitted and the summation over double indices will be implied. And besides, the symbol of exterior multiplication will be also omitted for the sake of convenience in account].

From the closure condition of exterior form θ^p

$$d\theta^p = 0 \tag{4}$$

one can see that the closed exterior form is conserved quantity. This means that it may correspond to the conservation law, namely, to some conservative quantity.

If the form is closed only on pseudostructure, i.e. this form is a closed inexact one, the closure condition is written as

$$d_\pi \theta^p = 0 \tag{5}$$

$$d_\pi {}^*\theta^p = 0 \tag{6}$$

where ${}^*\theta^p$ is the dual form. Condition (6) specifies the pseudostructure π.

From conditions (5) and (6) one can see that the form closed on pseudostructure is a conservative object, namely, this quantity conserves on pseudostructure. This can also correspond to some conservation law, i.e. to conservative object.

Thus one can see that the closure conditions of the exterior form are mathematical expressions of the exact conservation law.

The exact conservation laws are those for physical fields.

The closure conditions of exterior form, which are the mathematical expression of the exact conservation law, describe the differential-geometrical structure, namely, the pseudostructure with conservative quantity. The physical structures, which forms physical fields and corresponding conservation laws, are just such structures.

The mathematical principles of the theory of closed exterior forms, which correspond to conservation laws, lie at the basis of existing field theories describing physical fields. Gauge transformations of field theory are transformations of the theory of closed exterior differential forms. These are transformations that conserve the differential. From the closure conditions of exterior forms (condition (4) and conditions (5) and (6)) one can see that any closed form is a differential of the form of lower degree: the total one

$$\theta^p = d\theta^{p-1} \tag{7}$$

if the form is exact, or the interior one

$$\theta^p = d_\pi \theta^{p-1} \tag{8}$$

on pseudostructure if the form is inexact. Since the closed exterior differential forms are differentials, they are invariant under all transformations that conserve the differential. The unitary transformations (0-form), the tangent and canonical transformations (1-form), the gradient and gauge transformations (2-form) and so on are examples. Nondegenerate transformations are used in field theory. These are gauge transformations for spinor, scalar, vector, tensor (3-form) fields. The exterior differential forms enable one to work out a classification of gauge transformations.

From the closure conditions for exterior forms, which describe the conservation laws, and from relations (7) and (8), which relate the forms of sequential degrees, the identical relations, being the identical ones of field theory, are obtained. The Poincare invariant, vector and tensor identical relations, the Cauchi-Riemann conditions, canonical relations, the thermodynamic relations, the eikonal relations and so on are examples of identical relations. In general form identical relation can be written as

$$d_\pi \varphi = \theta^p_\pi \tag{9}$$

Below it will be shown a physical meaning of such a relation.

It can be shown that the equations of existing field theories are those obtained on the basis of the properties of the exterior form theory. The Hamilton formalism is based on the properties of closed exterior and dual forms of the first degree, quantum mechanics does on the forms of zero degree, the electromagnetic field equations are based on the forms of second degree. The third degree forms are assigned to the gravitational field.

Thus, one can see that the exact conservation laws are those for physical fields. They are described by closed exterior differential forms. The closure conditions of exterior inexact form and of corresponding dual form are a mathematical expression of the exact conservation law and they are the equations for differential-geometrical structures. The physical strictures, which form physical fields, are just such differential-geometrical structures.

And here the questions arise of: (a) how are closed exterior forms, which reflect the properties of exact conservation laws, obtained; (b) what generates physical structures corresponding to exact conservation laws; and (c) what is responsible for such processes?

The mathematical apparatus of evolutionary differential forms, which correspond to the balance conservation laws (conservation laws for material media), enables us to answer these questions.

3. The balance conservation laws

Evolutionary forms reflect the properties of conservation laws for material system (medium). These are balance conservation laws for energy, linear momentum, angular momentum, and mass (they establish a balance between the variation of physical quantity and the corresponding external action). From the equations, which describe the balance conservation laws, the evolutionary relation in differential forms is obtained. {A material system is a variety of elements that have internal structure and interact to one another. As examples of material systems it may be thermodynamic, gas dynamical, cosmic systems, systems of elementary particles and others.}

The evolutionary differential forms are skew-symmetrical differential forms defined on deforming manifolds.

An evolutionary differential form of degree p (p-form) is written similarly to exterior differential form. But the evolutionary form differential cannot be written similarly to that presented for exterior differential forms (see formula 3). In the evolutionary form differential there appears an additional term connected with the fact that the basis of the form changes. For the differential forms defined on the manifold with unclosed metric form one have $d(dx^{\alpha_1}dx^{\alpha_2}\cdots dx^{\alpha_p})\neq 0$. For this reason the differential of the evolutionary form ω can be written as

$$d\omega^p = da_{\alpha_1\cdots\alpha_p}dx^{\alpha_1}dx^{\alpha_2}\cdots dx^{\alpha_p} + a_{\alpha_1\cdots\alpha_p}d(dx^{\alpha_1}dx^{\alpha_2}\cdots dx^{\alpha_p}) \qquad (10)$$

where the second term is a differential of unclosed metric form being nonzero. Since the second term of the evolutionary form differential is nonzero, the evolutionary form differential cannot vanish. This means that every evolutionary form is an unclosed form.

That is, unlike to the exterior forms, which are defined on the manifolds with closed metric forms, the evolutionary form cannot be closed. This specific feature of the evolutionary form gives rise to the properties differing from that of the exterior form. Such properties of the evolutionary form reflect the properties of the balance conservation laws.

Let us analyze the equations that describe the balance conservation laws for energy and linear momentum.

In the accompanying frame (this frame is connected with the manifold built by the trajectories of the material system elements) the energy equation is written in the form

$$\frac{\partial\psi}{\partial\xi^1}=A_1 \qquad (11)$$

Here ψ is the functional specifying the state of material system (the action functional, entropy, wave function can be regarded as examples of the functional), ξ^1 is the coordinate along the trajectory, A_1 is the quantity that depends on specific features of the system and on external energy actions onto the system.

In a similar manner, in the accompanying frame of reference the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial\psi}{\partial\xi^\nu}=A_\nu, \qquad \nu=2,... \tag{12}$$

where ξ^ν are the coordinates in the direction normal to the trajectory, A_ν are the quantities that depend on the specific features of the system and external force actions.

Eqs. (11), (12) can be convoluted into the relation

$$d\psi = A_\mu d\xi^\mu, \qquad \mu = 1, \nu \tag{13}$$

Relation (13) can be written as

$$d\psi = \omega \tag{14}$$

Here $\omega=A_\mu d\xi^\mu$ is the differential form of the first degree.

Since the equations of the balance conservation laws are evolutionary ones, the relation obtained is also an evolutionary relation.

Relation (14) was obtained from the equation of the balance conservation laws for energy and linear momentum. In this relation the form ω is that of the first degree. If the equations of the balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this form in the evolutionary relation will be the form of the second degree. And in combination with the equation of the balance conservation law of mass this form will be the form of degree 3.

Thus, in the general case the evolutionary relation can be written as

$$d\psi = \omega^p, \quad p = 0,1,2,3 \tag{15}$$

(The evolutionary relation for $p = 0$ is similar to that in the differential forms, and it was obtained from the interaction of energy and time).

As it will be shown below, from this relation, obtained from the equations of balance conservation laws, it follows the relation, which contains the closed exterior form corresponding to the exact conservation law.

What are the properties and specific features of this relation?

The specific features of this relation are connected with the differential form ω^p. This differential form is an example of the evolutionary differential form, that is, this form is defined on the deforming manifold.

We will show that the manifold, on which the differential form ω^p is defined, is a deforming manifold, i.e. a manifold with unclosed metric form.

Let us consider relation (14), where $\omega=A_\mu d\xi^\mu$ (that is, $p = 1$). The differential of this form can be written as $d\omega = K_{\alpha\beta} d\xi^\alpha d\xi^\beta$, where $K_{\alpha\beta}$ are the components of commutator of the form ω. The components of commutator of the form $\omega=A_\mu d\xi^\mu$ can be written as follows:

$$K_{\alpha\beta}=\left(\frac{\partial A_\beta}{\partial x^\alpha}-\frac{\partial A_\alpha}{\partial x^\beta}\right) \tag{16}$$

(here the term connected with the nondifferentiability of the manifold has not yet been taken into account). The coefficients of the form $\omega=A_\mu d\xi^\mu$ have been obtained either from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator built of the derivatives of such coefficients is nonzero. This points to the fact that the differential of the forms is also nonzero. Thus, the form ω proves to be unclosed.

It turns out that the left-hand side of relation (14) involves the differential, which is a closed form, whereas the right-hand side involves the unclosed differential form, which cannot be a differential. Relation (14) proves to be not identical one.

What is a physical meaning of such a relation?

This relation obtained from the equations of the balance conservation laws involves the functional that specifies the material system state. However, since this relation turns out to be not identical, from this relation one cannot get the differential $d\psi$ that could point out to the equilibrium state of material system. The absence of differential $d\psi$ means that the system state is nonequilibrium. That is, in material system the internal force acts. This leads to distortion of trajectories of material system. A manifold made up by the trajectories (the accompanying manifold) turns out to be a deforming manifold. The differential form ω, as well as the forms ω^p, appear to be evolutionary forms.

What properties does the evolutionary differential form, i.e. the differential form defined on deforming manifold, possess.

A deforming manifold has unclosed metric form because the metric form commutator, which describes the manifold deformation, is nonzero. That is, the metric form differential is nonzero. Since the metric form differential of nonzero value enters into the evolutionary form differential (see, formula (10)), this means that the evolutionary form differential cannot vanish also. That is, the evolutionary form, which is defined on deforming manifold, cannot become a closed form (in any physical process).

If to express the evolutionary form differential in terms of the commutator, the metric form commutator of the manifold metric form with nonzero value will enter into the evolutionary form commutator. For example, in commutator of the differential form $\omega = A_\mu d\xi^\mu$ the second term connected with the metric form commutator with nonzero value will arise (see, formula (16)). (This additional term describes the manifold torsion).

Since the differential form in the relation obtained from the equations of the balance conservation laws cannot be closed, this means that such a relation cannot become an identical relation. And since this relation is evolutionary one, it appears to be nonidentical selfvarying relation (a variation of one term in the nonidentical relation leads to variation of another term of the nonidentical relation and so on).

The selfvarying nonidentical relation has a physical meaning. This relation describes a selfvariation of the nonequilibrium state of the material system.

Selfvarying evolutionary relation possesses one more property of physical significance. Under selfvariations of the evolutionary relations it can be realized the conditions of degenerate transformation when from nonidentical relation the relation that is identical on pseudostructure is obtained.

If the transformation is degenerate, from the evolutionary form ω^p, which is unclosed, namely, $d\omega^p \neq 0$, it can be obtained the differential form closed on pseudostructure. The differential of this form equals zero. That is, it is realized the transition

$$d\omega^p \neq 0 \;\rightarrow (\textit{degenerate transform}) \rightarrow \quad d_\pi \omega^p = 0, \;\; d_\pi{}^{\bullet}\omega^p = 0$$

The evolutionary relation on the pseudostructure π takes the form

$$d_\pi \psi = \omega_\pi^p \tag{17}$$

where the form ω_π^p is closed on the pseudostructure.

The degenerate transformation it must correspond vanishing of some functional expressions, such as Jacobians, determinants, the Poisson brackets, residues and others. Vanishing these functional expressions is the closure condition for a dual form. The conditions of degenerate transformation are connected with symmetries, which can be due to the degrees of freedom of material system and its elements. The translational degrees of freedom, internal degrees of freedom of the system elements, and so on can be examples of such degrees of freedom. The degenerate transformation is realized as the transition from the accompanying noninertial coordinate system to the locally inertial system.

Since the form ω_π^p is a closed form, this form is a differential of some differential form, and the relation (17) obtained turns out to be identical: in the left-hand and right-hand sides of this relation there are differentials. This means that under the degenerate transformation from the nonidentical evolutionary relation it follows the identical on pseudostructure relation.

One can see that obtained identical relation (17) is a relation of the same type as identical relation (9) that contains closed exterior forms.

Transition from nonidentical relation (15) obtained from the balance conservation laws to identical relation (17) means the following. Firstly, an existence of the state differential $d_\pi\psi$ (left-hand side of relation (17)) points to a transition of the material system to the locally-equilibrium state. And, secondly, an emergence of the closed (on pseudostructure) inexact exterior form $d_\pi\psi$ (right-hand side of relation (17)) points to an origination of the physical structure, namely, the conservative object demonstrating a fulfilment of the exact conservation law. This conservative object is a conservative physical quantity (the form ω_π^p) on the pseudostructure (the dual form ${}^*\omega^p$, which defines the pseudostructure).

4. Causality

It has been shown that the closed exterior forms, which correspond to exact conservation law and describe physical structures forming physical fields, are obtained from evolutionary forms, to which the balance conservation laws for material systems correspond. This proves that material media generate physical fields. Thus, it is disclosed a determinacy of physical processes and phenomena.

Nonidentity of the relation obtained from the equations of the balance conservation laws relates to the fact that the evolutionary form, which enters into this relation, appears to be unclosed, i.e. the commutator of this form is nonzero. As one can see from formula (15), the commutator contains contributions from quantities, which describe different external actions onto material system. They cannot compensate one another because they have different natures (for example, force and power actions). Transition from nonidentical relation to identical one means that the quantity stored by commutator due to external actions onto material system and acting like internal force converts into a measurable quantity and this is accompanied by emerging physical structure. The emergence of physical structures in the evolutionary process reveals in material system as an emergence of certain observable formations, which develop spontaneously. In this manner the causality of emerging various observable formations in material media is explained. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, creating massless particles and others.

Characteristics of the formations originated are defined by those of evolutionary forms and by the evolutionary form commutators, which depend on the properties of material system and external actions [2].

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Transformations of bispinors in the flat Finslerian event space with a partially broken 3D isotropy

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Within the framework of the Finslerian approach to the problem of violation of Lorentz symmetry, consideration is given to a flat axially symmetric Finslerian space of events, which is the generalization of Minkowski space. Such an event space arises from the spontaneous breaking of initial gauge symmetry and from the formation of anisotropic fermion-antifermion condensate. It is shown that the appearance of an anisotropic condensate breaks Lorentz symmetry; relativistic symmetry, realized by means of the 3-parameter group of generalized Lorentz boosts, remains valid here nevertheless. We have obtained the bispinor representation of the group of generalized Lorentz boosts, which makes it possible to construct the Lagrangian for an interaction of fundamental fields with anisotropic condensate.

1 . Introduction

It is known that the Einstein relativity principle is most fully reflected in the requirement of local Lorentz invariance of the laws of nature. Compared with the other physical symmetries, Lorentz space-time symmetry occupies a special place: since its discovery a hundred years ago it has determined the development of the theory of fundamental interactions. At present, however, owing to progress made in the construction of unified gauge theories, there have been reasons to consider Lorentz symmetry not as a strict but only as an approximate symmetry of nature.

Subsequent to the paper [1], the following point of view is becoming more and more popular: in this view even in a theory which has Lorentz invariance at the most fundamental level, this symmetry can be spontaneously broken if some (for example, vector) field acquires a vacuum expectation value which breaks the initial Lorentz symmetry. Certainly the question here concerns symmetry violation with respect to active Lorentz transformations of fundamental fields against the background of a fixed (in this case, vector) condensate. As for passive Lorentz transformations, under which the condensate is transformed as a Lorentz vector, the corresponding Lorentz covariance remains valid.

It is a noteworthy fact that the usual Standard Model of strong, weak and electromagnetic interactions does not possess the dynamics necessary to cause spontaneous breaking of Lorentz symmetry. In other words, the standard Higgs mechanism, which breaks the local gauge symmetry and gives rise to the scalar Higgs condensate, does not affect the initial Lorentz symmetry of the theory. However, the fact that in more complicated theories, in particular string theories, spontaneous breaking of Lorentz symmetry may well occur has given impetus to the construction of a phenomenological theory [2] referred to as the Standard Model Extension (SME). The Lagrangian of this theory includes all passive Lorentz scalars formed by combining standard-model fields with coupling coefficients having Lorentz indices. As for all dominant gravitational couplings in the SME action, such a problem has been studied in [3]. As a result the SME has made it possible to describe many possible effects caused by violation of active Lorentz invariance and to classify them as effects of Planck-scale physics, strongly suppressed at attainable energy scales.

In spite of the absence of direct evidence in favour of Lorentz symmetry violation, interest in the given fundamental problem is still increasing [4]. In this connection we also point out the publications [5,6] in which, along with experimental investigations inspired, in particular, by the SME, other theoretical approaches to Lorentz violation are also discussed. One of such approaches was proposed relatively long ago [7]; it is based on the more general, Finslerian geometrical model of space-time [8,9] rather than on the Riemannian one. Although from the phenomenological point of view the Finslerian approach is still in need of further development, potentially it is more appealing since it admits violation of Lorentz invariance with no breaking of relativistic invariance.

The key moment in the formation of the Finslerian approach was the fact that within the framework of the principle of correspondence with Minkowski metric it became possible to find two types of Finslerian metrics, which possess relativistic symmetry, i.e. symmetry corresponding to boosts. The first of the above-mentioned metrics describes a flat space of events with axial symmetry, i.e. with partially broken $3D$ isotropy [7] while the second one [10] exhibits an entirely broken $3D$ isotropy.

The appearance of $3D$ anisotropy of the event space with the preservation of its relativistic symmetry indicates that the spontaneous breaking of the initial gauge symmetry may be accompanied by a corresponding phase transition in the geometrical structure of space-time. In other words, spontaneous breaking of gauge symmetry may lead to a dynamic rearrangement of vacuum which results in the formation of a relativistically invariant anisotropic fermion-antifermion condensate, i.e. of a constant classical nonscalar field. This constant field physically manifests itself as a relativistically invariant anisotropic medium filling space-time. Such a medium, leaving space-time flat, gives rise to its anisotropy, that is, instead of Minkowski space there appears a relativistically invariant Finslerian event space. Since the relativistic symmetry of the Finslerian space is realized with the aid of the so-called generalized Lorentz transfomations, being different from the usual Lorentz boosts, this then is why one can speak of violation of Lorentz symmetry, too. However the remaining relativistic symmetry, now represented by a 3-parameter noncompact group of the generalized Lorentz transformations, still plays the important constructive role. In particular, it makes possible to take explicitly into account the influence of the anisotropic fermion-antifermion condensate on the dynamics of fundamental fields after spontaneous breaking of the initial gauge symmetry.

In view of the foregoing we call attention to the fact that, irrespective of the problem of violation of Lorentz symmetry, in the literature consideration has already been given to the mechanism of the dynamical breaking of the initial gauge symmetry which is alternative to the standard one; instead of the elementary Higgs condensate there appears a scalar fermion-antifermion condensate [11]. As for fermion-antifermion condensate, which would break $3D$ isotropy of event space in a relativistically invariant way, it should be noted that in terms of quantum theory such a problem has not yet been considered. However, within the framework of classical theory [12], relativistically invariant fermion-antifermion condensate, which leads at least to partial breaking of $3D$ isotropy, actually arises.

In the present work we confine ourselves to the case of flat space-time with partially broken $3D$ isotropy and again consider the group of its relativistic symmetry. Thereafter we shall construct the bispinor representation of the corresponding group.

In comparison with the 6-parameter homogeneous Lorentz group of Minkowski space, the homogeneous group of isometries of the flat Finslerian space with partially broken $3D$ isotropy is a 4-parameter group. Apart from 3-parameter boosts (generalized Lorentz transformations) it includes only axial symmetry transformations, i.e. the 1-parameter group of rotations about the preferred direction in $3D$ space; in this case this direction is determined by a spontaneously arising axially symmetric fermion-antifermion condensate.

It will be demonstrated below that the given 4-parameter group of Finslerian isometries is locally isomorphic to the corresponding 4-parameter subgroup of the Lorentz group. Since this fact is fundamental for the construction of the bispinor representation of the group of Finslerian isometries, we primarily consider the above-mentioned 4-parameter subgroup of the Lorentz group.

2. The 4-parameter subgroup of Lorentz group and its 3-parameter noncompact subgroup

In [13], in terms of Lie algebras all continuous subgroups of Lorentz group were classified. It turned out that the Lorentz group contains not a single 5-parameter subgroup and has a single (up to isomorphism) 4-parameter subgroup. This subgroup includes independent rotations about an arbitrarily selected axis, the direction of which will be denoted using a unit vector $\boldsymbol{\nu}$, and a 3-parameter group consisting of noncompact transformations only. Physically such noncompact transformations are realized as follows. First choose as $\boldsymbol{\nu}$ a direction towards a preselected star and then perform an arbitrary Lorentz boost by complementing it with such a turn of the spatial axes that in a new reference frame the direction towards the star remains unchanged.

The set of the transformations described, while linking the inertial reference frames, actually constitutes a 3-parameter noncompact group (in contrast to the usual Lorentz boosts). Let us write the corresponding 3-parameter transformations in the infinitesimal form

$$\begin{aligned} dx^0 &= -\boldsymbol{n}\boldsymbol{x}d\alpha, \\ d\boldsymbol{x} &= (-\boldsymbol{n}x^0 - [\boldsymbol{x}[\boldsymbol{\nu}\boldsymbol{n}]])d\alpha \end{aligned} \tag{1}$$

where $d\alpha$ is a rapidity, the unit vector $\boldsymbol{n}$ indicates a direction of the infinitesimal boost, so that $d\boldsymbol{v} = \boldsymbol{n}d\alpha$, and the meaning of $\boldsymbol{\nu}$ has been explained. Integration of equations (1) leads to final transformations which, at any fixed $\boldsymbol{\nu}$, belong to Lorentz group and themselves form a 3-parameter noncompact group with parameters $\boldsymbol{n}\,, \alpha$:

$$x'^i = \Lambda^i_k(\boldsymbol{\nu}; \boldsymbol{n}, \alpha)\, x^k\,, \tag{2}$$

where

$$\begin{aligned} \Lambda^0_0 &= 1 + \frac{\cosh \boldsymbol{\nu}\boldsymbol{n}\alpha - 1}{(\boldsymbol{\nu}\boldsymbol{n})^2}\,, \\ \Lambda^0_\beta &= \frac{1 - e^{-\boldsymbol{\nu}\boldsymbol{n}\alpha}}{\boldsymbol{\nu}\boldsymbol{n}}\, n_\beta + \frac{\cosh \boldsymbol{\nu}\boldsymbol{n}\alpha - 1}{(\boldsymbol{\nu}\boldsymbol{n})^2}\, \nu_\beta\,, \\ \Lambda^\rho_0 &= \frac{1 - e^{\boldsymbol{\nu}\boldsymbol{n}\alpha}}{\boldsymbol{\nu}\boldsymbol{n}}\, n^\rho + \frac{\cosh \boldsymbol{\nu}\boldsymbol{n}\alpha - 1}{(\boldsymbol{\nu}\boldsymbol{n})^2}\, \nu^\rho\,, \\ \Lambda^\rho_\beta &= \delta^\rho_\beta + \frac{1 - e^{\boldsymbol{\nu}\boldsymbol{n}\alpha}}{\boldsymbol{\nu}\boldsymbol{n}}\, n^\rho \nu_\beta + \nu^\rho \Lambda^0_\beta\,. \end{aligned}$$

Hereafter the latin indices take on values of 0, 1, 2, 3 while the greek ones, values of 1, 2, 3. Note also that the n^β and ν^β denote the Cartesian components of unit vectors $\boldsymbol{n}$ and $\boldsymbol{\nu}$, in which case $n_\beta = -n^\beta$, $\nu_\beta = -\nu^\beta$. The transformations inverse to (2) appear as

$$x^i = \Lambda^{-1}{}^i_k(\boldsymbol{\nu}; \boldsymbol{n}, \alpha)\, x'^k\,, \tag{3}$$

where

$$\Lambda^{-1}{}^i_k(\boldsymbol{\nu}; \boldsymbol{n}, \alpha) = \Lambda^i_k(\boldsymbol{\nu}; \boldsymbol{n}, -\alpha)\,. \tag{4}$$

Consider two arbitrary elements of the group (2). Let the first element g_1 be characterized by the parameters $\boldsymbol{n}_1, \alpha_1$, and the second one, g_2, by the parameters $\boldsymbol{n}_2, \alpha_2$. Then to the element $g = g_2 g_1$ there will correspond the parameters $\boldsymbol{n}, \alpha$, which are functionally dependent on the $\boldsymbol{n}_1, \alpha_1$ and $\boldsymbol{n}_2, \alpha_2$, i.e. $\Lambda^i_k(\boldsymbol{\nu}; \boldsymbol{n}, \alpha) = \Lambda^i_j(\boldsymbol{\nu}; \boldsymbol{n}_2, \alpha_2)\Lambda^j_k(\boldsymbol{\nu}; \boldsymbol{n}_1, \alpha_1)$. Using the explicit form of the matrix elements $\Lambda^i_k(\boldsymbol{\nu}; \boldsymbol{n}, \boldsymbol{\alpha})$ and making the corresponding calculations we arrive at the following relations:

$$\boldsymbol{n}\,\alpha = \frac{\boldsymbol{\nu}(\boldsymbol{n}_1\alpha_1 + \boldsymbol{n}_2\alpha_2)}{1 - e^{\boldsymbol{\nu}(\boldsymbol{n}_1\alpha_1 + \boldsymbol{n}_2\alpha_2)}} \left[\frac{1 - e^{\boldsymbol{\nu}\boldsymbol{n}_1\alpha_1}}{\boldsymbol{\nu}\boldsymbol{n}_1}\boldsymbol{n}_1 + \frac{e^{\boldsymbol{\nu}\boldsymbol{n}_1\alpha_1}(1 - e^{\boldsymbol{\nu}\boldsymbol{n}_2\alpha_2})}{\boldsymbol{\nu}\boldsymbol{n}_2}\boldsymbol{n}_2\right], \tag{5}$$

$$\boldsymbol{n}^2 = 1\,. \tag{6}$$

These relations essentially represent the law of group composition for the 3-parameter non-compact subgroup (2) of Lorentz group.

Since the group (2) links the coordinates of events in the initial and primed inertial reference frames, from the physical standpoint it is more natural to use as group parameters three velocity components, $\boldsymbol{v}$, of the primed reference frame rather than the $\boldsymbol{n}, \alpha$. In order to express the $\boldsymbol{v}$ in terms of the $\boldsymbol{n}, \alpha$ it is sufficient to consider motion in the initial frame of the origin of the primed frame, i.e. to write $x^\beta = \Lambda^{-1}{}^\beta_0\, x'^0$ and $x^0 = \Lambda^{-1}{}^0_0\, x'^0$. Then $v^\beta = x^\beta/x^0 = \Lambda^{-1}{}^\beta_0/\Lambda^{-1}{}^0_0$. Using now eqs. (2)–(4), we get as a result

$$\boldsymbol{v} = \left[\frac{1 - e^{-\boldsymbol{\nu}\boldsymbol{n}\alpha}}{\boldsymbol{\nu}\boldsymbol{n}}\boldsymbol{n} + \frac{\cosh \boldsymbol{\nu}\boldsymbol{n}\alpha - 1}{(\boldsymbol{\nu}\boldsymbol{n})^2}\boldsymbol{\nu}\right] \Big/ \left[1 + \frac{\cosh \boldsymbol{\nu}\boldsymbol{n}\alpha - 1}{(\boldsymbol{\nu}\boldsymbol{n})^2}\right]. \tag{7}$$

Hereafter we put $c = 1$. Since $\boldsymbol{n}^2 = \boldsymbol{\nu}^2 = 1$, eq. (7) yields the inverse relations:

$$\boldsymbol{n} = \frac{\boldsymbol{v}}{\sqrt{2(1 - \boldsymbol{v}\boldsymbol{\nu})(1 - \sqrt{1 - \boldsymbol{v}^2})}} - \sqrt{\frac{1 - \sqrt{1 - \boldsymbol{v}^2}}{2(1 - \boldsymbol{v}\boldsymbol{\nu})}}\,\boldsymbol{\nu}\,, \tag{8}$$

$$\alpha = \frac{\sqrt{2(1 - \boldsymbol{v}\boldsymbol{\nu})(1 - \sqrt{1 - \boldsymbol{v}^2})}}{\sqrt{1 - \boldsymbol{v}^2} + \boldsymbol{v}\boldsymbol{\nu} - 1} \ln\left(\frac{\sqrt{1 - \boldsymbol{v}^2}}{1 - \boldsymbol{v}\boldsymbol{\nu}}\right). \tag{9}$$

In terms of $\boldsymbol{v}$ the law of group composition (5),(6) takes the form

$$\boldsymbol{v} = \frac{(\boldsymbol{v}_1(1-\boldsymbol{v}_2\boldsymbol{\nu}) + \boldsymbol{v}_2\sqrt{1-\boldsymbol{v}_1^2})(1-\boldsymbol{v}_1\boldsymbol{\nu}) + \boldsymbol{\nu}(\boldsymbol{v}_1\boldsymbol{v}_2 + \boldsymbol{\nu}\boldsymbol{v}_2(\sqrt{1-\boldsymbol{v}_1^2} - 1))\sqrt{1-\boldsymbol{v}_1^2}}{1 - \boldsymbol{v}_1\boldsymbol{\nu} + \boldsymbol{v}_1\boldsymbol{v}_2\sqrt{1 - \boldsymbol{v}_1^2} + \boldsymbol{\nu}\boldsymbol{v}_2(1 - \boldsymbol{v}_1\boldsymbol{\nu} + \sqrt{1 - \boldsymbol{v}_1^2})(\sqrt{1 - \boldsymbol{v}_1^2} - 1)}, \tag{10}$$

whereby $\Lambda^i_k(\boldsymbol{\nu};\, \boldsymbol{v}) = \Lambda^i_j(\boldsymbol{\nu};\, \boldsymbol{v}_2)\Lambda^j_k(\boldsymbol{\nu};\, \boldsymbol{v}_1)$. It is thus clear that eq. (10) represents Einstein's law of addition of velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. In comparison with its usual form one should remember, however, that after the transformation $\Lambda^j_k(\boldsymbol{\nu};\, \boldsymbol{v}_1)$ the spatial axes, in which the

$\boldsymbol{v}_2$ is prescribed, appear now not to be parallel to the initial axes but turned so that the vector $\boldsymbol{\nu}$ relative to them maintains its initial orientation. It is just therefore, irrespective of the direction of $\boldsymbol{v}_1$, eq. (10) yields $\boldsymbol{v} = \boldsymbol{\nu}$ if $\boldsymbol{v}_2 = \boldsymbol{\nu}$.

3. Matrices of the finite bispinor transformations representing a 3-parameter group of the generalized Lorentz transformations

The 3-parameter group of the generalized Lorentz transformations, similarly to the subgroup (2) of Lorentz group, consists of noncompact transformations only. In the infinitesimal form the transformations belonging to it appear as

$$\begin{aligned} dx^0 &= (-r(\boldsymbol{\nu n})x^0 - \boldsymbol{n x})d\alpha\,, \\ d\boldsymbol{x} &= (-r(\boldsymbol{\nu n})\boldsymbol{x} - \boldsymbol{n}x^0 - [\boldsymbol{x}[\boldsymbol{\nu n}]])d\alpha\,. \end{aligned} \tag{11}$$

Here, as in the infinitesimal transformations (1) of the group (2), the $\boldsymbol{n}$ and α are group parameters while the $\boldsymbol{\nu}$ is a fixed unit vector. And the difference between (11) and (1) consists in the appearance of an additional generatior of the scale transformations, which is proportional to a new fixed dimensionless parameter r. Since the scale transformations commute with the Lorentz boosts and $3D$ rotations, the result of integration of eqs. (11) is a priori clear:

$$x'^i = D(r, \boldsymbol{\nu};\, \boldsymbol{n}, \alpha)\Lambda^i_k(\boldsymbol{\nu}; \boldsymbol{n}, \alpha)\, x^k\,, \tag{12}$$

where $D(r, \boldsymbol{\nu};\, \boldsymbol{n}, \alpha) = e^{-r\,\boldsymbol{\nu n}\,\alpha} I$, whereby I is a unit matrix while $\Lambda^i_k(\boldsymbol{\nu}; \boldsymbol{n}, \alpha)$ are matrices which make up the group (2). As for the law of group composition for the group (12), it is given by eqs. (5) and (6) obtained for the group (2). We note incidentally that eq. (5) yields the relation $\boldsymbol{\nu n}\,\alpha = \boldsymbol{\nu n}_1\alpha_1 + \boldsymbol{\nu n}_2\alpha_2$. Thus the group of the generalized Lorentz transformations (12), on the one hand, is locally isomorphic to the corresponding 3-parameter subgroup (2) of Lorentz group and, on the other hand, it is a homogeneous 3-parameter noncompact subgroup of the similitude group [14]. Since (according to (12)) in passing to the primed inertial frame the time (x^0) and space ($\boldsymbol{x}$) event coordinates are subjected to identical additional dilatations D, then the velocity $\boldsymbol{v}$ of the primed frame is related to the group parameters $\boldsymbol{n}, \alpha$ by the same eqs. (7)–(9) as in the case $r = 0$ where the group (12) coincides with the subgroup (2) of Lorentz group. For the same reason the transformations (12) retain valid Einstein's law of addition of 3-velocities, written as (10). As for the reparametrization of the group (12), then, for example, the matrix D, involved in (12), takes the following form in terms of $\boldsymbol{v}$:

$$D(r, \boldsymbol{\nu};\, \boldsymbol{v}) = \left(\frac{1 - \boldsymbol{v\nu}}{\sqrt{1 - \boldsymbol{v}^2}}\right)^r I\,. \tag{13}$$

Unlike the transformations (2), 3-parameter group of the generalized Lorentz boosts (12) conformly modifies Minkowski metric but leaves invariant the metric

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)\,. \tag{14}$$

The given Finslerian metric generalizes Minkowski metric and describes the relativistically invariant flat space of events with partially broken $3D$ isotropy. The inhomogeneous isometry group of space (14) is 8-parameter: four parameters correspond to space-time translations, one parameter, to rotations about the physically preferred direction $\boldsymbol{\nu}$, and three parameters, to the generalized Lorentz boosts.

Now turn to the construction of bispinor representation of the group of the generalized Lorentz boosts (12). Since the group (12) is locally isomorphic to the 3-parameter subgroup (2) of Lorentz group, its bispinor representation must also be locally isomorphic to the bispinor representation of the subgroup (2). This signifies that the transformations $x'^i = D\Lambda^i_k x^k$ of the event coordinates should be accompanied by the following transformations of a bispinor field: $\Psi'(x') = D^d S\Psi(x)$, $\overline{\Psi}'(x') = \overline{\Psi}(x)D^d S^{-1}$. Here the group of matrices S, operating on the bispinor indices, represents the subgroup (2) of the Lorents matrices Λ^i_k while D^d denotes the corresponding additional scale transformations of bispinors, in which case the unit matrix, involved in the definition (13), operates on the bispinor indices in this context. Since $d^4x' = |D\Lambda^i_k| d^4x = D^4 d^4x$ and matrices S satisfy the standard condition $S^{-1}\gamma^n S = \Lambda^n_m \gamma^m$, then, proceeding from the generalized Lorentz invariance of action for a free massless field Ψ, it is easy to show that $d = -3/2$. As a result the bispinor representation of the group of generalized Lorentz boosts (12) is realized by the transformations

$$\Psi' = D^{-3/2} S\Psi\,, \qquad \overline{\Psi}' = \overline{\Psi} D^{-3/2} S^{-1} \tag{15}$$

and it remains to find a 3-parameter group of the matrices $S = S(\boldsymbol{\nu}; \boldsymbol{n}, \alpha)$. For this purpose, using the 4-vectors $\nu^l = (1,\, \boldsymbol{\nu})$; $\nu_l = (1,\, -\boldsymbol{\nu})$; $n^l = (0,\, \boldsymbol{n})$; $n_l = (0,\, -\boldsymbol{n})$, first rewrite the infinitesimal transformations (1) in the form $dx^i = \omega^i{}_k x^k$, where $\omega^i{}_k = (\nu^i n_k - n^i \nu_k)d\alpha$, in which case $-\omega_{ki} = \omega_{ik} = (\nu_i n_k - \nu_k n_i)d\alpha$. Thus, in the vicinity of the identical transformation the matrices Λ^i_k take the form $\Lambda^i_k(d\alpha) = \delta^i_k + \omega^i{}_k$. Respectively, the $S(d\alpha) = 1 + \frac{1}{8}(\gamma^i\gamma^k - \gamma^k\gamma^i)\omega_{ik}$. Considering that $n_0 = 0$ and $\nu_0 = 1$, the latter relation leads to S in the form:

$$S(\boldsymbol{\nu}; \boldsymbol{n}, \alpha) = e^{\{\cdots\}\alpha/2}\,. \tag{16}$$

Here and below, we use the notation $\{\cdots\}$ for the sum of generators of Lorents boosts and 3-rotations about the vector $[\boldsymbol{\nu}\boldsymbol{n}]$, that is

$$\{\cdots\} = -\,\gamma^0\boldsymbol{\gamma}\boldsymbol{n} - i\,\boldsymbol{\Sigma}[\boldsymbol{\nu}\boldsymbol{n}]\,, \tag{17}$$

where $\gamma^0\,, \boldsymbol{\gamma}$ are the Dirac matrices, $\boldsymbol{\Sigma} = diag\,(\,\boldsymbol{\sigma}\,, \boldsymbol{\sigma}\,)$ and $\boldsymbol{\sigma}$ are the Pauli matrices. With the aid of the algebra of γ matrices one can find that

$$\begin{array}{lll} \{\cdots\}^2 = (\boldsymbol{\nu}\boldsymbol{n})^2 I\,, & \{\cdots\}^4 = (\boldsymbol{\nu}\boldsymbol{n})^4 I\,, & \cdots\,; \\ \{\cdots\}^3 = (\boldsymbol{\nu}\boldsymbol{n})^3\,\{\cdots\}\,/(\boldsymbol{\nu}\boldsymbol{n})\,, & \{\cdots\}^5 = (\boldsymbol{\nu}\boldsymbol{n})^5\,\{\cdots\}\,/(\boldsymbol{\nu}\boldsymbol{n})\,, & \cdots\,. \end{array}$$

These relations make it possible to represent (16) as

$$S(\boldsymbol{\nu}; \boldsymbol{n}, \alpha) = I\cosh\frac{\boldsymbol{\nu}\boldsymbol{n}\alpha}{2} - \frac{i\,\boldsymbol{\Sigma}[\boldsymbol{\nu}\boldsymbol{n}] + \gamma^0\boldsymbol{\gamma}\boldsymbol{n}}{\boldsymbol{\nu}\boldsymbol{n}}\sinh\frac{\boldsymbol{\nu}\boldsymbol{n}\alpha}{2}\,. \tag{18}$$

Now reparametrizing the $S(\boldsymbol{\nu}; \boldsymbol{n}, \alpha)$ with the aid of (8), (9) and using eqs. (15) and (13), we thus arrive at the following 3-parameter noncompact group of bispinor transformations in the axially symmetric flat Finslerian space (14):

$$\begin{aligned} \Psi' \;&=\; \frac{\left((1 - \boldsymbol{v}\boldsymbol{\nu})/\sqrt{1 - \boldsymbol{v}^2}\right)^{-3r/2}}{2\sqrt{(1 - \boldsymbol{v}\boldsymbol{\nu})\sqrt{1 - \boldsymbol{v}^2}}}\left\{\left(1 - \boldsymbol{v}\boldsymbol{\nu} + \sqrt{1 - \boldsymbol{v}^2}\right) I \right. \\ &\left. -\; i\,[\boldsymbol{\nu}\boldsymbol{v}]\boldsymbol{\Sigma} - \left(\boldsymbol{v} - (1 - \sqrt{1 - \boldsymbol{v}^2})\,\boldsymbol{\nu}\right)\gamma^0\boldsymbol{\gamma}\right\}\Psi\,. \end{aligned} \tag{19}$$

We note finally that an invariant of the transformations (19) is the Finslerian form $[(\nu_n\overline{\Psi}\gamma^n\Psi/\overline{\Psi}\Psi)^2]^{-3r/2}\overline{\Psi}\Psi$ but no longer the bilinear form $\overline{\Psi}\Psi$.

4. Conclusion

Having described the group of isometries of the axially symmetric Finslerian event space (14) and the bispinor representation (19) of its homogeneous 3-parameter subgroup, we have shown that in the case of spontaneous breaking of the initial gauge symmetry an axially symmetric fermion-antifermion condensate may arise. Such condensate violates Minkowski geometry and, hence, Lorentz symmetry. However relativistic symmetry remains valid. In the presence of condensate it turns out to be represented by the 3-parameter group of generalized Lorentz boosts. Since in all inertial reference frames, linked by the generalized Lorentz boosts, the parameters of the axially symmetric fermion-antifermion condensate remain invariant, any active generalized Lorentz transformation of fundamental fields against the background of such condensate is fully equivalent to the corresponding passive transformation at which the condensate together with fundamental fields is considered from another inertial frame. As a result the generalized Lorentz symmetry is relativistic symmetry of space-time filled with anisotropic fermion-antifermion condensate. In this case its role is equally constructive as the role of usual Lorentz symmetry in the standard theory of fundamental interactions. In particular, the principle of generalized Lorentz invariance makes it possible to take exactly into account the influence of axially symmetric condensate on the dynamics of fundamental fields. Phenomenologically this significantly decreases (compared with SME) the number of free parameters which describe possible effects of Lorentz symmetry violation.

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The Law of a Composition of Physical Velocities in Locally Anisotropic Finsler Space-Time

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The signal method of Poincare of clocks synchronization is considered and new transformations of a time interval and a spatial distance of local space-time are obtained. Four are found in essence various such as two-dimensional locally (flat) anisotropic Finsler geometries with two scalar parameters and some invariant. Group properties of the law of a composition of unidirectional physical anisotropic velocities of the arbitrary signals are investigated.

1. Introduction

To signal method of clocks synchronization in relativistic to mechanics it was executed 107 years in 2005. For the first time this method has offered A. Poincare [1] by reviewing a problem of a simultaneity of distant events in an inertial system of a reference. Also A. Poincare [2] for the first time has considered a formalism of a four-dimensional space-time and has found all invariants of a Lorentz group. At last, G. Minkovsky [3] used A. Poincare's formalism and has offered local isotropic four-dimensional pseudoeuclidean space-time. The works of the French scientists are of great importance for relativistic mechanics and now are unduly belittled. It in spite of the fact that in 1904 the Kazan Society Physics and Mathematics awarded A. Poincare Lobachevsky gold medal. N.I. Lobachevsky was the rector of the Kazan Imperial University and in 1826 for the first time has unclosed non Euclidean geometry [4]. In particular this geometry is realized in Fock three-dimensional space of velocities [5]. A. Poincare and N.I. Lobachevsky stated, that physical phenomena can be described in terms of various geometries. On one of methods of expansion of pseudoeuclidean geometry the Finsler geometry which reference property is presence in local space-time of an anisotropy is grounded. We shall mark only monographies [6-13] where detailed reviews of examinations in this direction are reduced. The aim of the present work is more deep study of problems of a simultaneity of distant events in local Finsler space-time that allows to justify strictly physically Finsler structure of geometry and to find new transformations of time intervals and spatial distances between inertial systems of a reference. Here the case of two-dimensional space - time is used.

2. Definition of a simultaneity of distant events and an anisotropy of a speed of light

Let's consider three events interdependent by a light signal in spatial points *A* and *B* local reference system. Light propagation in the systems takes place along a solid rod with physical length, equal to a spatial distance dL_{AB}. Observations of clocks in points *A* and *B* are physical times T_A and T_B. Let from point *A* through an interval of time dT_1^A the signal which through an interval of time dT_2^B will arrive to point *B* is sent. Further the signal reflex from point *B*, through an interval of time dT_3^A will arrive to point *A*. According to A. Poincare [1] for reference standard clocks synchronization it is necessary to definition the ration of a metric simultaneity of event in point *A* with event in point *B* in the middle of a time interval $dT_3^A - dT_1^A$. Thus, we have a relation

$$dT_2^B - dT_1^A = dT_3^A - dT_2^B, \tag{2.1}$$

which gives equality of unidirectional physical speed of light $c_{AB} = c_{BA} = c_0$ along a solid rod. Thus the postulate about equality of scales of the distance measuring length of a solid rod in direct and inverse directions is fulfilled, and the experimental fact of a constancy of a light speed averaged ever closed paths is used

$$dL_{AB} = dL_{BA}\,,\ c_0 = \frac{dL_{AB} + dL_{BA}}{dT_3^A - dT_1^A} = \frac{2dL_{AB}}{dT_3^A - dT_1^A}\,. \tag{2.2}$$

Unidirectional physical speed of light has isotropic and invariant magniture.

According to G.Reichenbach and A.Grünbaum [14, 15] the events in a time interval $dT_3^A - dT_1^A$ is topologically simultaneous events to event in point *B*. The definition of a metric simultaneity is determined convection by a select of the arbitrary event from topologically simultaneous events.

Signal method of the clocks synchronization A. Poincare suggested for the first time, gives observable intervals of time in point *A*

$$dT_1^A = dT_2^B - dL_{AB}/c_0\,,\ dT_3^A = dT_2^B + dL_{AB}/c_0\,. \tag{2.3}$$

We distinguish some cases. In the first case of a proper time interval in point *B* is defined by expression

$$\begin{gathered} dT_0^B = \frac{1}{2}\sqrt{\left(dT_1^A + dT_3^A\right)^2 - \left(dT_1^A - dT_3^A\right)^2} = \\ = \sqrt{dT_1^A dT_3^A} = \sqrt{\left(dT - dL/c_0\right)\left(dT + dL/c_0\right)}. \end{gathered} \tag{2.4}$$

Where used indexes are dropped. From (2.4) one gets form-invariant length elements

$$ds^2 = c_0^2 dT_0^2 = c_0^2 dT^2 - dL^2, \tag{2.5}$$

in local Michelson to a reference system. For a Riemann geometry with a signature $(+,-,-,-)$ and length elements

$$ds^2 = g_{ij} dx^i dx^j \tag{2.6}$$

we have

$$dT = \sqrt{g_{00}}\left(dx^0 + \frac{g_{0\alpha} dx^\alpha}{g_{00}}\right), \tag{2.7}$$

$$dL^2 = \left(-g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}}\right) dx^\alpha dx^\beta, \tag{2.8}$$

where $dL = |dL| = \sqrt{dL^2}$, $x_0 = c_0 t$ and values of indexes $i = (0,1,2,3)\ \alpha = (1,2,3)$. In half-geodesist coordinates we have $g_{0\alpha} = 0$, $g_{00} = \pm 1$. For a determinant validly an inequality $|g_{ij}| < 0$.

For space-time of Minkowski in Galilean coordinates we have values

$$dT = dt\,,\ dL^2 = (d\vec{r})^2 = dx^2 + dy^2 + dz^2\,. \tag{2.9}$$

Here the interval of physical time coincides with an interval of coordinate time and the physical length is length of a local radius-vector $(d\vec{r})^2$ with coordinates (dx, dy, dz). Physical velocities of the arbitrary signals are equaled coordinates velocities. In a case from (2.7) and (2.8) these equalities are not fulfilled.

In the second case we shall consider a Riemann geometry variety with the length elements (2.6) having a signature $(+,+,+,+)$. Then we have relations

$$\begin{gathered} dT_0^B = \frac{1}{2}\sqrt{\left(dT_1^A + dT_3^A\right)^2 + \left(dT_1^A - dT_3^A\right)^2} = \\ = \sqrt{\frac{1}{2}\left[\left(dT_1^A\right)^2 + \left(dT_3^A\right)^2\right]} = \sqrt{dT^2 + dL^2/c_0^2}\,, \end{gathered} \tag{2.10}$$

$$ds^2 = c_0^2 dT_0^2 = c_0^2 dT^2 + dL^2, \tag{2.11}$$

$$dL^2 = \left(g_{\alpha\beta} - \frac{g_{0\alpha} g_{0\beta}}{g_{00}} \right) dx^\alpha dx^\beta . \tag{2.12}$$

And for a determinant validly an inequality $\left| g_{ij} \right| > 0$.

In the third case we shall put $dT_1^A = dT_3^A = dT_2^B$ and we get

$$dT_0^B = dT,\, ds^2 = c_0^2 dT_0^2 = c_0^2 dT^2 = c_0^2 \left[g_{00} \left(dx^0 + \frac{g_{0\alpha} dx^\alpha}{g_{00}} \right) \right]^2 . \tag{2.13}$$

The geometry variety with a determinant $\left| g_{ij} \right| = 0$ has a signature with some zero values.

One more case of Riemann geometry variety with a signature $(+,+,-,-)$demands separate reviewing.

The most general connection between time intervals will be noted so

$$\varepsilon_{12} dT_3^A + \varepsilon_{23} dT_1^A + \varepsilon_{31} dT_2^B = 0, \tag{2.14}$$

where ε_{12}, ε_{23} also ε_{31} there are constant elements of an antisymmetric time matrix of transition between events. At limit a point B we have

$$\lim_{A \to B} \left(\varepsilon_{12} dT_3^A + \varepsilon_{23} dT_1^A + \varepsilon_{31} dT_2^B \right) = \left(\varepsilon_{12} + \varepsilon_{23} + \varepsilon_{31} \right) dT_1^A = 0 . \tag{2.15}$$

As dT_1^A there is the arbitrary value we obtain

$$\varepsilon_{12} + \varepsilon_{23} + \varepsilon_{31} = 0 . \tag{2.16}$$

Thus, we have two independent parameters.

From (2.14) and (2.16) it is discovered the following equality

$$\frac{dT_2^B - dT_1^A}{\varepsilon_{12}} = \frac{dT_3^A - dT_2^B}{\varepsilon_{23}} = \frac{dT_1^A - dT_3^A}{\varepsilon_{13}} = \frac{dL_{AB}}{c_0}, \tag{2.17}$$

from which we shall receive values of the unidirectional anisotropic physical and a average speed of light

$$c_{AB} = c_+ = \frac{c_0}{\varepsilon_{12}},\; c_{BA} = c_- = \frac{c_0}{\varepsilon_{23}},\; c = \gamma c_0 = \frac{2c_0}{\varepsilon_{13}}, \tag{2.18}$$

$$\frac{1}{c_+} - \frac{1}{c_-} = \frac{2\varepsilon}{c},\; \frac{1}{c_+} + \frac{1}{c_-} = \frac{2}{c},\; \varepsilon = \frac{\varepsilon_{12} - \varepsilon_{23}}{\varepsilon_{13}}, \tag{2.19}$$

where ε there is a scalar parameter of a time anisotropy and γ - scalar parameter describing "index of refraction" for light. For average speed over closed paths the limit $\lim_{c_0 \to \infty} c/c_0 = 1$should be fulfilled. Unidirectional physical speed of light has non-isotropic and non-invariant magniture. The case with $\varepsilon_{12} = \varepsilon_{23} = 0$ corresponds to a absolute simultaneity of classical physics in which the signal method of Poincare misses.

Observable time intervals in a point A are equaled

$$dT_1^A = dT_2^B - dL_{AB}/c_+ ,\; dT_3^A = dT_2^B + dL_{AB}/c_- . \tag{2.20}$$

The value $c_{AB} = c_+$ defines a speed of light sent of a point A in a point B, and $c_{AB} = c_+$ - a speed of light sent from a point B in a point A of a solid rod. It means, that in a point A the speed of light sent from a point A in an opposite direction from a point B is not defined. Similarly, in a point B the speed of light sent from a point A in an opposite direction from a point A is not defined.

At $\varepsilon_{13}=2$ and $\varepsilon_{12}=\varepsilon_{23}=1$ from considered general nonstandard of clock synchronization we have standard of clock synchronization on Poincare. Transformations of an grief of space-time coordinates do not eliminate a physical anisotropy of a speed of light. The coordinate anisotropy of a velocity dx^{α}/dx^{0} in a Riemann geometry variety with (2.6) for isotropic geodesic is eliminated by transformations of a grief of space-time coordinates if dT there is a total differential. As against works [16-18] where are reduced for the first time a relation (2.14) for instants time, here we have a relation (2.14) for time intervals.

3. Types of Finsler geometries

Let's consider transformations of a time interval and a spatial distance at transition between local systems (K) and (K'). In a system (K') we have speeds of a light

$$c'_{+}=\frac{c_0}{\varepsilon'_{12}},\ c'_{-}=\frac{c_0}{\varepsilon'_{23}},\ c'=\gamma' c_0=\frac{2c_0}{\varepsilon'_{13}}, \tag{3.1}$$

$$\frac{1}{c'_{+}}-\frac{1}{c'_{-}}=\frac{2\varepsilon'}{c'},\ \frac{1}{c'_{+}}+\frac{1}{c'_{-}}=\frac{2}{c'},\ \varepsilon'=\frac{\varepsilon'_{12}-\varepsilon'_{23}}{\varepsilon'_{13}}. \tag{3.2}$$

For obviousness we shall accept, that the element of a solid rod is located along the positive direction $dx^{1'}$ $\left(dx^{2'}=dx^{3'}=0\right)$. Consider system (K') which moves relative to systems (K). The physical length of a device of the element of a solid rod located along the positive direction $dx^{1'}$ is absolute value $dX'=\left|dX'\right|$. The direction $dx^{1'}$ coincides with a direction dx^{1}.

Let's consider the first case. The transformation can be defined by the "k" coefficient method [18]. Let us write the relations

$$\left(c'/c_0\right)^{1/2}\left(dT'-dX'/c'_{+}\right)=k_{+}\left(c/c_0\right)^{1/2}\left(dT-dX/c_{+}\right), \tag{3.3}$$

$$\left(c'/c_0\right)^{1/2}k'_{-}\left(dT'+dX'/c'_{-}\right)=\left(c/c_0\right)^{1/2}\left(dT+dX/c_{-}\right). \tag{3.4}$$

In other cases of a disposition of a element of a solid rod in systems (K) and (K') in the relations (3.3) and (3.4) we obtain other values a speed of light also. Coefficients $k_{+}\left(c/c'\right)^{1/2}$ also $k'_{-}\left(c'/c\right)^{1/2}$ describe the Doppler Effect in direct and inverse directions. From (3.3) and (3.4) one can get the following relations

$$\begin{aligned} k'_{-}\frac{c'}{c_0}&\left[dT'^2-\left(\frac{1}{c'_{+}}-\frac{1}{c'_{-}}\right)dT'dX'-\left(\frac{1}{c'_{+}c'_{-}}\right)dX'^2\right]=\\ &=k_{+}\frac{c}{c_0}\left[dT^2-\left(\frac{1}{c_{+}}-\frac{1}{c_{-}}\right)dTdX-\left(\frac{1}{c_{+}c_{-}}\right)dX^2\right], \end{aligned} \tag{3.5}$$

$$k_{+}k'_{-}\frac{dT'+dX'/c'_{-}}{dT'-dX'/c'_{+}}=\frac{dT+dX/c_{-}}{dT-dX/c_{+}}. \tag{3.6}$$

At $dX'=0$ also $dX=0$ we have, accordingly, $dX=v_{+}dT$ and $dX'=-v'_{-}dT'$ where $v_{+}=\left|v_{+}\right|$ and $v'_{-}=\left|v'_{-}\right|$ there are the relative unidirectional velocities of systems. From (3.6) one gets the equality

$$k_{+}k'_{-}=k^2=\frac{1+v_{+}/c_{-}}{1-v_{+}/c_{+}}=\frac{1+v'_{-}/c'_{+}}{1-v'_{-}/c'_{-}}, \tag{3.7}$$

to define correlation between velocities

$$c\left[\frac{1}{v_+}-\frac{1}{c_+}\right]=c'\left[\frac{1}{v'_-}-\frac{1}{c'_-}\right]. \tag{3.8}$$

At $dX/dT = u_+ = |u_+|$ and $dX'/dT' = u'_+ = |u'_+|$ we obtain equality

$$\frac{1-v_+/c_+}{1+v_+/c_-}\,\frac{1-u'_+/c'_+}{1+u'_+/c'_-}=\frac{1-u_+/c_+}{1+u_+/c_-}. \tag{3.9}$$

Consider nonstandard clock synchronization at $\varepsilon=\varepsilon'$. From (3.9) we have the law of a composition of the dimensionless unidirectional anisotropic velocities

$$\left(\frac{u_+}{c}\right)=\left(\frac{u'_+}{c'}\right)\circ\left(\frac{v_+}{c}\right)=\frac{u'_+/c'+v_+/c-2\varepsilon u'_+v_+/c'c}{1+\left(1-\varepsilon^2\right)u'_+v_+/c'c}, \tag{3.10}$$

which set forms Abelian group.

Determinants direct and inverse transformations from relations (3.3) and (3.4), are equaled $A=k_+/k'_-$ and $A'=k'_-/k$ $(AA'=1)$. Taking into account (3.7), we have values

$$k_+=\sqrt{A}\,k,\; k'_-=\sqrt{A'}\,k, \tag{3.11}$$

where $A=A(v_+)$, as well as $A'=A(v'_-)$, has group property

$$A(u_+)=A(u'_+)A(v_+). \tag{3.12}$$

Using the law of a composition as (3.9) and equality (3.12), we obtain the equation

$$\left(1-v_+/c_+\right)\left(1+v_+/c_-\right)\frac{d\ln A}{dv_+}=-2r. \tag{3.13}$$

The invariant parameter r can depend on invariant values c_+ and c_-. Integrating (3.13) under condition of $A(0)=1$, we have expression $A(v_+)$, transformations and quadrate of form-invariant metric function in the following types of local Finsler geometries.

Type I $(\varepsilon_{12}\neq\varepsilon_{23})$.

$$A(v_+)=\left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r}, \tag{3.14}$$

$$\frac{dX'}{\sqrt{c'}}=\sqrt{\frac{A(v_+)}{c}}\,\frac{dX-v_+dT}{\alpha_+}\qquad \alpha_+=\left[\left(1-\frac{v_+}{c_+}\right)\left(1+\frac{v_+}{c_-}\right)\right]^{1/2}, \tag{3.15}$$

$$\sqrt{c'}\,dT'=\frac{\sqrt{A(v_+)c}}{\alpha_+}\left\{dT\left[1-\frac{(\varepsilon+\varepsilon')v_+}{c}\right]-dX\left[\frac{v_+}{c_+c_-}+\frac{\varepsilon-\varepsilon'}{c}\right]\right\}, \tag{3.16}$$

$$F^2=cc_0\left(\frac{dT-dX/c_+}{dT+dX/c_-}\right)^r\left(dT-dX/c_+\right)\left(dT+dX/c_-\right)=$$

$$=cc_0\left(\frac{dT-(1+\varepsilon)dX/c}{dT+(1-\varepsilon)dX/c}\right)^r\left[dT^2-\frac{2\varepsilon dTdX}{c}-\frac{\left(1-\varepsilon^2\right)dX^2}{c^2}\right],$$

(3.17)

Type II $(\varepsilon_{12}+\varepsilon_{23}=\ \varepsilon'_{12}+\varepsilon'_{23}=0)$.

$$A(v_+)=\exp\left(-\frac{2rv_+}{1-v_+/c_+}\right), \tag{3.18}$$

$$dX' = \sqrt{A(v_+)}\left(dX - v_+ dT\right)/\alpha_+ , \ \alpha_+ = 1 - v_+/c_+ , \tag{3.19}$$

$$dT' = \frac{\sqrt{A(v_+)}}{\alpha_+}\left\{dT\left[1 - v_+\left(\frac{1}{c_+} + \frac{1}{c'_+}\right)\right] - dX\left[-\frac{v_+}{c_+^2} + \left(\frac{1}{c_+} - \frac{1}{c'_+}\right)\right]\right\}, \tag{3.20}$$

$$F^2 = c_0^2 \exp\left(\frac{2rdX}{dT - dX/c_+}\right)\left(dT - dX/c_+\right)^2 . \tag{3.21}$$

Value $A(v_+)$ and transformations to type II imply from formulas (3.13), (3.15)-(3.17) in type I it is formal at $c_+ = -c_-$.

Type III $(\varepsilon_{12} \neq \varepsilon_{23})$.

$$A(v_+) = \exp\left(-2r \cdot \mathrm{arctg}\frac{v_+/c}{1 - \varepsilon v_+/c}\right), \tag{3.22}$$

$$\frac{dX'}{\sqrt{c'}} = \sqrt{\frac{A(v_+)}{c}}\frac{dX - v_+ dT}{\alpha_+}, \ \alpha_+ = \left[1 - \frac{2\varepsilon v_+}{c} + \frac{\left(1+\varepsilon^2\right)v_+^2}{c^2}\right]^{1/2}, \tag{3.23}$$

$$\sqrt{c'}\,dT' = \frac{\sqrt{A(v_+)c}}{\alpha_+}\left\{dT\left[1 - \frac{(\varepsilon + \varepsilon')v_+}{c}\right] - dX\left[-\frac{v_+\left(1+\varepsilon^2\right)}{c^2} + \frac{\varepsilon - \varepsilon'}{c}\right]\right\}, \tag{3.24}$$

$$F^2 = \frac{1}{2}cc_0\left\{\exp\left[2r\,\mathrm{arctg}\frac{\left(dT + dX/c_-\right) - \left(dT - dX/c_+\right)}{\left(dT + dX/c_-\right) + \left(dT - dX/c_+\right)}\right]\right\}\times$$
$$\times\left[\left(dT - dX/c_+\right)^2 + \left(dT + dX/c_-\right)^2\right] = \quad . \tag{3.25}$$
$$= cc_0\left[\exp\left(-2r\,\mathrm{arctg}\frac{dX}{cdT - \varepsilon dX}\right)\right]\left[dT^2 - \frac{2\varepsilon dTdX}{c} + \frac{\left(1+\varepsilon^2\right)dX^2}{c^2}\right]$$

Type IV $(\varepsilon_{12} = \varepsilon_{23} = \varepsilon'_{12} = \varepsilon'_{23} = 0)$.

$$A(v) = \exp(-2rv), \tag{3.26}$$

$$dX' = \sqrt{A(v)}\,(dX - vdT), \ dT' = \sqrt{A(v)}\,dT , \tag{3.27}$$

$$F^2 = c_0^2\left[\exp(2r\,dX/dT)\right]dT^2 . \tag{3.28}$$

Formulas for type III are obtained on the results of work [18]. Formulas for type IV are obtained from relations (3.18)-(3.21) in type II at $\varepsilon_{12} = \varepsilon_{23} = 0$. At $\varepsilon' = \varepsilon$ and $c' = c$ the first three types correspond for defined values r and c to three types of local Finsler geometries with an indicatrix of a constant value of the curvature, surveyed in work [19].

Consider a case with $r = r(c_+, c_-)$ for which of proper time interval in type I has a form

$$dT_0 = \left(dT - dX/c_+\right)^{\frac{1+r}{2}}\left(dT + dX/c_-\right)^{\frac{1-r}{2}} . \tag{3.29}$$

The equality $dT_0 = dT$ corresponds to Galilean geometry and takes place at $\varepsilon_{12} = \varepsilon_{23} = 0$ if relations are fulfilled

$$\frac{1+r}{2} = \frac{c_+}{c_+ + c_-} = \frac{c}{2c_-}, \ \frac{1-r}{2} = \frac{c_-}{c_+ + c_-} = \frac{c}{2c_+} . \tag{3.30}$$

From (3.30) we obtain invariant parameter

$$r = \frac{c_+ - c_-}{c_+ + c_-} = -\varepsilon . \tag{3.31}$$

Hence, the proper time interval will become

$$dT_0 = \left(\frac{dT - dX/c_+}{dT + dX/c_-} \right)^{(c_+ - c_-)/2(c_+ + c_-)} \sqrt{(dT - dX/c_+)(dT + dX/c_-)} = \tag{3.32}$$
$$= (dT - dX/c_+)^{c_+/(c_+ + c_-)} (dT + dX/c_-)^{c_-/(c_+ + c_-)}.$$

The quadrate of Finsler metric function will be noted so

$$F^2 = \frac{2c_0^2}{\varepsilon_{13}} (dT_1)^{2c_+/(c_+ + c_-)} (dT_3)^{2c_-/(c_+ + c_-)} = cc_0 dT_0^2 = \frac{2c_+ c_-}{c_+ + c_-} c_0 dT_0^2 . \tag{3.33}$$

In work [13], [19] and [20] the anisotropy of a physical velocity of light ($c_+ \neq c_- \neq c_0$, $c_+ c_- = c_0^2$, $r = -\varepsilon$) for quadrate of Finsler metric function (3.33) without coefficient $2/\varepsilon_{13}$ is considered. The case of an anisotropy of a coordinate speed of light with $r = -\varepsilon$ is explored in [21].

For a case $r = 0$ in type I we have

$$F^2 = \frac{2c_0^2}{\varepsilon_{13}} (dT_1)(dT_3) = cc_0 dT_0^2 = \frac{2c_+ c_-}{c_+ + c_-} c_0 (dT - dX/c_+)(dT + dX/c_-). \tag{3.34}$$

In a case $r \neq 0$ also $c_+ = c_- = c_0$ we shall receive

$$F^2 = \left[\frac{(c_0 dT - dX)^2}{c_0^2 dT^2 - dX^2} \right]^r (c_0^2 dT^2 - dX^2). \tag{3.35}$$

Generalization of expression (3.35) with the account (1.6) - (1.8) for a four-dimensional space-time is

$$F^2 = \left[\frac{(c_0 dT - dL)^2}{c_0^2 dT^2 - dL^2} \right]^r (c_0^2 dT^2 - dL^2) = \left[\frac{(c_0 dT - dL)^2}{g_{ij} dx^i dx^j} \right]^r g_{ij} dx^i dx^j . \tag{3.36}$$

As against work [22] in (3.36) is not a four-dimensional vector ν_i with $\nu_i \nu^i = 0$. Generalization of results of work [22] on a case of an anisotropy of a coordinate speed of light is given in [32]. It is necessary to note, that adding to surveyed transformations of two $dY'/\sqrt{c'} = dY \sqrt{A(v_+)/c}$ more and $dZ'/\sqrt{c'} = dZ \sqrt{A(v_+)/c}$ do not reduce in replacement $dX \to dL$ in the metric functions.

In Galilean coordinates we have quadrate of Finsler metric function

$$F^2 = \left[\frac{\left(c_0 dt - \sqrt{d\vec{r}^{\,2}}\right)^2}{c_0^2 dt^2 - d\vec{r}^{\,2}} \right]^r (c_0^2 dt^2 - d\vec{r}^{\,2}), \tag{3.37}$$

demanding separate reviewing.

4. The law of a composition of physical unidirectional anisotropic velocities

Let in systems the invariant anisotropy of speeds of a light signal is fulfilled. ($\varepsilon_{12} = \varepsilon'_{12}$, $\varepsilon_{23} = \varepsilon'_{23}$ and $\varepsilon_{13} = \varepsilon'_{13}$) From (3.14)-(3.16) we have direct and inverse transformations in type I

$$dX' = \left(\frac{1 + v_+/c_-}{1 - v_+/c_+} \right)^{-r/2} \frac{dX - v_+ dT}{\alpha_+}, \quad \alpha_+ = \left[\left(1 - \frac{v_+}{c_+}\right)\left(1 + \frac{v_+}{c_-}\right) \right]^{-1/2}, \tag{4.1}$$

$$dT' = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r/2} \frac{1}{\alpha_+}\left\{dT\left[1-\frac{2\varepsilon v_+}{c}\right] - dX\frac{v_+}{c_+c_-}\right\}, \qquad (4.2)$$

$$dX = \left(\frac{1-v'_-/c_-}{1+v'_-/c_+}\right)^{-r/2} \frac{dX' + v'_- dT'}{\alpha'_-}, \; \alpha'_- = \left[\left(1+\frac{v'_-}{c_+}\right)\left(1-\frac{v'_-}{c_-}\right)\right]^{1/2}, \qquad (4.3)$$

$$dT = \left(\frac{1-v'_-/c_-}{1+v'_-/c_+}\right)^{-r/2} \frac{1}{\alpha'_-}\left\{dT'\left[1+\frac{2\varepsilon v'_-}{c'}\right] + dX'\frac{v'_-}{c_+c_-}\right\}, \qquad (4.4)$$

where the relative velocities satisfy to equality

$$\frac{1}{v_+} - \frac{1}{v'_-} = \frac{1}{c_+} - \frac{1}{c_-}. \qquad (4.5)$$

The law of a composition of unidirectional absolute anisotropic velocities looks like

$$u_+ = u'_+ \circ v_+ = \frac{u'_+ + v_+ - 2\varepsilon u'_+ v_+/c}{1+\left(1-\varepsilon^2\right)u'_+ v_+/c^2}. \qquad (4.6)$$

Let's consider a third system (K'') which moves along the positive direction with a velocity w_+ and z'_+ concerning systems (K) and (K'), accordingly. Then using transformations between (K'') and (K'), we shall finally receive the law of a composition of the absolute unidirectional anisotropic velocities

$$w_+ = z'_+ \circ v_+ = \frac{z'_+ + v_+ + z'_+ v_+\left(1/c_- - 1/c_+\right)}{1+z'_+ v_+/c_+c_-} = \frac{z'_+ + v_+ - 2\varepsilon z'_+ v_+/c}{1+\left(1-\varepsilon^2\right)z'_+ v_+/c^2}. \qquad (4.7)$$

The set of absolute velocities forms Abelian group with the commutative law of a composition of elements of group $z'_+ \circ v_+ = v_+ \circ z'_+$.

For the law property of an associativity is fulfilled

$$r''_+ \circ z'_+ \circ v_+ = \left(r''_+ \circ z'_+\right) \circ v_+ = r''_+ \circ \left(z'_+ \circ v_+\right) =$$
$$= \frac{r''_+ + z'_+ + v_+ - 2\varepsilon\left(r''_+ z'_+ + z'_+ v_+ + v_+ r''_+\right)/c + 4\varepsilon^2 r''_+ z' v_+/c^2}{1-\left(1-\varepsilon^2\right)\left(r''_+ z'_+ + z'_+ v_+ + v_+ r''_+\right)/c^2 - 2\varepsilon\left(1-\varepsilon^2\right) r''_+ z' v_+/c^3}. \qquad (4.8)$$

The unity element of group is discovered from the formula

$$E \circ v_+ = \frac{E + v_+ + \left(1/c_- - 1/c_+\right)E v_+}{1+E v_+/\left(c_+ c_-\right)} = v_+. \qquad (4.9)$$

Thus, the unity element corresponds to a value $v_+ = 0$.

From the law of a composition

$$v_+ \circ v_+^{-1} = \frac{v_+ + v_+^{-1} - 2\varepsilon v_+ v_+^{-1}/c}{1+\left(1-\varepsilon^2\right)v_+ v_+^{-1}/c^2} = E, \; \left(1+\left(1-\varepsilon^2\right)v_+ v_+^{-1}/c^2 \neq 0\right) \qquad (4.10)$$

expression of an inverse element follows

$$v_+^{-1} = -\frac{v_+}{1-2\varepsilon v_+/c}. \qquad (4.11)$$

Elements of group are selfconjugat.

Using (4.5)-(4.11) we shall write out some equalities

$$\frac{1}{\left(-v_+\right)} - \frac{1}{\left(v_+^{-1}\right)} = \frac{1}{c_-} - \frac{1}{c_+}, \; \left[1-\frac{2\varepsilon v_+}{c}\right]\left[1+\frac{2\varepsilon\left(v_+^{-1}\right)}{c}\right] = 1, \qquad (4.12)$$

$$\frac{1}{1+\left(1-\varepsilon^2\right)v_+v_+^{-1}/c^2}=\frac{1-2\varepsilon v_+/c}{1-2\varepsilon v_+/c+\left(1-\varepsilon^2\right)\left(v_+\right)^2/c^2}=\frac{1-2\varepsilon\left(v_+^{-1}\right)/c}{1-2\varepsilon\left(v_+^{-1}\right)/c+\left(1-\varepsilon^2\right)\left(v_+^{-1}\right)^2/c^2},\tag{4.13}$$

$$\left(-c_-\right)\circ v_+=\left(-c_-\right),\ c_+\circ v_+=c_+,\tag{4.14}$$

$$1-2\varepsilon w_+/c-\left(1-\varepsilon^2\right)w_+^2/c^2=$$
$$=\frac{\left[1-2\varepsilon z'_+/c-\left(1-\varepsilon^2\right)\left(z'_+\right)^2/c^2\right]\left[1-2\varepsilon v_+/c-\left(1-\varepsilon^2\right)\left(v_+\right)^2/c^2\right]}{\left[1+\left(1-\varepsilon^2\right)z'_+v_+/c^2\right]^2},\tag{4.15}$$

$$\frac{w_+}{\sqrt{1-2\varepsilon w_+/c-\left(1-\varepsilon^2\right)w_+^2/c^2}}=$$
$$=\frac{z'_++v_+-2\varepsilon z'_+v_+/c}{\sqrt{1-2\varepsilon z'_+/c-\left(1-\varepsilon^2\right)\left(z'_+\right)^2/c^2}\sqrt{1-2\varepsilon v_+/c-\left(1-\varepsilon^2\right)\left(v_+\right)^2/c^2}},\tag{4.16}$$

$$1-2\varepsilon z'_+/c-\left(1-\varepsilon^2\right)z'_+w_+=\frac{1-2\varepsilon z'_+/c-\left(1-\varepsilon^2\right)\left(z'_+\right)^2/c^2}{1+\left(1-\varepsilon^2\right)z'_+v_+/c^2},\tag{4.17}$$

$$1-2\varepsilon v_+-\left(1-\varepsilon^2\right)v_+w_+=\frac{1-2\varepsilon v_+/c-\left(1-\varepsilon^2\right)\left(v_+\right)^2/c^2}{1+\left(1-\varepsilon^2\right)z'_+v_+/c^2},\tag{4.18}$$

$$1-2\varepsilon w_+/c-\left(1-\varepsilon^2\right)w_+^2/c^2=$$
$$=\left[1-2\varepsilon z'_+/c-\left(1-\varepsilon^2\right)z'_+w_+/c^2\right]\left[1-2\varepsilon v_+/c-\left(1-\varepsilon^2\right)v_+w_+/c^2\right],\tag{4.19}$$

$$\left[1+\left(1-\varepsilon^2\right)\left(z'_+\right)^2/c^2\right]\left[1+\left(1-\varepsilon^2\right)\left(v_+\right)^2/c^2\right]\left[1+\left(1-\varepsilon^2\right)\left(z'_+\circ z'_+\right)\left(v_+\circ v_+\right)/c^4\right]=$$
$$=\left[1+\left(1-\varepsilon^2\right)z'_+v_+/c^2\right]^2\left[1+\left(1-\varepsilon^2\right)w_+^2/c^2\right].\tag{4.20}$$

The parameter of an anisotropy ε demonstrated difference of an inverse device v_+^{-1} from opposite $\left(-v_+\right)$. Speeds of light c_+ also c_- have no inverse elements c_+^{-1} and c_-^{-1} by virtue of violation of a side condition in (4.10). Therefore they do not enter a set of velocities and (4.14) there is a formal equaility. From (4.6) we get the law of a composition

$$u'_+=u_+\circ v_+^{-1}=\frac{u_+-v_+}{1-2\varepsilon v_+/c-\left(1-\varepsilon^2\right)u_+v_+/c^2}.\tag{4.21}$$

Also it is represented in direct transformations through velocities u_+ and v_+. In inverse transformations, according to (3.15), the equality $v_+^{-1}=-v'_-$ is valid. From (4.6) we have relations

$$\left(1+\frac{u_+}{c_-}\right)=\frac{\left(1+u'_+/c_-\right)\left(1+v_+/c_-\right)}{1+u'_+v_+/\left(c_+c_-\right)},\ \left(1-\frac{u_+}{c_+}\right)=\frac{\left(1-u'_+/c_+\right)\left(1-v_+/c_+\right)}{1+u'_+v_+/\left(c_+c_-\right)}.\tag{4.22}$$

In work [20, 21, 23-25] the law of a composition in form (4.7) from the various points of view was considered.

The law of a composition of the unidirectional anisotropic velocities (4.7) for type I one gets from equality (3.9). In case of types II and III we have equalities

$$\frac{u'_+}{1-u'_+/c_+}+\frac{v_+}{1-v_+/c_+}=\frac{u_+}{1-u_+/c_+},\tag{4.23}$$

$$\frac{\frac{u'_+}{1-\varepsilon u'_+/c}+\frac{v_+}{1-\varepsilon v_+/c}}{1-\frac{u'_+\quad v_+}{(1-\varepsilon u'_+/c)(1-\varepsilon v_+/c)}}=\frac{u_+}{1-\varepsilon u_+/c}, \tag{4.24}$$

from which the following laws of compositions imply

$$u_+=u'_+\circ v_+=\frac{u'_+ + v_+ - 2u'_+v_+/c_+}{1-u'_+\,v_+/c_+^2}, \tag{4.25}$$

$$u_+=u'_+\circ v_+=\frac{u'_+ + v_+ - 2\varepsilon u'_+v_+/c}{1+(1-\varepsilon^2)u'_+\,v_+/c^2}. \tag{4.26}$$

For type IV we have the traditional law of a velocity addition in classical physics

$$u=u'+v. \tag{4.27}$$

At $r=0$ also $\varepsilon=0$ in types I, III and IV we have laws of a composition of velocities in flat geometries to which the postulate parallel is maintained [26].

5. Conclusions

In work the locally anisotropic (flat) Finsler geometry with two scalar parameters $\varepsilon c_0/c=(\varepsilon_{12}-\varepsilon_{23})/2$ and $c_0/c=(\varepsilon_{12}+\varepsilon_{23})/2$, depending from elements of a time matrix of transition between events, and also from an invariant r is considered. Four types are found in essence various such as two-dimensional Finsler space-time. Group properties of a composition of equally directional anisotropic velocities of the arbitrary signals are explored. The anisotropy of physical speeds of light is not eliminated by any transformations of a grief of space-time coordinates, than the obtained new transformations of a time interval and a spatial distance differ from some known transformations for coordinate representation of an anisotropy [25, 27-32]. To a problem of a simultaneity work [33] is devoted to the review of such approaches. It is necessary to note also and attempt of the experimental detection of the relative anisotropy of unidirectional velocities of light and neutrons [34].

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CMC surfaces in Finsler framework - the DPW approach

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Abstract. The first section of the paper contains a brief outlook on Finsler spaces and recent advances in the theory of Finsler CMC and minimal surfaces. §2 presents a recent approach which aims to find moduli for minimal and CMC surfaces in the Euclidean space, using techniques of complex analysis and loop groups, called the DPW method. A first step to develop this method for imbedded surfaces in Finsler 3-dimensional spaces (M, F), is to derive the equations of the moving frames and the corresponding integrability conditions. This is done by the present paper for the case of mean curvature in the sense of H. Rund.

In §3, after describing the induced connection coefficients, the shape operators (horizontal and vertical) and the $h-$ and $v-$ parts of the second fundamental form are obtained the equations of moving frames and the three sets of integrability conditions are explicitely derived in terms of the deflection tensors. The obtained results are shown to hold true more generally, for ambient generalized Lagrange spaces, and for Lagrange spaces $(M, \mathcal{L})$ ([10]), where the metric is provided by L via $g_{ij}(x, y) = \frac{1}{2}\frac{\partial^2\mathcal{L}}{\partial y^i\partial y^j}$ endowed with the Kern the non-linear connection.

A particular case is when $(M, g_{ij}(x))$ is a (pseudo-/)Riemannian space. A fulfilled natural claim is that, applied to the Euclidean case $(M = R^3, \delta_{ij})$, the moving frame equations obtained provide the known corresponding result of the DPW method. In this flat case one can apply the DPW method for obtaining minimal, CMC or constant Gauss curvature surfaces ([7], [8], [6]). Open problems in the general DPW Finsler case are still: the characterisation of minimality in terms of harmonicity of a convenient adapted Gauss map to a homogeneous reductive space G/K, the construction of the lift of frames to the universal cover $\tilde{G}$ of G, and the Birkhoff and Iwasawa-type decompositions which are prerequisites to finding the moduli of CMC/minimal surfaces and to their construction from moduli.

In §4 are presented alternatives for defining the mean curvature, and advances in the theory of minimal surfaces within the Finslerian framework.

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Observations support spherically closed dynamic space without dark energy

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The Dynamic Universe theory [1,2] describes space as the surface of a 4-sphere expanding in zero-energy balance between the energies of motion and gravitation. Such an approach re-establishes Einstein's original view of space as the surface of a four dimensional sphere but converts the Einsteinian spacetime to dynamic space in absolute coordinates. That is how the description of space and time most probably would have been formulated if modern atomic clocks, GPS satellites and supernova observations — or at least Edwin Hubble's observations — had been available in early 1900's. In such an approach the rest energy of matter appears as the energy mass has due to the motion of space in the direction of the 4-radius of the structure and the velocity of light in space becomes fixed to the velocity of space in the fourth dimension. Motion *in* space becomes related to the motion *of* space and the local reference at rest to the local energy system the where the motion has been obtained. Accordingly, a local state of rest appears as a property of an energy system rather than the state of an inertial observer. Predictions obtained are supported by experiments from physical experiments at Earth laboratories and satellite systems to distant cosmological observations. Also, recent observations on the magnitude versus redshift of supernova explosions give strong support to the closed dynamic space approach without an inclusion of dark energy or adjustable density parameters.

1. Introduction

In early 1900's when the theory of relativity was formulated the view of the structure of space was quite limited. The expansion of space had not been detected and the galactic structures were unknown. It was natural to think space as static entity without a specific center or a universal reference at rest. When Einstein in 1917 published his view of the cosmological structure of space as the "surface" of a 4-sphere, he needed the famous cosmological constant to prevent the collapse of space into singularity [3].

In static space the interpretation of the observed constancy of the velocity of light led to spacetime concept with time-like fourth dimension and variable distance and time coordinates characterized as proper time and proper distance. Dilated time was explained as a consequence of the velocity the object relative to the observer, and through the curved spacetime, a property of the spacetime geometry.

Allowing spherically closed space contract and expand in a zero-energy balance of motion and gravitation the Einsteinian time-like fourth dimension becomes replaced by a purely metric dimension in the direction of the motion of space along the 4-radius of the structure. The center of symmetry and the reference at rest for the expansion and contraction of spherically closed space is in the center of the 4-sphere. Expansion of spherically closed space does not create motion within space; the momentum of the expansion appears only in the direction of the 4-radius perpendicular to all space directions. The related energy of motion appears as the rest energy of matter in space. In kinematic sense, homogeneous expansion of the 4-sphere is observed as recession of objects in space at a velocity proportional to their distance from the observer.

2. Zero-energy balance in a 4-sphere

In spherically closed space a natural solution is not static space but space subject to contraction and expansion. Dynamics based on a zero-energy principle shows the rest energy of matter as the energy of motion mass has due to the contraction or expansion of space in the fourth dimension, in the direction of the 4-radius. As a consequence of the conservation of the primary energy created in the contraction-expansion process the velocity of space in the fourth dimension set the upper limit to velocities obtainable in space. The "great mystery" of the equality of the rest energy and the gravitational energy of all mass in space is direct indication of the zero-energy balance of motion and gravitation in space [4].

In contraction started from the state of rest at infinity in the past motion is gained against release of gravitational energy, in expansion motion works against gravitation resulting in gradual deceleration of expansion until rest at infinity (see Figure 1).

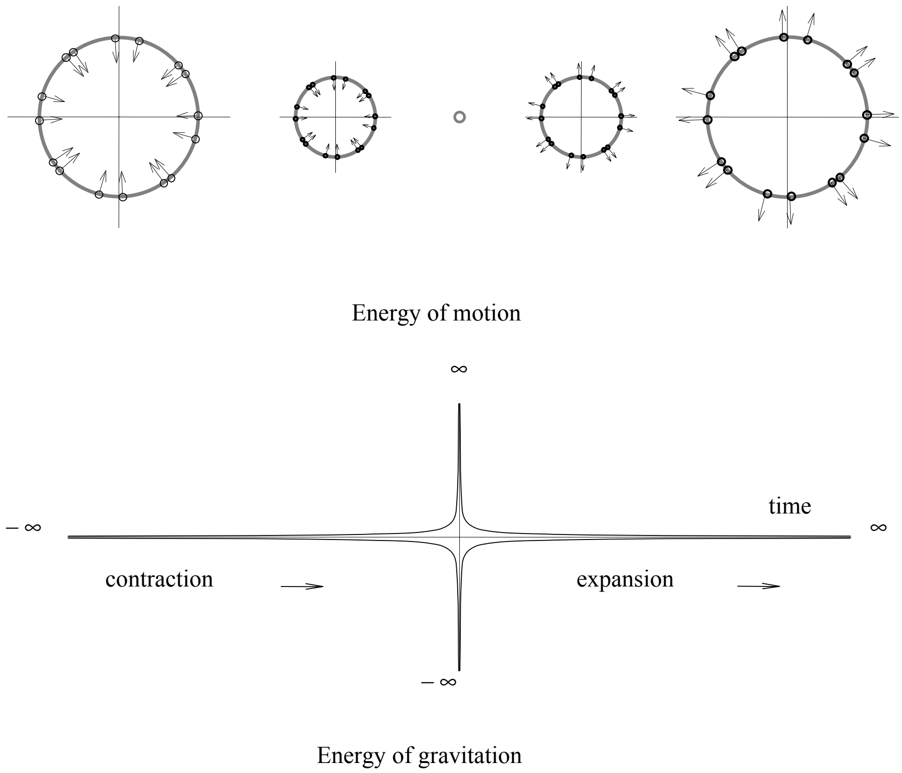

Figure 1. Energy buildup and release in spherical space. In the contraction phase, the velocity of space in the direction of the 4-radius increases due to the energy gained from loss of gravitation. In the expansion phase, the velocity gradually decreases, while the energy of motion gained in contraction is returned to gravity.

A detailed analysis of the intrinsic forms of the energies of motion and gravitation in a homogeneous 4-shere allows the expression of the zero energy condition as

$$M_\Sigma c_0^2 - \frac{G M_\Sigma M''}{R_4} = 0 \quad (1)$$

where G is the gravitational constant, c_0 the velocity in the direction of the radius R_4 of the 4-sphere, and $M'' = 0.776 \cdot M_\Sigma$ the mass equivalence of the total mass M_Σ in space. The factor 0.776 comes from the integration of the gravitational energy of the 4-sphere. Equation (1) links the velocity of the contraction or expansion along the 4-radius R_4, to the gravitational constant, the total mass in space, and the 4-radius as

$$c_0 = \pm\sqrt{\frac{GM''}{R_4}} \quad (2)$$

Applying a mass density $\rho \approx 0.55 \cdot \rho_c$, where ρ_c is the Friedmann critical mass density, 4-radius $R_4 = 14 \cdot 10^9$ light years (present estimate of the Hubble radius), and the gravitational constant $G = 6.7 \cdot 10^{-11}$ [N m^2/kg^2] equation (2) gives $c_0 = 300\,000$ km /s. Conservation of energy in interactions in space requires that c_0 is the maximum velocity obtainable in space, which confirms the interpretation of c_0 as the velocity of light in homogeneous space.

When solved for time t since singularity, the expansion velocity, and the velocity of light in space obtains the form

$$c = \frac{dR_4}{dt} = \left(\frac{2}{3}\frac{GM''}{t}\right)^{1/3} \quad (3)$$

Time t from singularity can be expressed as

$$t = \frac{2}{3}\frac{R_4}{c} \qquad (4)$$

which means that for Hubble radius 14 billion light years [corresponding to Hubble constant $H_0 = 70$ [(km/s)/Mpc] the age of the expanding universe since singularity is 9.3 billion years.

3. Unified expressions of energy

Following the zero-energy principle in the buildup of mass centers in space the velocity of free fall of mass becomes related to the local bary center. For conserving the energy of motion related to the expansion *of* space, local space becomes tilted resulting in a reduction in the momentum in the local fourth dimension and the locally available rest energy of matter. Accordingly, the velocity of free fall in space is obtained against a reduction of the local velocity of space in the fourth dimension, which also means that the velocity of light is a function of the tilting angle and, accordingly, the gravitational potential and the distance from a local mass center (see Figure 2).

Due to the nature of the rest energy of matter as the energy of motion due to the motion in space, mass should not be considered as a form of energy but, instead, the substance for the expression of energy. Such an approach leads to unified expressions of energy and relates all forms of the energy of matter to the energy matter has at rest in hypothetical homogeneous space expanding at velocity c_0.

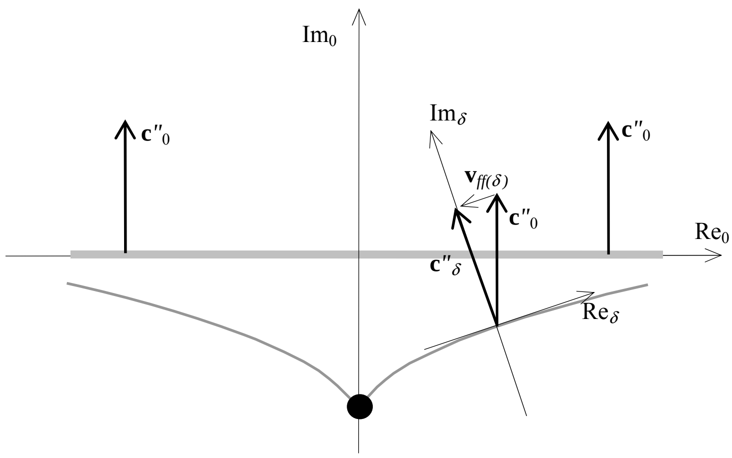

Figure 2. As a consequence of the conservation of the primary energies of motion and gravitation, the buildup of a mass center in space bends the spherical space locally causing a tilting of space near the mass center. The local imaginary axis is always perpendicular to local space. As a consequence, the local imaginary velocity of space, and accordingly the local velocity of light, is reduced in tilted space.

In a detailed analysis of free fall it can be shown that in space expanding at velocity c_0 in the direction of the 4-radius the kinetic energy obtained in free fall by conserving the total energy of motion is

$$E_{kin(ff)} = c_0 m(c_0 - c_\delta) = c_0 m\Delta c = c_0 |\Delta \mathbf{p}| \quad (5)$$

where c_0 is the imaginary velocity (the velocity in the fourth dimension) of space in the non-tilted space.

Buildup of kinetic energy by conserving the total energy at a constant gravitational potential, where the imaginary velocity of space is unchanged the expression of kinetic energy obtains the form

$$E_{kin(\delta)} = c_0 \left(m_{eff} - m\right)c = c_0 \Delta m \cdot c = c_0 |\Delta \mathbf{p}| \quad (6)$$

which is equal to the expression of kinetic energy in the theory of special relativity.

The source for the increased mass (the buildup of effective mass) may be Coulomb energy which, by applying the vacuum permeability μ_0 rather than the vacuum permittivity ε_0, obtains the form

$$E_{\mathrm{EM}} = -\frac{q_1 q_2 \mu_0}{4\pi r} c_0 c = -m_{\mathrm{EM}} c_0 c \quad (7)$$

where the m_{EM} is denoted as the mass equivalence of electromagnetic energy with the dimensions of kilogram [kg]. "Free fall" of a charge particle from distance r_1 to r_2 in Coulomb field releases the energy

$$\Delta E_{EM} = \frac{q_1 q_2 \mu_0}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) c_0 c = c_0 \left(m_{EM(1)} - m_{EM(2)}\right) c = \Delta m_{EM} c_0 c \quad (8)$$

which now obtains the form of mass release from the Coulomb field to the object accelerated.

We can extend the unified expression of energy to the energy of electromagnetic radiation by first solving the minimum dose of electromagnetic radiation as the energy of radiation emitted by a single oscillation cycle of a unit charge in a dipole. Following the standard procedure of solving Maxwell's equations, and again, by applying the vacuum permeability μ_0 rather than the vacuum permittivity ε_0, we obtain

$$E_\lambda = \frac{P}{f} = \frac{e^2 z_0^2 \chi \mu_0 16\pi^4 f^4}{12\pi c f} = \left(\frac{z_0}{\lambda}\right)^2 \frac{2}{3}\left(2\pi^3 e^2 \mu_0 c_0\right) f \quad (9)$$

which essentially has the form of Planck's equation $E = hf$.

In dynamic space moving at velocity c in the fourth dimension, a point source like an emitting atom can be considered as a dipole in the fourth dimension. In one cycle, a point charge at rest in space moves the distance of a wavelength in the fourth dimension perpendicularly to all space directions. Accordingly, for a point emitter all space directions are in the "normal plane" of a dipole, and the energy of one cycle obtains the form

$$E_\lambda = \left(1.1049 \cdot 2\pi^3 e^2 \mu_0 c_0\right) f = hf = h_0 f c = \frac{h_0}{\lambda} c_0 c = m_\lambda c_0 c \quad (10)$$

where m_λ is the mass equivalence of electromagnetic radiation with dimensions of kilogram [kg]. The factor 1.1049 needed to match equation (9) to the precise value of the Planck constant consists of the ratio c_0/c (c is the local velocity of light on the Earth) and a possible geometrical factor resulting from the application of Maxwell's equations in a dipole in the fourth dimension. Accordingly, the Planck constant can be expressed as

$$\begin{aligned} h &= 1.104905316 \cdot 2\pi^3 e^2 \mu_0 c_0 \\ &= \left[1.104905316\right] \cdot \left(5.99695618 \cdot 10^{-34}\right) \cdot f \qquad (11) \\ &= 6.626068765 \cdot 10^{-34} \qquad \left[\mathrm{kgm^2/s}\right] \end{aligned}$$

It is important to note that the expansion velocity of space (the velocity of light in hypothetical homogeneous space) is a hidden factor in the Planck constant. It is therefore necessary to define the intrinsic Planck constant $h_0 = h/c$ which allows the unified format of the energy of a quantum of electromagnetic energy (the energy of one cycle of radiation emitted by a single electron transition in a point source)

$$E_{0\lambda} = hf = h_0 f c_0 = \frac{h_0}{\lambda} cc_0 = m_{0\lambda} cc_0 \quad (12)$$

where $m_{0\lambda}$ is the mass equivalence of a quantum with dimensions of kilogram [kg]. By applying equation (11) the fine structure constant obtains the form

$$\alpha = \frac{e^2\mu_0}{1.104905316 \cdot 2\pi^3 e^2 \mu_0} = \frac{1}{1.104905316 \cdot 4\pi^3} \simeq 7.297352533 \cdot 10^{-3} \simeq \frac{1}{137} \quad (13)$$

which show the fine structure constant as a purely mathematical, dimensionless constant without connections to any physical constants. Application of the fine structure constant to the Coulomb energy between unit charges $q_1 = q_2 = e$ in equation (7) the Coulomb energy obtains the form

$$E_{\mathrm{EM}(0)} = -\frac{e^2\mu_0}{4\pi r} c_0 c = \alpha \frac{h_0}{2\pi r} c_0 c = -m_{\mathrm{EM}(0)} c_0 c \quad (14)$$

which demonstrates the close connection between the Coulomb energy and the energy of electromagnetic radiation.

As a consequence of the conservation of energy in free fall in the buildup of mass centers in space the local velocity of light was found to be a function of the local gravitational potential. In a detailed analysis taking into account the chain of inbuilt cascaded gravitational systems (mass centers) the local velocity of light in the n:th gravitational frame can be expressed as

$$c = c_0 \prod_{i=1}^{n} (1 - \delta_i) \quad (15)$$

where the gravitational factors δ_i can be expressed as

$$\delta_i = \frac{M_i}{M''} \frac{R''}{r_i} = \frac{GM_i}{c_0 c r_i} = \frac{GM_i}{c_0 c_{0\delta} r_{0\delta}} \approx \frac{GM_i}{r_i c^2} \quad (16)$$

where M_i is the central mass of the local system i, and r_i is the distance to the barycenter of the system.

Conservation of energy in the buildup of kinetic energy in local energy systems in a constant gravitational potential was expressed via the transfer of mass from the potential energy of the system to the object accelerated. Such an energy transfer occurs through the buildup of momentum in a space direction. As a part of the conservation of the zero energy balance in space the buildup of momentum in space reduces the momentum of the object in the fourth dimension. This is described as a reduction of the internal mass of the moving object so that the product of the effective mass and the internal mass is conserved at any velocity. Internal mass reduces the momentum balancing the gravitational force by all mass in space on the moving mass object. In other words, the overall conservation of energy means that: “Expression of energy through motion in space reduces the energy the object expresses in the fourth dimension”.

As a demand of the energy balance of an object moving in a local energy system n the internal mass of the object obtains the form

$$m_{I(n)} = m\sqrt{1 - \beta_n^2} \quad (17)$$

where m is the rest mass and β_n the velocity of the object in the local system. The rest mass is subject to similar reduction due to the motion of the local system in its parent system and, further, due the motions of the parent systems in their parent systems as

$$m = m_0 \prod_{i=1}^{n-1} \sqrt{1 - \beta_i^2} \quad (18)$$

where m_0 is the rest mass of the object at rest in hypothetical homogeneous space.

By applying equations (15) and (18) the rest energy of mass m in a local energy frame can be expressed as

$$E = c_0 mc = mc_0^2 \left(1-\delta_n\right) \prod_{i=1}^{n-1} \left(1-\delta_i\right) \sqrt{1-\beta_i^2} \quad (19)$$

As a consequence of the conservation of the total energy in closed dynamic space the rest energy of mass objects appear a function of the gravitational state and motion of the object. Motion of a local energy system reduces the rest energy available for mass within the local system (see Figure 3).

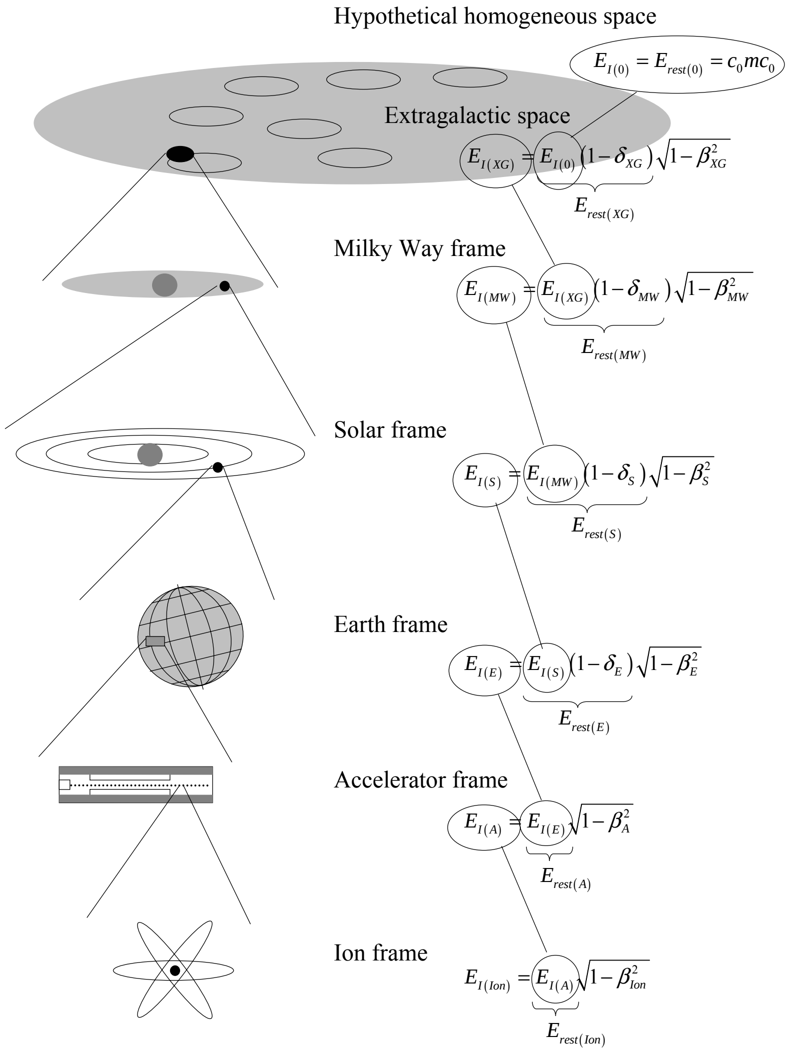

Figure 3. The rest energy of an object in a local frame is determined by the internal energy of the local frame in its parent frame. The internal energy is the imaginary component of the rest energy. The system of cascaded energy frames relates the internal energy of an object in a local frame to the rest energy of the object in hypothetical homogeneous space.

4. The effect of motion and gravitation on characteristic frequencies

By applying the intrinsic Planck constant defined in equation (12) the standard non-relativistic expression of energy states of electrons in a hydrogen-like atom solved from Schrödinger's equation can be expressed in form

$$E_{Z,n} = \frac{e^4 \mu_0^2}{8h_0^2} \left(\frac{Z}{n}\right)^2 m_e cc_0 = \frac{\alpha^2}{2} \left(\frac{Z}{n}\right)^2 E_e \quad (20)$$

where E_e is the rest energy of electron in the nucleus energy frame. With reference to equation (19) the energy states of hydrogen like atoms are functions of the gravitational state and motion of the atom. By applying equations (19) and (20) Balmer's formula for the characteristic frequencies of hydrogen like atoms obtains the form

$$f_{(n1,n2)} = \frac{\Delta E_{(n1,n2)}}{h_0 c_0} = Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \frac{\alpha^2}{2h_0} m_e c = f_{0(n1,n2)} \prod_{i=1}^{n} (1-\delta_i) \sqrt{1-\beta_i^2} \quad (21)$$

where factors δ_i and β_i define the state of gravitation and motion of the atom

$$f_{0(n1,n2)} = Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \frac{\alpha^2}{2h_0} m_{e(0)} c_0 \quad (22)$$

where c_0 is the expansion velocity of space and $m_{e(0)}$ the mass of electron at rest in hypothetical homogeneous space. As shown by equations (21) and (22) the characteristic frequency of a specific transition in an atom is a function of both the motion and gravitational state of the atom.

Equation (21) combines the coordinate time scales in different frames like the Earth Centered Inertial Frame applied in satellite systems and the Solar Barycenter Frame applied in observations in the solar system and extends the coordinate time structure to laboratory frames on the rotating Earth and anywhere in space.

Because the expansion velocity of space is subject to gradual decrease with the expansion of space, the reference frequency $f_{0(n1,n2)}$ in equation (22) declines with time as

$$f_{0(n1,n2)} = Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \frac{\alpha^2 m_{e(0)}}{2h_0} 0.803 \cdot (GM_\Sigma)^{1/3} t^{-1/3} \quad (23)$$

where G is the gravitational constant and M_Σ the total mass in space. As shown by equation (21) the characteristic frequency is directly proportional to the local velocity of light, c, which means that in local measurements based on atomic clocks, the velocity of light is observed as constant.

When solved for characteristic wavelength, Balmer's formula obtains the form

$$\lambda_{(n1,n2)} = \frac{c}{f_{(n1,n2)}} = \frac{\lambda_{0(n1,n2)}}{\prod_{i=1}^{n} \sqrt{1-\beta_i^2}} \quad (24)$$

where

$$\lambda_{0(n1,n2)} = \frac{c_0}{f_{0(n1,n2)}} \quad (25)$$

Accordingly, the characteristic wavelength is subject to increase due to the motion of the emitting atom but it is not affected by the gravitational state (or the velocity of light).

The reduction of the frequencies of atomic clocks in motion is a consequence of the energetic state, the state of gravitation and motion (velocity) of the clock. The frequency is neither a function of the velocity of the clock relative to an observer nor a function of the acceleration the clock. There is no place for the Lorentz transformation or the equivalence principle in the Dynamic Universe.

All energetic states in space are related to the reference at rest in hypothetical homogeneous space. The unified expressions of energy apply in all local frames in space and manifest the zero energy principle as a universal law of nature. There is no place for the principle of relativity in the Dynamic Universe.

5. Cosmological appearance of spherically closed space

As a consequence of the identical velocities of space along the 4-radius and electro magnetic radiation in space, the optical distance D, which is the distance traveled by light from object A_1 to object B in space (in the tangential direction), is equal to the corresponding change of the radius (see Figure 4).

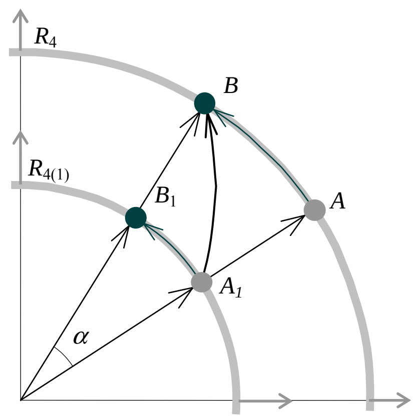

Figure 4. Propagation of electromagnetic radiation from object A_1 in Universe state $R_{4(1)}$ to object B in Universe state R_4.

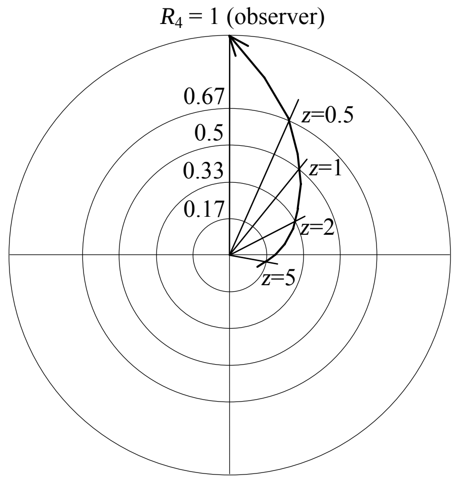

Figure 5. Redshift as a function of optical distance versus Hubble radius (D/R_H) according to the predictions of the linear Hubble law, the general theory of relativity (27), and the Dynamic Universe (26), respectively. The GR prediction is calculated for $q_0 = 0$, $q_0 = 0.5$, and $q_0 = 1$. For $H_0 = 70$ [(km/s)/Mpc], distance $R_H = R_4 = 14 \cdot 10^9$ l.y.

The wavelength of electromagnetic radiation propagating in expanding space is subject to an increase in direct proportion to the expansion of space. In spherically closed space the Hubble law obtains the form

$$z = e^{\alpha} - 1 = \frac{D}{R_4} e^{\alpha} = \frac{D/R_4}{1 - D/R_4} \quad (26)$$

The maximum optical distance of an object in space is $D = R_4$. Figure 6 illustrates the development of the optical path and the redshift from objects at different R_4 radii of space (see Figure 5).

The optical angle θ subtended by an object can be expressed as the ratio of a standard rod r_s and the optical distance. When normalized to (r_s/R_4), we get

$$\frac{\theta_{\text{DU(rod)}}}{r_s/R_4} = \frac{z+1}{z} \quad (27)$$

(see Figure 6). As a major difference to the standard cosmology model, in the Dynamic Universe the orbital radii of local gravitational systems are subject to the expansion of space. The radii of planetary systems as well as the radii of galaxies expand in direct proportion to the expansion of the 4-radius R_4. Accordingly, out of the 3.8 cm annual increase of the Earth to Moon distance 2.8 cm comes from the expansion of space and only 1 cm from the tidal effects.

The observation angle of expanding objects obtains the Euclidean form

$$\frac{\theta_{\text{DU(exp.obj.)}}}{r_s/R_4} = \frac{1}{z} \quad (28)$$

The prediction of equation (28) is strongly supported by observations of angular sizes of radio sources (see Figure 7).

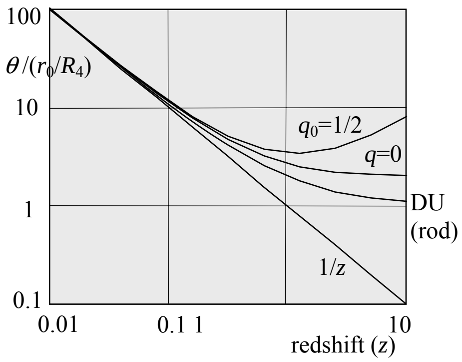

Figure 6. Normalized observation angle of a standard rod as a function of redshift in the DU model and in the standard model with expansion parameter q_0 as $q_0 = ½$ and $q_0 = 0$. The reference line 1/z shows the observation angle of the standard rod in Euclidean space. For objects expanding in space, the prediction for the observation angle obtains the Euclidean 1/z form.

DU: $$\frac{\theta_{\mathrm{DU(rod)}}}{r_s/R_4} = \frac{z+1}{z} \quad ; \quad \frac{\theta_{\mathrm{DU(exp.obj.)}}}{r_s/R_4} = \frac{1}{z}$$

Standard cosmology: $$\frac{\theta_{\mathrm{F}}}{r_s/R_H} = \frac{q_0^2\left(z+1\right)}{q_0 z + \left(q_0 - 1\right)\left(-1 + \sqrt{1+2q_0 z}\right)}$$

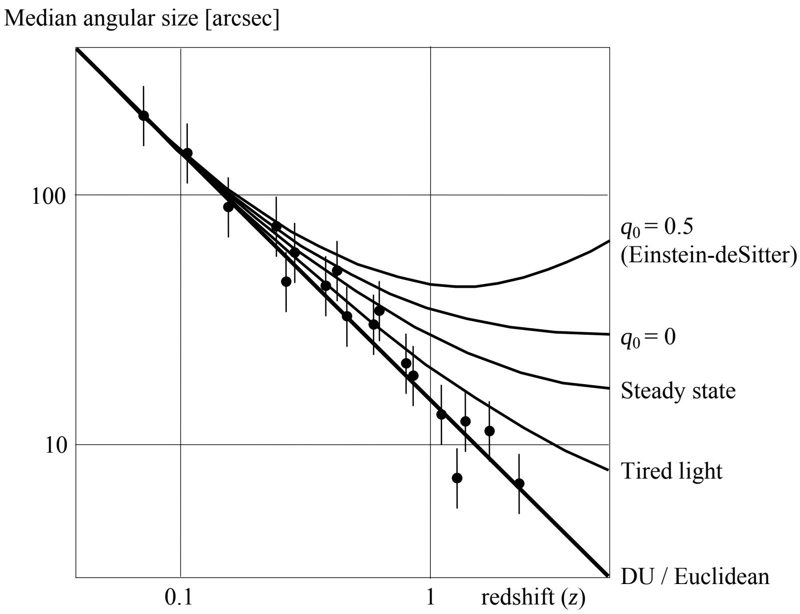

Figure 7. Comparison of equation (28) with the predictions of the standard cosmology model for various q_0 values (without evolution) and the tired light model [A. Sandage: The Deep Universe, original data for the median angular sizes (arcsec) and redshifts for radio-sources by Kapahi's (1987)]. The prediction given by equation (28) is the straight line with Euclidean appearance showing a considerable match with observations.

Dilution of the rest energy of matter with the decreasing velocity of light in expanding space means that the rate of all internal atomic processes slows down with the expansion. Also the rate of radioactive decay decreases with the expansion, which means that the results of radioactive dating shall be corrected for higher decay rate in the past. As demonstrated in Figure 8, a dating result of 14 billion years with a constant decay rate is reduced to about 9 billion years when taking into account the decreasing decay rate. The reduction solves the presently recognized problem of the age of oldest stars which look like exceeding the age of expanding space.

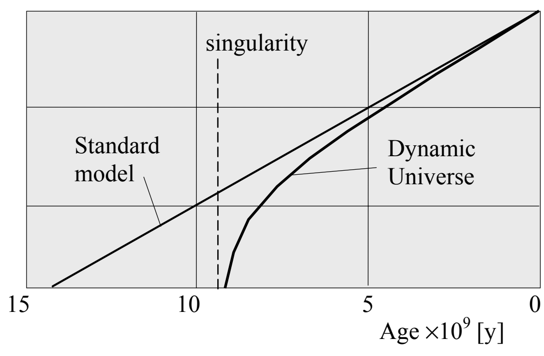

Figure 8. Accumulation of decay products according to the standard model with a constant decay rate and the DU model with a decreasing decay rate.

Perhaps the most striking recent cosmology observation is the magnitude versus redshift of supernova explosions [5,6,7]. When interpreted with the standard cosmology model, the observations meant that the expansion of space is accelerating instead of decelerating as could be expected due the work done against gravitation. For motivating the acceleration, dark energy in the form of Ω_λ has been added to the expression of the magnitude in the standard model

$$m = M + 5\log\left[\frac{c(1+z)}{H_0}\int_0^z \frac{1}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\lambda}}dz\right] + 25 \quad (29)$$

In the Dynamic Universe the expression of the magnitude obtains the form

$$m = m_0 + 5\log(z) + 2.5\log(z+1) \quad (30)$$

which, without free parameters, agrees with observations at least as well as the standard model with optimized Ω_m, Ω_λ, and H_0. The excellent fit of equation (30) means strong support to the zero energy balance of closed spherical space (see Figure 9) [8]. In equation (30) the only parameter is the reference magnitude m_0, whereas in the standard cosmology prediction (29) there are, additionally, two density parameters and the Hubble constant as parameters to be optimized.

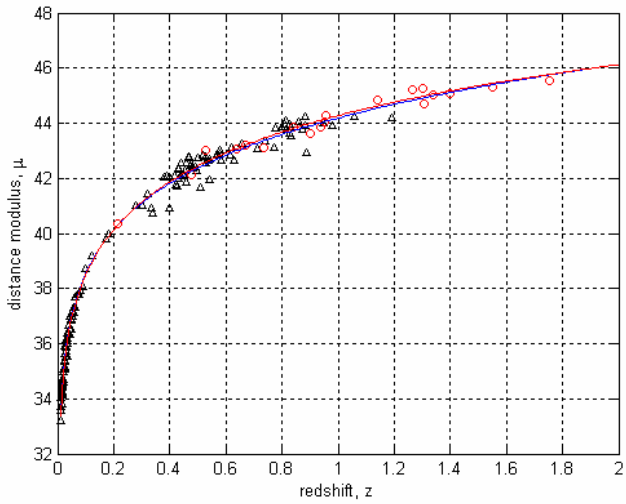

Figure 9. Distance modulus $\mu = m - M$, vs. redshift for Riess et al's gold dataset and the data from the HST. The triangles represent data obtained via a ground-based observations, and the circles represent data obtained by the HST. The optimum fit for the standard cosmology equations is shown in red, and the optimum fit for the equation $m = m_0 + 5\log z + 2.5\log(z+1)$ is shown, slightly below, in blue.

In spherically closed space the background radiation appears as radiation propagated trough a full 360° path around the expanding sphere. With reference to equation (26) the red shift of background radiation is

$$z = e^{2\pi} - 1 = 534.5 \quad (31)$$

The 4-radius of space at the time of the emission of the background radiation was $R_{4(0)} = R_4/535.5 \approx 26$ million light years, which occurred about 750 000 years after the singularity.

6. Summary

Some important physical and cosmological consequences of the zero energy balance in spherically closed space can be summarized as follows:

- universal, absolute time applies to all phenomena in space
- a local state of rest is a property of a local energy system instead of a property of an inertial observer
- the rest energy of matter is the energy of motion mass possesses due to the motion of space in the fourth dimension; conservation of the total rest energy in interactions in space relates any state of motion in space to the state of rest in hypothetical homogeneous space

– the buildup and release of the rest energy of matter can be described as a zero energy process from infinity in the past through singularity to infinity in the future
– the characteristic emission and absorption frequency due to an electron transition in atomic objects is a function of the velocity and gravitational potential of the atom in the local energy system and the parent systems; as a consequence coordinate time scales in cascaded gravitational frames and proper times in systems of motion can all be linked to universal absolute time
– electromagnetic resonators can be studied as closed energy systems; as an implication Michelson–Morley type experiments in moving frames show zero result
– precise predictions for Shapiro-delay, perihelion advance of planetary orbit and the bending of light path near mass centers can be expressed in closed mathematical form
– annual Doppler of pulsars, the Roemer–effect and the aberration of starlight get their natural solutions
– the radius of local gravitational systems expand in direct proportion to the expansion of the 4-radius of space
– distant space is observed in Euclidean geometry (e.g. the angular sizes of radio sources)
– prediction derived for the magnitude versus redshift of a standard emission source gives a perfect fit to recent supernova observations without an assumption of dark energy [8]
– the age of the expanding space obtains the form $t = 2/3\ R_4/c$
– age estimates obtained by radioactive dating are reduced due to the higher decay rate in the young universe (the decay rate is inversely proportional to $t^{1/3}$)
– the expansion of space continues to infinity ; the energy of matter, material, and radiation diminishes in the course of the expansion until it becomes zero at infinity, thus completing the cycle of observable physical existence

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Basic Statements Required for a Minimum Contradictions Aether-Everything

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Any Physics theory is stated through the basic communication system. However, by means of a theorem, it can be proved that this system is contradictory; this theorem has similarities with Gödel's inference which is the basis for his theorems proof. Thus, a least contradictory physics theory can be stated only on the basis of a claim for minimum contradictions. According to previous work this physics describes a Minimum Contradictions Aether-Everything and it is compatible, under certain simplifications, either with Newtonian Mechanics or Relativity Theory or QM. Purpose of this paper is to present the basic points of the previous work and to prove the basic statements required for a Minimum Contradictions Physics of Aether-Everything. This work is funded by EU and Program Arximedes II.

1. Previous Work [1-10]

From Aristotle it is known that the way in which we communicate and prove various statements obeys the rules of classical logic i.e. the propositional and the predicate logic[1,2,3,4]. For the purposes of this paper Classical Logic is denoted as Principle I or P_I.

Apart from these rules Aristotle also stated the causality principle according to which for everything a reason-cause is needed. Leibniz expanded the causality principle and claimed more generally that something is valid if it can be logically proved by something else that is valid. So, Leibniz' Sufficient Reason Principle could be written in the following form[1,2,3,4]:

Principle II (P_{II}): "No statement is valid if it cannot be logically proved through some valid statements different from it."

We name logic Λ the system which includes principles I and II i.e.;

$$\Lambda \equiv P_I \cdot P_{II}$$

It can be proved the following:

Theorem I: " Any system that includes logic Λ and a statement that is not theorem of logic Λ leads to contradiction."

In previous works efforts have been made to prove this Theorem; purpose of this paper is to prove it in a more integrated way.

It can be shown that the anterior-posterior axiom is not theorem of Λ. Thus, the following can be proved:

Statement I: " Any system that includes logic Λ and the anterior-posterior axiom leads to contradiction."

where the anterior – posterior axiom is stated as follows.

Anterior – Posterior Axiom:

a. There is a physical state named Anterior.

b. If there is Anterior then there is a sequent different state named Posterior.

Our basic communication system includes logic Λ and the anterior-posterior axiom; in fact, in our language for everything we seek the reason of its power (P_{II}); we put a phrase after another phrase, a word after another word e.t.c. (anterior – posterior axiom). Thus according to Statement I it is proved that this basic communication system is contradictory. However, we notice that Statement I cannot be stated because it is based on the basic communication system which, according to Statement I itself, is contradictory. *Thus,*

Statement I imposes silence. When we communicate, we use a hidden claim according to which *"what is accepted as valid is what includes the minimum possible contradictions"* since the contradictions cannot be vanished[4]. According to this hidden claim we obtain a logical and an

illogical dimension. In fact, through this axiom we try to approach logic (minimum possible contradictions) but at the same time we expect something illogical since the contradictions cannot be vanished.
The systems of axioms we use in Physics include the communication system and, therefore, their contradictions are minimized when they are reduced to the communication system itself; because of theorem I further axioms - beyond the ones of logic Λ - cause further contradictions.
Therefore, we have minimum contradictions in Physics when it is based only on the basic communication system, i.e. on logic Λ and on the "anterior-posterior axiom" .
In order that such physics be valid, a unifying principle is needed, since everything, *i.e.* matter, field, and space-time, needs to be described in anterior-posterior terms.
Thus, *in a first sight*, for a least contradictory physics we can make the following statement:

Statement II: Any matter space-time system can be described in anterior–posterior terms.
It is noted that time implies the existence of anterior and of posterior; space does, too. If I say 10cm, I mean the existence of anterior-posterior measuring states corresponding to 1,2,3….10cm Therefore, the existence of anterior and posterior is the condition for space and time to exist and *vice-versa*. Thus, because of Statement II, for a least contradictory physics we can state the following statement:

Statement III: Any matter system can be described in space-time terms.
Since everywhere there is space-time and not something else, *Space-Time-Everything* can be regarded as *Matter-Aether*. A matter system, in general, has differences within its various areas. This means that a matter system, in general, is characterized by different rates of anterior - posterior (time) within its various points. Since space is also locally affected by the local rate of anterior-posterior, it can be expected to deform due to different rates of anterior-posterior. This means that time can be regarded as a 4^{th} dimension which implies Lorentz transformations and in extension a relativistic theory[1,2,3,4].
In a second sight, taking into account the above-mentioned, and applying the claim of the minimum contradictions, we conclude that *Matter-Space-Time-Everything-Aether* can have logical and contradictory behavior at the same time; *this can be valid only if space-time is stochastic.* This is in contrast with the GRT; according to A.Pais, Einstein had said:
"I consider it quite possible that physics cannot be based on the field concept; i.e., on continuous structures. In that case nothing remains of my entire castle in the air, gravitation theory included, and the rest of modern physics"[5,6].
According to a previous work [7], statement III in combination with the claim for minimum contradictions leads to a Minimum Contradictions Physics of Aether-Everything. This physics can imply the principles of QM, and under some simplifying hypothesis (continuity of space-time), it can imply the GRT; of course without this simplifying hypothesis, it is in contrast with the GRT. On this basis, the hypothesis of the Quantum Space Time [8] can be mentally verified. The hypothesis of the quantum space-time i.e. of the *unified space-time-matter-field*, is based on *the unification of the physical meaning of the notions which derive either from the GRT or the QM.* According to the GRT, a particle field consists of a particle mass and a spacetime continuum which surrounds this mass. According to the QM, a particle field is described by means of a De Broglie matter wave, which includes the notion of a particle mass. Therefore, the following question arises: is an infinitesimal part of a field spacetime or is it an area which is described by a matter wave? If we want to achieve the unification mentioned, the following principle should be valid[7]:
"Any infinitesimal spacetime can be regarded as a matter wave".
We may notice that this principle is compatible with Statement III on condition that space time is stochastic.
Basic tool for minimum contradictions physics description is the Hypothetical Measuring Field (HMF)[7,8].

*As Hypothetical Measuring Field (*HMF) is defined *a hypothetical field, which consists of a Euclidean reference space time, in which at every point* A_0 *-* $(\vec{r},t)$ *- the real characteristics of the corresponding – through deformity transformations - point* A *of the real field exist.*

In a HMF, *we define as mean relative space time magnitude* $\overline{sr}$ *the ratio of the mean real infinitesimal space time magnitude* $\overline{ds}$ *to the corresponding infinitesimal magnitude* ds_0 *of the reference space time*: i.e. $\overline{sr} = \overline{ds} / ds_0$. This can apply to any magnitude as follows :

α) Relative time $\overline{tr} = \overline{dt} / dt_0$,

where dt is an infinitesimal time of comparison at a given position of the HMF.

b) *Relative length* in a direction $\vec{n}$ $\overline{lr}_n = \overline{dl}_n / dl_{n0}$,

where dl_n is an infinitesimal length of comparison in a direction $\vec{n}$ and at a given time of the HMF.

Concerning the notion of time we have the *internal time* of an infinitesimal space time element and the *sensible time which expresses an irreversible passage from an earlier to a posterior*. According to the spirit of this work *internal time of an infinitesimal space time element is equivalent to its energy*; it can apply both to (g) space-time and to (em) space-time. *Sensible time is closed to the notion "arrow of time" and it expresses a passage from (g) to (em) space-time*[3,7]; see more in the Appendix A.

With the aid of the HMF minimum contradictions aether-space-time geometry can be defined by means of an equation system defining a Ψ wave function[7,8]; this geometry derives from the distribution of the properties of a flat relativistic space-time based on the probability density $P(\vec{r},t)$ of Schroendinger relativistic equation; the validity of this equation can be proved. Aether-space-time as a whole has both gravitational (g) and electromagnetic (em) dimensions; the (g) and the (em) space-time coexist and interact. The electromagnetic (em) space-time is a space-time whose all magnitudes are considered imaginary and behave exactly like the gravitational (g). Minimum Contradictions Aether Everything Equations are shown in Appendix A[3,7,10].

Minimum contradictions aether physics can be the basis for explanation of laws and of various phenomena that cannot be explained through a classical approach [3,7,8,9,10]. Thus, forces unification can be achieved, arrow of time, electric clusters stability, cold fusion, *asymmetric capacitor propulsion* can be explained. It is shown that Minimum Contradictions Aether Physics leads to chaotic self-similar quantum matter space-time systems[8,10].

All these are based on two statements (Theorem I and statement I) proving that the basic communication system is contradictory and on the claim for minimum contradictions. Thus it might be constructive a question to the scientific community of whether these statements proof, which constitutes the main part of this paper, is valid or not.

2. Proof of Theorem I and of Statement I

2.1 Symbols

For the purpose of this paper we use the symbolic logic *not only through the frame of the propositional and predicate logic, but through the frame of logic* Λ. Thus we have:

Principle I (P_I): The symbols of Classical Logic are used[11,12].

Principle II (P_{II}): This principle which expresses Leibniz' Sufficient Reason Principle[13] can be stated through the following statements.

$$P_{IIa}: \qquad \sim prov_{\Lambda}(p,p) \tag{1}$$

This Principle states that it is not valid that statement- or set of statements- p can prove itself on the basis of logic Λ i.e. on the basis of a system including the classical logic P_I and the principle P_{II}.

$$P_{IIb}: \quad p \Rightarrow \wp \cdot prov_{\Lambda}(\wp, p) \tag{2}$$

This Principle states that if p is valid then statement-or set of statements- $\wp$ is valid so that p can be proved by means of $\wp$ through logic Λ.

Applying Classical Logic we have the following property of logic Λ.

$$prov_{\Lambda}(p,q) \cdot pov_{\Lambda}(q,r) \Rightarrow prov_{\Lambda}(p,r) \tag{3}$$

i.e.: if p proves statement-or set of statements- q (through Λ) and p proves statement-or set of statements- r, then p proves r.

Notice:

$prov_{\Lambda}(A,B)$ is not a simple logical proof of B through A; it implies that:

$$\sim prov_{\Lambda}(A,A)$$

i.e. A can not prove itself.

Thus Pythagorean Theorem denoted as P can be proved by means of Euclidean Axiom denoted as E i.e.:

$$prov_{\Lambda}(E,P)$$

However we have:

$$\sim prov_{\Lambda}(E,E)$$

i.e. E cannot be self-proved and therefore is not a priori valid.

2.2 Theorem I: *" Any system that includes logic Λ and a statement that is not theorem of logic Λ leads to contradiction."*

Proof:

We feel that logic Λ is valid, but we don't know a priori whether it is valid or not. When we already speak logically it means that we have decided to communicate and we cannot but, most generally, think -according to P_I - that:

$$\Lambda \vee \sim \Lambda \tag{4}$$

which means that either logic Λ is valid or logic Λ is not valid. So, our consideration takes the widest credibility. Therefore, we can look into the following cases:

2.2.1. Logic Λ is non valid.

It is obvious that if a system includes Λ this system is contradictory since it must be valid Λ and ($\sim \Lambda$) at the same time.

2.2.2. Logic Λ is valid.

If $R_{\Lambda 1}, R_{\Lambda 2}, ...R_{\Lambda N}$ are the statements-reasons for Λ validation, then, since any proof requires Λ, we will have that $R_{\Lambda 1}, R_{\Lambda 2}, ...R_{\Lambda N} \cdot \Lambda \Rightarrow \Lambda$. Since $\Lambda \Rightarrow \Lambda$, we conclude that Λ is valid due to Λ itself, and does not require any further reason. This is not in contrast with principle II, since in this case, Λ is regarded as valid, due to a hypothesis (case 2.2.2 instead of 2.2.1) .

We consider the system:

$$\Pi \equiv \Lambda \cdot p \cdot q \equiv \Lambda \cdot p' \tag{5}$$

We symbolize as Π_c the system Π when it is complete that is when the validity of p, q is due to Π_c itself. According to P_I we have:

$$\Pi_c \vee \sim \Pi_c \tag{6}$$

As long as Π is valid according to P_{IIb} it must be provable. Thus we will have.

$$\Pi_c \vee \sim \Pi_0 \tag{7}$$

that is either Π is complete (Π_c), or Π is open (Π_0) that is p, q are provable not through Π. Thus we have the following cases:

2.2.2.a. Π_c (Π is complete)

In this case p, q must be provable through Λ, p, q. Because of principle P_{IIb} we will have:

$$p \Rightarrow prov_{\Lambda}(\Lambda, p) \vee prov_{\Lambda}(q, p) \quad (8)$$
$$q \Rightarrow prov_{\Lambda}(\Lambda, q) \vee prov_{\Lambda}(p, q) \quad (9)$$
By hypothesis there is a statement of Π which is not theorem of Λ; let be p this statement. Thus we will have:
$$\sim prov_{\Lambda}(\Lambda, p) \quad (10)$$
Thus, because of statements (9,10,11) we obtain:
$$p \cdot q \Rightarrow prov_{\Lambda}(\Lambda, q) \cdot prov_{\Lambda}(q, p) \vee prov_{\Lambda}(q, p) \cdot prov_{\Lambda}(p, q) \quad (11)$$
both terms of right part express impossibility; in fact applying statement (3) we have:
$$prov_{\Lambda}(\Lambda, q) \cdot prov_{\Lambda}(q, p) \Rightarrow prov_{\Lambda}(\Lambda, p) \quad (12)$$
i.e. if Λ proves q and q proves p then Λ proves p; this is in contrast with statement (10). Working in the same way we have that :
$$prov_{\Lambda}(q, p) \cdot prov_{\Lambda}(p, q) \Rightarrow prov_{\Lambda}(q, q) \quad (13)$$
which is in contrast with Principle P_{IIa}.
Thus, because of statements (10,11,12,13) and since Λ is by hypothesis valid we have:
$$\Pi_c \equiv \Lambda \cdot p \cdot q \Rightarrow contr. \quad (14)$$
where by the term $contr.$ the existence of contradiction is symbolized. Thus because of statement (14) we can state the following :

Statement IV: " If logic Λ is by hypothesis valid, then any system that includes this logic Λ and a statement that is not a theorem of logic Λ cannot be complete and consistent at the same time."
2.2.2.b. Π_0 (Π is open-non complete)
According to principle II(P_{II}), Λ and $p \cdot q \equiv p'$ must be provable through some valid statements different from them. These statements- reasons must be concrete final valid statements ; if there are not concrete final valid statements then there is not proof for p' validity and this in contrast with P_{II}. As was mentioned, Λ is by hypothesis valid.
According to P_{II} it is valid that:
$$p' \Rightarrow \wp' \cdot prov_{\Lambda}(\wp', p') \quad (15)$$
where $\wp'$ is the set of statements-reasons for p' validity. The system:
$$\Lambda \cdot \wp' \cdot p' \quad (16)$$
must be complete and consistent since it includes all related to p' finally provable statements. This system includes p' and therefore p; thus according to statement I this system leads to contradiction; i.e.:
$$\Lambda \cdot \wp' \cdot p' \Rightarrow contr. \quad (18)$$
Taking into account principle P_{II} we obtain:
$$\Pi \equiv \Lambda p' \Rightarrow \Lambda \wp' p' \Rightarrow contr. \quad (19)$$
Therefore, in general, the system Π leads to contradiction regardless of whether it is complete or not; thus taking into account what was mentioned in case 2.1 and statement (19) we can state Theorem I since it is valid without any restriction for Λ.

2.3 Statement I: *" Any system that includes logic Λ and the anterior-posterior axiom leads to contradiction."*
Proof:
We correspond numbers 1,2,…,x to various sequent states mentioned in the Anterior – Posterior Axiom. By S_x is denoted a state which corresponds to number x.
Because of the Anterior – Posterior Axiom we have:
$$(\forall x) \quad (S_x \Rightarrow \sim S_{x+1}) \quad (20)$$

i.e. if S_x is valid then no other state is valid and therefore state S_{x+1} is not valid as well.

$$(\forall x) \quad (S_x \Rightarrow \exists S_x \Rightarrow \exists S_{x+1}) \tag{21}$$

i.e. if S_x is valid then S_x exists; according to Anterior – Posterior Axiom if S_x exists then S_{x+1} exists as well.
Because of statements (20) and (21) we obtain:

$$(\forall x) \quad (S_x \Rightarrow (\sim S_{x+1}) \cdot \exists S_{x+1}) \tag{22}$$

The 2nd part of statement (22) is not always consistent; in fact $\exists S_{x+1}$ implies that it is possible for S_{x+1} to be valid which is in contrast with the statement "$\sim S_{x+1}$". In extension the 1st part of statement (22) and therefore the Anterior – Posterior Axiom is not always consistent. Thus, the Anterior – Posterior Axiom is not compatible with Classical Logic i.e. with principle P_I; in extension this axiom is non compatible with logic Λ which includes principle P_I. Therefore we can state that *the Anterior – Posterior Axiom is not theorem of* Λ.
Applying Theorem I for systems including the Anterior – Posterior Axiom we obtain Statement I.

3. Gödel's Work

Gödel's Theorem can be stated in the form of the following statement [14,16]:

Statement V: "A consistent system including Peano's arithmetic cannot be complete".
It is noted that this statement was proved on the basis of the following:
Gödel's Hypothesis: "There is an algorithm that permits the derivation of only true statements".
Of course this hypothesis is arbitrary. According to Hillary Putnam, Gödel's second incompleteness theorem states that if a system 'S' of formalized mathematics – that is, a set of axioms and rules so precisely described that a computer could be programmed to check proofs in the system for correctness – is strong enough for us to do number theory in it, then a certain well-formed statement of the system, one which implies that the system is consistent, cannot be proved within the system. [15]. As Putnam noticed, this Gödel's theorem had been misinterpreted; statement V has not been proved in spite of efforts made by Church, Schröter and others [17]. Roger Penrose investigated the 2nd Gödel's theorem and, taking into account the fact that it is not completely valid in the form of Statement V, concluded that[16]:

Conclusion I: There is a part of our thinking which cannot be computational; this part could be investigated by laws of physics.
There are doubts that there is a possibility for non-computational thinking able to be investigated by the laws of physics to exist [15]; however, Penrose's conclusion completely takes into account what exactly has until now been proved [16]. Thus, if we were to prove Statement V and more generally statement IV, theorem I and statement I, we should find another way beyond Gödel's work; this is the subject of this paper.
It is noted that statement IV can be regarded as a generalized case of Gödel's theorem [14]; this theorem requires in order to derive Aristotlean logic (Mathematica Principia) and axioms that are not theorems of this logic (Peanno's axioms); besides, statement IV requires the Sufficient Reason Principle (P_{II}) which has similarities with Gödel's hypothesis mentioned.
It is also noted that Statement I has similarities with Gödel's inference which is the basis for his theorems proof. In fact according to Gödel's work and to J. Barkley Rosser Theorem there is a type G (Gödel type) which is provable only on condition that $\sim G$ is provable as well [14]; one can see similarities between Peanno's axioms and the anterior posterior axiom as it is stated in this work.

Appendix A [3,7,10]

1. Minimum Contradictions Aether-Everything Equations

A minimum contradictions space-time-aether field in general, behaves locally as a particle-space-time field; if we put:
$\Box = \partial^2 / \partial t^2 - c^2 \nabla^2$
the following equations are valid.

$$\partial_{xi} \frac{\Box \Psi_g(\mathbf{r},t)}{\Psi_g(\mathbf{r},t)} = 0 \, , \quad \partial_{xi} \frac{\Box \Psi^g_{em}(\mathbf{r},t)}{\Psi^g_{em}(\mathbf{r},t)} = 0 \tag{A.1}$$

$$\partial_t \left(\frac{\partial_t \Psi_g(\mathbf{r},t)}{\Psi_g(\mathbf{r},t)} + \alpha \frac{\partial_t \Psi^g_{em}(\mathbf{r},t)}{\Psi^g_{em}(\mathbf{r},t)} \right) = 0 \tag{A.2}$$

$$\partial_t \left(\frac{\nabla \Psi_g(\mathbf{r},t)}{\Psi_g(\mathbf{r},t)} + \alpha \frac{\nabla \Psi^g_{em}(\mathbf{r},t)}{\Psi^g_{em}(\mathbf{r},t)} \right) = 0 \tag{A.3}$$

$$\mathbf{g}(\mathbf{r},t) = \frac{c^2 \nabla \left(\Psi_g{}^* \partial_t \Psi_g - \Psi_g \partial_t \Psi_g{}^* \right)}{\left(\Psi_g{}^* \partial_t \Psi_g - \Psi_g \partial_t \Psi_g{}^* \right)} \tag{A.4}$$

$$\mathbf{g}_{\mathrm{em}}(\mathbf{r},t) = \frac{c^2 \nabla \left(\Psi^g_{em}{}^* \partial_t \Psi^g_{em} - \Psi^g_{em} \partial_t \Psi^g_{em}{}^* \right)}{\left(\Psi^g_{em}{}^* \partial_t \Psi^g_{em} - \Psi^g_{em} \partial_t \Psi^g_{em}{}^* \right)} \tag{A.5}$$

$$\overline{tr}(\mathbf{r},t) = \frac{ic}{2h} \frac{\partial_t \Psi}{(\Psi \Box \Psi)^{1/2}} \left(\Psi^* \partial_t \Psi - \Psi \partial_t \Psi^* \right) \tag{A.6}$$

$$\overline{lr}_n(\mathbf{r},t) = -\frac{ih}{2} \frac{\Psi}{\Box \Psi} \left(1 - c^2 \frac{\partial^2 \Psi / \partial x_n{}^2}{\partial^2 \Psi / \partial t^2} \right)^{1/2} \left(\Psi^* \partial_t \Psi - \Psi \partial_t \Psi^* \right) \tag{A.7}$$

where α is the fine structure constant, Ψ_g , Ψ^g_{em} are the gravitational and the electromagnetic space-time wave functions, which are identical with equivalent local particle Ψ functions, and $(\mathbf{r}, t)$ is a point of the hypothetical measuring field (HMF). Eqs. (A.1) describe Schrödinger's relativistic equations.
Eq. (A.2) describes the energy conservation principle.
Eq. (A.3) describes the momentum conservation principle.
Eqs. (A.4, A.5) describe the gravitational acceleration of (g) and (em) space-time at point $(\mathbf{r}, t)$.
Eqs(A.6, A.7) describe the mean relative time and the mean relative length in a direction $\vec{n}$ of (g) space-time; this can be extended to the (em) space-time.
It is noted that the electromagnetic (em) field for the same reasons as the (g) does, is described with the aid of an electromagnetic (em) hypothetical measuring field through electromagnetic coordinates $(\mathbf{r}_{em}, t_{em})$. However the (em) HMF coexists with the (g) HMF while $(\mathbf{r}_{em}, t_{em})$ corresponds to $(\mathbf{r}, t)$ through a scale so that:

$$\frac{\partial x_{ig}}{\partial x_{iem}} = i\alpha \qquad (i = 1,2,3,4) \tag{A.8}$$

If $\Psi_{em}(\mathbf{r}_{em}, t_{em})$ is the (em) space-time wave function we define as function $\Psi^g_{em}(\mathbf{r},t)$ a function for which is valid that:

$$\Psi_{em}(\mathbf{r}_{em}, t_{em}) = \Psi^g_{em}(\mathbf{r},t) \tag{A.9}$$

This is the reason why spacetime as a whole i.e. Minimum Contradictions Aether Everything can be described by means only of coordinates $(\mathbf{r}, t)$ of (g) space-time.
Eqs. (A.2, A.3) describe any kind of energy and momentum interactions between the (g) and the (em); on this basis we can get useful information for explaining gravielectric phenomena.

2. Conservation Principle – Notion of Time Flow

In a closed system regarded as a whole, the energy conservation principle can be applied as follows:

$$\overline{E}_g + \overline{E}_{em-g} = \text{constant} \tag{A.10}$$

where $\overline{E}_{em} = i\overline{E}_{em-g}$ and the dash ($^{-}$) indicates the mean value.

If the closed system of Eq. (A.10) is the Universe and the constant is zero, we have another point of view for the creation and the evolution of Universe; it can be proved that $\overline{V}_g \uparrow \Rightarrow \overline{E}_g \downarrow$, where $\overline{V}_g$ is volume which contains energy $\overline{E}_g$; thus, the expansion of Universe implies a continuing irreversible conversion of $\overline{E}_g$ into $\overline{E}_{em-g}$ and - as was mentioned in the text (previous work) - because of equivalence of energy and time [3,7,10] an irreversible conversion of (g) into (em) time which can be regarded as related to the arrow and the flow of sensible time [18].

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On “gauge renormalization” in classical electrodynamics

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We pay attention to the inconsistency in the derivation of the symmetric electromagnetic energy-momentum tensor for a system of charged particles from its canonical form, when the homogeneous Maxwell’s equations are applied to the symmetrizing gauge transformation, while the non-homogeneous Maxwell’s equations are used to obtain the motional equation. Applying the appropriate non-homogeneous Maxwell’s equation to both operations, we have revealed an additional symmetric term in the tensor, named as “compensating term”. Analyzing the structure of this “compensating term”, we suggested a method of “gauge renormalization”, which allows transforming the divergent terms of classical electrodynamics (infinite self-force, self-energy and self-momentum) to converging integrals. The motional equation obtained for a non-radiating charged particle does not contain its self-force, and the mass parameter includes the sum of mechanical and electromagnetic masses. The motional equation for a radiating particle does not yield any “runaway solutions”. It has been shown that the energy flux in a free electromagnetic field is guided by the Poynting vector, whereas the energy flux in a bound EM field is described by the generalized Umov’s vector, defined in the paper.

1. Introduction

The problem of infinite electromagnetic (EM) mass of the electron and self-forces of charged particles has continued to be one of the central issues of classical electrodynamics during more than a century [1-10]. One of the reasons, explaining such a great attention to these problems, is their persistence in quantum electrodynamics [8, 9]. The simplest method to avoid the infinite EM mass of an electron is to add a compensating infinite negative mass. However, such a method does not overcome all difficulties, in particular, the “runaway solutions” (*e.g.*, a “self-acceleration” of radiating electron). In the present paper we omit a review of these problems, insofar as we will apply a primary modification of the energy-momentum tensor to remove an inconsistency, which seems not to have been revealed before.

It is known (see, *e.g.* [5, 6]) that the motional equation for an EM field with the Lagrangian density $-F_{\mu\nu}F^{\mu\nu}/16\pi$ gives the following expression for the canonical energy-momentum tensor of the electromagnetic (EM) field

$$T_{EM}{}^{\mu\nu} = -\partial^{\mu}A^{\gamma}F^{\nu}{}_{\gamma}/4\pi + g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}/16\pi \,, \tag{1}$$

where $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ is the tensor of EM field, A^{μ} is the four-potential, $g^{\mu\nu}$ is the metric tensor, and μ, ν=0…3. In order to transform Eq. (1) into a symmetric form, the gauge transformation

$$T_{EM}{}^{\mu\nu} \to T_{EM}{}^{\mu\nu} + \partial_{\gamma}\psi^{\mu\nu\gamma} \text{ (where } \psi^{\mu\nu\gamma} = -\psi^{\mu\gamma\nu}\text{)}, \tag{2}$$

should be applied. Choosing

$$\psi^{\mu\nu}{}_{\gamma} = A^{\mu}F^{\nu}{}_{\gamma}/4\pi \tag{3}$$

and writing

$$\partial^{\gamma}\psi^{\mu\nu}{}_{\gamma} = \left[\left(\partial^{\gamma}A^{\mu}\right)F^{\nu}{}_{\gamma}\right]/4\pi + \left[A^{\mu}\left(\partial^{\gamma}F^{\nu}{}_{\gamma}\right)\right]/4\pi \,, \tag{4}$$

we can transform the tensor (1) to the symmetric form

$$T_{EM}{}^{\mu\nu} = \left(1/4\pi\right)\left(-F^{\mu\gamma}F^{\nu}{}_{\gamma} + g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}/4\right), \tag{5}$$

if we recognize that

$$\partial_{\gamma}F^{\nu\gamma} = 0 \tag{6}$$

(the field equation in the absence of source charges). Eq. (5) represents the conventional expression for the tensor of EM field.

Further, it is known that the energy-momentum tensor for matter has the form

$$T_M^{\mu\nu} = mc\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}, \qquad (7)$$

where m is the mass density, and τ is the proper time. Then the total energy-momentum tensor is defined as the sum of Eqs. (5) and (7):

$$T^{\mu\nu} = T_M{}^{\mu\nu} + T_{EM}{}^{\mu\nu} = mc\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau} + \left(-F^{\mu\gamma}F_\gamma^\nu/4\pi + g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}/16\pi\right). \quad (8)$$

The energy-momentum conservation law requires that the four-divergence of $T^{\mu\nu}$ should vanish:

$$\partial_\mu\left[(T_M)^\mu{}_\nu + (T_{EM})^\mu{}_\nu\right] = 0. \qquad (9)$$

Using the Maxwell' equations $\partial_\gamma F_{\mu\nu} = -\partial_\mu F_{\nu\gamma} - \partial_\nu F_{\gamma\mu}$, and

$$\partial_\gamma F^{\gamma\nu} = \frac{4\pi j^\nu}{c} 4\pi j^\nu/c \qquad (10)$$

(j^ν is the four-current), we find the motional equation in the form

$$mc^2\, dv_\nu/dt = F_{\nu\gamma}j^\gamma, \qquad (11)$$

where v_ν is the four-velocity.

Eqs. (1)-(11) briefly reproduce the derivation of the energy-momentum tensor and motional equation from [5, 6], which are widely accepted. Then applying Eq. (11) to a single isolated charged particle we obtain the spatial components of this equation in the form

$$d\vec{p}/dt = q\vec{E} + q\vec{v}\times\vec{B}/c, \qquad (12)$$

where q is the charge of particle, $\vec{p}$ is its momentum, $\vec{v}$ is the velocity, and $\vec{E}$, $\vec{B}$ are **its own** electric and magnetic fields. Furthermore, the requirement $\partial_\mu T^{\mu 0} = 0$ gives the following energy balance equation:

$$\partial u/\partial t + \nabla\cdot\vec{S} + \vec{E}\cdot\vec{j} = 0, \qquad (13)$$

where

$$u = E^2 + B^2/8\pi, \quad \vec{S} = c\left(\vec{E}\times\vec{B}\right)/4\pi \qquad (14), (15)$$

are the energy density of EM field and the Poynting vector of the particle, correspondingly. The term $\vec{E}\cdot\vec{j}$ in Eq. (13) describes **the self-action** of charged particle.

In this paper we intend to resolve the problems of self-action and infinite self-energy, applying a procedure of renormalization of the EM energy-momentum tensor under its proper gauge transformation. First of all we pay attention to a lack of logic in the derivation of Eq. (5) and further calculation of the four-divergence of $T^{\mu\nu}$. Namely, under the gauge transformation from Eq. (1) to Eq. (5) the homogeneous Maxwell equation (6) was used, while in proving the equality (11) the non-homogeneous Maxwell equation (10) was used. Thus, two different equations have been applied to the same physical entity, the EM energy-momentum tensor. This inconsistency prompts a closer look at the symmetrization of EM tensor, which is done below.

2. The electromagnetic energy-momentum tensor for a system of charged particles and its "gauge renormalization"

Consider a system of N>1 charged particles, where the total tensor of the EM field $F^{\mu\nu}$ represents the sum of corresponding tensors $\underset{(k)}{f}{}^{\mu\nu}$ for each particle

$$F^{\mu\nu} = \sum_k \underset{(k)}{f}{}^{\mu\nu} \qquad (16)$$

(k=1…N) due to the superposition principle. The mechanical energy-momentum tensor (7) is properly modified as

$$T_M^{\mu\nu}=\sum_k \underset{(k)}{m}\, c\frac{d\underset{(k)}{x}^{\mu}}{dt}\frac{d\underset{(k)}{x}^{\nu}}{d\tau}, \tag{17}$$

where the mass density is defined by the equation $\underset{(k)}{m}=\underset{(k)}{M}\,\delta(\vec{r}-\vec{r}_k)$, $\underset{(k)}{M}$ being the mass of particle k.

Determining the EM energy-momentum tensor for this system, we again proceed from the canonical form (1) and apply the gauge transformation (2). We use the gauge function (3) modified for the discrete system of N particles: $\psi^{\mu\nu\gamma}=\frac{1}{4\pi}\sum_k \underset{(k)}{A}^{\mu}\sum_l \underset{(l)}{f}^{\nu\gamma}$ (l=1…N). Noting that

$$\partial^{\gamma}\psi^{\mu\nu}{}_{\gamma}=\frac{1}{4\pi}\left(\sum_k\partial^{\gamma}\underset{(k)}{A}^{\mu}\right)\left(\sum_l \underset{(l)}{f}^{\nu}{}_{\gamma}\right)+\frac{1}{4\pi}\left(\sum_k \underset{(k)}{A}^{\mu}\right)\left(\sum_l\partial^{\gamma}\underset{(l)}{f}^{\nu}{}_{\gamma}\right),$$

and carrying out the gauge transformation (2) for the tensor (1), we obtain:

$$T_{EMG}{}^{\mu\nu}=\frac{1}{4\pi}\left(-\sum_k \underset{(k)}{f}^{\mu\gamma}\sum_l \underset{(l)}{f}^{\nu}{}_{\gamma}+\frac{1}{4}g^{\mu\nu}\sum_k \underset{(k)}{f}_{\gamma\alpha}\sum_l \underset{(l)}{f}^{\gamma\alpha}\right)+\frac{1}{4\pi}\sum_k \underset{(k)}{A}^{\mu}\sum_l\partial^{\gamma}\underset{(l)}{f}^{\nu}{}_{\gamma}. \tag{18}$$

Eq. (18) differs from Eq. (5) by the second term in *rhs*, which was omitted in Eq. (5) due to the condition (6), which cannot be accepted for the system of charged particles. In order to distinguish the tensor (18) from the conventional tensor (5), we name it as the "generalized electromagnetic energy-momentum tensor" (EMG).

Now let us apply the non-homogeneous Maxwell's equation (10) for any particle l. Outside of this particle $\partial^{\gamma}\underset{(l)}{f}^{\nu}{}_{\gamma}=0$, while at its location $\partial^{\gamma}\underset{(l)}{f}^{\nu}{}_{\gamma}{}^{\nu}{}_{\gamma}=-(4\pi/c)j_l{}^{\nu}$. Hence at this point the last term in *rhs* of Eq. (18) is equal to

$$(l)^{\mu\nu}=-\frac{1}{c}\left(\sum_k \underset{(k)}{A}^{\mu}\right)\underset{(l)}{j}^{\nu}=-\frac{1}{c}\underset{(l)}{A}^{\mu}\underset{(l)}{j}^{\nu}, \tag{19}$$

insofar as the four-potential $\underset{(l)}{A}^{\mu}$ dominates over all other $\underset{(k)}{A}^{\mu}$ at the location of l^{th} particle. Note that the tensor (19) is symmetrical, because $\underset{(l)}{A}^{\mu}$ is proportional to $\underset{(l)}{v}^{\mu}$. We name the tensor (19) as **"compensating term"** for the reasons clarified below. Defining the same compensating term for each particle from the considered ensemble, we write the EMG tensor in the form:

$$T_{EMG}{}^{\mu\nu}=\frac{1}{4\pi}\left(-\sum_k \underset{(k)}{f}^{\mu\gamma}\sum_l \underset{(l)}{f}^{\nu}{}_{\gamma}+\frac{1}{4}g^{\mu\nu}\sum_k \underset{(k)}{f}_{\gamma\alpha}\sum_l \underset{(l)}{f}^{\gamma\alpha}\right)-\frac{1}{c}\sum_k \underset{(k)}{A}^{\mu}\underset{(k)}{j}^{\nu}, \tag{20}$$

The first term in *rhs* of Eq. (20) can be written in the form

$$T_{EM}{}^{\mu\nu}=T_{(EM)\mathrm{ex}}{}^{\mu\nu}+\frac{1}{4\pi}\sum_k\left(-\underset{(k)}{f}^{\mu\gamma}\underset{(k)}{f}^{\nu}{}_{\gamma}+\frac{1}{4}g^{\mu\nu}\underset{(k)}{f}_{\gamma\alpha}\underset{(k)}{f}^{\gamma\alpha}\right),$$

where the tensor $T_{(EM)\mathrm{ex}}{}^{\mu\nu}$ is defined by the equation

$$T_{(EM)\mathrm{ex}}{}^{\mu\nu}=\frac{1}{4\pi}\left(-\sum_k \underset{(k)}{f}^{\mu\gamma}\sum_{l\neq k}\underset{(l)}{f}^{\nu}{}_{\gamma}+\frac{1}{4}g^{\mu\nu}\sum_k \underset{(k)}{f}_{\gamma\alpha}\sum_{l\neq k}\underset{(l)}{f}^{\gamma\alpha}\right). \tag{21}$$

The introduced subscript "ex" indicates that the terms of "self-action", containing $(f_k)(f_k)$ (k=1…N), have been excluded from the tensor $\underset{(EM)\mathrm{ex}}{T}{}^{\mu\nu}$. One can see that at the location of any particle l, this tensor satisfies the equality

$$\partial_{\mu}\left(T_{(EM)\mathbf{ex}}\right)^{\mu}{}_{\nu}=-\left(F_{\nu\gamma}\right)_{\mathbf{ex}(l)}\underset{(l)}{j}^{\gamma}\Big/c, \tag{22}$$

where $\left(F_{\nu\gamma}\right)_{\mathrm{ex}(l)}$ does not contain $\left(\underset{(l)}{f}\right)_{\nu\gamma}$. Then Eq. (20) acquires the form

$$T_{EMG}{}^{\mu\nu}=T_{(EM)\mathrm{ex}}{}^{\mu\nu}+\sum_k \underset{(k)}{T}{}_{EEM}{}^{\mu\nu}\,, \tag{23}$$

In the latter equation we have introduced a new tensor

$$\underset{(k)}{T}{}_{EEM}{}^{\mu\nu}=\frac{1}{4\pi}\left(-\underset{(k)}{f}{}^{\mu\gamma}\underset{(k)}{f}{}^{\nu}{}_{\gamma}+\frac{1}{4}g^{\mu\nu}\underset{(k)}{f}{}_{\gamma\alpha}\underset{(k)}{f}{}^{\gamma\alpha}\right)-\frac{1}{c}\underset{(k)}{A}{}^{\mu}\underset{(k)}{j}{}^{\nu}\,, \tag{24}$$

which describes only the properties of particle k, but not its interaction with other particles. That is why we can name it as the **Eigen ElectroMagnetic (EEM) energy-momentum tensor** of charged particle, supplying it by the subscript "EEM".

Eqs. (20) and (23) can be derived in another way, using the energy-momentum tensor, defined according to Hilbert [6]:

$$\frac{1}{2}\sqrt{-g}\,T_{\mu\nu}=\frac{\partial\sqrt{-g}L}{\partial g^{\mu\nu}}-\frac{\partial}{\partial x^{\gamma}}\frac{\partial\sqrt{-g}L}{\partial\left(g^{\mu\nu}/\partial x^{\gamma}\right)}, \tag{25}$$

where L is the electromagnetic Lagrangian density. Taking L in the form

$$L=-\frac{1}{c}\sum_k \underset{(k)}{A}{}_{\mu}\sum_l \underset{(l)}{j}{}^{\mu}-\frac{1}{16\pi}\sum_k \underset{(k)}{f}{}_{\gamma\alpha}\sum_l \underset{(l)}{f}{}^{\gamma\alpha}\,, \tag{26}$$

with inclusion of both "interaction part" (the first term in *rhs* of Eq. (26)) and "field part" (the second term in *rhs* of Eq. (26)), and inserting L from Eq. (26) into Eq. (25), we obtain the EMG tensor in the form

$$T_{EMG}{}^{\mu\nu}=\frac{1}{4\pi}\left(-\sum_k \underset{(k)}{f}{}^{\mu\gamma}\sum_l \underset{(l)}{f}{}^{\nu}{}_{\gamma}+\frac{1}{4}g^{\mu\nu}\sum_k \underset{(k)}{f}{}_{\gamma\alpha}\sum_l \underset{(l)}{f}{}^{\gamma\alpha}\right)-\frac{1}{c}\sum_k \underset{(k)}{A}{}^{\mu}\sum_l \underset{(l)}{j}{}^{\nu}\,. \tag{27}$$

We again see that outside the particles ($\underset{(l)}{j}=0$) the second term in *rhs* of Eq. (27) vanishes, while at the location of each l^{th} particle, $\underset{(l)}{A}$ dominates over the four-potentials of all other particles. Hence $\sum_k \underset{(k)}{A}{}^{\mu}\sum_l \underset{(l)}{j}{}^{\nu}=\sum_l \underset{(l)}{A}{}^{\mu}\underset{(l)}{j}{}^{\nu}$, and Eq. (27) agrees with Eqs. (20) and (23).

Using the tensor (23) and taking into account the matter tensor (17), we write the total energy-momentum tensor as

$$T^{\mu\nu}=\sum_{k=1}^{N}\left(\underset{(k)}{m}c\frac{d\underset{(k)}{x}{}^{\mu}}{dt}\frac{d\underset{(k)}{x}{}^{\nu}}{d\tau}+\underset{(k)}{T}{}_{EEM}{}^{\mu\nu}\right)+T_{(EM)\mathrm{ex}}{}^{\mu\nu}\,. \tag{28}$$

The above-introduced EEM tensor (24) represents the difference of two divergent terms and, in fact, is uncertain. Nevertheless, we can examine its general properties, considering first an isolated charged particle, moving at the constant velocity $\vec{v}$ in the frame of observation. For such a particle $T_{(EM)\mathrm{ex}}{}^{\mu\nu}$=0 by definition, and its total energy-momentum tensor acquires the form

$$T^{\mu\nu}=mc\frac{dx^{\mu}}{dt}\frac{dx^{\nu}}{d\tau}+T_{EEM}{}^{\mu\nu}\,, \tag{29}$$

where its rest mechanical mass density is denoted as m. For the total energy-momentum tensor $\partial_{\mu}T^{\mu\nu}=0$. Since for a freely moving particle $\partial_{\nu}\left(mc\frac{dx^{\mu}}{dt}\frac{dx^{\nu}}{d\tau}\right)=0$, then

$$\partial_{\mu}T_{EEM}{}^{\mu\nu}=0\,, \tag{30}$$

too. Hence we get the energy balance equation for a bound EM field of an isolated charged particle:

$$\frac{\partial T_{EEM}{}^{0\nu}}{\partial x^{\nu}}=\frac{1}{4\pi}\frac{\partial}{\partial x^{\nu}}\left[-\left(f_s\right)^{0\gamma}\left(f_s\right)^{\nu}{}_{\gamma}+\frac{1}{4}g^{0\nu}\left(f_s\right)_{\gamma\alpha}\left(f_s\right)^{\gamma\alpha}\right]-\frac{1}{c}\frac{\partial}{\partial x^{\nu}}\left[\left(A_s\right)^{0}\left(j_s\right)^{\nu}\right]=0\,.$$

where the subscript "*s*" refers to an isolated charged particle. From there we derive

$$\frac{\partial u_s}{\partial t}+\nabla\cdot\vec{S}_s-\frac{\partial}{\partial x^\mu}(A_s)^\mu\rho=0. \tag{31}$$

where ρ is the charge density of the particle. Using the vector identity $(\vec{E}_s\times\vec{B}_s)=\vec{B}_s\cdot(\nabla\times\vec{E}_s)-\vec{E}_s\cdot(\nabla\times\vec{B}_s)$ as well as the Maxwell's equations $(\nabla\times\vec{E}_s)=-\partial\vec{B}_s/c\partial t$, $(\nabla\times\vec{B}_s)=(4\pi/c)\vec{j}_s+\partial\vec{E}_s/c\partial t$, we can transform Eq. (31) to the form

$$\vec{j}_s\cdot\vec{E}_s+\frac{d}{dt}(\rho_s\varphi_s)=0. \tag{32}$$

Outside the charged particle both terms in *lhs* of Eq. (32) disappear. Thus, the equality $\partial T_{EEM}{}^{0\nu}/\partial x^\nu=0$ is valid in the whole free space. However, at the location of the particle the terms of Eq. (32) trend to infinity. Their vanishing sum signifies that the "self-work" done $\vec{j}_s\cdot\vec{E}_s$ is compensated by the change of the "potential energy" of particle $U_{ps}=\rho_s\varphi_s$. Noting that $\vec{j}_s\cdot\vec{E}_s=dE_{ks}/dt$, E_{ks} being the kinetic energy, we arrive at the equality $\frac{d}{dt}(E_{ks}+U_{ps})=0$, which means the conservation of the sum of kinetic and potential energy. For an isolated charged particle both components of energy do not depend on time, and the particle moves at a constant velocity, as it should be.

In a similar way we analyse the spatial components of Eq. (30). Outside the charged particle we get $\partial_\mu T_s{}^{\mu i}=0$ for i=1…3. At the location of the particle $d(\vec{S}_s-\rho_s\vec{A}_s)/dt=0$, which means that the time rate of the divergent Poynting vector $\vec{S}_s$ is compensated by the corresponding time rate of the divergent "potential momentum" $\rho_s\vec{A}_s$ of the particle.

Nevertheless, cancelling a self-action for an isolated charged particle with the help of EEM tensor (24), we have still failed to determine unambiguously the total energy and momentum of such a particle. Indeed, Eq. (24) yields the following energy density $T_{EEM}{}^{00}$ and momentum density $\vec{p}_{EMs}$ at the location of the particle

$$T_{EEM}{}^{00}=\frac{E_s{}^2+B_s{}^2}{8\pi}-\rho_s\varphi_s,\ \vec{p}_{EMs}=\frac{c}{4\pi}(\vec{E}_s\times\vec{B}_s)-\rho_s\vec{A}_s. \tag{33), (34}$$

A vagueness of these quantities means the impossibility of determining the total energy and momentum of the EM field of a single particle.

Under these conditions we can carry out a suitable gauge modification of the EEM tensor (24), in order to escape the mentioned shortcomings. This mathematical problem can be much more easily solved physically, if we introduce a new tensor satisfying the conservation law (30). Namely, let us use a natural assumption that the total mass of a charged particle M_t is composed from its mechanical mass M and the mass M_{EM} of its EM field. Denoting as m and m_{EM} the corresponding rest mass densities, we transform the matter tensor (7) to the form

$$T_M{}^{\mu\nu}=(m+m_{EM})c\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}, \tag{35}$$

where for an isolated charged particle $\partial_\mu T_M{}^{\mu\nu}=0$. Owing to the law of charge conservation, the mechanical mass cannot be transformed into EM mass and vice versa. Therefore, the vanished four-divergence is derived independently for the mechanical and EM parts of the tensor (35), and

$$\partial_\mu\left(m_{EM}c\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}\right)=0. \tag{36}$$

We see that the symmetric tensor

$$T_{mass}{}^{\mu\nu}=m_{EM}c\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}, \tag{37}$$

named by us as **the tensor of EM mass**, also satisfies Eq. (30). Hence it is connected with the tensors $T_{EEM}{}^{\mu\nu}$ by the gauge transformation (2). Therefore, we can replace $T_{EEM}{}^{\mu\nu}$ by $T_{mass}{}^{\mu\nu}$ in equation (28) for the total energy-momentum tensor of the system of charged particles:

$$T^{\mu\nu} = c\sum_k \left(\underset{(k)}{m} + \underset{(k)EM}{m} \right) \frac{d \underset{(k)}{x}^{\mu}}{dt} \frac{d \underset{(k)}{x}^{\nu}}{d\tau} + T_{(EM)\mathrm{ex}}{}^{\mu\nu} . \tag{38}$$

It is known that the gauge transformation (2) does not influence the total energy and momentum, and

$$\int_V T_{mass}{}^{00} dV = \int_V T_{EEM}{}^{00} dV \,;\; \int_V T_{mass}{}^{0i} dV = \int_V T_{EEM}{}^{0i} dV \qquad (39), (40)$$

(the integration is carried out over the whole 3-space *V*). These equalities allow us to establish a relationship between the introduced EM mass of particle and its electric and magnetic fields. In particular, combining Eqs. (33), (37), and (39), we get for the rest frame of the charged particle:

$$M_{EM} = \int_V \frac{E_s^{\,2}}{8\pi c^2} dV - \int_V \frac{\rho_s \varphi_s}{c^2} dV , \tag{41}$$

while combining Eqs. (34), (37), (40) we arrive at

$$\gamma M_{EM} \vec{v} = \int_V \frac{1}{4\pi c} \left(\vec{E}_s \times \vec{B}_s \right) dV - \int_V \frac{\vec{v} \rho_s \varphi_s}{c^2} dV \,,\; \gamma = 1 \big/ \sqrt{1 - v^2/c^2} \tag{42}$$

These equations state that the difference of two divergent integrals in their *rhs* must be finite and equal to the EM mass of particle (Eq. (41)) and EM momentum of particle (Eq. (42)). Such statements are sufficient for further development of classical theory.

Thus, the obtained tensor (38) contains single-valued quantities and does not include a self-action of charged particles due to Eq. (22). The method proposed can be termed a “gauge renormalization”. We have to emphasize that this method has been applied to a bound EM field of a non-radiating isolated charged particle. If a particle moves in the external EM fields, and its EM radiation is not negligible, we have to proceed from the general tensor (23) for description of its EM field. Then the total energy-momentum tensor acquires the form

$$T^{\mu\nu} = \sum_{k=1}^{N} \left(c \underset{(k)}{m} \frac{d \underset{(k)}{x}^{\mu}}{dt} \frac{d \underset{(k)}{x}^{\nu}}{d\tau} + \underset{(k)}{T}{}^{\mathbf{r}}{}_{EEM}{}^{\mu\nu} \right) + T_{(EM)\mathbf{ex}}{}^{\mu\nu} , \tag{43}$$

where the superscript “r” indicates that the EEM tensor includes the radiation of each particle *k*. In order to write this tensor explicitly, we use the superposition principle, whence the EM energy-momentum tensor of each particle represents the sum of components with a bound f_{b} and free f_{r} EM fields, with $\partial_\mu (f_{\mathrm{b}})^{\mu\nu} = 4\pi j^\nu / c$, $\partial_\mu (f_{\mathrm{r}})^{\mu\nu} = 0$. Then

$$\left(\underset{(l)}{T}{}^{\mathbf{r}}{}_{EEM} \right)^{\mu\nu} = \frac{1}{4\pi} \left[- \left(\underset{(l)\mathbf{b}}{f} + \underset{(l)\mathbf{r}}{f} \right)^{\mu\gamma} \left(\underset{(l)\mathbf{b}}{f} + \underset{(l)\mathbf{r}}{f} \right)^{\nu}{}_{\gamma} + \frac{1}{4} g^{\mu\nu} \left(\underset{(l)\mathbf{b}}{f} + \underset{(l)\mathbf{r}}{f} \right)_{\gamma\alpha} \left(\underset{(l)\mathbf{b}}{f} + \underset{(l)\mathbf{r}}{f} \right)^{\gamma\alpha} \right] - \frac{1}{c} \left(\underset{(l)\mathbf{b}}{A} + \underset{(l)\mathbf{r}}{A} \right)^{\mu} \underset{(l)}{j}{}^{\nu} . \tag{44}$$

In the next section we analyze some physical consequences, resulting from the application of the tensor (43) and its particular form (38) to radiating and non-radiating charged particles.

3. Classical electrodynamics after “gauge renormalization”: basic points

3.1. Motional equation for a charged particle

The motional equation is derived from the equality

$$\partial_\mu T^{\mu\nu} = 0 , \tag{45}$$

If a particle does not radiate, we insert the tensor (38) into the conservation law (45). Then we obtain

$$\partial_\mu T^{\mu\nu} = \sum_k \left(c\partial_\mu \left[\left(\underset{(k)}{m} + \underset{(k)EM}{m} \right) \frac{d \underset{(k)}{x}^{\mu}}{dt} \right] v_l^{\ \nu} + c\left(\underset{(k)}{m} + \underset{(k)EM}{m} \right) \frac{dv_{k\nu}}{dt} + \left(\partial_\mu T_{EM(\mathrm{ex})}^{\ \ \mu\nu} \right) \right) = 0 . \quad (46)$$

The latter equation is implemented, if and only if

$$\left(\underset{(k)}{m} + \underset{(k)EM}{m} \right) \frac{dv_{l\nu}}{dt} = \frac{1}{c^2} \left(F_{ex} \right)_{\nu\gamma} j_k^{\ \gamma} \text{, and } \partial_\mu \left(\left(\underset{(k)}{m} + \underset{(k)EM}{m} \right) \frac{dx_k^{\ \mu}}{dt} \right) = 0 . (47), (48)$$

Now consider the motion of a single non-radiating charged particle q with the mechanical rest mass M in an external EM field. Proceeding from continuous to discrete distributions of masses and charges, we obtain from Eq. (47)

$$\left(M + M_{EM} \right) \frac{dv_\nu}{dt} = \frac{q}{c^2} \left(F_{\nu\gamma} \right)_{\mathrm{ex}} v^\gamma , \quad (49)$$

Eq. (49) has two differences from the conventional motional equation (11). First it shows that a particle experiences the forces only due to the external EM fields, and a self-action is impossible. This result reflects our original exclusion of self-action from the electromagnetic energy-momentum tensor under the “gauge renormalization”. Secondly, the EM mass of the particle is explicitly added to its mechanical mass. Of course, the idea to include the EM mass in the total mass of charged particles is as old as the classical model of the electron. However, it seems that this idea was usually forgotten, when the electromagnetic energy-momentum tensor and the motional equation were derived. The continuity equation (48) is common for both masses, and hence it is impossible to determine the relative contribution of M and M_{EM} to the total mass within classical electrodynamics.

When a particle radiates, we use the tensor (43) to get its motional equation. Then the straightforward calculations give the following expression for the force of radiation reaction:

$$\vec{F}_{\mathrm{r}} = -q\nabla\left(\varphi_{\mathrm{r}} - \left(\vec{v} \cdot \vec{A}_{\mathrm{r}} \right) / c \right) = -q\left(\nabla \varphi'_{\mathrm{r}} \right) / \gamma , \quad (50)$$

where φ'_{r} is the scalar potential of EM radiation in the rest frame of particle. Note that $\nabla\varphi'_{\mathrm{r}}$ and φ'_{r} have the same sign, because the electric field of EM radiation falls as $1/r$. Hence no “runaway solutions”, like a self-acceleration of radiating particle, is appeared.

3.2. Energy flux in free and bound electromagnetic fields

First consider a free EM field in the absence of charged particles. Then the electromagnetic energy-momentum tensor (43) takes its usual form (5), and the equality $\partial_\mu T^{\mu 0}$=0 yields $\frac{\partial u}{\partial t} + \nabla\vec{S} = 0$, where the Poynting vector $\vec{S}$ is given by Eq. (15). If the EM radiation falls on a system of charged particles, then the latter equation transforms to Eq. (13).

Now let us determine the energy balance equation for a bound EM field with the total energy-momentum tensor (38). The equality $\partial_\mu T^{\mu 0} = 0$ yields:

$$\left(\vec{j}\vec{E} \right)_{\mathrm{ex}} + \frac{\partial u_{\mathrm{ex}}}{c\partial t} + \nabla \cdot S_{\mathrm{ex}} = 0 , \quad (51)$$

where $\left(\vec{j}\vec{E} \right)_{\mathrm{ex}} = \left(F_{0\gamma} \right)_{\mathrm{ex}} j^\gamma$ is the time rate of work done (without the self-forces), $u_{\mathbf{ex}} = \left(-F^{0\gamma} F_\gamma^0 / 4\pi + F_{\gamma\alpha} F^{\gamma\alpha} / 4 \right)_{\mathbf{ex}}$ is the part of energy density of EM field, where the “self-action” components $\vec{E}_l\vec{E}_l$ and $\vec{B}_l\vec{B}_l$ are excluded, and $\vec{S}_{\mathrm{ex}}$ is the portion of Poynting vector, where the “self-action” components $\vec{E}_l \times \vec{B}_l$ are also excluded. It is given by the equation $S^i{}_{\mathrm{ex}} = c\left(-F^{i\gamma} F_\gamma^0 \right)_{\mathrm{ex}} / 4\pi$.

Eq. (51) does not yet determine the total flow of energy in a bound EM field, because the flow of EM masses should be added. As we mentioned above, due to the fixed ratio of mechanical to EM mass, the continuity equation (48) is separately valid for the density of EM mass u_s/c^2:

$$\partial_\mu\left(u_s \frac{dx_s^{\ \mu}}{dt}\right)=0\,, \tag{52}$$

and the total flow of EM energy is determined by summing up of Eqs. (51) and (52). Then simple, but extensive calculations give for the system of N non-radiating charged particles:

$$\frac{\partial u_\Sigma}{\partial t}+\nabla_{-\Sigma}\cdot\vec{U}_G=0\,, \tag{53}$$

where we have introduced the vector

$$\vec{U}_G=\sum_{k=1}^{N}\vec{v}_k\left(\vec{E}_\Sigma\cdot\vec{E}_k+\vec{B}_\Sigma\cdot\vec{B}_k\right), \tag{54}$$

named as the generalized Umov vector. Here $\vec{E}_\Sigma=\sum_k \vec{E}_k$ and $\vec{B}_\Sigma=\sum_k \vec{B}_k$ are the resultant electric and magnetic fields. The operator $\nabla_{-\Sigma}$ acts only on $\vec{E}_k$, $\vec{B}_k$, but not on the resultant fields.

Thus, we have got the energy balance equation (53), which determines the energy flux in a bound EM field. We see that it does not contain the term of dissipation of EM energy $\vec{j}\cdot\vec{E}$. In this connection we mention that the term $\vec{j}\cdot\vec{E}$ describes a time derivative of the kinetic energy of particles, which is equal with the opposite sign to the time rate of change of potential energy of particles in the bound EM field. In turn, the change of potential energy is already included in the partial time derivative $\partial u/\partial t$. Hence, in comparison with the energy balance equation (13) for free EM field, the term $\vec{j}\cdot\vec{E}$ does not appear for the bound fields. Inasmuch as Eq. (53) represents the sum of Eqs. (51), (52), it incorporates two different effects: the flow of EM masses of all individual particles, as well as the superposition of bound EM fields of the particles. We notice that in the particular case, where the instantaneous velocities of all particles are equal to each other ($\vec{v}_k=\vec{v}$ for any k), Eq. (53) acquires the form

$$\frac{\partial u_\Sigma}{\partial t}+\nabla\cdot\left(\vec{v}u_\Sigma\right)=0\,. \tag{55}$$

This equation shows that the resultant EM field rigidly moves together with the source particles. It is interesting that each individual particle carries its EM mass independently of other particles, but the superposition of bound EM fields from all particles transforms the sum of these individual motions into a common motion of the resultant bound EM field at the same velocity $\vec{v}$.

The results obtained in this sub-section indicate that free and bound EM fields have substantially different physical properties. It warrants their primary distinction in the original energy-momentum tensor (43).

3.3. The momentum of free and bound EM fields

The momentum density of the EM field is the component $T_{EM}^{\ \ i0}/c$ (i=1…3) in the EM energy-momentum tensor. For electromagnetic radiation it is written in the known form $\vec{p}=\vec{E}\times\vec{B}/4\pi c$. For a bound EM field we determine the EM energy-momentum tensor as

$$T_{EM}^{\ \ \mu\nu}=\sum_k m_{(k)EM}\frac{d\underset{(k)}{x}^{\mu}}{dt}\frac{d\underset{(k)}{x}^{\nu}}{d\tau}+\left(-F^{\mu\gamma}F_\gamma^{\nu}/4\pi+\frac{1}{4}g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}/4\right)_{\mathbf{ex}}.$$

which is derived from the tensor Eq. (38) by the exclusion of its mechanical part. Then the momentum density as a function of velocities of particles and their EM fields is

$$\vec{p}=\sum_k\left[\vec{v}_k\gamma_k\left(E_k^{\ 2}+B_k^{\ 2}\right)\right]/c^2+\sum_{k\neq k'}\vec{E}_k\times\vec{B}_{k'}/4\pi\,. \tag{56}$$

The total momentum of a bound EM field is computed by integration of Eq. (56) over the whole 3-space:

$$\vec{P}_{EM}=\int_V\sum_k\left[\vec{v}_k\gamma_k\left(E_k^{\ 2}+B_k^{\ 2}\right)\right]/c^2dV+\int_V\sum_{k\neq k'}\left(\vec{E}_k\times\vec{B}_{k'}/4\pi\right)dV\,. \tag{57}$$

It consists of two parts: the momentum density, associated with the EM mass of charged particles, and the momentum density, resulting from the superposition of EM fields of different particles. We emphasize that the first term in *rhs* of Eq. (56) represents the sum of contributions of EM momenta of the particles, associated with their EM mass, to the total momentum of that particles. Therefore, the time rate of the first term in *rhs* of Eq. (57) is rather the consequence than the cause of the force experienced by the particles. Hence the external forces, acting on charged particles, are determined by the time rate of the second term in *rhs* of Eq. (57).

Let us consider an isolated system, consisting of two non-radiating charged particles q_1 and q_2, and determine a total force exerted on this system. In general, it does not vanish, owing to violation of Newton's third law in EM interactions. Adding the mechanical momenta of both particles to Eq. (57), we obtain

$$\vec{P}_{EM} = \gamma_1 (M_1 + M_{EM1})\vec{v}_1 + \gamma_2 (M_2 + M_{EM2})\vec{v}_2 + \int_V (\vec{E}_1 \times \vec{B}_2 + \vec{E}_2 \times \vec{B}_1) dV .$$

The resulting force, acting on the particles, is

$$\vec{F} = \frac{d}{dt}[\gamma_1 (M_1 + M_{EM1})\vec{v}_1 + \gamma_2 (M_2 + M_{EM2})\vec{v}_2] = -\frac{d}{dt}\int_V (\vec{E}_1 \times \vec{B}_2 + \vec{E}_2 \times \vec{B}_1) dV . \quad (58)$$

If the particles are non-relativistic, then [11]

$$\int_V (\vec{E}_1 \times \vec{B}_2) dV = q_1 \vec{A}_{21}/c , \quad \int_V (\vec{E}_2 \times \vec{B}_1) dV = q_2 \vec{A}_{12}/c ,$$

where $\vec{A}_{21}$ is the vector potential produced by the particle 2 at the location of particle 1, and $\vec{A}_{12}$ is the vector potential of particle 1 at the location of particle 2. Hence

$$\vec{F} = \frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt} = -\frac{q_1}{c}\frac{d\vec{A}_{21}}{dt} - \frac{q_2}{c}\frac{d\vec{A}_{12}}{dt} . \quad (59)$$

This equation reflects the law of conservation of the canonical momentum

$$\vec{P}_C = (\vec{P}_1 + q_1 \vec{A}_{21}/c) + (\vec{P}_2 + q_2 \vec{A}_{12}/c) = \text{const}$$

for the considered non-radiating non-relativistic system. Eq. (59) has also been derived in [12] within the Lagrangian formalism.

Without the "gauge renormalization", the conventional Poynting vector would determine the resultant force:

$$\vec{F} = \frac{d}{dt}(\gamma_1 M_1 \vec{v}_1 + \gamma_2 M_2 \vec{v}_2) = -\frac{d}{dt}\int_V (\vec{E}_1 \times \vec{B}_1 + \vec{E}_1 \times \vec{B}_2 + \vec{E}_2 \times \vec{B}_1 + \vec{E}_2 \times \vec{B}_1) dV , \quad (60)$$

and instead of Eq. (59), we would obtain

$$\vec{F} = \frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt} = -q_1 \frac{d\vec{A}_{21}}{dt} - q_2 \frac{d\vec{A}_{12}}{dt} - \frac{d}{dt}\int_V (\vec{E}_1 \times \vec{B}_1) dV - \frac{d}{dt}\int_V (\vec{E}_2 \times \vec{B}_2) dV . \quad (61)$$

which does not agree with the law of conservation of the canonical momentum. Moreover, at the location of point-like charges the third and fourth integrals in *rhs* of Eq. (61) diverge. The difference between Eqs. (59) and (61) reflects a physical meaning of the "gauge renormalization", when the time rates of the terms, taken from the same source particles ($(\vec{E}_1 \times \vec{B}_1)$ and $(\vec{E}_2 \times \vec{B}_2)$) contribute to their own EM momentum, associated with the EM mass, and thus represent the consequences of an action of the external forces, but not their cause.

Finally, for an isolated charged particle, moving at the constant velocity $\vec{v}$ in a laboratory, the momentum density of the bound EM field is determined as

$$\vec{p}_{EM} = \gamma \vec{v} u(v=0)/c^2 = m_{EM}(\vec{v})\vec{v} , \quad (62)$$

where $m_{EM}(\vec{v}) = \gamma u(v=0)/c^2$ is the density of velocity-dependent EM mass of the particle. Since the equality $u(\vec{v}) = m_{EM}(\vec{v})c^2$ is implemented by definition, then the known problem "4/3" is formally eliminated in Eq. (62). It does not mean that the problem is resolved: it is simply relocated from Eq. (62) into Eqs. (41) and (42). It reflects the obvious fact that any gauge operation does not change the total energy of charged particles, which includes the energy that provides the stability of the electron ("Poincaré stresses" [13]). A detailed analysis of this problem falls outside the scope of the present paper.

4. Conclusions

1. In this paper we have removed the inconsistency that existed up to now in classical electrodynamics. Namely, in the gauge transformation of canonical energy-momentum tensor (1) to the symmetric form, we applied the non-homogeneous Maxwell equation (10) instead of the irrelevant homogeneous equation (6). As a result, the symmetric "generalized" energy-momentum tensor acquired the additional "compensating term" (19). This allows a gauge transformation, converting the divergent terms of classical electrodynamics to converging integrals. This operation was named as "gauge renormalization".
2. The obtained energy-momentum tensor has been applied to derive the motional equation, the energy balance equation, and the momentum conservation law for the system of moving charged particle. The motional equation for a non-radiating charged particle does not contain its self-force, and the mass parameter represents the sum of mechanical and electromagnetic masses. The motional equation for a radiating particle does not yield any "runaway solutions". The energy flux in a free EM field is guided by the Poynting vector. The energy flux in a bound EM field is described by the generalized Umov vector, defined in the paper.

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Учет поляризации вакуума в рамках стандартных КЭД и КХД приводит к асимптотической свободе на малых расстояниях

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Трудность в квантовой и классической электродинамике – электростатическая энергия заряженной частицы (электрона) оказывается бесконечной - известна давно.[1 –7]. Формальное устранение трудностей – теория перенормировок (1948) -отбрасывание бесконечных величин – не нравилось многим выдающимся физикам, например Р.Фейнману [8] и Л.Д.Ландау [9]. Мы покажем, что корректный учет реального физического эффекта - поляризации вакуума позволяет устранить появления бесконечных величин в электродинамике.

Задача о поляризации вакуума - неоднократно рассматривалась классиками квантовой электродинамики[1-9] Первый цикл работ появился за несколько лет до второй мировой войны, до появления метода перенормировок, авторы перечислены в сборнике [3]. Мы использовали работы Вайсскопфа [7,8], они вошли в учебник [1 § 47.3]. Для света $\Box A = 0$, и ток поляризации исчезает. Поэтому ток поляризации отличен от нуля только вблизи заряженных частиц и на внутренних фотонных линиях диаграмм Фейнмана.

Квантовый расчет поляризации электронного вакуума был сделан Вайсскопфом [7,8], все необходимые формулы есть в книге [1,§47.3]. Мы используем обозначения [1], в книге h=1, c=1. Рассматривается общая задача. Имеется ток $j^{(e)}=-\Box A^{(e)}$ Кроме того, имеется и ток поляризации электронного вакуума. Переходя к обычным единицам [1, формула 47,40]:

$$j^{(e)}+j_0 = -\Box A\,,\quad j_{(пол)} = -\frac{1}{15\pi}\frac{e^2}{hc}\left(\frac{h}{mc}\right)^2\Box j^{(e)} = \frac{1}{15\pi}\frac{e^2}{hc}\left(\frac{h}{mc}\right)^2\Box\Box A \qquad (1)$$

где j_0 - ток источника – протона. Дрожание протона мало, его рассматриваем как точечный заряд, расположенный в точке r=0. Вне этой точки есть только ток поляризации вакуума, и нас интересуют уравнения поля в вакууме. Из (1) видно, что в уравнения для потенциала вводятся высшие производные. Именно высшие производные приводят к конечности энергии поля [9§ 33, 10,11] .В (1) надо добавлен еще и внешний ток, создающий поле - $j^{(0)}$ - неподвижный заряд.

Оператор $\left(\frac{h}{mc}\right)^2\Box$ имеет нулевую размерность. Поэтому ряд, описывающий связь между током поляризации и вектор потенциалом A_i может содержать любые степени этого оператора. В общем случае для тока поляризации электронного вакуума будет ряд:

$$j_{i(пол)} = \frac{1}{15\pi}\frac{e^2}{hc}\,\Sigma k_n\left[\left(\frac{h}{mc}\right)^2\Box\right]^n\Box A_i = L\Box A_i,\ \ L=L(\Box),\ \ n\geq 1.\ \ k_1 = 1 \qquad (2)$$

где L – линейный дифференциальный оператор с постоянными коэффициентами. Подчеркнем, что в (2) отсутствует слагаемое n=0. Коэффициенты должны постоянными быть в силу инвариантности относительно сдвига. Ряд (2) удовлетворяет требованиям релятивистской инвариантности и поэтому теория релятивистки инвариантна. Напомним, что теории, в которых вводится элементарная длина и или размазывание заряда [3,12-14], нарушают релятивистскую инвариантность. Ряд (2) линеен по А, поэтому его можно перенести в левую часть уравнения $\Box A=-j$. Получаем уравнение для потенциала А, которое также содержит высшие производные. Производные более высокого порядка $\approx\square^3$ мы в настоящей работе опускаем. Полный ряд, включая производные более высокого порядка, был рассмотрен [10,11], где была доказана конечность пропагатора фотона при больших k и конечность собственной энергии .для точечного заряда. Это приводит конечным интегралам и хорошо сходящимся рядам. Уравнения поля будут:

$$\square[1+L(\square)]\,A = -j_0 \tag{3}$$

где – j_0 – источник поля – неподвижный заряд. Необходимо оговорить, что в точном ряде (2) коэффициенты k_n могут отличаться от значений, полученных методом последовательных приближений. Если метод последовательных приближений не достоверен, то k_n надо рассматривать как числовые параметры, с которыми можно провести вычисления до конца. Тем самым можно выйти за пределы теории возмущений. Электромагнитное поле характеризуется также и поляризацией, так как оператор $\square$ по отношению к поляризации является диагональным и при взаимодействии с электроном поляризация не меняется, то уравнение (3) нужно написать для каждой поляризации. В статике достаточно рассматривать только электростатический потенциал φ, для заряда источника поля ρ_0 и заряда поляризации вакуума имеем, используя (3) [1,7,8]:

$$\square=\Delta; \quad \rho_0=-\Delta\varphi \quad \rho_{(\text{пол})} = -\frac{1}{15\pi}\frac{e^2}{hc}\left(\frac{h}{mc}\right)^2\Delta\Delta\varphi \tag{4}$$

$$\Delta[1-(\Delta/k_0^2)]\,\varphi = -\rho_0 \quad \lambda^2=(k_0)^{-2} = \frac{1}{15\pi}\frac{e^2}{hc}\left(\frac{h}{mc}\right)^2,$$

Можно получить все соотношения непосредственно из электростатики материальных сред, и можно вести диэлектрическую проницаемость ε [10,11] :

$$D = \varepsilon E\ ;\ \operatorname{div} D= -\rho_0;\ \operatorname{div} E = -(\rho_{(\text{пол})} + \rho_0)$$

$$\varepsilon = \varepsilon(\omega,k)=(\rho_{(\text{пол})} + \rho_0)/\rho_0= 1+\left(\frac{k}{k_0}\right)^2,\ k \ll \omega/c, \tag{5}$$

$$\lambda = 1/k_0 = \sqrt{\frac{1}{15\pi 137}}\frac{h}{mc} = \sqrt{\frac{137}{15\pi}}\frac{e^2}{mc^2};\ \alpha = \frac{e^2}{hc} = \frac{1}{137}$$

$\lambda= 1/k_0 = 4{,}80\ 10^{-13}$ см. Ток поляризации протонного вакуума меньше на 6 порядков. В работах [10,11] была ошибка - приведено другое численное значение k_0, в настоящей работе это значение уточнено. Вывод работ [10,11] о конечности собственной энергии остается в силе. Обсудим структуру формулы (4) для λ^2. В нее входят два множителя – безразмерная константа взаимодействия электромагнитного поля и электрона и квадрат комптоновской длины электрона – частиц, движение которых вызывает поляризацию вакуума.

Уравнения электростатики в представлении Фурье имеют два полюса:

$$\varphi= \frac{1}{4\pi}\frac{e}{k^2\varepsilon},\quad \text{полюс } k=0$$

$$\varepsilon = 0,\ \text{полюс: } k=\pm ik_0$$

Волновой вектор становится чисто мнимым.

Для перехода к координатному представлению надо применить обратное преобразование Фурье. После интегрирования по углам в k-пространстве возникает множитель $(e/4\pi r)$.Интеграл по k в пределах $(0<k<+\infty)$можно свести к интегралу по вещественной оси комплексной плоскости k $(-\infty<k<+\infty)$ и интегралу по полуокружности вокруг точки k=0. После чего контур интегрирования замыкается в верхней полуплоскости и сводится к вычету в точке $k=ik_0$. Подчеркнем, что в статике никаких разрезов не возникает.

Поэтому электростатический потенциал φ точечного заряда равен

$$\varphi = \frac{e}{4\pi}\frac{1-e^{-k_0 r}}{r}, \tag{6}$$

Рассмотрим точку r = 0, числитель (6) можно разложить в ряд по степеням k_0:

$$\varphi(0) = \frac{e}{4\pi}k_0 < \infty, \quad E(0) = 0. \;\; k_0 \approx \sqrt{\alpha} \tag{7}$$

электростатический потенциал не имеет особенности при r=0. На расстояниях, меньших $1/k_0$, меняется структура статического электрического поля. Электростатический потенциал слабее, чем это следует из закона Кулона. Пробная частица двигается в слабом поле, как свободная. Это относится к электрическому полю, создаваемому любой заряженной частицей. Модель приводит к асимптотической свободе на малых расстояниях. Ядерные взаимодействия происходят в области, где электрические поля подавлены поляризацией электронного вакуума. Можно сказать, что эффективный заряд уменьшается на малых расстояниях.

Это качественно согласуется с тем, что учет поляризации вакуума улучшает сходимость интегралов для собственной энергии.[2,3, 10-12]. Наличие плато электростатического потенциала на расстояниях порядка нескольких ферми, является аргументом в пользу линейного рассмотрения. Напомним, что нелинейность проявляется на существенно меньших расстояниях [14]

Мы использовали формулы Вейсскопфа для уточнения уравнений электродинамики, вводили поправки к коэффициентам уравнений, но не искали поправки к потенциалу φ.

Свойства потенциала φ согласно формуле (6) радикально отличаются от свойств нулевого приближения φ= (e/4πr), поэтому решения в виде ряда теории возмущений, которые имеются в литературе [1, §50.3] и которые содержат особенности при r=0, не достоверны. Первый член ряда в точке r=0 должен быть бесконечным и отрицательным, чтобы уничтожить бесконечную особенность нулевого приближения. Отметим, что на нежелательность использования рядов теории возмущений указывал в 1954 году Л.Д.Ландау [13,14], и он оказался прав. «Размазывание» источника поля [13] по области, имеющей линейный размер a≠0, нарушает релятивистскую инвариантность теории, и при предельном переходе (a→0) она не восстанавливается. Подчеркнем, что исходные уравнения теории (1,2) релятивистки инвариантны. Обратим внимание, что $k_0 \approx \sqrt{\alpha}$ и поэтому формула (4) не может быть получена в виде ряда по степеням α. Поэтому рассмотрение области малых r требует выхода за рамки теории возмущений.

Обратим внимание еще на одно обстоятельство. Исходное уравнение (1) для потенциала $\Box A=0$ допускало решение только в виде поперечных волн, уравнение с высшими производными (3) допускает еще одну ветвь – продольные затухающие решения, аналог экранировки Дебая в плазме – ближние поля, которые локализованы вблизи источников поля. По аналогии с КХД [15] эту ветвь можно назвать «духами», в нашем случае она имеет ясный физический смысл, и избавляться от нее не надо. Вопрос о том, является ли масса электрона электромагнитной или механической, ставился давно – более 100 лет тому назад. Трудности состояли в том, что для существования электрона в статике нужно притягивающее поле, энергия которого отрицательна и для точечного электрона бесконечная. Энергия электрического поля положительна и бесконечна. Выражение ∞–∞ неопределенное и расшифровать его невозможно. Учет поляризации вакуума приводит к конечной и положительной величине, которая может оказаться другой для другой модели. Полевая энергия только электрического поля меньше энергии покоя электрона m_0c^2 , что, на наш взгляд, связано с тем, что есть и другие поля, в том числе и сегодня не известные.

В классической электродинамике [9] энергия точечного заряда можно сделать конечной двумя способами [9]:

- введением производных более высокого порядка, которые сглаживают разрыв потенциала при r=0. Этот вариант получается из квантовой теории, коэффициент при вторых производных имеет квантовую природу. Этот вариант в КЭД является, по нашему мнению, достаточным. Вариант приводит к асимптотической свободе на малых расстояниях.
- введением нелинейности в функцию Лагранжа, которые запрещают бесконечные поля. Пример для сферически симметричного случая приведен в [9 § 36]. Однако этот пример нельзя нарушает принцип суперпозиции и его трудно обобщить на квантовый случай.

Таким образом, эффект асимптотической свободы имеет место не только в КХД [15-17], но и в стандартной линейной квантовой электродинамике.

Рассмотрим теперь сильные взаимодействия, в рамках квантовой хромодинамики – КХД [15].Сходство между КХД и КЭД неоднократно обсуждалась в литературе, ниже эти результаты будут получены в рамках КХД.

Глюонное поле в КХД имеет векторный потенциал B_a^μ, латинский индекс означает цвет [15]. Ковариантный ротор поля B имеет вид [15] :

$$(D^\mu \times B^\nu)_a \equiv G_a^{\mu\nu} = \partial^\mu B_a^\nu - \partial^\nu B_a^\mu + g\ \Sigma f_{abc} B_b^\mu B_c^{\ \nu} \tag{8}$$

где f_{abc} - структурные константы, антисимметричные по цветовым индексам. Оператор дифференцирования – линейный оператор по отношению к вектору B, а формула (8) содержит произведения компонент B. Казалось бы, они нарушают линейность. Однако линейность остается благодаря свойствам антисимметрии по цветовым индексам структурных констант – в последнее слагаемое B_a вообще не входит. Тензор $B_b^\mu B_c^\nu$ при фиксированных координатных индексах симметричен по цветовым индексам, как произведение, и поэтому его свертка с антисимметричными по цвету структурными константами дает нуль. Поэтому, на наш взгляд, утверждения в литературе о принципиальной нелинейности [15 стр. 5] глюонного поля, которая проявляется во взаимодействии глюонов, требуют более подробного обсуждения. В формуле (8) g – универсальная константа сильного взаимодействия, для безразмерной константы имеем:

$$\alpha_s = \frac{g^{\ 2}}{hc} \approx 1 \tag{9}$$

Большое значение константы (9) делает невозможным разложение в ряд теории возмущений по степеням α_s.Для перехода к волновому уравнению с производными второго порядка надо к (8) еще раз применить линейный оператор $(D \times B)$.Однако, кратное применение линейных операторов не может привести к нелинейности.

Поскольку нелинейности нет, то остается возможность линейной связи между разными глюонами. Если это так, то в рамках КХД задача о глюонах может быть решена точно. В уравнениях для глюонов фиксированного цвета входят компоненты вектор потенциала других цветов и при этом линейно. Всего имеется 8 типов глюонов. Таким образом, получаем линейную систему из 8 уравнений, и всегда можно найти нормальные волны [18], каждая нормальная волна распространяется независимо.

При переходе от КЭД к КХД имеет место соответствие $A \Leftrightarrow B_s$, причем индекс указывает тип глюона. При этом имеет место соответствие и для констант:

$$\alpha_{(\text{элм})} = \frac{e^2}{3\pi} \approx 10^{-3} \Leftrightarrow \alpha_s \approx 1\ ;\ h=1,\ c=1;\ \varphi = A_0 \Leftrightarrow B_{s0}, \tag{10}$$

В формуле (5) для элементарной длины $\alpha_{(\text{элм})}$ - безразмерная постоянная электромагнитного взаимодействия в рационализированных единицах должна быть заменена на безразмерную постоянную α_s кварк-глюонного взаимодействия (10) [16].

Кроме того, массу электрона m=0,5 Мэв надо заменить на массу кварков, надо брать массу самых легких кварков m=10 Мэв. И есть еще одно обстоятельство. При взаимодействии с вакуумом поляризация фотона не меняется, а цвет глюона может измениться. При выводе в КХД уравнения типа (3) нужно учесть, что объединять в левой части можно только глюоны одного цвета. Поэтому возникнет добавочный множитель, равный вероятности сохранения цвета. Учитывая, что всего имеется 8 сортов глюонов, этот множитель равен примерно 1/8. Для элементарной длины в КХД получаем оценочную формулу:

$$a_s^2 = \frac{1}{8}\frac{g^2}{hc}\frac{h}{m_1 c}\frac{h}{m_2 c} \tag{11}$$

Как и в КЭД, из релятивистских уравнений в статическом случае [19] появляется элементарная длина и асимптотическая свобода при малых расстояниях. При вычислениях исчезают бесконечности и необходимость проведения перенормировок. Элементарная длина имеет разумный порядок около 1 ферми, сегодня эта область может быть исследована экспериментально. Асимптотическая свобода определяется для статического поля, как уменьшение эффективного заряда, более точное количественное определение зависит от формы потенциала.

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From Laue's stress-energy tensor to Maxwell's Equations and the implications for Einstein's GTR.

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In this paper I will connect the electrodynamic stress energy tensor, in its electrodynamic version that dates back to Gustav Mie's 1912 papers, to the anti-symmetric electromagnetic field tensor and to the Lorentz Force Law and Maxwell's Equations. Important insight concerning the *mathematical* connection between Einstein's GTR and Maxwell-Lorentz ED will result. Backed by these results I will put forward the claim that in the regions where Maxwell-Lorentz's electrodynamics can be successfully applied, Einstein's restrictions put on his GTR stress-energy tensor are such that they cannot be fulfilled and so GTR cannot be applied.

I. INTRODUCTION

These days, the aspirations of grand unification in physics are defined as the wish to unify the Standard Model with Einstein's General Theory of Relativity into one theory called GUT. As a philosopher specialized in ontology however, I feel it difficult to accept the Standard Model as a *complete* theory. The Standard Model is composed of impressive mathematics and complicated high-tech and high-cost experimentations, but its ontological part looks rather poor. Its internal consistency can also be criticized to because the integration of its mayor ingredient, Quantum Chromodynamics, with electrodynamics and quantum mechanics has more the character of a wishful proclamation than a smooth thing. To bring some clarity in the *ontological* aspects of the project of unification, I found it necessary to go back to the foundations of the three ontologically better defined theories of relativistic Maxwell-Lorentz electrodynamics (RED), Einstein's GTR and Copenhagen QM. After some study, it seemed to me that a general component in all three was relativistic tensor dynamics (RTD), as defined first by Max von Laue [1].

So I tried to link the three theories, GTR, QM and ED, to one single stress energy tensor $T_{\mu\nu}$ and its relativistic derivations. In an earlier article I connected the quantized action to the source of gravity, the trace of the stress energy tensor $T_{\mu\nu}$, a connection that resulted in a principle of equivalence, of the phases instead of the masses, for Quantum Gravity [2]. In the article presented at the 2004 PIRT conference in London I tried to translate the mechanical stress energy tensor $T_{\mu\nu} = V_\mu G_\nu$ in terms of the electrodynamic current and potential $T_{\mu\nu} = J_\mu A_\nu$ [3]. In this article I will first summarize the 2004 PIRT result and then try to connect the electrodynamic stress energy tensor, in its electrodynamic version $J_\mu A_\nu$ that dates back to Gustav Mie's 1912 papers ([4], p. 525), to the anti-symmetric electromagnetic field tensor $B_{\mu\nu}$. Important insight concerning the *mathematical* connection between Einstein's GTR and Maxwell-Lorentz ED will result. If this mathematical link is embodied in real physics, if it has an appropriate ontology, is a subject of further research.

II. BASIC RESULTS PRESENTED AT THE 2004 LONDON PIRT.

A. Max von Laue's relativistic conservation laws.

1. *The mechanical stress energy tensor*

The stress energy tensor introduced by Max von Laue in 1911 can be written as $T_{\mu\nu} = V_\mu G_\nu$ [1], [5]. With

$$G_\nu = \begin{bmatrix} \frac{i}{c}u \\ \boldsymbol{g} \end{bmatrix} \text{ and } V_\mu = \begin{bmatrix} ic \\ \boldsymbol{v} \end{bmatrix},$$

we can write the SE-tensor in one single velocity field as

$$T_{\mu\nu} = \begin{bmatrix} ic \\ \boldsymbol{v} \end{bmatrix} \begin{bmatrix} \frac{i}{c}u \\ \boldsymbol{g} \end{bmatrix} = \begin{bmatrix} -u & ic\boldsymbol{g} \\ \frac{i}{c}u\boldsymbol{v} & \boldsymbol{v}\otimes\boldsymbol{g} \end{bmatrix}. \tag{1}$$

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In the paper I will as much as possible omit the multiplier γ, defined as

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{2}$$

This seems to imply $\gamma \approx 1$ and $v \ll c$, but there is an other, more ontological reason for this approach, the use of the concept of a *velocity field* (as in Dirac-1951, [6], [7]). The velocity four vector is not connected to one localized particle or to a particle seen as a specific reference frame, but it is defined as a field existing in all space-time. The velocity field as a whole can seen from the perspective of one single observer in his frame. When a particle moves in this field, it has at every world-point the velocity connected to this localization in space-time. The Lorentz Transformations do not connect specific velocities of one field, but relate the fields as seen by different observers in their specific reference frames. The Lorentz Transformation multiplication factor meanly appears when we map the velocity field V_μ of observer A onto the velocity field V'_μ of observer B moving with velocity w relative to A.

2. The conservation laws for energy and momentum

For closed systems energy and momentum are conserved and the conservation laws can be expressed as $\partial_\mu T_{\mu\nu} = 0$ with the four vector partial derivative defined as

$$\partial_\mu = \begin{bmatrix} -\frac{i}{c}\partial_t \\ \nabla \end{bmatrix}. \tag{3}$$

This leads to

$$\begin{gathered}\partial_\mu T_{\mu\nu} = \begin{bmatrix} -\frac{i}{c}\partial_t \\ \nabla \end{bmatrix} \begin{bmatrix} -u & ic\boldsymbol{g} \\ \frac{i}{c}u\boldsymbol{v} & \boldsymbol{v}\otimes\boldsymbol{g} \end{bmatrix} = \\ \begin{bmatrix} \frac{i}{c}(\partial_t u + \nabla\cdot u\boldsymbol{v}) \\ \partial_t\boldsymbol{g} + \nabla(\boldsymbol{v}\otimes\boldsymbol{g}) \end{bmatrix} = \begin{bmatrix} \frac{i}{c}\mathcal{P} \\ \boldsymbol{f} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{0} \end{bmatrix}.\end{gathered} \tag{4}$$

We have used $\mathcal{P}$ for the power density and $\boldsymbol{f}$ for the force density. If we write $\boldsymbol{S} = u\boldsymbol{v}$ for the energy density current or Umov's vector, this results in the conservation equation for energy

$$\nabla\cdot\boldsymbol{S} + \partial_t u = 0, \tag{5}$$

which can also be written as $\partial_\mu S_\mu = 0$, and the conservation equation for momentum

$$\partial_t\boldsymbol{g} + \nabla(\boldsymbol{v}\otimes\boldsymbol{g}) = 0. \tag{6}$$

Of course, if the system is not completely closed, we have a net power density

$$\mathcal{P} = \partial_\mu S_\mu = \nabla\cdot\boldsymbol{S} + \partial_t u. \tag{7}$$

3. The conservation law for angular momentum

A relativistic system also has a mechanical torque-tensor

$$N_{\mu\nu} = T_{\mu\nu} - T_{\nu\mu} = V_\mu G_\nu - V_\nu G_\mu. \tag{8}$$

If we use the abbreviation $_a$ for anti-symmetric we can define $\boldsymbol{n}_a = \boldsymbol{v}\times\boldsymbol{g}$ and $\boldsymbol{g}_a = \frac{1}{c^2}u\boldsymbol{v} - \boldsymbol{g}$, we can write the torque-tensor in full as:

$$N_{\mu\nu} = \begin{bmatrix} 0 & -icg_{a,1} & -icg_{a,2} & -icg_{a,3} \\ icg_{a,1} & 0 & n_{a,3} & -n_{a,2} \\ icg_{a,2} & -n_{a,3} & 0 & n_{a,1} \\ icg_{a,3} & n_{a,2} & -n_{a,1} & 0 \end{bmatrix}. \tag{9}$$

A closed system, for example a free particle, should not acquire any additional angular momentum, free as it is from external influences. The conservation of the intrinsic angular momentum requires the stress-energy tensor to be symmetric $V_\mu G_\nu = V_\nu G_\mu$ and the torque-density tensor and six-vector to vanish, so $N_{\mu\nu} = 0$. This leads to $\boldsymbol{n}_a = \boldsymbol{v} \times \boldsymbol{g} = 0$ and $\boldsymbol{g}_a c^2 = u\boldsymbol{v} - \boldsymbol{g}c^2 = 0$, which gives

$$\boldsymbol{g} = \frac{1}{c^2} u\boldsymbol{v} \tag{10}$$

or $\mathbf{g} = \rho_i \mathbf{v}$ and $u = \rho_i c^2$ for a moving free particle. These equations are very familiar, the last is the density expression of Einstein's famous $U = mc^2$ and $\boldsymbol{g} = \rho_i \boldsymbol{v}$ is nothing but the density version of Newton's *definition* of momentum $\boldsymbol{p} = m_i \boldsymbol{v}$.

B. Translating these results into EM potential terms.

1. The electromagnetic stress energy tensor

The electromagnetic system can be given a stress energy tensor in terms of the charge density current and the electric potential, as first done by Gustav Mie in 1912 ([4], p. 525):

$$T_{\mu\nu} = V_\mu G_\nu = V_\mu \rho_{em} A_\nu = \rho_{em} V_\mu A_\nu = J_\mu A_\nu. \tag{11}$$

This makes it possible to achieve a direct translation of von Laue's relativistic tensor dynamics, including the conservation laws, into electrodynamic terms.

The stress energy tensor introduced by Max von Laue in 1911 can be written as $T_{\mu\nu} = J_\mu A_\nu$, with

$$A_\nu = \begin{bmatrix} \frac{i}{c}\phi \\ \boldsymbol{A} \end{bmatrix} \;\; and \;\; J_\mu = \begin{bmatrix} ic\rho \\ \boldsymbol{J} \end{bmatrix} \tag{12}$$

so

$$T_{\mu\nu} = \begin{bmatrix} ic\rho \\ \boldsymbol{J} \end{bmatrix} \begin{bmatrix} \frac{i}{c}\phi \\ \boldsymbol{A} \end{bmatrix} = \begin{bmatrix} -\rho\phi & ic\rho\boldsymbol{A} \\ \frac{i}{c}\phi\boldsymbol{J} & \boldsymbol{J} \otimes \boldsymbol{A} \end{bmatrix}. \tag{13}$$

2. The torque tensor and conservation law for angular momentum

A relativistic system also has a mechanical torque-tensor

$$N_{\mu\nu} = T_{\mu\nu} - T_{\nu\mu} = J_\mu A_\nu - J_\nu A_\mu. \tag{14}$$

If we use the abbreviation $_a$ for anti-symmetric we can define $\boldsymbol{n}_a = \boldsymbol{J} \times \boldsymbol{A}$ and $\boldsymbol{g}_a = \frac{1}{c^2}\phi\boldsymbol{J} - \rho\boldsymbol{A}$, we can write the torque-tensor in full as:

$$N_{\mu\nu} = \begin{bmatrix} 0 & -icg_{a,1} & -icg_{a,2} & -icg_{a,3} \\ icg_{a,1} & 0 & n_{a,3} & -n_{a,2} \\ icg_{a,2} & -n_{a,3} & 0 & n_{a,1} \\ icg_{a,3} & n_{a,2} & -n_{a,1} & 0 \end{bmatrix}. \tag{15}$$

A closed system, for example a free particle, should not acquire any additional angular momentum, free as it is from external influences. The conservation of the intrinsic angular momentum requires the stress-energy tensor to be symmetric $J_\mu A_\nu = J_\nu A_\mu$ and the torque-density tensor and six-vector to vanish, so $N_{\mu\nu} = 0$. This leads to $\boldsymbol{n}_a = \boldsymbol{J} \times \boldsymbol{A} = 0$ and $\boldsymbol{g}_a = \frac{1}{c^2}\phi\boldsymbol{J} - \rho\boldsymbol{A} = 0$ which gives

$$\boldsymbol{A} = \frac{1}{c^2}\phi\boldsymbol{v} \tag{16}$$

for a moving free particle. In four vector terms this implies, for closed systems,

$$A_\mu = \begin{bmatrix} \frac{i}{c}\phi \\ \boldsymbol{A} \end{bmatrix} = \begin{bmatrix} \frac{i}{c}\phi \\ \frac{1}{c^2}\phi\boldsymbol{v} \end{bmatrix} = \frac{1}{c^2}\phi \begin{bmatrix} ic \\ \boldsymbol{v} \end{bmatrix} = \frac{1}{c^2}\phi V_\mu. \tag{17}$$

3. The conservation laws for electromagnetic energy and momentum

For closed systems energy and momentum are conserved and the conservation laws can be expressed as $\partial_\mu T_{\mu\nu} = 0$. This leads to

$$\begin{gathered}\partial_\mu T_{\mu\nu} = \begin{bmatrix} -\frac{i}{c}\partial_t \\ \nabla \end{bmatrix} \begin{bmatrix} -\rho\phi & ic\rho\boldsymbol{A} \\ \frac{i}{c}\phi\boldsymbol{J} & \boldsymbol{J}\otimes\boldsymbol{A} \end{bmatrix} = \\ \begin{bmatrix} \frac{i}{c}(\partial_t(\rho\phi) + \nabla\cdot(\phi\boldsymbol{J})) \\ \partial_t(\rho\boldsymbol{A}) + \nabla(\boldsymbol{J}\otimes\boldsymbol{A}) \end{bmatrix} = \begin{bmatrix} \frac{i}{c}\mathcal{P} \\ \boldsymbol{f} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{0} \end{bmatrix}.\end{gathered} \tag{18}$$

This results in the conservation equations for energy

$$\nabla\cdot(\phi\boldsymbol{J}) + \partial_t(\rho\phi) = 0 \tag{19}$$

and momentum

$$\partial_t(\rho\boldsymbol{A}) + \nabla(\boldsymbol{J}\otimes\boldsymbol{A}) = 0. \tag{20}$$

Of course, if the system is not completely closed, we have a net power density

$$\mathcal{P} = \partial_\mu S_\mu = \partial_t(\rho\phi) + \nabla\cdot(\phi\boldsymbol{J}). \tag{21}$$

We will investigate this last term further. We write it out as

$$\begin{aligned}\mathcal{P} = \partial_\mu S_\mu = \partial_t(\rho\phi) + \nabla\cdot(\phi\boldsymbol{J}) = \rho\partial_t\phi + \phi\partial_t\rho + \phi\nabla\cdot\boldsymbol{J} + \boldsymbol{J}\nabla\cdot\phi = \\ \phi(\nabla\cdot\boldsymbol{J} + \partial_t\rho) + \boldsymbol{J}\nabla\cdot\phi + \rho\partial_t\phi = \phi(\partial_\mu J_\mu) + J_\mu\partial_\mu\phi.\end{aligned} \tag{22}$$

Let's study the last term $\partial_\mu\phi$ a bit closer. If we use $A_\mu = \partial_\mu\chi$ and $\partial_t A_\mu = E_\mu$ for symmetric electric forces, we can write it as

$$\partial_\mu\phi = -\partial_\mu\partial_t\chi = -\partial_t\partial_\mu\chi = -\partial_t A_\mu = -E_\mu \tag{23}$$

So if we use $E_\mu = -\partial_\mu\phi$ for symmetric systems, we get

$$\mathcal{P} = \partial_\mu S_\mu = \phi(\partial_\mu J_\mu) - J_\mu E_\mu. \tag{24}$$

If the charge can be considered as a conserved quantity we have $\partial_\mu J_\mu = 0$ and get a net power density equation

$$\mathcal{P} = \partial_\mu S_\mu = -J_\mu E_\mu \tag{25}$$

4. The process of symmetry breaking results in an equality needed for the derivation of Maxwell's Equations

Let's consider the last equation, which can be applied to open systems that lose electromagnetic energy but maintain charge conservation. Suppose we can separate it in $\nabla\cdot\boldsymbol{S} = -\boldsymbol{J}\cdot\boldsymbol{E} \neq 0$ and $\partial_t u = \rho\partial_t\phi = 0$. We will not justify the physics of this situation, but simply claim the *possibility* that this describes *mathematically* the anti-symmetric character of a photon emitting system. Once it radiates, the symmetry of the EM system is broken. We hypothesize that the time-like parts of the four vectors S_μ and E_μ of a radiating system are zero. For such a system we can deduce a simple relation between the acceleration and the partial space derivative.With

$$-\boldsymbol{J}\cdot\boldsymbol{E} = -\boldsymbol{v}\cdot\rho\boldsymbol{E} = -\boldsymbol{v}\cdot\boldsymbol{f} = -\rho_i\boldsymbol{v}\cdot\boldsymbol{a} = -\boldsymbol{g}\cdot\boldsymbol{a} = -\boldsymbol{a}\cdot\boldsymbol{g} \tag{26}$$

and

$$\nabla\cdot\boldsymbol{S} = c^2\nabla\cdot\boldsymbol{g} \tag{27}$$

we get

$$c^2\nabla\cdot\boldsymbol{g} = -\boldsymbol{a}\cdot\boldsymbol{g} \tag{28}$$

so

$$\boldsymbol{a} = -c^2\nabla. \tag{29}$$

This equation is a crucial step in the derivation of the anti-symmetric Maxwell-Lorentz theory from Laue's symmetrical Relativistic Tensor Dynamics. The physical justification of $\boldsymbol{a} = -c^2\nabla$ lies in *the process of symmetry breaking*. We cannot investigate this crucial symmetry breaking process at this stage of analysis. For the time being, we use the relation $\boldsymbol{a} = -c^2\nabla$ as a mathematical necessity in the route from Laue's symmetrical RTD towards Maxwell-Lorentz' anti-symmetrical RED. Discussing the physics incorporated in the road map from Laue to Maxwell-Lorentz makes sense after we showed the existence of a mathematical connection.

C. The antisymmetric potential field tensor

In our attempt to connect Laue's symmetric Relativistic Tensor Dynamics to Maxwell-Lorentz anti-symmetric relativistic Electrodynamics, we introduced the JA-stress energy tensor and the anti-symmetric torque tensor. As a next step we introduced, in our London PIRT paper, the antisymmetric potential field tensor $A_{\mu\nu}$ as

$$N_{\mu\nu} = -\rho c^2 A_{\mu\nu} \tag{30}$$

With the EM torque-tensor as

$$N_{\mu\nu} = J_\mu A_\nu - J_\nu A_\mu \tag{31}$$

we get for $A_{\mu\nu}$

$$A_{\mu\nu} = -\frac{1}{c^2}(V_\mu A_\nu - V_\nu A_\mu) \tag{32}$$

and

$$A_{\mu\nu} = -\frac{1}{\rho c^2} N_{\mu\nu} \tag{33}$$

We want to define the reduced-torque potential tensor in full as:

$$A_{\mu\nu} = \begin{bmatrix} 0 & \frac{i}{c}A_{a,1} & \frac{i}{c}A_{a,2} & \frac{i}{c}A_{a,3} \\ -\frac{i}{c}A_{a,1} & 0 & \widetilde{A}_{a,3} & -\widetilde{A}_{a,2} \\ -\frac{i}{c}A_{a,2} & -\widetilde{A}_{a,3} & 0 & \widetilde{A}_{a,1} \\ -\frac{i}{c}A_{a,3} & \widetilde{A}_{a,2} & -\widetilde{A}_{a,1} & 0 \end{bmatrix}. \tag{34}$$

With the abbreviation $_a$ for anti-symmetric this gives us the reduced torque potential or magnetic potential as

$$\widetilde{\boldsymbol{A}} = -\frac{1}{\rho c^2}\boldsymbol{n}_a = -\frac{1}{\rho c^2}\boldsymbol{J} \times \boldsymbol{A} \tag{35}$$

so

$$\boldsymbol{n}_a = -\rho c^2 \widetilde{\boldsymbol{A}} = \boldsymbol{J} \times \boldsymbol{A} \tag{36}$$

and

$$\widetilde{\boldsymbol{A}} = -\frac{1}{c^2}\boldsymbol{v} \times \boldsymbol{A}, \tag{37}$$

and the electric potential as

$$\boldsymbol{g}_a = \rho \boldsymbol{A}_a = \frac{1}{c^2}\boldsymbol{J}\phi - \rho \boldsymbol{A}, \tag{38}$$

so

$$\boldsymbol{A}_a = \frac{1}{c^2}\boldsymbol{v}\phi - \boldsymbol{A}. \tag{39}$$

We introduced the the antisymmetric potential field tensor $A_{\mu\nu}$ as a mathematical entity that can be defined in a consistent uniform manner. Its physical existence depends on the reality, or not, of the breaking of symmetry. The symmetry is broken once the torque tensor $N_{\mu\nu}$ is not equal to zero. The question if the mathematical road from Laue's RTD to Maxwell-Lorentz' RED corresponds to some physical reality can be pinpointed to the breaking of symmetry. Mathematically, this means the existence of a non-zero torque tensor. Does this non-zero torque tensor correspond to some real ontology? Before we investigate this, we will show how to derive, mathematically, the Lorentz Force Law and Maxwell's Equations from Laue's RTD.

III. FROM LAUE'S TENSOR DYNAMICS TO THE RELATIVISTIC MAXWELL-LORENTZ THEORY

A. From the anti-symmetric potential field tensor to the electromagnetic field tensor

Using the relation $\boldsymbol{a} = -c^2\nabla$ or $-\frac{1}{c^2}\boldsymbol{a} = \nabla$ makes it easy to derive the electromagnetic field tensor $B_{\mu\nu}$ from the potential field tensor $A_{\mu\nu}$. In the process a rest tensor will appear. We will name this rest tensor $E_{\mu\nu}$ and derive its properties. We then have the simple relation

$$\partial_t A_{\mu\nu} = B_{\mu\nu} + E_{\mu\nu}. \tag{40}$$

We will start with the anti-symmetric electric potential first.

$$\begin{aligned}\partial_t \boldsymbol{A}_a = \partial_t(\frac{1}{c^2}\boldsymbol{v}\phi - \boldsymbol{A}) = \frac{1}{c^2}\boldsymbol{a}\phi - \partial_t\boldsymbol{A} + \frac{1}{c^2}\boldsymbol{v}\partial_t\phi = \\ (-\nabla\phi - \partial_t\boldsymbol{A}) + \frac{1}{c^2}\boldsymbol{v}\partial_t\phi = \boldsymbol{E}_a + \boldsymbol{E}_r\end{aligned} \tag{41}$$

So we get to parts, the usual anti-symmetric electric field $\boldsymbol{E}_a$ and a rest anti-symmetric electric field $\boldsymbol{E}_r$

Then we differentiate the anti-symmetric magnetic potential to get

$$\begin{aligned}\partial_t\widetilde{\boldsymbol{A}} = \partial_t(-\frac{1}{c^2}\boldsymbol{v}\times\boldsymbol{A}) = -\frac{1}{c^2}\boldsymbol{a}\times\boldsymbol{A} - \frac{1}{c^2}\boldsymbol{v}\times\partial_t\boldsymbol{A} = \\ \nabla\times\boldsymbol{A} - \frac{1}{c^2}\boldsymbol{v}\times\boldsymbol{E} = \boldsymbol{B}_a + \boldsymbol{B}_r.\end{aligned} \tag{42}$$

In six-vector terms this gives

$$\partial_t(\widetilde{\boldsymbol{A}} - \frac{i}{c}\boldsymbol{A}_a) = (\boldsymbol{B}_a - \frac{i}{c}\boldsymbol{E}_a) + (\boldsymbol{B}_r - \frac{i}{c}\boldsymbol{E}_r). \tag{43}$$

We can split this up and write $B_{\mu\nu}$ in six-vector notation as

$$\boldsymbol{B}_a - \frac{i}{c}\boldsymbol{E}_a = \nabla\times\boldsymbol{A} - \frac{i}{c}(-\nabla\phi - \partial_t\boldsymbol{A}) \tag{44}$$

and $E_{\mu\nu}$ as

$$\boldsymbol{B}_r - \frac{i}{c}\boldsymbol{E}_r = (-\frac{1}{c^2}\boldsymbol{v}\times\boldsymbol{E}) - \frac{i}{c}(\frac{1}{c^2}\boldsymbol{v}\partial_t\phi). \tag{45}$$

In tensor formulation we have for $B_{\mu\nu}$

$$B_{\mu\nu} = \begin{bmatrix} 0 & \frac{i}{c}E_{a,1} & \frac{i}{c}E_{a,2} & \frac{i}{c}E_{a,3} \\ -\frac{i}{c}E_{a,1} & 0 & B_{a,3} & -B_{a,2} \\ -\frac{i}{c}E_{a,2} & -B_{a,3} & 0 & B_{a,1} \\ -\frac{i}{c}E_{a,3} & B_{a,2} & -B_{a,1} & 0 \end{bmatrix}. \tag{46}$$

For $E_{\mu\nu}$ we can write the same tensor.

B. The Lorentz Force Law

The relativistic formulation of the Lorentz Force Law is

$$B_{\mu\nu}J_\nu = f_\mu \tag{47}$$

When we apply it to our potential formulation we get

$$(\partial_t A_{\mu\nu})J_\nu = B_{\mu\nu}J_\nu + E_{\mu\nu}J_\nu. \tag{48}$$

We have several possibilities to interpret this result. The most simple thing to do is to give every term its own force, like in

$$f^A_\mu = (\partial_t A_{\mu\nu})J_\nu = B_{\mu\nu}J_\nu + E_{\mu\nu}J_\nu = f^B_\mu + f^E_\mu. \tag{49}$$

We do not however want to discuss the complications of our derivations, we just want to show how to derive the LFL from Laue's tensor dynamics in JA formulation.

If we go back to the torque tensor we have

$$(\partial_t(-\frac{1}{\rho c^2}N_{\mu\nu}))\rho V_\nu = B_{\mu\nu}J_\nu + E_{\mu\nu}J_\nu, \tag{50}$$

and, if we assume ρ to be time independent, we get

$$-\frac{1}{c^2}(\partial_t N_{\mu\nu})V_\nu = B_{\mu\nu}J_\nu + E_{\mu\nu}J_\nu, \tag{51}$$

giving

$$-\frac{1}{c^2}(\partial_t N_{\mu\nu})V_\nu = f_\mu. \tag{52}$$

If we go all the way back to the stress energy tensor, written as $T_{\mu\nu} = V_\mu G_\nu = J_\mu A_\nu$ we can write

$$-\frac{1}{c^2}(\partial_t T_{\mu\nu} - \partial_t T_{\nu\mu})V_\nu = f_\mu, \tag{53}$$

for the Lorentz Force Law with a time independent charge density.

C. Gauss' Law and Ampère's Law as Maxwell's Equations

We will not derive all four of Maxwell's Equations but restrict ourselves to Gauss' Law and Ampère's Law. But because the other two laws are just there dual parts in case of the homogenous forms, we talk about the two as Maxwell's Equations. They can be formulated as

$$\partial_\nu B_{\mu\nu} = \mu_0 J_\mu. \tag{54}$$

Thus the homogenous Maxwell Equations, when $\mu_0 J_\mu = 0$, can be derived from the potentials as

$$\partial_\nu(\partial_t A_{\mu\nu}) = \partial_\nu B_{\mu\nu} + \partial_\nu E_{\mu\nu} = 0. \tag{55}$$

If we go back to the torque tensor we have

$$\partial_\nu(\partial_t(-\frac{1}{\rho c^2}N_{\mu\nu})) = \partial_\nu B_{\mu\nu} + \partial_\nu E_{\mu\nu} = 0. \tag{56}$$

And if we go all the way back to the stress energy tensor

$$\partial_\nu(\partial_t(-\frac{1}{\rho c^2}(T_{\mu\nu} - T_{\nu\mu}))) = \partial_\nu B_{\mu\nu} + \partial_\nu E_{\mu\nu} = 0. \tag{57}$$

In the case of the inhomogeneous equations we could perhaps have

$$\partial_\nu(\partial_t(-\frac{1}{\rho c^2}(T_{\mu\nu} - T_{\nu\mu}))) = \mu_0 J_\mu. \tag{58}$$

IV. COMPARING EINSTEIN'S GTR WITH THE RELATIVISTIC MAXWELL-LORENTZ EM

Einstein's GTR has the stress energy tensor as a fundamental input. The stress energy tensor has to have two basic properties, it must be symmetric and it's divergence must be zero ([8], p. 58). So Einstein's Equations are valid for those stress energy tensors who have

$$N_{\mu\nu} = T_{\mu\nu} - T_{\nu\mu} = 0 \tag{59}$$

and

$$\partial_\mu T_{\mu\nu} = 0. \tag{60}$$

If we look at the Lorentz Force Law as

$$(\partial_t(-\frac{1}{\rho c^2}N_{\mu\nu}))J_\nu = B_{\mu\nu}J_\nu + E_{\mu\nu}J_\nu, \tag{61}$$

and at Maxwell's Equations as

$$\partial_\nu(\partial_t(-\frac{1}{\rho c^2}(T_{\mu\nu} - T_{\nu\mu}))) = \mu_0 J^A_\mu. \tag{62}$$

then it is immediately clear that these two theories cannot possibly be unified. The restrictions put on Einstein's GTR are axiomatic in character, which implies that the unification of the phenomena of gravity with the phenomena of electrodynamics can only be achieved by abandoning GTR. The General Theory of Relativity as a theory of gravity seems much to *restricted* to be unifiable with Maxwell-Lorentz electrodynamics. I emphasized the word restricted because I do not claim GTR to be a *wrong* theory. In those regions of reality where the axioms of GTR are valid, the theory of GTR, being bases on those axioms, is valid. What I claim is that in the regions where Maxwell-Lorentz's electrodynamics can be successfully applied, Einstein's restrictions put on his stress-energy tensors are such that they cannot be fulfilled and so GTR cannot be applied.

There are however two severe restriction put on this conclusion. First of all, we have shown a mathematical possibility, not a physical reality. The existence of a mathematical derivation of Maxwell-Lorentz' RED from Laue's RTD does not automatically imply it's physical reality. The equation $\boldsymbol{a} = -c^2\nabla$ for example is a mathematical tool we need in the derivation, but its physical relevance is unclear. Asking if the mathematical derivation makes physical sense is, however, not a simple question but contains an entire research program. The secondly restriction put on our conclusion comes from the use of the pseudo gravitational stress energy tensor $t_{\mu\nu}$ in GTR. This pseudo tensor functions as a *save the theory tensor* in those cases where the conservation of energy and the being zero of $\partial_\mu T_{\mu\nu}$ are not fulfilled. The pseudo tensor does not have to be symmetric, nor does $\partial_\mu t_{\mu\nu}$ have to be zero, so without doubt the pseudo gravitational stress energy tensor $t_{\mu\nu}$ can also be made flexible enough to "save" the unification project of GTR with Maxwell-Lorentz electrodynamics from our objections. (As such, the pseudo tensor $t_{\mu\nu}$ strongly resembles the Poincaré stress energy tensor as it was and still is used in relativistic electrodynamics for exactly the same reasons, to save the theory in case of violation of the conservation of energy [3].)

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On possibility of a new 3D experimental test of moving media electrodynamics

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Invariance of electrodynamics equations relative to the group of transformations should have close connection with non-invariant properties of transformations for partial differentials of space-time. If the connection has observed consequences, they should appear in moving media electrodynamics: in tasks, where a source, a receiver, boundaries between media and the media move with different velocities.

A fundamental aspect of the question is that moving media electrodynamics equations were tested in some special cases and they weren't tested for 3-dimensional tasks. An applied aspect is an answer on the question: How do readings of an interferometer moving around the Earth depend on its position and orientation? It's cost to be noticed that dependence on a Earth rotation rate was found in an Sagnac-type interferometer.

An analysis of Michelson-Morley-type experiments allow to assert that invariance of the result is provided with very high degree of accuracy. One can made the conclusion after calculating with account: terms of second order smallness β^2, contraction of interferometer length and its elements, alteration of source frequency, and radiation frequency when it reflects from moving elements, variation of reflection angles from moving elements.

The Fizeau's interferometer (fig.1a) is more interesting for analysis. There is no an unique inertial reference frame (IRF), in which all elements rests, therefore, in the case we cannot pass from a rest IRF to a moving IRF. As composition of velocities for the interferometer and the medium should satisfy to relativistic law, it is intrinsic to assume that if non-invariant properties of partial differentials can have observed appearances, they will be found in non-linear terms of a solution of the dispersion equation.

Let us to consider the Fizeau's interferometer in the IRF, in which the interferometer rests, that is $\beta = v/c = 0$, where c is light velocity in vacuum. $\vec{u}$ is water velocity in the interferometer IRF, and $\beta_{2n} = \pm u/c$. The invariants $I_t = k_t = k_0 \sin\vartheta_0 = 0$, $-I_1 = \omega_0(1-\beta) = \omega_0$ corresponds to normal incident beams [1]. In the case parameters $d = \dfrac{I_t}{I_1} = 0$, $Q = n_2^2$ are contained in the solution of the dispersion equation.

Then a wave vector for a refractive beam is

$$k_{2n} = \frac{\omega_0}{c}\frac{-\beta_{2n}(n_2^2-1) + n_2(1-\beta_{2n}^2)}{1-n_2^2\beta_{2n}^2}. \tag{1}$$

Difference between beam passages will depend on time of light propagation in opposite directions:

$$\Delta_0 = \frac{c}{\lambda}(t_2 - t_1) = \frac{lc}{\lambda\omega_0}(k_{2n,2} - k_{2n,1}) = \frac{4l}{\lambda}\frac{\beta_{2n}(n_2^2-1)}{1-n_2^2\beta_{2n}^2}. \tag{2}$$

For parameters of the Fizeau's experiment [2] $l = 1{,}4875\,\text{m}$, $u = 7{,}059\,\text{m/s}$, $\lambda = 0{,}526\,\text{mkm}$, $n_2 \approx 1{,}33$ we have got $\Delta_0 = 0{,}170$. A shift $\Delta = 0{,}23$ was observed in the Fizeau's experiment, the value is explained with the fact that water velocity along an axis of the tube was more than an average value for u which was used in calculations. A second order term $n_2^2\beta_{2n}^2 = 1{,}7\times10^{-15}$ is very small and it doesn't influence on results.

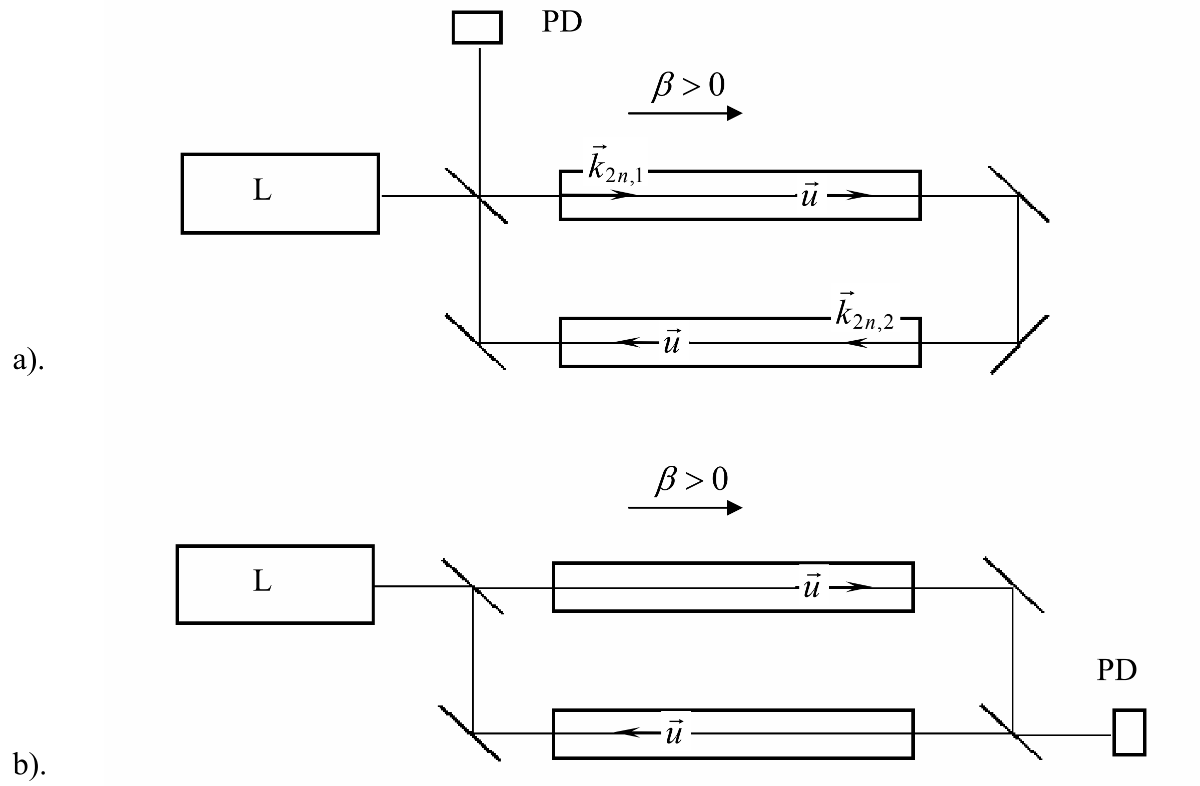

Fig. 1. *Schemes of interferometers with two passages (a) and single passage (b), in the interferometers light from a laser L propagates in a moving medium with velocity $\vec{u}$. A photodetector PD register interference fringes (IF). Velocity $\vec{u}$ is given in an observer IRF. The interferometer moves with velocity $\vec{v}$ to the right ($\beta = v/c > 0$) or to the left ($\beta < 0$) relative to the observer IRF.*

Let us consider the interferometer moving with velocity v relative to the IRF. First of all e will consider the case when light beams pass a tube one time (fig.1.b). Then, $-I_1 = \omega_1(1-\beta)$, here ω_1 is a source frequency in a n observer IRF. The expression (1) will have a view

$$k_{2n} = \frac{\omega_1}{c}(1-\beta)\frac{\beta + \left(n_2^2 - 1\right)\dfrac{\beta - \beta_{2n}}{1-\beta_{2n}^2} + n_2}{1 - \beta^2 - \left(n_2^2 - 1\right)\dfrac{\left(\beta - \beta_{2n}\right)^2}{1-\beta_{2n}^2}}. \tag{3}$$

Here u is velocity in IRF where the interferometer moves. The velocity in an interferometer IRF is u', in the case

$$\beta_{2n} = \frac{\beta + \beta'_{2n}}{1 + \beta\beta'_{2n}}. \tag{4}$$

By substituting (4) in (3), we will get

$$k_{2n,1} = \frac{\omega_1}{c}\frac{\beta + \beta'_{2n} - n_2^2\beta'_{2n}(1+\beta\beta'_{2n}) + n_2(1-\beta'^2_{2n})}{(1+\beta)\left(1 - n_2^2\beta_{2n}^2\right)}, \tag{5}$$

A sign before β_{2n} changes in the expression. Path difference will be calculated

$$\Delta = \frac{2l}{\lambda_1} \frac{\beta'_{2n}(n_2^2 - 1)}{(1+\beta)\left(1 - n_2^2 \beta'^2_{2n}\right)}. \tag{6}$$

Due to Doppler's effect a wave length is equal to $\lambda_1 = \lambda_0 \sqrt{\frac{1-\beta}{1+\beta}}$. Taking into account a kinematical shift of the interferometer, a path in a medium increases $l = l_1/(1-\beta)$, and also there is the contraction effect $l_1 = l_0\sqrt{1-\beta^2}$. A resulting shift of IF is equal to

$$\Delta = \frac{2l_0}{\lambda_0} \frac{\beta'_{2n}(n_2^2 - 1)}{(1-\beta)\left(1 - n_2^2 \beta'^2_{2n}\right)}. \tag{7}$$

Difference in interferometer readings when $\beta = 0$ and $\beta \neq 0$ will be equal to

$$\Delta - \Delta_0/2 \approx \beta\Delta_0. \tag{8}$$

Thus, maximal variations for the IF shift in the interferometer moving relative to the Sun with $\beta \cong 10^{-4}$ and with different orientations of the interferometer to velocity vector would have order of a value $\delta\Delta = \pm\beta\Delta_0 = \pm 1{,}7 \times 10^{-5}$ (of fringe).

Let us consider a complete scheme of the Fizeau's interferometer and estimate an IF shift with account dispersion in a material. In the case we have

$$\Delta = \frac{l}{\lambda k_0}\left(\frac{1}{1-\beta}k_{2n,1} + \frac{1}{1+\beta}k_{2n,2} - \frac{1}{1-\beta}k_{1n,1} - \frac{1}{1+\beta}k_{1n,2}\right). \tag{9}$$

In limit $\beta'^2_{2n} \to 0$ and $\beta\beta'_{2n} \to 0$ we have

$$k_{2n,1} = k_{01}\frac{n_{2,1} + \beta + \beta'_{2n}\left(n_2^2 - 1\right)}{1+\beta}, \tag{10a}$$

$$k_{2n,2} = k_{02}\frac{n_{2,2} - \beta + \beta'_{2n}\left(n_2^2 - 1\right)}{1-\beta}, \tag{10b}$$

$$k_{1n,1} = k_{01}\frac{n_{1,1} + \beta - \beta'_{2n}\left(n_2^2 - 1\right)}{1+\beta}, \tag{10c}$$

$$k_{1n,2} = k_{01}\frac{n_{1,2} - \beta - \beta'_{2n}\left(n_2^2 - 1\right)}{1-\beta}. \tag{10d}$$

Here are $k_{01} = k_0\sqrt{\frac{1+\beta}{1-\beta}} \approx k_0(1+\beta)$, $\quad k_{02} = k_0\sqrt{\frac{1-\beta}{1+\beta}} \approx k_0(1-\beta)$. (11)

For the case when $\beta >> \beta_{2n}$ refractive indexes are $n_{2,2} \approx n_{1,2}$, $n_{2,1} \approx n_{1,1}$. Then substitution to (9) gives

$$\Delta = \frac{2l\beta'_{2n}}{\lambda} \frac{\left(n_{1,1}^2 - 1\right)(1+\beta) + \left(n_{1,2}^2 - 1\right)(1-\beta)}{1-\beta^2}. \tag{12}$$

In non-dispersion approximation when $n_{1,1} = n_{1,2} = n$ we will get

$$\Delta = \frac{4l\beta'_{2n}}{\lambda}\frac{\left(n^2-1\right)}{1-\beta^2}. \tag{13}$$

As the expression was received for limits $\beta'^2_{2n} \to 0$ and $\beta\beta'_{2n} \to 0$, the term β^2 can be taken into account, hence, it gives slight contribution to interference fringe shift and it is equal to $\beta^2\Delta_0$.

Influence of dispersion may be estimated in the first approximation in the following way

$$n_{2,2} \approx n_{1,2} \cong n+\delta,\;\; n_{2,1} \approx n_{1,1} \cong n-\delta,\quad \delta = \Delta n - \delta n, \tag{14}$$

where Δn is variation of refractive index n due to motion of a boundary between two media, δn is variation of refractive index n due to length difference of waves which are incident onto the boundary.

By taking into account the dispersion from the expression (12) we have

$$\Delta = \frac{4l\beta'_{2n}}{\lambda}\frac{\left(n^2+\delta^2-2\beta n\delta-1\right)}{1-\beta^2}. \tag{15}$$

As the variations Δn and δn have different signs, we can neglect δ^2, more over $\delta^2 << n^2$ and the expression (15) can be reduced to the classical result (13).

Let us write down exact expressions for wave vectors and frequencies, which contain β'^2_{2n} and $\beta\beta'_{2n}$ to estimate influence of dispersion more precisely. Also, we will use an experimental tested dependence for a refractive index of optical glass on wave length of radiation.

In the case the expressions (10) will take a view

$$k_{2n,1} = k_{01}\frac{\beta-\beta'_{2n}+n^2_{2,1}\beta'_{2n}\left(1-\beta\beta'_{2n}\right)+n_{2,1}\left(1-\beta'^2_{2n}\right)}{\left(1+\beta\right)\left(1-n^2_{2,1}\beta'^2_{2n}\right)}, \tag{16a}$$

$$k_{2n,2} = k_{02}\frac{-\beta-\beta'_{2n}+n^2_{2,1}\beta'_{2n}\left(1+\beta\beta'_{2n}\right)+n_{2,1}\left(1-\beta'^2_{2n}\right)}{\left(1-\beta\right)\left(1-n^2_{2,1}\beta'^2_{2n}\right)}, \tag{16b}$$

$$k_{1n,1} = k_{01}\frac{\beta+\beta'_{2n}-n^2_{1,1}\beta'_{2n}\left(1+\beta\beta'_{2n}\right)+n_{1,1}\left(1-\beta'^2_{2n}\right)}{\left(1+\beta\right)\left(1-n^2_{1,1}\beta'^2_{2n}\right)}, \tag{16c}$$

$$k_{1n,2} = k_{02}\frac{-\beta+\beta'_{2n}-n^2_{1,2}\beta'_{2n}\left(1-\beta\beta'_{2n}\right)+n_{1,1}\left(1-\beta'^2_{2n}\right)}{\left(1-\beta\right)\left(1-n^2_{1,2}\beta'^2_{2n}\right)}. \tag{16d}$$

Wave numbers are defined with a method of successive approximations. First of all an refractive index, which was measured in a IRF where a medium rests, is substituted in the expression (16). Moreover frequency of an incident radiation is defined. Then corresponding wave lengths are calculated in a moving medium.

$$\lambda_{1,1} = \frac{2\pi c}{k_{1n,1}v+\omega_1\left(1-\beta\right)},\quad \lambda_{1,2} = \frac{2\pi c}{-k_{1n,2}v+\omega_1\sqrt{\frac{1-\beta}{1+\beta}}}, \tag{17a}$$

$$\lambda_{2,1} = \frac{2\pi c}{k_{2n,1}v+\omega_1\left(1-\beta\right)},\quad \lambda_{2,2} = \frac{2\pi c}{-k_{2n,2}v+\omega_1\sqrt{\frac{1-\beta}{1+\beta}}}. \tag{17b}$$

A refractive index is found for each wave length, for example, for $n_{1,1}$ the expression will correspond to:

$$n_{1,1}^{2} = A_1 + A_2\lambda_{1,1}^{2} + A_3\lambda_{1,1}^{-2} + A_4\lambda_{1,1}^{-4} + A_5\lambda_{1,1}^{-6} + A_6\lambda_{1,1}^{-8}. \qquad (18)$$

Coefficients A_i are selected with respect to experimental results. The indexes $n_{1,2}$, $n_{2,1}$, $n_{2,2}$ are analogically calculated. Then the indexes are substituted into (16) for the second time and wave numbers are calculated. When it is needed to increase accuracy of results t.e procedure can be repeated. The results of numerical experiments are presented in a table. Common parameters for all schemes were $l = 1{,}4875$ ì , $u = 7{,}059$ ì/ñ , $\beta = (V_z + V_s)/c$, here V_z and V_s are a daily velocity of the Earth, and an orbital velocity of the Sun. An refractive index for water $n_2 = 1{,}3314$ was taken in the experiment. Thus, water dispersion wasn't taken into account.

When we used the glass LK5, a refractive index was calculated with the formula (18) for each beam and passage in dependence on a motion direction and a frequency of incident radiation, respectively. It has average value $n_2 = 1{,}476615$. Approximation without dispersion meant that refraction onto a moving boundary between two media was calculated for a refractive index and. In a real case after refraction onto a moving boundary between two media a frequency of incident light changed that leads to recalculate a refractive index for a moving medium. The results of calculations with dispersion on a boundary between two media are collected in the third and sixth rows of the table. Values Δ and Δ' are presented as absolute those.

Type of interferometer, Its parameters	**Shift of interference fringes**		$\Delta' - \Delta$	$\Delta' + \Delta$
	$\beta > 0$	$\beta < 0$		
	Δ	Δ'		
1. One-passage, $\lambda = 0{,}526$ mkm, water without dispersion	$1{,}0283246 \times 10^{-1}$	$1{,}0288774 \times 10^{-1}$	$5{,}53 \times 10^{-5}$	$2{,}057202 \times 10^{-1}$
2. One-passage, $\lambda = 0{,}6328$ mkm, LK5, approximation without dispersion	$1{,}3054644 \times 10^{-1}$	$1{,}3062296 \times 10^{-1}$	$7{,}65 \times 10^{-5}$	$2{,}611694 \times 10^{-1}$
3. One-passage, $\lambda = 0{,}6328$ mkm, LK5 with dispersion	$1{,}3056019 \times 10^{-1}$	$1{,}3060920 \times 10^{-1}$	$4{,}90 \times 10^{-5}$	$2{,}611694 \times 10^{-1}$
4. Two-passages, $\lambda = 0{,}526$ mkm, water without dispersion	$1{,}7094844 \times 10^{-1}$	$1{,}7094844 \times 10^{-1}$	0	$3{,}4189688 \times 10^{-1}$
5. Two-passages, $\lambda = 0{,}6328$ mkm, LK5, approximation without dispersion	$2{,}6117071 \times 10^{-1}$	$2{,}6116809 \times 10^{-1}$	$-2{,}62 \times 10^{-6}$	$5{,}2233879 \times 10^{-1}$
6. Two-passages, $\lambda = 0{,}6328$ mkm, LK5 with dispersion	$2{,}6117186 \times 10^{-1}$	$2{,}6116694 \times 10^{-1}$	$-4{,}92 \times 10^{-6}$	$5{,}2233879 \times 10^{-1}$

First of all it is necessary to notice that the sum $\Delta' + \Delta$ is equal to the value given in the corresponding column for all schemes with $\beta = 0$. Therefore, resulting shift of IF doesn't depend on the fact an interferometer moves or doesn't move when a direction of motion is changed. Moreover, the difference $\Delta' - \Delta$ is equal to zero in the case.

It can be noticed from the given values Δ and Δ' in the table that the values Δ and Δ' have some difference for different signs $\beta > 0$ or $\beta < 0$.

In the first scheme the difference $\Delta' - \Delta$ is equal to $2\delta\Delta \cong 2\beta\Delta_0$, which was obtained from the expression(8). The magnitude $5{,}53\times10^{-5}$ is less than an error in the Fizeau's experiment in three orders. The result was received without account dispersion in moving water. As it is difficult to take into account dispersion in water, we used light glass LK5, for which dispersion coefficients were experimentally defined.

In the second row of the table results are given in approximation without dispersion. The difference $\Delta' - \Delta$ increased due to a coefficient $(n^2 - 1)$ was larger for the glass.

In the third row of the table results are given with dispersion. The dispersion in material of moving glass decreased the difference $\Delta' - \Delta$ on 36%. Estimation of dispersion influence was carried out for stationary glass and dispersion coefficients provided calculation error $\pm1\times10^{-5}$ which was calculated with the expression (18).

In two passages schemes the difference $\Delta' - \Delta$ decreased due to compensation of light dragging effects in a moving medium for opposite direction of motion. Really, the difference is equal to zero in the fourth row, the value in the fifth and sixth rows is considerable less than in the one-passage scheme with dispersion. In whole, it can be concluded that variations of Δ are slight and the maximal value $\Delta' - \Delta$ is equal to $4{,}9\times10^{-5}$. We can notice that that variations of $\Delta' - \Delta$ depend on refractive index n_2, length l and velocity $\vec{u}$ and $\vec{\beta}$. Carrying out a similar experiment can allow to find out is there a dependence $\Delta' - \Delta$ on spatial orientation of an interferometer. If a result is zero, we can define maximal limit for $\vec{\beta}$ in the case.

The given schemes of an interferometer are not optimal from a view point of experiment. The one-passage scheme is not stable to perturbing factors; the two-passage scheme has low sensibility. Besides, there are interferometers which have several orders higher measurement accuracy, for an example, interferometers for gravitational wave detection [3]. But distinctive peculiarity of interferometers, which are interesting for us, is availability of a moving medium. The medium will bring in vibrations in an interferometer, hence, as measured value of IF shift is not connected with motion of an element, we can use a compensation scheme when motion of any element leads to the same influence on each interfering beam.

A scheme with a rotating disc, close to given that [4], can be alternative. As light propagates in a rotating disc with 3-dimational presentation of velocity, we can use corresponding integral equations [5] for precise description. In such schemes we can observe violation of the Snell's law, which can considerably influence on results, especially with account dispersion. Carrying out the experiment could provide testing electrodynamics equations with 3-dimational presentation of velocity law.

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The estimation of parameters of the accelerated expansion of the Universe with use of photometry of quasars and the factor of gravitational self-lensing

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According to Hubble's law

$$cz = Hr \quad , \tag{1}$$

where c – speed of light in vacuum; z – red shift; H – Hubble's parameter, r – distance up to object.

Believing as a first approximation, that

$$\lg r = 0{,}2(m - M) + 1 \quad , \tag{2}$$

Where m – relative, and M – absolute star magnitude of object.

The joint decision (1) - (3) gives the linear Hubble's $m = m(\lg z)$ in the form:

$$m = 5\lg z + M - 5\lg H + 5\lg c - 5 \quad . \tag{3}$$

The linear law (3) is not traced on the diagram $m \leftrightarrow \lg(cz)$ in the field of quasars because of a high dispersion of absolute star sizes of quasars.

In [1] the phenomenon gravitational self-lensing is described, according to which light beams from large space object, deviating in a gravitation field of the same object, cause amplification its luminosity, that leads to growth of star magnitude m of object against valid for size:

$$\Delta m = -5\lg r - 5\lg r_g + 10\lg R \quad , \tag{4}$$

where R – geometrical, and r_g – gravitational radius of object.

The size m can be corrected by corrective value $-\Delta m$ (4), therefore the linear law $m = m(\lg z)$ (3) in view of (1) is led to a kind:

$$m - 5\lg cz = 5\lg cz + M - 10\lg R + 5\lg r_g - 10\lg H - 5 \quad . \tag{5}$$

For the comparative analysis (3) and (5) sample of 60.000 quasars and active galactic kernels of a database [2] has been used. It is established{installed}, that the factor of correlation between the left and right parts of dependences increases from 0,002 in case of (3) up to 0,675 in case of (5) at 5 %-th level of the statistical importance according to w-criterion. Thus the estimation of parameter of Hubble H appears still impossible because of absence of authentic data on parameters R, r_g, and M of quasars.

However, the narrowness of statistical intervals for estimations of parameters of the equation of regression (5) testifies to an opportunity of an estimation of relative change of parameter of Hubble eventually, i.e. about an opportunity of an estimation of parameter q_0 of accelerations of expansion of the Universe according to the nonlinear equation of Hubble

$$cz = Hr + \frac{1}{2c} \cdot (1 + q_0) H^2 R^2 \quad , \tag{6}$$

that can be reached by introduction in (5) a member $1{,}086 \cdot (1 - q_0) \cdot z$, nonlinear on $\lg cz$.

Besides using available estimations of parameter of Hubble H, thus the radius and time of expansion of the Universe can be estimated.

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Experimental study of advanced nonlocal correlation of large-scale dissipative processes

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Macroscopic nonlocality has its origin in quantum nonlocality and consists in correlation of different dissipative processes without any local carriers of interaction. Nonlocal correlation obeys weak causality principle. It involves the possibility of superluminar and even advanced transaction between the random dissipative processes without violation of relativity principle. The experiments on observation of nonlocal transaction of the lab probe-processes with the large-scale natural source-processes, in particular, the solar activity, have been performed. As a result advanced correlation has been reliably detected and its nonlocal nature has been proved by Bell-type inequality violation. Due to high level of this correlation and large time of advancement, forecasting applications proved to be possible.

1. Introduction

Apparent violation of relativity principle for the entangled states, which had been formulated as well known Einstein-Podolsky-Rosen paradox, at present is quite understood in the framework of quantum nonlocality concept. Superluminar communication is possible namely due to absence of any local carriers of interaction between the particles which are in the entangled states. But it is possible to use this nonlocal channel for transmission only unknown information (information about unknown states) and therefore «superluminar telegraph» is impossible, according to relativity theory again. In spite of this restriction, phenomenon of quantum nonlocality is such surprising, that attracts increasing attention, in particular, in relation to its possible macroscopic manifestation.

On the other hand, more than 30 years ago N.A. Kozyrev had suggested causal mechanics theory and conducted the various experiments [1,2], which originated from the idea of fundamental time asymmetry, but led to macroscopic phenomena similar to microscopic nonlocal ones (with exception of dissipation role). Specifically, he had observed correlation of the probe dissipative process (in the telescope detector) with large-scale ones of the stars with three time shifts, corresponding classical retardation, symmetrical advancement and zero between them, i.e. instantaneous [3,4]. According to causal mechanics such correlation of different dissipative processes were explained by not any local carriers of interaction, but by some physical properties of time as an active substance. Kozyrev's theoretical and experimental conclusions were so unexpected (with weakly formalized theory and not too strict performance of the experiment), that they could not be accepted in due course.

But in 1990s similarity of the results of causal mechanics and some recent ones of quantum mechanics had become obvious. Understanding of causal mechanics effects as possible manifestation of quantum nonlocality at the macro-level had allowed to perform the experiments showed availability of advanced correlation [5-13]. In this article we generalize obtained results and pay particular attention to the most recent ones. In Sec. 2 we shortly describe theoretical ideas and in Sec. 3 – experimental ones. In Sec.4 we summarize the main previous experimental results, and in Sec.5 – the most recent ones. We conclude in Sec.6.

2. Heuristic model of nonlocal transaction

By the early 1990s formalization of axiomatic of causal mechanics, including the concept of causality itself had been gained [8]. Furthermore, an analysis had shown that properties of Kozyrev's correlation of the dissipative processes were similar to ones of quantum nonlocal correlation [5,6,13,15]. In particular, interpretation of quantum nonlocality in the framework of Wheeler-Feynman action-at-a-distance electrodynamics [10] substantiates existence of the signal in reverse time. Interference of the retarded and advanced luminar signals may lead to apparent superluminar velocity (more precisely: any effective time shift between cause and effect inside of space-like interval is allowed). According to J.G. Cramer weak causality principle [16] it leads to observability of advanced correlation of the unknown states [17] or, in other words, the random processes. Next, the idea of asymptotic persistence of quantum correlation in the strong macro-limit had appeared, which had been verified in the numerical [18] in real [19] experiments. A new way of entanglement formation via a common thermostat (which the electromagnetic field could be served} had been discovered [20] and this way required dissipativity of quantum-correlated processes. It means that dissipation may not only lead to decoherence, but on the contrary, play a constructive role.

Our idea consisted in inclusion of dissipation in the framework of quantum action-at-a-distance electrodynamics [21], axiomatic of which is akin to axiomatic of causal mechanics [6,9,15]. It allowed to suggest the following equation of macroscopic nonlocality, describing factual Kozyrev's results [5-7,9,10]:

$$\dot{S} = \sigma \int \frac{\dot{s}}{x^2} \delta(t^2 - \frac{x^2}{v^2}) dV , \tag{1}$$

where $\dot{S}$ is the rate of entropy production in the absorber (probe-process in the detector), $\sigma \approx \hbar^4 / m^2 e^4$, m is electron mass, $\dot{s}$ is density of entropy production in the sources, velocity v is subluminar: $v^2 \leq c^2$, and integral extends over infinite volume V. δ-function shows that transaction progresses with finite symmetrical retardation and advancement. If the transaction occurs through a medium by interpapticle chains via microscopic Wheeler-Feynman fields, resulting time shifts of both signs are large.

The simplest Eq.(1) does not take into account absorption by the intermediate medium. Its influence, however, is very peculiar. In Ref. [21] it has been proved, that although action-at-a-distance electrodynamics equations are time symmetrical, fundamental time asymmetry manifests itself via absorption efficiency: if the retarded field is perfectly absorbed, then absorption of the advanced field, by contrast, must be imperfect. It may lead to that level of advanced correlation of the probe-process with source-processes proved to be higher than retarded one.

3. Performance of the experiments

The task of experiment is to relate the entropy change in the probe-process and source-processes according to Eq.(1) under condition that all classical local influences (temperature, pressure, electromagnetic field, etc.) are suppressed.

As of now there are two experimental setups- GEMRI (Geoelectromagnetic Research Institute RAS) and CAP (Center of Applied Physics BMSTU). In the former nonlocal correlation detectors based on spontaneous self-potential variations of weakly polarized electrodes in an electrolyte and on spontaneous dark current variations of the photomultiplier are used. In the latter the detector based on spontaneous dispersion variations of ion mobility fluctuations in a small electrolyte volume is used. Theory of detectors [5,13,22] allows to relate the measured signal with the rate of entropy production in the probe-process, i.e. to calculate left-hand side of Eq. (1) and consciously to take exhaustive steps on local influences (noises) suppressing. All technical details about design of detectors and their parameters are presented in Ref. [5-9].

As a source-processes the large-scale helio-geophisical processes with big random component , and in contrast – determined lab processes (phase transitions) were used. Since for the latter only retarded correlations is observed [23], only the former are considered next.

The experiments with natural helio-geophisical processes were long-term (with duration not less than several months). They were conducted in 1993-96 with the electrode detector; in 1996-97 with four detectors: the electrode and photocathode detectors of GEMRI setup, spaced at 300*m* one more electrode detector and spaced at 40 *km* (ion mobility detector of CAP setup; and in 2001-2004 with described above GEMRI and CAP setups.

4. Main previous experimental results

Below we summarize the results of long-term natural experiments described in detail in Ref. [5-13]. These results were repeatedly reproduced by different time series with all types of detectors (though the most of them were obtained with the electrode detector, which turned out most reliable one).

The signals of all detectors are synchronously correlated. Analysis has shown that signals are formed by some common causes, but their influence can not be local.

Such common causes proved to be (in order of decreasing influence): solar, synoptic, geomagnetic and ionospheric activity. Advanced response of the detector signals to these processes has been revealed reliably. Retarded response is always less than advanced one. Order of value of advancement (and retardation) is large – from 10 hours to 100 days. Magnitude of response and time advancement increases along with the space scale. For relatively small space scales advancement and retardation times are symmetrical and in such cases the synchronous response is added. For relatively large space scales retardation time is more than advanced one.

Nonlocal nature of correlation of the probe-processes with the source-processes has been proved by Bell-type inequality violation by analyzing of two kings of local causal chains: external temperature-interval temperature-detector signal [5,7-9] and solar activity-geomagnetic activity-detector signal [12].

Heuristic model of nonlocal transaction has been verified quantitatively by the example of geomagnetic activity source-process, which admits relatively simple calculation of right-hand side of Eq. (1). In particular, experimental estimations of cross-section σ proved to be of order 10^{-2} m^2, i.e. of order of an atom cross-section in agreement with theoretical expectation.

The level of advanced correlation proved to be enough for the employment of macroscopic nonlocality effect for geomagnetic, synoptic and solar activity forecast.

5. New results

Among considered before source-processes the solar activity is most important as the most large-scale one and the primary cause of many other ones. As convenient index of the solar activity the radio wave flux was chosen, because the detectors were perfectly insensitive to its local influence. On the other hand, the radio wave flux is well qualitative measure of the entropy production in its source, that is solar atmosphere. Furthermore, it turned out that detector signal was most correlated with the solar radio flux in the center of range of 9 standard frequencies (254…15400 *MHz*), around the frequency 1415 *MHz* [8,9]. This frequency corresponds to emission from the upper chromosphere - lower corona level, that is just from the level of maximal dissipation the magneto-sound waves energy. Next, it was found that detector response on solar activity is advanced with several time shifts around τ = 130 days for frequency 1415 *MHz.* Lastly, violation of Bell-type inequality was verified with the geomagnetic activity as an intermediate source-process (as an index of which *Dst*-index, reflecting the most large-scale dissipative processes in the magnetosphere, was used). Taking into account responsibility of those conclusions, their new experimental corroboration is necessary.

For this purpose we used data of the most recent experiment, namely data of continuous measurements with the electrode detector of GEMRI setup. As compared with the previous experiments, the system of its temperature stabilization was improved and thus signal/noise ratio was magnified. Duration of time series was 1 year (10/19/2002 – 10/18/2003). The detector signal (potential difference) U was measured accurate to 0.5 μV with data sampling 1 hour.

As solar activity data we took daily solar radio flux R (published in «Solar-Geophysical Data» at mentioned above frequency 1415 *MHz* and two adjacent ones: 610 and 2800 *MHz*. Time series was taken for about 3 years (beginning 371 days before and finishing 371 days after the ends of U series). As geomagnetic activity data we took international hourly *Dst*-index for the same time as R. For correlation with R, U, and *Dst* data were previously daily averaged.

Whereas in processing of earlier experimental data [5-11] we had used, besides traditional statistical method, the causal analysis as the most sophisticated informational-statistical method, the achieved level of knowledge allowed, beginning from Ref. [12] and ending with this study, to restrict our self by usual correlation analysis (with exception of Bell-type inequality problem). But at the same time we have been considering problem of detection of advanced correlation in more distilled performance. The matter of fact is, advanced correlation is physical property only the random processes. If the determined, that is in given case periodic, components of variations are not suppressed, then advanced cross-correlation could be amplified by auto-correlation. It would be useful in forecasting practice, but here we are going to investigate namely advanced cross-correlation. Therefore we have to suppress the periodic components. The main periodicity in R (having a response in U [10,11]) is synodic solar rotation period. In addition, a lot of geophysical processes have annual period. For these reasons U and R data were wide-band filtered in the period range $183^d > T > 28^d$. (For *Dst* because of splitting of the spectral line corresponding to the solar rotation period, optimal lower bound of the wide-band filtration was more: 32^d [13]).

After this filtration the correlation function r_{UR} was calculated in the time shift range $\tau = \pm\, 371^d$ ($\tau<0$ corresponds to retarded correlation r^{ret} , $\tau > 0$ – advanced one r^{adv}). The result for frequency of R 1415 *MHz* is presented in Fig.1. Correlation time asymmetry $\max|r_{UR}^{adv}|/\max|r_{UR}^{ret}| = 1.18 \pm 0.06$, that is quite reliable. Maximal correlation $r_{UR}^{adv} = 0.92 \pm 0.03$, is at advancement $\tau = 130^d$. At the adjacent frequencies the main maximum is also at $\tau = 130^d$, but level of correlation is slightly less: for 610 *MHz* $r_{UR}^{adv} = 0.88 \pm 0.04$ and for 2800 *MHz* $r_{UR}^{adv} = 0.90 \pm 0.03$. That is the frequency 1415 *MHz* is optimal.

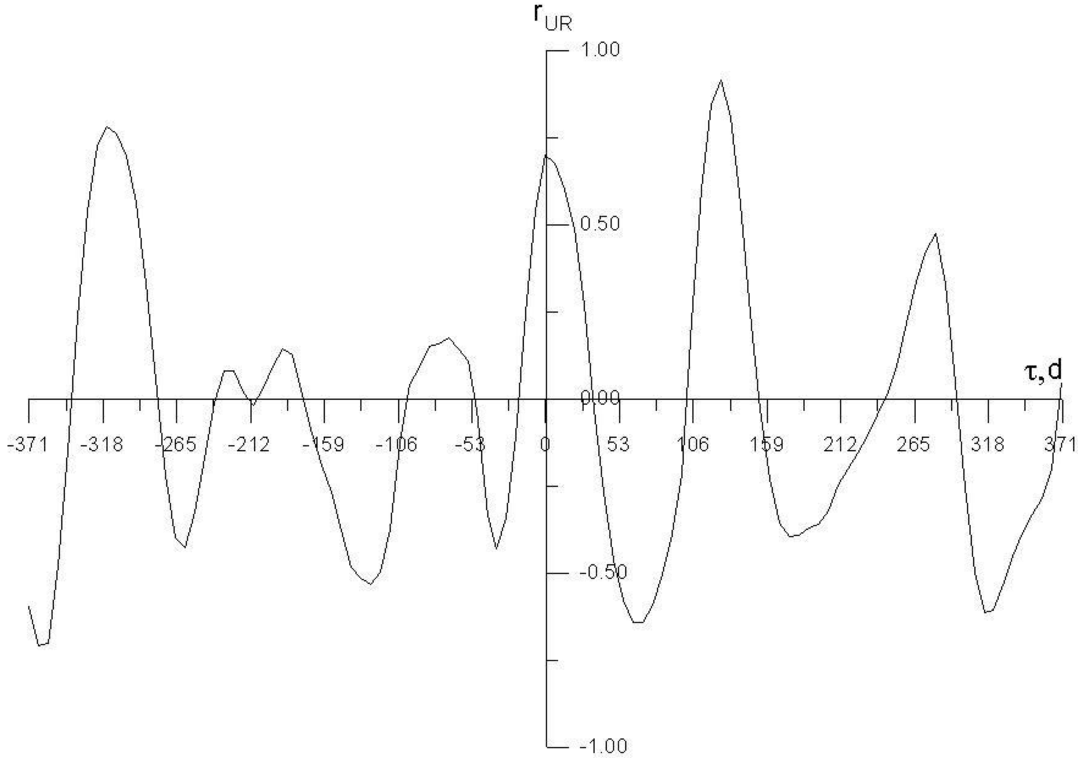

Fig. 1 *Correlation function r_{UR} of the detector signal U and solar radio wave flux R. Negative time shift τ corresponds to retardation U relative to R, positive one-to advancement.*

But the solar activity excites much more close (to the detector) the process of geomagnetic activity and it is legitimately to speculate that latter is direct cause of U variation. In Fig.2 the correlation function r_{UDst} is shown. The main extremum of correlation is almost at the same time (about 10^{d} more), but it is weaker: $r_{UDst}^{adV} = -0.87 \pm 0.04$. Correlation time asymmetry is also weaker: $\max|r_{UDst}^{adV}|/\max|r_{UDst}^{ret}| = 1.11 \pm 0.06$. On the other hand, though the *Dst*- variations are excited just by solar activity, due to complexity of their relation, their correlation (negative by nature of *Dst*-index) is rather weak. For given serieses *Dst* and *R* at 1415 *MHz* the main extremum $r_{DstR} = -0.38 \pm 0.07$ is observed at τ = - 10^{d} (*Dst* is retarded relative to R).

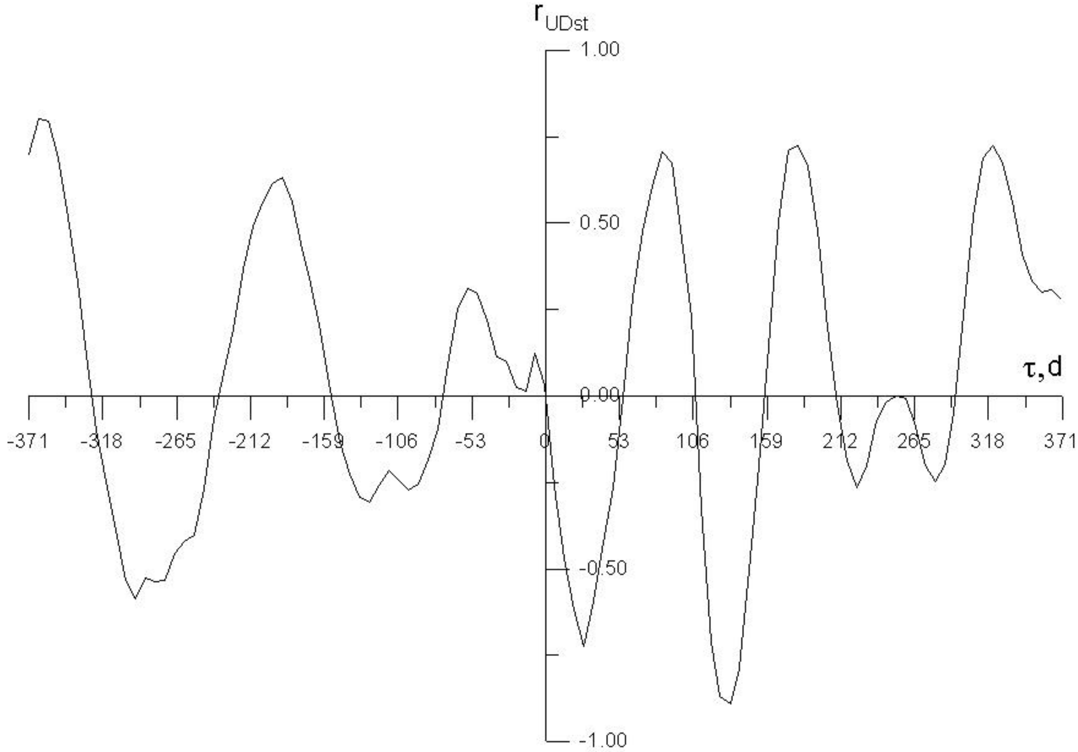

Fig. 2 *Correlation function* r_{UDst} *of the detector signal U and Dst-index of the geomagnetic activity. Negative time shift τ corresponds to retardation U relative to Dst, positive one – to advancement.*

Thus we have $r_{UR} = 0.92 \pm 0.03$, $r_{UDst} = -0.87 \pm 0.04$ (both advanced) and $r_{DstR} = -0.38 \pm 0.07$ (retarded). Such relationship suggests that connection of U and R is direct, i.e. nonlocal. But all three links might be nonlinear. Indeed nonlolinearity of (classical local) *R-Dst* link is well known, U is related with left-hand side of Eq.(1) in nonlinear manner [8,9,13], as well as *Dst* and ,probably, *R* [5-9] - with its right-hand side.

As correlation function is not representative for nonlinear dependence, adopt more strict way for evidence and consider the following Bell-type inequality [7-9,12]:

$$i_{U|R} \geq \max(i_{U|Dst}, i_{Dst|R}), \tag{2}$$

were i are the independence functions. The independence functions are terms of causal analysis (e.g.[14]) and defined as $i_{Y|X} = H(Y|X)/H(Y)$, where $H(Y|X)$ is conditional Shannon entropy and $H(Y)$ is marginal one of the variables X and Y. $0 \leq i_{Y|X} \leq 1$; $i_{Y|X} = 0$ if Y is one-valued function of X, $i_{Y|X} = 1$ if Y is not depended on X. Value of $i_{Y|X}$ is equally fit for linear or any nonlinear type of dependence Y on X. The fulfillment of Ineq.(2) is sufficient condition for locality of connection along the causal chain *R→Dst→U* (since any local solar influence on the detector can not come avoiding the magnetosphere that is source of *Dst* variations, and connection between the origin and end of the chain can not be stronger than in the weakest of two intermediate links).

All three independence functions of Ineq. (2) were calculated with mentioned above time shifts. For estimation of their stability all three serieses were alternately noised by 21% (by power) flicker-noise [8].

The results are: $i_{U|R} = 0.46^{+0.01}_{-0.02}$, $i_{U|Dst} = 0.51^{+0.00}_{-0.02}$, $i_{Dst|R} = 0.83^{+0.00}_{-0.02}$. Ineq. (2) is reliably violated, therefore connection $R \rightarrow U$ is nonlocal. Even choice of optimal frequency of R 1415 *MHz* is not crucial: for 610 *MHz* $i_{U|R} = 0.50^{+0.03}_{=0.01}$, for 2800 *MHz* $i_{U|R} = 0.49^{+0.02}_{-0.01}$, Ineq.(2) is violated, though slightly less.

It is possible to utilize advanced nonlocal correlation for the forecast of solar activity. As connection of U and R is far from δ-correlated the plural (and probably nonlinear) regression is necessary for correct forecasting. But for demonstration of the principal possibility we may simply shift corresponding annual segment of R series (at 1415 *MHz*) forward relative to U one by $\tau = 130^d$. The result is shown in Fig.3. The forecasting effect is evident quite clearly. The peculiarity of this forecasting picture is that U curve is smoother than R one (with the same filtration). Therefore U responses mainly on long term and, correspondingly, large-scale disturbances of R. It should be emphasized that U forecasts namely random component of R, which is eluded forecasting by any classical methods.

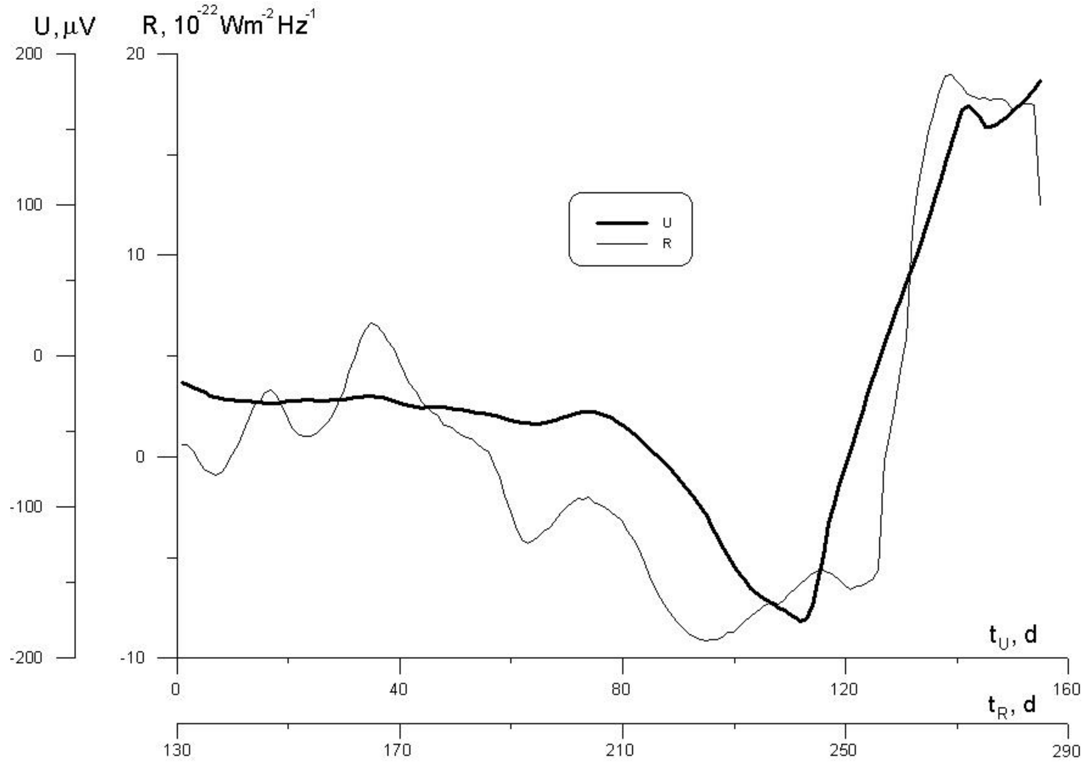

Fig. 3 *The detector signal U forecasts the solar radio wave flux R with advancement 130 days. Origin of time axis corresponds to 01/07/2003.*

6. Conclusion

The experiments on the modern level of strictness confirm earlier Kozyrev's results on surprising manifestation of reversibility in irreversible time – the possibility of observation of future random states (undetermined by previous evolution). Advanced nonlocal (violating Bell-type inequality) correlation of the practically insulated macroscopic dissipative processes has been detected quite reliably. It can be utilized, in particular, for long-term solar activity forecasting.

But the most important, although very difficult, problem at present is development of the theory of mechanism of persisting of quantum nonlocality on the macro-level, because our heuristic model is, of course, very rough and might be naive approximation of reality.

Acknowledgement
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Non-impactive Transformation of the Motion by Leier Constraint in the Newtonian Force Field

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The motion of a material point relatively of the extended massive body moving on a circular orbit in the Newtonian force field is considered. The possibility of unimpactive transformation of any motion of this point to periodic by using a cord with both ends that are fixed on a body is studied. Such cord may be named 'leier' (it's the dutch sea term meaning 'handrail'). The algorithm of such transformation is constructed and conditions of its practicability are deduced.

1. Introduction

Now frequently there is a question about protection of orbital stations from 'space garbage', i.e. parts of spacecrafts and missiles that lost control and freely moves on a space. The catching of such bodies before their collision with station is the cardinal solution of this problem. We shall notice, however, that change of motion by jump at the moment of the capture can be not less dangerous, than direct collision. The similar problem arises in a case when it is necessary 'to pull down from an orbit' the satellite or the spacecraft for its repair, rescue of crew, removing of the old equipment, etc. - in this case the capture with shock can be catastrophic for grasped object too. In this paper the possibility of the soft (non-impactive) transformation of motion of the free-moving object with the help of special tethered system is proved.

Motion of an uncontrollable object of enough small mass relatively of the massive station taking place on a circular geocentric orbit are considered. The station is supplied by 'leier constraint', that is a cable which both ends are fixed on the station (from the Dutch 'leier' - a cord with fixed ends) . The device, capable to grasp such object can move along the cable. The possibility of such grasping without jumps of relative velocity and acceleration of the object is studied. The motion along a cable both up to, and after gripping should be non-impactive and occur only by inertia (in some cases the initial impulse for the gripper's motion is necessary).

2. Statement of the problem

We consider the massive body O, moving on the Kepler's circular orbit in Newtonian central force field. The body is supplied with spar AB along which two sliding piece C and D can move. Weightless inextensible cable can be reeled up the bobbins E and F which are fixed on the sliding pieces. The gripper G can move along a cable (fig. 1.). We assume that the mass of G is so small, that its influence on the motion of O is insignificant.

If the sliding pieces don't moves along the spar and the bobbins does not rotate, motion of the gripper G is limited to an ellipse with focuses that are in points C and D and with the big semi-axis that equals half of length of the unwrapped part of a cable. We shall examine only those situations when one of axes of the ellipse is directed to attractive center (i.e. if the spar is "vertical" or "horizontal"). Motion on the border of such ellipse or "constrained" motion is investigated in [1,2] in the assumption, that the size of the ellipse are small in comparison with radius of the orbit. Besides in [3] are classified the periodic trajectories consisting of the free motion segment (the motion with non-tense cable) and of the arc of the ellipse, such that transition between parts occurs unimpactively. In particular, it is shown, that any of such periodic trajectory is symmetric about a "vertical" axis of an ellipse.

Let Oxy and $O_1x_1y_1$ is two coordinates systems with accordingly parallel axes, such that Ox is directed as a tangent to an orbit, Oy is directed to attractive center and O_1 is the center of the ellipse. Let's assume, that uncontrollable object H moving on a Kepler orbit is found near the body O and its trajectory is unlimited in frame of reference Oxy.

3. Algorithm of motion transformation

Let's try to determine such positions of points C and D and such length of the unwrapped part of a cable that the part of the object's H trajectory that located in the ellipse limiting gripper's G motion, coincided with a part of free motion of a unimpactive periodic trajectory. According to classification of unimpactive periodic trajectories [3], two types of motion transformation of object H are possible. It is "oscillatory" type (at the left on fig. 2) and "rotary" type (on the right on fig. 2). The following sequence of actions results in transformation:

- Calculation by elements of an orbit of point H of initial position of gripper (point K) and its keeping in this position till some moment,
- Motion of gripper along a cable by inertia with fixed focuses C and D along the tense cable up to a point M in which gripper meets object H and "goes from constraint", i.e. the cable weakens. "Oscillatory" motion begins with zero velocity. Some initial impulse is necessary for "rotary" motion,
- Free motion of object and gripper during which linking can be carried out, up to point L in which jointing *G-H* "unimpactively goes to constraint » ,
- "Constraint motion" of jointing at the fixed bobbins and the tense cable up to point L (trajectory *L-N-L-P-M-K-M* or *L-K-M*) then free movement from M to L again begins, etc.

Such algorithm possesses the following important qualities:

- During all time of motion velocity and acceleration of object H varies continuously,
- Gripper's motion occurs on inertia (can the initial impulse is required),
- Process of joining should not be in a moment.

4. The law of free motion and the condition of leaving from constraint and an entrance to constraint

Passing to dimensionless variables, we shall consider, that the period of rotation of O around attractive center is equal 2π and the maximal length of a cable is equal 2. Since we examine a case when the distance OH is small enough in comparison with radius of the circular orbit, for the description of motion of point H is possible to use known V.V.Beletsky model [4,5]. Exchanging scale of dimensionless variables and saving for them the used earlier designations, we shall write down the law of motion of H and of free motion of G as

$$\begin{cases} x_1 = 3/2 \cdot f\, t + \sin t \\ y_1 = 1/2 \cdot \cos t + f - \nu \end{cases},$$

Where ν characterizes a distance between the center of the ellipse restricting the motion of a gripper, and the circular orbit, and the parameter f is expressed through elements of an orbit of object H by the formula

$$|f| = \frac{|1 - r_0 / a_0|}{2\varepsilon},$$

r_0 - the radius of a circular orbit, a_0, ε - big semi-axis and eccentricity of object H orbit. The moment $t = 0$ corresponds to crossing by point H of axis $O_1 y_1$.

Moving of gripper on leier can be considered as motion on the unilateral constraint determined by an inequality

$$f(x_1, y_1) = x_1^2 + d\, y_1^2 - a^2 \le 0, \qquad d = a^2 / b^2,$$

a and b are "horizontal" and "vertical" semi-axis of the ellipse. Let's consider the function $\varphi(t) = f(x_1(t), y_1(t))$. Then, similarly to [6], in the point of leaving from constraint (M) and in the point of unimpactive entrance to constraint (L) the equations

$$\varphi(t_{L,M}) = 0, \quad \dot\varphi(t_{L,M}) = 0, \quad \ddot\varphi(t_{L,M}) = 0 \tag{1}$$

and inequalities

$$\dddot\varphi(t_M) < 0, \quad \dddot\varphi(t_L) > 0 \tag{2}$$

are fair.

5. Details of the algorithm.

Equations (1) allow to express the relation of semi-axises of the ellipse and ν through f and t_M by formulas

$$d := \frac{36(t_M \cos\tau_M - \sin t_M)f^2 + 12(2t_M - \sin 2t_M)f + 12\sin t_M - 4\sin 3t_M}{3\sin t_M - \sin 3t_M},$$

$$\nu = \frac{\left(36(\cos t_M - \sin t_M)f^2 + 3(6t_M \cos 2t_M + 8t_M - 7\sin 2t_M)f - 4\sin 3t_M + 9t_M \cos t_M + 3t_M \cos 3t_M\right)f}{36f^2(t_M \cos t_M - \sin t_M) + 12f(2t_M - \sin 2t_M) + 12\sin t_M - 4\sin 3t_M}$$

Coordinates of the point K, its velocity and also the moment of the beginning of gripper's motion can be calculated by Jacobi's integral from [3]. It is necessary to notice, however, that there are some restrictions on value t_M. First $t_M < 0$, second $d > 0$, thirdly, from inequalities (2) follows, that

$$\left(9(3\sin 2t_M - 2t_M \cos 2t_M - 4t_M)f + 3(\sin 3t_M + 9\sin t_M - 4t_M \cos t_M)\right)f < 0$$

.

It is possible to show, that for some set of values of parameters f and t_M after passage of the linking *G-H* through the point L again there will be a weakening of the cable and a new entrance to constraint will be impactive.

Let's notice also, that at $d > 1$ spar AB should be horizontal, and at $d < 1$ vertical.

The motion transformation type of the object H can be determined on a value of a constant of Jacobi's integral of the motion under constraint. Thus if $f < 2/3$ the constant of Jacobi's integral is so big, that joint movement of points H and G will be rotary. If $f > 2/3$, rotary and oscillatory motion is possible, and if $f = 2/3$ unimpactive transformation of motion is impossible.

The diagram of various types of transformation is represented on fig. 3. In this figure: I - area of transformation to rotary type of movement with "horizontal" spar AB, II - the same but with "vertical" AB, III - area of transformation to oscillatory type from motion with "horizontal" spar AB, IV - the same, but with "vertical" AB. (Areas II and IV represent narrow strips on marked borders of areas I and II). Weak shading marks that part areas III in which after grasp there is an impact about the cable, by black color - a similar part of area I.

As the length of a cable cannot be infinite, relative velocity of free-moving object H at the moment of gripping is limited. Dependence of the maximal relative velocity at the moment of grasp from parameter f is represented on fig. 4. In this figure the values of α_i are determined by the equations

$$\alpha_i = \arctan f_i, \quad f_i = \frac{2}{3\sqrt{1+\tau_i^2}}, \quad \tau_i = \tan\tau_i$$

6. About practical realization of the algorithm.

Literal practical realization of the described algorithm in most cases is impossible, since, apparently from fig. 4, capture of the object moving even with small velocity, needs big enough length of a cable. If value f is close to any of f_i, on a segment of free motion (*ML*) the object H can leave on big enough distance from O, and the gripper cannot move together with him. However, if to consider only a problem of grasp, actually it is necessary to provide motion in a vicinity of a point of an entrance to constraint L. It is possible, if the cable has length, a little bit big, than double distance *OL*. Thus the motion under constraint after passage of point L can occur on an ellipse to the focuses which not necessarily lay in points C and D. It is enough to provide such motion of these points that true focuses of an ellipse laid on straight lines *GC* and *GD* during all time of " the constraint's motion", and the sum of distances from G to focuses does not vary. In particular, at oscillatory type of capture such approach allows at many times to reduce necessary length of a cable.

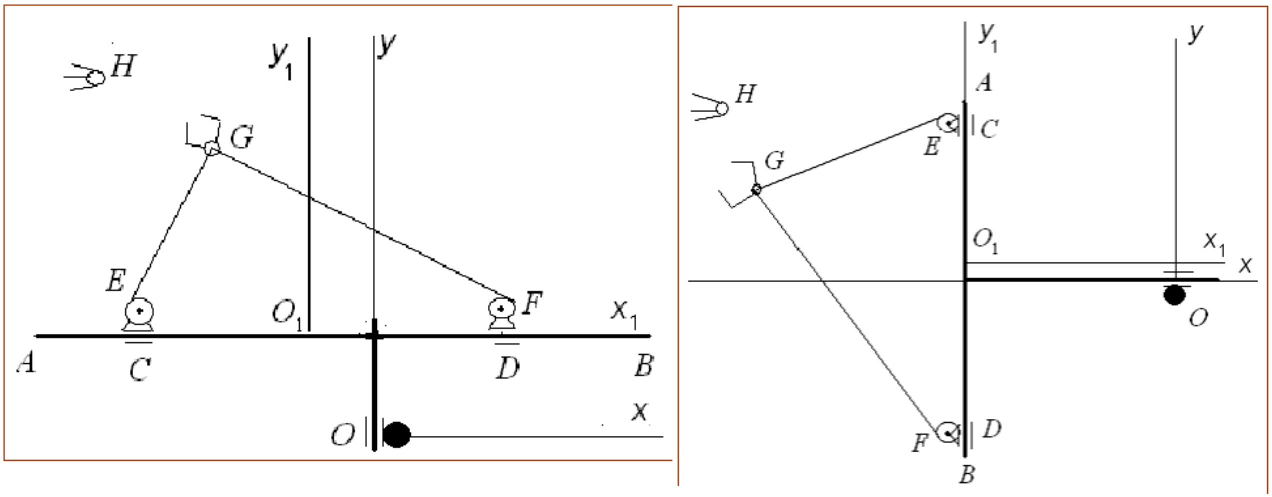

Fig.1

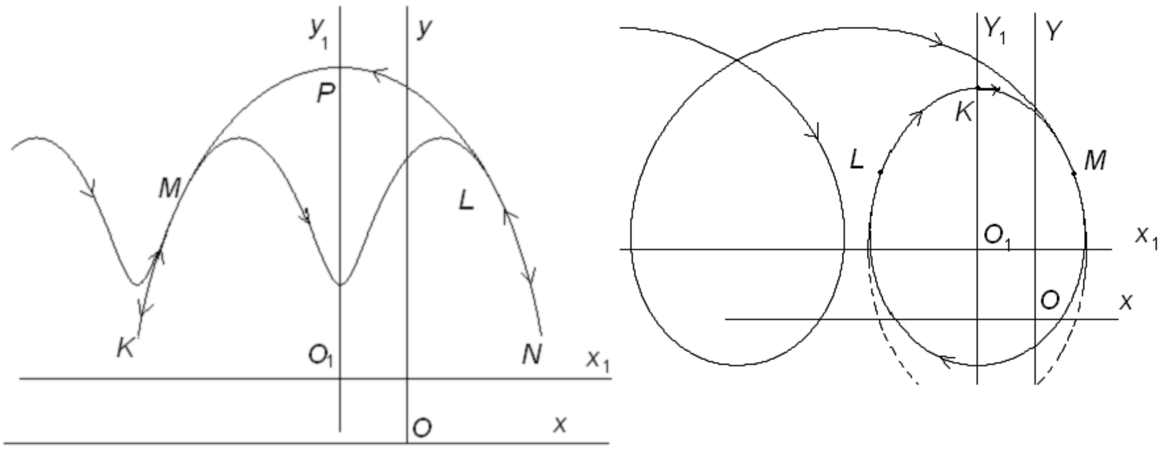

Fig.2

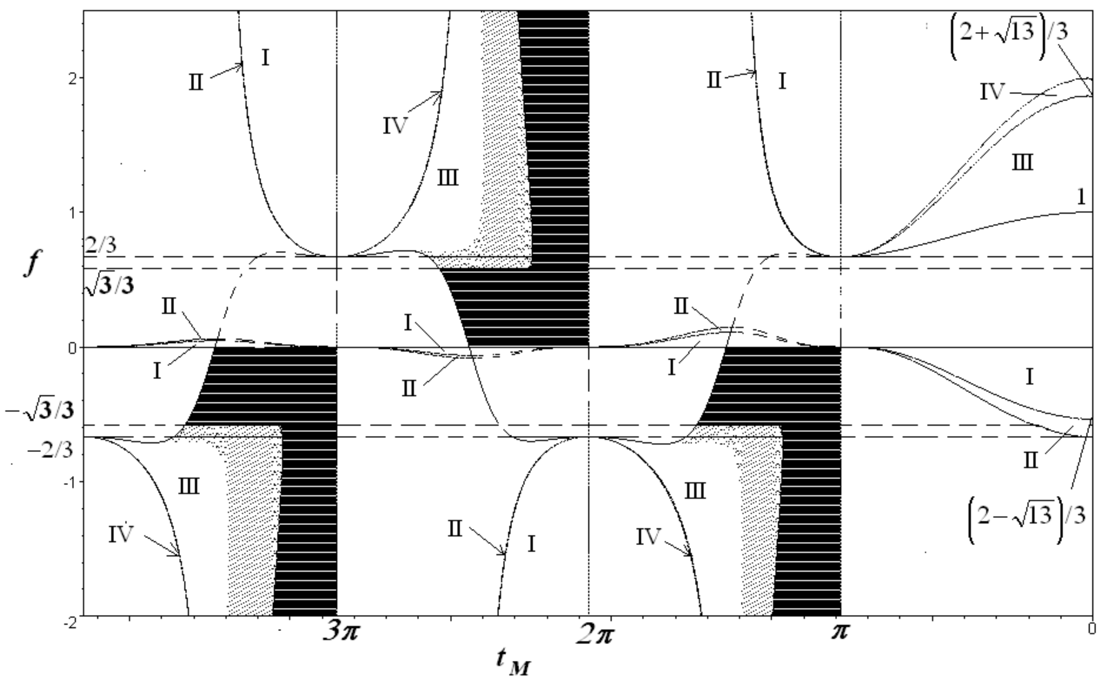

Fig.3

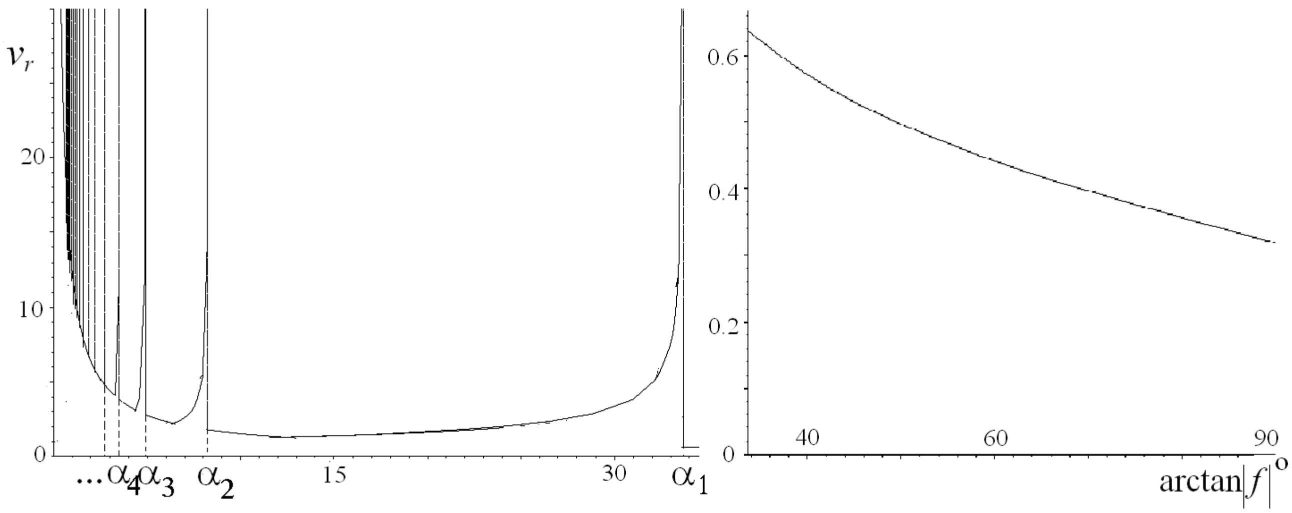

Fig.4

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Исследование орбитальной устойчивости пробной частицы в задаче трех тел на основе модифицированной модели Калуцы-Вессона

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В работе показывается возможности полевой теории Калуцы-Вессона (пятимерной теории гравитации [1]), моделирующей консервативную систему двойных звезд, взаимодействующих между собой; при этом переток массы-энергии оболочки звезд осуществляется через лагранжеву точку L_1; пренебрегается унос массы-энергии через лагранжеву точку L_2 [2]. Оценка возмущения звезды 2 от звезды 1 осуществляется посредством решения уравнения девиации методом М.Ф. Широкова на основе известной метрики возмущающей звезды; в качестве теста рассматривается сколлапсированная звезда 2 (черная дыра, нейтронная звезда), с сильным гравитационным полем, окруженная аккреционным диском. Таким образом, масса звезд считается переменной. В этих условиях орбитальная устойчивость пробной частицы, движущейся вокруг черной дыры, зависит и от возмущения звезды 1, дополняя результаты [3].

1. Построение функции Лагранжа-Дирихле на основе теории Калуцы-Вессона

Рассмотрим обобщенную статическую с условием цилиндричности по x^5 метрику черной дыры вида

$$ds^2 = f\left(1-\frac{\alpha}{r}\right)^a \left(dx^0\right)^2 - f\frac{dr^2}{\left(1-\frac{\alpha}{r}\right)^{b+a}} - fr^2\left(1-\frac{\alpha}{r}\right)^{1-a-b}\left(d\theta^2 + r^2\sin^2\theta d\phi^2\right) - f\gamma_{55}\left(dx^5\right)^2, \quad (1)$$

где $\gamma_{55} = \left(1-\frac{\alpha}{r}\right)^b$, конформный параметр f есть отношение $f = \frac{ds^2}{dS^2}$, ds^2 – инвариант в 4-х мерном пространстве-времени (4D); $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$, $\alpha, \beta = 0,1,2,3$; dS^2 – инвариант в 5-и мерном пространстве-времени (5D): $dS^2 = g_{AB}dx^A dx^B$, $A, B = 0,1,2,3,5$.

В работах [1], [4] показано, что

$$f = 1 + B^2/\gamma_{55}\,,\ f = f(\gamma_{55}), \quad (2)$$

где B^2 идентифицируется как удельный заряд материи; для простоты положим $B^2/\gamma_{55} = k = const$, и значит, $f = const$.

Скалярное поле γ_{55} будем определять выше как возмущение полной энергии поля. Константы a, b являются результатом уравнения вакуумного поля в пятимерном пространстве 5D и удовлетворяют условию совместности

$$a^2 + ab + b^2 = 1. \quad (3)$$

Таким образом, метрика (1) определяется константами a, b, α, f, где α – гравитационный радиус, в общем случае зависящего от времени x^0, $\alpha = \alpha\left(x^0\right)$; по нашей гипотезе

$$\frac{d\alpha}{ds} = -\frac{d\alpha^*}{ds}, \quad (4)$$

где $\alpha^* = \alpha^*\left(x^0\right)$– гравитационный радиус возмущающей звезды 1; именно (4) указывает на принятый постулат консервативности изучаемой системы.

В случае $a=1$, $b=0$, $\gamma_{55}=0$ мы имеем стандартное решение Шварцшильда общей теории относительности (ОТО); для него в работе [3] получен результат о существовании класса устойчивых круговых орбит при $r=3\alpha$. В нашем случае из-за возмущения от звезды 1 результат может отличаться, что и будет показано ниже. В случае Шварцшильда имеем интеграл вида

$$h=\frac{\sqrt{1-\alpha/r}}{\sqrt{1-q^2/c^2}}\,, \tag{5}$$

где q^2– квадрат синхронизированной скорости; указанный интеграл представляет полную энергию системы; в ньютоновском случае малых скоростей и гравитационного потенциала имеем интеграл энергии вида

$$\mathrm{v}^2-\frac{2\mu}{r}=\widetilde{h}=\left(h^2-1\right)c^2=const\,, \tag{6}$$

где v– трехмерная скорость, μ– гравитационная постоянная массивного тела.

Указанный аналог интеграла (5) имеем из (1)

$$f\left(1-\frac{\alpha}{r}\right)^a \dot{x}^0=H=const\,, \tag{7}$$

H будем считать полной энергией звезды 2 вместе с аккреционным диском и учитывающим через γ_{55} возмущение от звезды 1. Поэтому $H=V_{\text{ýô}}$ можно принять за "эффективную потенциальную энергию" (функцию Лагранжа-Дирихле) как инструмент для анализа допустимых областей движения.

Итак, запишем (1) в виде ($\dot{r}=0$, $\dot{\phi}=0$)

$$L\equiv 1=f\left(1-\frac{\alpha}{r}\right)^a\left(\dot{x}^0\right)^2-fr^2\left(1-\frac{\alpha}{r}\right)^{1-a-b}\dot{\theta}^2-f\gamma_{55}\left(\dot{x}^5\right)^2. \tag{8}$$

Отсюда следуют интегралы вида

$$\frac{\partial L}{\partial \dot{x}^0}=f\left(1-\frac{\alpha}{r}\right)^a\dot{x}^0=H=const\,, \tag{9}$$

$$\frac{\partial L}{\partial \dot{\theta}}=fr^2\left(1-\frac{\alpha}{r}\right)^{1-a-b}\dot{\theta}=l=const\,, \tag{10}$$

$$\frac{\partial L}{\partial \dot{x}^5}=f\gamma_{55}\dot{x}^5=h_5=const\,. \tag{11}$$

Подставляя (9)-(11) в (8), получим выражение для H^2

$$V_{\text{ýô}}^2=H^2=f\left(1-\frac{\alpha}{r}\right)^a+\frac{l^2\left(1-\frac{\alpha}{r}\right)^{2a-1+b}}{r^2}+\frac{h_5^2\left(1-\frac{\alpha}{r}\right)^a}{\gamma_{55}}\,. \tag{12}$$

Положим, в соответствии с [3]

$$\beta=1-\frac{\alpha}{r}\,,\ \frac{\alpha}{r}=1-\beta\,. \tag{13}$$

Тогда имеем (учитывая $\gamma_{55}=\left(1-\frac{\alpha}{r}\right)^b$)

$$V_{\text{ýô}}^2=f\beta^a+\frac{l^2\beta^{2a-1+b}(1-\beta)^2}{\alpha^2}+h_5^2\beta^{a-b}\,. \tag{14}$$

Определим теперь экстремальное значение момента количества движения $l_{\text{э}}$:

$$\frac{\partial V^2_{\text{ýô}}}{\partial \beta} = 0\,,\; l^2_{\text{ý}} = \frac{\left[af - (a-b)\beta^{-b} h_5^2\right]\alpha^2 \beta^{1-a-b}}{(1-\beta)\left[2\beta - (2a-1+b)(1-\beta)\right]}. \tag{15}$$

Пренебрегая величиной $\frac{\alpha b}{r} << 1$ примем $\beta^{-b} \to 1$; положим выражение в числителе (15) равным единице $af - (a-b)h_5^2 = 1\,,\; h_5^2 = \frac{af-1}{a-b}$, (16)

что соответствует выражению для момента $l_{\text{э}}$ (15) в метрике Шварцшильда при $a \to 1$, $b \to 0$:

$$l^2_{\text{ýø}} = \frac{\alpha^2}{(1-\beta)(3\beta-1)}\,,\; h_5^2 \to f - 1 = k\,. \tag{17}$$

Таким образом, мы дали интерпретацию константы h_5 как величины, связанной с конформным параметром $f\left(B^2/\gamma_{55}\right)$. Итак,

$$l^2_{\text{ý}} = \frac{\alpha^2 \beta^{1-a}}{(1-\beta)(\mu_1\beta + \mu_2)}, \tag{18}$$

где $\mu_1 = 1 + b + 2a$, $\mu_2 = -(2a - 1 + b)$. Имеем

$$1 \ge \beta \ge \frac{2a-1+b}{1+b+2a}, \tag{19}$$

в случае (17) $1 \ge \beta \ge \frac{1}{3}$; таким образом, между $r = \alpha$ и $r = \frac{1+2a+b}{2}\alpha$ вообще нет действительных круговых орбит. Подставляя (18) в (14), получим

$$V^2_{\text{ýô}} = \left(f + \frac{(1-\beta)}{(\mu_1\beta + \mu_2)} + \frac{(af-1)}{(a-b)}\right)\beta^a,$$

или

$$V^2_{\text{ýô}} = \frac{\left[((2a-b)f - 1)(\mu_1\beta + \mu_2) + (1-\beta)(a-b)\right]\beta^a}{\mu_1\beta + \mu_2}. \tag{20}$$

Отсюда видно, что на радиусе, равном $r = \frac{\mu_1 \alpha}{\mu_1 + \mu_2}$ эффективная энергия равна ∞; пробная частица должна на этой орбите двигаться со скоростью света; для решения Шварцшильда это будет $r = \frac{3\alpha}{2}$ ($\mu_1 = 3\,, \mu_2 = -1$). (21)

Величина f в (20) должна удовлетворять согласно (16) условию $f > \frac{1}{a_{\text{ïð}}}$, (22)

где в соответствии с (3) $b_{\text{ïð}} = -\frac{2}{\sqrt{3}}$, $a_{\text{ïð}} = \frac{1}{\sqrt{3}}$ $\left(b^2 \le \frac{4}{3}\right)$.

Для указанного классического решения при $a \to 1$, $b \to 0$ из (20) следует, что между сферами с $r = \frac{3\alpha}{2}$ и $r = 2\alpha$ энергия круговых орбит больше, чем энергия на бесконечности, и поэтому эти орбиты неустойчивы. После определения b (см. (1)) по заданному γ_{55} следует аналогичный анализ для общего случая (20).

Экстремальное значение для $V^2_{\text{эф}}$ получим из (20)

$$\frac{\partial V_{\text{ýô}}^2}{\partial \beta}=0\,,\ \beta_{\text{ý}}=\frac{-w_2\pm\sqrt{w_2^2-4w_1w_3}}{2w_1}\,, \tag{23}$$

который определяет устойчивую орбиту, где $w_1=a\mu_1n_1+n_4$,

$$w_2=a\mu_1n_2+a\mu_2n_1+n_3+n_5\,,\quad w_3=a\mu_2n_2\,,\ n_1=\mu_1\big((2a-b)f-1\big)-(a-b),$$
$$n_2=\mu_2\big((2a-b)f-1\big)-(a-b),\ n_3=\big((2a-b)f-1\big)\mu_1-(a-b),$$
$$n_4=\mu_1\big((2a-b)f-1\big)-(a-b),\ n_5=\mu_2\big((2a-b)f-1\big)+(a-b).$$

Тогда

$$V_{\text{ýô}\min}=\sqrt{\frac{\left[f(a-b)(\mu_1\beta_{\text{ý}}+\mu_2)+(1-\beta_{\text{ý}})(a-b)+(af-1)(\mu_1\beta_{\text{ý}}+\mu_2)\beta_{\text{ý}}^a\right]}{(a-b)(\mu_1\beta_{\text{ý}}+\mu_1)}}\,, \tag{24}$$

$$l_{\min}=\frac{\alpha\beta_{\text{ý}}^{(1\text{-a})/2}}{\sqrt{(1-\beta_{\text{ý}})\left[2\beta_{\text{ý}}-(2a-1+b)(1-\beta_{\text{ý}})\right]}}\,. \tag{25}$$

Для случая $a=1$, $b=0$, $f=1$ устойчивая орбита реализуется при $\beta_{\text{ý}}=2/3$, $r=3\alpha$, $V_{\text{ýô}\min}=\sqrt{8/9}$. Отсюда для ОТО следует, что при $1\geq V_{\text{ýô}}>V_{\text{ýô}\min}$ существуют две круговые орбиты с разными l, но с одинаковыми $V_{\text{ýô}}$; одна из них - вне сферы с $r=3\alpha$ – устойчивая, а другая, внутри сферы с $r=3\alpha$ – неустойчивая.

2. Определение возмущающего поля γ_{55}

Нами рассматривается движение пробной частицы в поле двух тел - бинарной системы, состоящей из двух звезд, внешние оболочки которых взаимодействуют между собой, т.е. массы обоих звезд меняются. Именно для такой задачи рассматривается задача трех тел - пробная частица и две звезды. При этом основное внимание уделяется звезде с более мощным гравитационным потенциалом - нейтронной звезде, черной дыре. В этом случае вторая компонента является возмущением для пробной частицы и черной дыры; целью является определение этого возмущения.

Вся классическая и релятивистская небесная механика посвящена проблеме 3-х и более тел. В этом случае допускается пертурбационная функция $\widetilde{R}=\mu'\left(\frac{1}{\Delta}-\frac{\vec{r}\vec{r}\,'}{r'^2}\right)$, (26)

где Δ– взаимное расстояние между звездами, $\vec{r},\vec{r}\,'$– их радиусы - векторы, μ'– гравитационная постоянная звезды. Проблема решается разложением $\widetilde{R}$ в ряд, используя задачу двух тел с последующим интегрированием уравнений Лагранжа для оскулирующих элементов [5].

Нами предлагается более простое получение возмущения методом, предложенным М.Ф. Широковым [6]; затем, в метрике пятимерного пространства (5D) метрическую компоненту γ_{55} при избыточном измерении x^5 соотносим с возмущением полной энергии из теории М.Ф. Широкова. При этом используем естественно возникающий в теории Калуцы-Вессона (5D) индуцированный тензор энергии-импульса для получения закона изменения массы возмущающей звезды $\alpha^*(x^0)$ за счет перетекания массы к аккреционному диску черной дыры. Полагаем при этом бинарную систему консервативной., т.е. равенство расхода масс между компонентами $\frac{d\alpha^*}{ds}=-\frac{d\alpha}{ds}$, (27)

где $\alpha^*(x^0)$ – гравитационный радиус возмущающей звезды, $\alpha(x^0)$– гравитационный радиус черной дыры. При этом в выбранной бинарной системе пренебрегается унос массы-энергии из лагранжевой точки L_2 [2]. Таким образом, пятимерная метрика черной дыры представляет собой консервативную систему, позволяющую аналитически решить поставленную задачу. Параметры орбиты пробной частицы зависят от времени и представляют аналог оскулирующих элементов в небесной механике.

Итак, рассмотрим в общем виде внешнее поле возмущающей звезды, зависящей от гравитационного и электромагнитного потенциалов; таким может быть решение уравнений Эйнштейна-Максвелла (метрика Эрнста [7]) имеющего вид (4D)

$$ds^2 = \left\{\Lambda^2\left(1-\frac{\alpha^*}{r}+\frac{Q^2}{r^2}\right)-\frac{r^2\omega'^2}{\Lambda^2}\right\}\left(dx^0\right)^2 - \frac{\Lambda^2}{1-\frac{\alpha^*}{r}+\frac{Q^2}{r^2}}dr^2 + \frac{2r^2\omega'}{\Lambda^2}d\phi dx^0 - \frac{r^2 d\phi^2}{\Lambda^2}, \quad (28)$$

$$\omega' = -2B_0 Q r^{-1} + B_0^3 Q r + \frac{1}{2}B_0^3 Q^3 r^{-1} - \frac{1}{2}B_0^3 Q r^{-1}\left(r^2 - \alpha^* r + Q^2\right) + const, \Lambda^2 = 1 + \frac{B_0^2 r^2}{4} \quad (29)$$

Здесь B_0, Q – параметры, пропорциональные магнитному и электрическому зарядам; видно, что при наличии магнитного поля решение – стационарное. Запишем для (28) функцию Лагранжа для случая орбитального движения пробной частицы $\left(\dot{r}=0\right)$

$$L \equiv 1 = g_{00}\left(\dot{x}^0\right)^2 + 2g_{03}\dot{x}^0\dot{\phi} + g_{33}\dot{\phi}^2, \quad (30)$$

где $g_{00} = \left\{\Lambda^2\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)-\frac{r^2\omega'^2}{\Lambda^2}\right\}$, $g_{03} = \frac{r^2\omega'}{\Lambda^2}$, $g_{33} = -\frac{\Lambda^2}{1-\frac{\alpha}{r}+\frac{Q^2}{r^2}}$.

Тогда для случая $\frac{\partial L}{\partial \dot{r}} \equiv 0$ следует выражение $\frac{\partial L}{\partial r} = g_{00,1}\left(\dot{x}^0\right)^2 + 2g_{03,1}\dot{x}^0\dot{\phi} + g_{33,1}\dot{\phi}^2 = 0$, (31)

(($_{,1}$) означает производную по r).

Решая совместно (30) и (31), получим выражения для $\dot{x}^0$ и $\dot{\phi}$ как функции от r

$$\begin{aligned}\left(\dot{x}^0\right)_1^2 = g_{33,1}^2 \Big/ \Big\{ & g_{00}g_{33,1}^2 - \left(2g_{03}g_{03,1} + g_{33}g_{00,1}\right)g_{33,1} + \\ & + 3g_{03,1}^2 g_{33} + 2\left(g_{03}g_{33,1} - g_{03,1}g_{33}\right)\sqrt{g_{03,1}^2 - g_{33,1}g_{00,1}}\Big\}\end{aligned} \quad (32)$$

$$\begin{aligned}\left(\dot{x}^0\right)_2^2 = g_{33,1}^2 \Big/ \Big\{ & g_{00}g_{33,1}^2 - \left(2g_{03}g_{03,1} + g_{33}g_{00,1}\right)g_{33,1} + \\ & + 3g_{03,1}^2 g_{33} - 2\left(g_{03}g_{33,1} - g_{03,1}g_{33}\right)\sqrt{g_{03,1}^2 - g_{33,1}g_{00,1}}\Big\}\end{aligned} \quad (33)$$

$$\dot{\phi}_1 = \frac{-\left(g_{03,1} - \sqrt{g_{03,1}^2 - g_{33,1}g_{00,1}}\right)}{g_{33,1}}\cdot \dot{x}_1^0, \quad (34) \qquad \dot{\phi}_2 = \frac{-\left(g_{03,1} + \sqrt{g_{03,1}^2 - g_{33,1}g_{00,1}}\right)}{g_{33,1}}\cdot \dot{x}_2^0. \quad (35)$$

Полученные результаты входят в коэффициенты в эффекте Широкова. Полагая возмущения ξ^i пробной частицы равными $\xi^0 = x_0^0 - x^0$, $\xi^1 = r_0 - r$, $\xi^3 = \phi_0 - \phi$, запишем согласно [1] уравнения возмущенного движения в виде

$$\frac{d^2\xi^i}{ds^2} + 2\Gamma_{jk}^i U^j \frac{d\xi^k}{ds} + \Gamma_{jl,k}^i U^j U^l \xi^k = 0, \quad (36)$$

(символы Кристоффеля Γ_{jk}^i приведены в Приложении), U^j – 4-х скорости.

Имеем согласно (36) систему дифференциальных уравнений второго порядка с постоянными коэффициентами

$$\frac{d^2\xi^1}{ds^2}+a_1\frac{d\xi^3}{ds}+a_2\frac{d\xi^0}{ds}+a_3\xi^1=0, \tag{37}$$

$$\frac{d^2\xi^3}{ds^2}+b\frac{d\xi^1}{ds}=0, \quad (38) \qquad \frac{d^2\xi^0}{ds^2}+C\frac{d\xi^1}{ds}=0, \quad (39)$$

где
$$a_1=\Gamma^1_{03}\dot{x}^0+2\Gamma^1_{33}\dot{\phi},\ a_2=2\left(\Gamma^1_{00}\dot{x}^0+\Gamma^1_{30}\dot{\phi}\right), \tag{40}$$

$$a_3=\Gamma^1_{00,1}\left(\dot{x}^0\right)^2+\Gamma^1_{33,1}\dot{\phi}+2\Gamma^1_{03,1}\dot{x}^0\dot{\phi}, \tag{41}$$

$$b=2\left(\Gamma^3_{01}\dot{x}^0+\Gamma^3_{31}\dot{\phi}\right), \quad (42) \qquad C=2\left(\Gamma^0_{01}\dot{x}^0+\Gamma^0_{31}\dot{\phi}\right). \quad (43)$$

Здесь принято $r=R=const$; значения $\dot{x}^0$, $\dot{\phi}$ см. в (32)-(35). Полагая

$$\xi^1=\xi^0_1e^{i\omega s},\ \xi^0=\xi^0_0e^{i\omega s},\ \xi^3=\xi^3_0e^{i\omega s}, \tag{44}$$

получаем из (37)-(39) условие нетривиальной совместимости однородной системы линейных уравнений

$$\begin{vmatrix} a_3-\omega^2 & ia_1\omega & ia_2\omega \\ ib\omega & -\omega^2 & 0 \\ iC\omega & 0 & -\omega^2 \end{vmatrix}=0, \tag{45}$$

откуда получим
$$\omega^4\left(a_2C-a_3+a_1b+\omega^2\right)=0; \tag{46}$$

отличное от нуля решение есть
$$\omega^2=a_3-a_2C-a_1b, \tag{47}$$

что формально совпадает с решением Широкова для поля Шварцшильда; в данном случае параметры a_1, a_2, a_3, C, b зависят еще от электромагнитного поля; выражение для (47) приведено в Приложении.

Возмущенная функция в данном случае получается из решения для ξ^0_0

$$\xi^0_0=\frac{C}{\omega}\xi^0_1\cos\omega s, \ (48) \qquad \gamma_{55}=\left(\frac{d\xi^0_0}{ds}\right)=\left|C\left(\xi^0_1\right)\sin\omega s\right|, \ (49) \qquad \Phi=\sqrt{\left|C\left(\xi^0_1\right)\sin\omega s\right|}.$$

Рассмотрим теперь подробнее выражение для γ_{55} (49). Очевидно, что при значениях $\omega s=k\pi$, $k=0,\pm1,\pm2...$ отсутствует возмущение полной энергии, значит, при этих значениях ωs метрика - четырехмерная, для которой решается лишь задачи 2-х тел [5].

Вернемся к исходной метрике (1), где $\left(1-\frac{\alpha}{r}\right)^b=\gamma_{55}=C\xi^0_1\sin\omega s$.

Очевидно,
$$b=\frac{\ln\gamma_{55}}{\ln\left(1-\frac{\alpha}{r}\right)}=\frac{\ln C\xi^0_1\sin\omega s}{\ln\left(1-\frac{\alpha}{r}\right)},\ \alpha=\alpha\left(x^0\right), \tag{50}$$

где $x^0=ct$, t – время в лабораторной системе; в общем случае полагаем, что $C=C\left(x^0\right)$, $\omega=\omega\left(x^0\right)$. Для метрики Шварцшильда имеем $b=0$, следовательно, из (50) следует, что $C\left(x^0=0\right)\xi^0_1\sin\omega\left(x^0=0\right)s=1$ и постоянная ξ^0_1 выбирается в виде $\xi^0_1=\dfrac{1}{C(0)\sin\omega(0)s}$.

Подставляя это выражение в (50), получим

$$
b=\frac{\ln\dfrac{C\left(x^0\right)}{C}\cdot\dfrac{\sin\omega\left(x^0\right)s}{\sin\omega s}}{\ln\left(1-\dfrac{\alpha}{r}\right)}. \tag{50'}
$$

Из (3) для постоянной b имеем ограничение вида $b^2\le\frac{4}{3}$. Оценки для a и b показывают, что по мере продвижения константы b к нулю (уменьшению возмущения от звезды 1) устойчивая орбита приближается к классической $r_{\min}=3\alpha$ $\left(\beta_{\min}=2/3\right)$; покажем это. Пусть γ_{55} удовлетворяет значениям $b=b_{\text{ïð}}=-2/\sqrt{3}$, $a_{\text{ïð}}=1/\sqrt{3}$, $b=-0.01/\sqrt{3}$, $a=1.0029$; имеем для $f=1.8$ ($h_5^2>0$) следующие таблицы:

Таблица 1.

$a=1/\sqrt{3}$

β	0.9	0.7	2/3	0.5	0.4
$V_{\text{эф}}$	2.03	1.88	1.86	1.72	1.73

Откуда видно, что $\beta_{\min}\sim 0.5,\ r_{\min}\sim 2\alpha$.

Таблица 2.

$a=1.0029$

β	0.9	0.7	2/3	0.5	0.4
$V_{\text{эф}}$	1.55	1.43	1.33	1.35	1.5

$\beta_{\min}\to 2/3,\ r_{\min}\to 3\alpha$.

Итак, согласно влиянию третьего тела (звезды 2) возможно расширение областей устойчивости движения. Выражению (49) соответствует индуцированный тензор энергии – импульса

$$
T_\beta^\mu=\frac{1}{8\pi}g^{\mu\alpha}\left(\Phi_{\alpha.\beta}-\Gamma_{\alpha\beta}^\sigma\Phi_\sigma\right)/\Phi,\qquad \Phi=\sqrt{\gamma_{55}} \tag{51}
$$

$$
T_{\beta,\beta}^\mu=0=T_{\beta,\beta}^\mu-\Gamma_{\beta\beta}^\rho T_\rho^\mu+\Gamma_{\rho\beta}^\mu T_\beta^\rho. \tag{52}
$$

Для случая $\alpha^*=\alpha^*\left(x^0\right)$ для поля (28) получим из (51), (52) выражения

$$
T_0^3=h_t=const,\ (53)\quad \gamma_{55,0}=2\gamma_{55}\Gamma_{01}^0, \tag{54}
$$

$$
h_t\frac{16\pi\gamma_{55}}{g^{03}}=-2\left\{\Gamma_{01,0}^0\gamma_{55}+\Gamma_{01}^0\gamma_{55,0}+2\left(\Gamma_{01}^0\right)^2\gamma_{55}\right\}. \tag{55}
$$

Значение Γ_{01}^0 см. в Приложении.

Из (55) имеем дифференциальное уравнение первого порядка для определения $\alpha^*=\alpha^*\left(x^0\right)$

$$
\frac{d\alpha^*}{dx^0}\cong\widetilde{b}\left(\widetilde{a}\,\alpha^*+\xi\right)/\left(1+3\alpha^*/2R\right), \tag{56}
$$

$$
x^0-x_0^0\cong\left(\frac{1}{\widetilde{a}\widetilde{b}}-\frac{3\xi}{2R\widetilde{b}\widetilde{a}^2}\right)\ln\frac{\left(\widetilde{a}\,\alpha^*+\xi\right)}{\left(\widetilde{a}\,\alpha_0^*+\xi\right)}+\frac{3\alpha^*}{2R\widetilde{b}\widetilde{a}}-\frac{3\alpha_0^*}{2R\widetilde{b}\,\widetilde{a}}, \tag{57}
$$

где

$$\widetilde{a}=\frac{1}{R}+\frac{B_0^3QR}{2},\ \widetilde{b}=-\frac{16\pi h_t R^3}{2B_0Q_0},\ \xi=-\frac{2Q^2}{R^2}+B_0^2R^2-2B_0Q+B_0^3QR^2. \quad (58)$$

Здесь R – расстояние пробной частицы от возмущающего тела (звезды). Для примера приведем данные по взаимодействующей системе "Лебедь Х-1" [2] – это пара, состоящая из звезды с массой $M_0=20M_\oplus$, $M_\oplus$ – масса Солнца с гравитационным радиусом $\alpha^*=60$ км, радиусом $1.9\cdot10^6$ км и черной дырой с массой $M_\circ=6M_\oplus$ и гравитационным радиусом $\alpha=18$ км, удаленным от звезды примерно на ~47 млн. километров. Учитывая эти параметры, из (56) можно приближенно получить линейный закон изменения массы звезды от времени в лабораторной системе отсчета $\alpha^*=Ax^0+\alpha_0^*,\ A<0$. (59)

Принимая равенство расходов массы–энергии указанной пары, получим

$$\sigma=\sigma_0-Ax^0-\alpha_0^*. \quad (60)$$

Тогда интеграл энергии, получаемого из (1), будет зависеть от времени x^0; первые два интеграла получаются сразу - это $\dfrac{\partial L}{\partial\dot\theta}=const=l$ – интеграл момента количества движения

$$fr^2\left(1-\frac{\sigma}{r}\right)^{1-a-b}\dot\theta=l, \quad (61)$$

интеграл для пятой координаты $f\gamma_{55}\dot x^5=h_5$. (62)

Поскольку $\sigma=\sigma\left(x^0\right)$, дифференциальное уравнение для $\dot x^0$ имеет вид

$$2f\left(1-\frac{\sigma\left(x^0\right)}{r}\right)^a\ddot x^0+\frac{afA}{r}\left(1-\frac{\sigma}{r}\right)^{a-1}\left(\dot x^0\right)+frA\dot\theta^2+f\gamma_{55,0}\left(\dot x^5\right)^2=0, \quad (63)$$

или, учитывая малость α/r, и интегралы (61), (62), имеем

$$\ddot x^0+\frac{A}{2r}\left(1+\frac{\sigma\left(x^0\right)}{r}\right)^a\left(\dot x^0\right)^2+\frac{aAl^2}{r^3}\left(1+\frac{3\sigma\left(x^0\right)}{r}\right)+\frac{\gamma_{55,0}}{\gamma_{55}^2f}\left(1-\frac{\sigma\left(x^0\right)}{r}\right)=0. \quad (64)$$

Полагая $x^0=y$, $\dot x^0=y'(s)$, $p(y)=y'(s)$, получим уравнение типа Бернулли для фиксированного значения $r=r_\phi$: $\dfrac{dp}{dy}+P(y)p=-Q(y)p^{-1}$,

где $$P(y)=\frac{A}{2r_\phi}\left(1+\frac{\sigma(y)}{r_\phi}\right),\ Q=\frac{Al^2}{2r_\phi^3}\left(1+\frac{3\sigma(y)}{r_\phi}\right)+\frac{\gamma_{55,y}h_5^2}{\gamma_{55}^2(y)f}\left(1-\frac{\sigma(y)}{r_\phi}\right).$$

Далее, обозначая $z=p^2=\left(\dot x^0\right)^2$, получим искомое решение в виде

$$h^2=e^{\int\widetilde{P}dx^0}\left(\dot x^0\right)^2+\Psi,\ \left(\dot x^0\right)^2=W, \quad (65)$$

где $\widetilde{P}=2P$, $\widetilde{Q}=-Q/2$, $\Psi=\int\widetilde{Q}e^{\int\widetilde{P}dx^0}dx^0$, откуда видно, что интеграл энергии зависит явно от времени x^0.

В случае движения, отличного от кругового, из (1) при $\dot\phi=0$ и (9)÷(11) следует уравнение траектории $(\rho=1/r)$:

$$\left(\frac{d\rho}{d\theta}\right)^2=\sigma\left(\rho^3-a_1\rho^2+a_2\rho+a_3\right); \quad (66)$$

где $a_1 = -1/\sigma$, (67) $a_2 = \frac{1}{l^2}\left[h_5^2(2-a-2b) - f^2(2-b)W + f(2-b-a)\right]$, (68)

$$a_3 = \frac{1}{\sigma l^2}\left(f^2 W - h_5^2 - f\right). \tag{69}$$

С учетом свойств корней правой части уравнения (66), получим

$$\rho_3 = -a_1 - (\rho_1 + \rho_2) = \frac{1}{\sigma} - (\rho_1 + \rho_2). \tag{70}$$

Согласно методу Чандрасекхара и Брумберга [5], представим правую часть (66) в виде (для примера рассмотрим гиперболу)

$$\left(\frac{d\rho}{d\theta}\right)^2 = \sigma\left[\rho - \frac{1}{\tilde{\alpha}(e+1)}\right]\left[\frac{1}{\tilde{\alpha}(e-1)} - \rho\right]\left[\frac{1}{\alpha} - \frac{2e}{\tilde{\alpha}(e^2-1)} - \rho\right]; \tag{71}$$

Раскрывая (48) и сравнивая с (45) получим

$$\sigma\left\{\rho^3 - \rho^2 - \frac{(2\tilde{\alpha}-\sigma)(e^2-1) - 4\sigma}{\tilde{\alpha}^2(e^2-1)^2}\rho + \frac{e^2 - 1 - 2\sigma/\tilde{\alpha}}{\tilde{\alpha}^2(e^2-1)^2}\right\} = \left(\frac{d\rho}{d\theta}\right)^2. \tag{72}$$

Сравнивая (72) и (66), получим уравнения для определения e, $\tilde{\alpha}$ через интегралы движения, зависящие от времени x^0.

Решение (71) ищем в виде $\frac{1}{r} = \rho = \frac{1 + e\cos V}{\tilde{\alpha}(e^2-1)}$. (73)

3. Эффект М.Ф.Широкова для решения Керра

В случае вращения возмущающего тела (звезды) скалярное поле передает возмущения вращательного момента аккреционноиу диску черной дыры; рассмотрим для простоты случай отсутствия электромагнитного поля у исследуемой звезды 1. Метрика Керра на экваториальной орбите ($\dot{r}=0$)запишется в виде [8]

$$ds^2 = \left(1 - \frac{\alpha}{r}\right)(dx^0)^2 - \frac{r^4 + r^2a^2 + \alpha a^2 r}{r^2}d\phi^2 + \frac{2a\alpha}{r}d\phi dx^0, \tag{74}$$

где a – величина, пропорциональная $\tilde{w}$ – угловой скорости на экваторе

$$a = -\frac{2I\tilde{w}}{cM}, \tag{75}$$

I – момент инерции звезды, M – ее масса, c – скорость света, $\alpha = \frac{2\mu}{c^2}$ – гравитационный радиус планеты, μ – ее гравитационная постоянная; ϕ – угол поворота спутника на экваторе; выражения для компонентов гравитационного потенциала метрики (74) приведены в Приложении. Разделив (74) на ds^2, получим функцию Лагранжа

$$L = \left(1 - \frac{\alpha}{r}\right)(\dot{x}^0)^2 - \frac{r^3 + ra^2 + \alpha a^2}{r}\dot{\phi}^2 + \frac{2a\alpha}{r}\dot{\phi}\dot{x}^0 \equiv 1. \tag{76}$$

Из решения (76) следуют для импульсов $\frac{\partial L}{\partial \dot{x}^0}$ и $\frac{\partial L}{\partial \dot{\phi}}$ интегралы h и l

$$h = \left(1 - \frac{\alpha}{r}\right)\dot{x}^0 + \frac{a\alpha}{r}\dot{\phi}, \tag{77}$$

$$l = \frac{r^3 + ra^2 + \alpha a^2}{r}\dot{\phi} - \frac{a\alpha}{r}\dot{x}^0. \qquad (78)$$

Записывая уравнения Лагранжа для импульса $\frac{\partial L}{\partial \dot{r}}$ и учитывая, что $\dot{r} = 0$, получим из $\frac{\partial L}{\partial r} = 0$ соотношение между скоростями $\dot{x}^0$ и $\dot{\phi}$:

$$\frac{\alpha}{r}\left(\dot{x}^0\right)^2 - \left(2r^2 + \frac{\alpha a^2}{r}\right)\dot{\phi}^2 - \frac{2a\alpha}{r}\dot{\phi}\dot{x}^0 = 0. \qquad (79)$$

Решая совместно (79) и (76), получим выражение $\left(\dot{x}^0\right)^2$ через r, a, α:

$$\left(\dot{x}^0\right)^2 = \frac{1}{\left\{\left(1 - \frac{\alpha}{r}\right) - \frac{\left(\alpha a/r + A\right)\left[\left(3r^2 - a^2 + 3\alpha a^2/r\right)\alpha a + \left(r^3 + ra^2 - \alpha a^2\right)A\right]}{r\left(2r^2 + \alpha a^2/r\right)^2}\right\}}, \qquad (80)$$

где $A = \sqrt{2\alpha r + 2\alpha^2 a^2/r^2}$.

При отсутствии вращения звезды $a = 0$ мы имеем результат Широкова [2] для поля Шварцшильда

$$\left(\dot{x}^0\right)^2 = \frac{1}{\left(1 - \frac{3}{2}\frac{\alpha}{r}\right)}. \qquad (81)$$

Запишем уравнения возмущенного движения пробной частицы [2]

$$\frac{d^2\xi^i}{ds^2} + 2\Gamma^i_{jk}U^j\frac{d\xi^k}{ds} + \Gamma^i_{jl,k}U^jU^l\xi^k = 0, \quad i, j, k, l = 0,1,3. \qquad (82)$$

Здесь U^j – компоненты скорости частицы; вычисляя символы Кристоффеля Γ^i_{jk} на основе метрических тензоров g_{jk}, g^{jk} (см. Приложение), получим систему дифференциальных уравнений

$$\frac{d^2\xi^1}{ds^2} + a_1\frac{d\xi^3}{ds} + a_2\frac{d\xi^0}{ds} + a_3\xi^1 = 0, \qquad (83)$$

$$\frac{d^2\xi^3}{ds^2} + b\frac{d\xi^1}{ds} = 0, \ (84) \qquad \frac{d^2\xi^0}{ds^2} + C\frac{d\xi^1}{ds} = 0, \qquad (85)$$

где ξ^0 – возмущение $x^0 = ct$, ξ^1 – возмущение радиуса r, ξ^3 – возмущение угла ϕ.

Представляя решение системы (83)÷(85) в виде

$$\xi^1 = \xi^0_1 e^{i\omega s}, \ \xi^0 = \xi^0_0 e^{i\omega s}, \ \xi^3 = \xi^3_0 e^{i\omega s}, \qquad (86)$$

где ω – угловая скорость центра масс спутника, $s = c\tau$, τ – собственное время, ξ^0_i – комплексные амплитуды, получим условие нетривиальной совместимости однородной системы линейных уравнений

$$\begin{vmatrix} a_3 - \omega^2 & ia_1\omega & ia_2\omega \\ ib\omega & -\omega^2 & 0 \\ ic\omega & 0 & -\omega^2 \end{vmatrix} = 0, \qquad (87)$$

откуда следует выражение

$$\omega^4\left(a_2C - a_3 + a_1b + \omega^2\right) = 0. \qquad (88)$$

Очевидно, что отличное от нуля решение есть

$$\omega^2 = a_3 - a_2C - a_1b. \qquad (89)$$

Выражения для a_3, a_2, C, a_1, b даны в Приложении.

Для $\dot{x}^0$ и $\dot{\phi}$ используются формулы (79), (80); константы ξ_0^3 и ξ_0^0 можно связать с ξ_0^1 соотношениями $\xi_0^3 = \dfrac{b}{r\omega}\xi_0^1 e^{i\pi/2}$, (90) $\xi_0^0 = \dfrac{C}{\omega}\xi_0^1 e^{i\pi/2}$. (91)

Таким образом, получим решения о колебании пробной частицы относительно центра инерции

$$\xi^1 = \xi_0^1 \sin\omega s,\ \xi^3 = \frac{b}{r\omega}\xi_0^1 \cos\omega s, \quad (92) \qquad \xi^0 = \frac{C}{\omega}\xi_0^1 \cos\omega s. \qquad (93)$$

Ньютоновский период обращения спутника есть $T_0 = 2\pi\sqrt{\dfrac{2r^3}{\alpha c^2}}$. (94)

Тогда фактический период за счет гравитационных колебаний равен $T = T_0\dfrac{b}{\omega}$ (95)

и будет учитывать вращение a звезды (Земли) на экваторе.

Примем вновь в качестве источника указанного возмущения приток (убыль) дополнительной массы; в качестве энергетической меры возмущений примем как прежде, производную (93) для (8) $\gamma_{55} = \dfrac{d\xi^0}{ds} = \dot{\xi}^0 = \left|\tilde{N}\xi_0^1 \sin\omega s\right|$, (96)

где выражения для C и ω связаны с параметром вращения и приведены в Приложении; закон изменения $\alpha\left(x^0\right)$ определяется аналогично методике гл.2, см. (8)÷(25), (51)÷(52).

4. Заключение

Наряду с известными параметрами, характеризующими возмущающее тело (звезду 1), возможны экспериментально не обнаруженные, но теоретически обоснованные частицы, например, магнитный заряд (гравитационный монополь); известно решение ОТО с учетом магнитного заряда m_0^* как решение NUT (Newman, Unti, Tamburino) [9]

$$L = 1 = U\left(\dot{x}^0\right)^2 - U^{-1}\dot{r}^2 - \left(r^2 + m^{*2}\right)\dot{\theta}^2,$$

$$ds^2 = U\left(dx^0\right)^2 - U^{-1}dr^2 - \left(r^2 + m^{*2}\right)d\theta^2, \qquad U = 1 - \frac{\alpha r + 2m^{*2}}{r^2 + 2m^{*2}}.$$

Согласно нашему подходу, методом М.Ф. Широкова может быть определено возмущение полной энергии, включая энергию монополя и его учет в скалярном поле γ_{55}; тогда с помощью функции Лагранжа-Дирихле можно будет анализировать влияние на орбитальную устойчивость магнитного заряда (монополя).

В ряде работ [10-11] проведены исследования (задачи 2-х тел) орбитальной устойчивости пробной частицы в рамках общей теории относительности (4D); в частности, в работе [11] использована глобальная аналитическая аппроксимация метрики внешнего поля быстровращающейся нейтронной звезды точным решением уравнений Эйнштейна (4D) в пустоте, которое определяется заданием массы звезды 2, ее углового момента и квадроупольного распределения масс $\tilde{b}$. Это решение при $\tilde{b} = 0$ переходит в метрику Керра. Подобные исследования в рамках теории Кленйа-Вессона в 5D в настоящей работе не затронуты. Наш подход с использованием (5D) позволяет решать аналитически задачу 3-х и более тел. Действительно, если имеем звездной скопление, то можно положить $\gamma_{55}=\sum_{i=1}^{n}\gamma_{55_i}$, где γ_{55_i} – возмущение энергии от i – ой звезды (планеты), имеющую свою метрику. Легко учесть и сплюснутость возмущающего тела, учитываемая решением М.Ф. Широкова [12].

Предложенную модель можно интерпретировать как управляемую систему с переменной структурой, определяемой размерностью пространства-времени: при $b=0$ или $\gamma_{55}=0$ при $\omega s=\lambda\pi$, $\lambda=0,\pm1,\ldots$ осуществляется переход к 4-х мерной метрике, решаемой с помощью нелинейных уравнений Эйнштейна $R^{\alpha\beta}-g^{\alpha\beta}R=T^{\alpha\beta}$, и уравнениями девиации, решаемыми методом М.Ф. Широкова; при $\omega s\neq\lambda\pi$, $b\neq0$ восстанавливается переход к 5-ти мерной метрике с условием цилиндричности по пятой координате; тогда изменение масс звездной пары определяется из закона сохранения индуцированного тензора энергии-импульса, возникающего из вакуумного решения теории Калуцы-Вессона ${}^5R_{AB}=0$.

Итак, мы в настоящей работе дали физическую интерпретацию задачи об орбитальной устойчивости в рамках обобщенной теории гравитации (5D), что является стимулом учета при изучении гравитационных волн [13] и для детального математического анализа устойчивости указанных физических задач в рамках римановой геометрии, которая возникла одновременно с гениальной работой по устойчивости А.М. Ляпунова.

5. Приложение

Для метрики Керра имеем:

$$g^{11}=\frac{1}{g_{11}},\ g^{00}=\frac{g_{33}}{g_{33}g_{00}-g_{03}^2},\ g^{33}=\frac{g_{00}}{g_{33}g_{00}-g_{03}^2},\ g^{03}=-\frac{g_{03}}{g_{33}g_{00}-g_{03}^2},$$

$$\begin{aligned}\omega=&\left(\Gamma^1_{00,1}-4\Gamma^1_{00}\Gamma^0_{01}-2\Gamma^1_{03}\Gamma^3_{01}\right)\left(\dot{x}^0\right)^2+\left(\Gamma^1_{33,1}-2\Gamma^1_{33}\Gamma^3_{13}\right)\dot{\phi}^2+\\&+\left(2\Gamma^1_{03,1}-4\Gamma^1_{00}\Gamma^0_{13}-2\Gamma^1_{03}\Gamma^0_{01}-\Gamma^1_{03}\Gamma^3_{13}-4\Gamma^1_{33}\Gamma^3_{01}\right)\dot{x}^0\dot{\phi}.\end{aligned}$$

$$g_{00}=1-\frac{\alpha}{r},\ g_{11}=-\frac{r^2}{r^2+a^2-\alpha r},\ g_{03}=\frac{\alpha a}{r},\ g_{33}=-\frac{r^3+ra^2+\alpha a^2}{r},$$

$$g^{00}=\frac{r^3+ra^2+\alpha a^2}{r\left(r^2+a^2-r\alpha\right)},\ g^{11}=-\frac{r^2+a^2-\alpha r}{r^2},\ g^{03}=\frac{\alpha a}{r\left(r^2+a^2-r\alpha\right)},$$

$$g^{33}=-\frac{r^2+a^2-\alpha r-\dfrac{\alpha^2a^2}{r^2}}{\left(r^2+a^2-r\alpha\right)\left(r^2+a^2+\dfrac{\alpha a^2}{r}\right)},$$

$$a_3=\left\{\Gamma^1_{00,1}\left(\dot{x}^0\right)^2+2\Gamma^1_{03,1}\dot{x}^0\dot{\phi}+\Gamma^1_{33,1}\dot{\phi}^2\right\},\ a_2=\frac{r^2+a^2-\alpha r}{r^4}\left\{\alpha\dot{x}^0-2\alpha a\dot{\phi}\right\},$$

$$C=\frac{\alpha}{r^2\left(r^2+a^2-r\alpha\right)}\left[\left(r^2+a^2\right)\dot{x}^0+a\dot{\phi}\left(3r^2+a^2\right)\right],$$

$$a_1=-\frac{r^2+a^2-\alpha r}{r^4}\left[\alpha a\dot{x}^0+\left(2r^3+\alpha a^2\right)\dot{\phi}\right],$$

$$b=\frac{\alpha a}{r^2}\frac{\dot{x}^0}{\left(r^2+a^2-r\alpha\right)}+\frac{ra^2\left(1-\frac{\alpha a^2}{r^3}\right)\left(1-\frac{\alpha^2}{r^2}\right)+2r^3\left(1-\frac{\alpha}{r}\right)}{\left(r^2+a^2-r\alpha\right)\left(r^2+a^2+\frac{\alpha a^2}{r}\right)}\dot{\phi}.$$

Для метрики Эрнста:

$$\Gamma^1_{00,1}=-\frac{\alpha\left(2r^2+a^2\right)}{2r^5},\ \Gamma^1_{03,1}=\frac{\alpha a}{2r^5}\left(2r^2+4a^2-3\alpha r\right),$$

$$\Gamma^1_{33,1}=-\frac{2r^5+2\alpha a^2r^2-3\alpha^2ra^2+4\alpha a^4-2r^3a^2}{2r^5}.\ \omega=\sqrt{a_3-a_2C-a_1b}.$$

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On the Possibility of Instant Displacements in the Space-Time of General Relativity

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Employing the mathematical apparatus of chronometric invariants (physical observable quantities), this study founds a theoretical possibility to displace particles instantly in the space-time of the General Theory of Relativity.

1. The teleportation condition in General Relativity

As it is known, the basic space-time of the General Theory of Relativity is a four-dimensional pseudo-Riemannian space, which is, in general, inhomogeneous, curved, rotating, and deformed. There the square of the space-time interval $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$, being expressed in the terms of physical observable quantities — chronometric invariants [1, 2], takes the form

$$ds^2 = c^2 d\tau^2 - d\sigma^2 .$$

Here the quantity

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2} v_i dx^i ,$$

is an interval of physical observable time, $w = c^2 \left(1 - \sqrt{g_{00}}\right)$ is gravitational potential, $v_i = -c \frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation, $d\sigma^2 = h_{ik} dx^i dx^k$ is the square of a spatial observable interval, $h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k$ is the metric observable tensor, g_{ik} are spatial components of the fundamental metric tensor $g_{\alpha\beta}$ (space-time indices are Greek $\alpha, \beta = 0,1,2,3$, while spatial indices — Roman $i,k = 1,2,3$).

Following this way we consider a particle displacing at ds in the space-time. We write ds^2 down as follows

$$ds^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right),$$

where $\mathrm{v}^2 = h_{ik} \mathrm{v}^i \mathrm{v}^k$, and $\mathrm{v}^i = \frac{dx^i}{d\tau}$ is the three-dimensional observable velocity of the particle. So ds is: (1) substantial quantity under $\mathrm{v} < c$; (2) zero quantity under $\mathrm{v} = c$; (3) imaginary quantity under $\mathrm{v} > c$.

Particles of non-zero rest-masses $m_0 \neq 0$ (substance) can be moved: (1) along real world-trajectories $cd\tau > d\sigma$, having real relativistic masses $m = \frac{m_0}{\sqrt{1 - \mathrm{v}^2 / c^2}}$; (2) along imaginary world-trajectories $cd\tau < d\sigma$, having imaginary relativistic\linebreak masses $m = \frac{i m_0}{\sqrt{\mathrm{v}^2 / c^2 - 1}}$ (tachyons). World-lines of the both kinds are known as *non-isotropic trajectories*.

Particles of zero rest-masses $m_0 = 0$ (massless particles), having non-zeroes relativistic masses $m \neq 0$, move along world-trajectories of zero four-dimensional lengths $cd\tau = d\sigma$ at the light velocity. They are known as *isotropic trajectories*. To massless particles are related light-like particles — quanta of electromagnetic fields (photons).

A condition under which a particle may realize an instant displacement (*teleportation*) is as equality to zero of the observable time interval $d\tau = 0$ so that the *teleportation condition* is

$$\mathrm{w} + v_i u^i = c^2,$$

where $u^i = \dfrac{dx^i}{dt}$ is its three-dimensional coordinate velocity.

2. Teleportation of mass-bearing particles and massless particles

From here the square of that space-time interval this particle displaces instantly takes the form

$$ds^2 = -d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik} dx^i dx^k,$$

where $1 - \dfrac{\mathrm{w}}{c^2} = \dfrac{v_i u^i}{c^2}$ in this case, because of $d\tau = 0$.

Actually being the signature $(+---)$ in the space-time area of a regular observer, the signature becomes $(-+++)$ in that space-time area where particles may be teleported. So the terms "time" and "three-dimensional space" change each other in that area. "Time" of teleporting particles is "space" of the regular observer, and vice versa "space" of teleporting particles is "time" of the regular observer.

At first, let us consider substantial particles. As it easy to see, instant displacements (teleportation) of such particles realize itself along world-trajectories in which $ds^2 = -d\sigma^2 \neq 0$ is true. So the trajectories represented in the terms of observable quantities are pure spatial lines of imaginary three-dimensional lengths $d\sigma$, although being taken in ideal world-coordinates t and x^i the trajectories are four-dimensional. In a particular case, where the space is free of rotation ($v_i = 0$) or its rotation velocity v_i is orthogonal to the particle's coordinate velocity u^i (so that $v_i u^i = |v_i||u^i|\cos(v_i;u^i) = 0$), substantial particles may be teleported if only gravitational collapse occurs ($\mathrm{w} = c^2$). In this case world-trajectories of teleportation taken in ideal world-coordinates become also pure spatial $ds^2 = g_{ik} dx^i dx^k$.

Second, massless light-like particles (photons) may be teleported along world-trajectories located in a space of the metric

$$ds^2 = -d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik} dx^i dx^k = 0,$$

because for photons $ds^2 = 0$ by definition. So the space of photon teleportation characterizes itself by the conditions $ds^2 = 0$ and $d\sigma^2 = c^2 d\tau^2 = 0$.

The obtained equation is like the "light cone" equation $c^2 d\tau^2 - d\sigma^2 = 0$ ($d\sigma \neq 0$, $d\tau \neq 0$), elements of which are world-trajectories of light-like particles. But, in contrast to the light cone equation the obtained equation is built by ideal world-coordinates t and x^i — no this equation in the terms of observable quantities. So teleporting photons move along trajectories which are elements of the world-cone (like the light cone) in that space-time area where substantial particles may be teleported (the metric inside that area has been obtained above).

Considering the photon teleportation cone equation from viewpoint of a regular observer, we can see that the spatial observable metric $d\sigma^2 = h_{ik}dx^i dx^k$ becomes degenerated $h = \det\|h_{ik}\| = 0$ in the space-time area called that cone. Taking the relationship $g = -hg_{00}$ [1, 2] into account, we arrive to that the four-dimensional metric $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ degenerates as well $g = \det\|g_{\alpha\beta}\| = 0$ there. The last fact implies that signature conditions defining pseudo-Riemannian spaces are broken. So that photon teleportation realizes itself outside the basic space-time of the General Theory of Relativity. Such fully degenerated space was considered in [3, 4], it was referred as *zero-space* because from viewpoint of a regular observer all spatial intervals and time intervals are zeroes there.

At $d\tau = 0$ and $d\sigma = 0$ observable relativistic mass m and the frequency ω become zeroes. So from viewpoint of a regular observer all particles located in zero-space (in particular, teleporting photons) having zero rest-masses $m_0 = 0$ are looking of zero relativistic masses $m = 0$ and the frequencies $\omega = 0$. Therefore particles of such kind may be assumed the ultimate case of massless light-like particles.

We will refer to all particles located in zero-space as *zero-particles*.

In the frames of the particle-wave concept each particle is given by its own wave world-vector $K_\alpha = \dfrac{\partial\psi}{\partial x^\alpha}$, where ψ is the wave phase (eikonal). Eikonal equation $K_\alpha K^\alpha = 0$ [5], setting forth that the length of the wave vector remains unchanged*, for regular massless light-like particles (regular photons) becomes travelling wave equation

$$\frac{1}{c^2}\left(\frac{^*\partial\psi}{\partial t}\right)^2 + h^{ik}\frac{^*\partial\psi}{\partial x^i}\frac{^*\partial\psi}{\partial x^k} = 0,$$

that may be obtained after taking $K_\alpha K^\alpha = g^{\alpha\beta}\dfrac{\partial\psi}{\partial x^\alpha}\dfrac{\partial\psi}{\partial x^\beta} = 0$ in the terms of physical observable quantities [1, 2], where we formulate regular derivatives through chronometrically invariant (physical observable) derivatives $\dfrac{^*\partial}{\partial t} = \dfrac{1}{\sqrt{g_{00}}}\dfrac{\partial}{\partial t}$ and $\dfrac{^*\partial}{\partial x^i} = \dfrac{\partial}{\partial x^i} + \dfrac{1}{c^2}v_i\dfrac{^*\partial}{\partial t}$ and we use $g^{00} = \dfrac{1}{g_{00}}\left(1 - \dfrac{1}{c^2}v_i v^i\right)$, $\mathrm{v}_k = h_{ik}\mathrm{v}^i$, $v^i = -cg^{0i}\sqrt{g_{00}}$, $g^{ik} = -h^{ik}$.

Eikonal equation in zero-space takes the form

$$h^{ik}\frac{^*\partial\psi}{\partial x^i}\frac{^*\partial\psi}{\partial x^k} = 0,$$

because of there is $\omega = \dfrac{^*\partial\psi}{\partial t} = 0$ putting the equation time term into zero. It is standing wave equation. So, from viewpoint of a regular observer, in the frames of the particle-wave concept all particles located in zero-space are looking *standing light-like waves*, so that all zero-space is

* According to Levi-Civita's rule, in a Riemannian space of n dimensions the length of any n-dimensional vector remains unchanged in its parallel transfer. So it is true for the four-dimensional wave vector in a four-dimensional pseudo-Riemannian space — the basic space-time of the General Theory of Relativity. As it is well-known, because all isotropic trajectories have zero four-dimensional length, the length of any isotropic vector is zero, of the wave vector included.

looking filled with a system of light-like standing waves — a light-like hologram. This implies that an experiment discovering non-quantum teleportation of photons should be linked to stop of light.

There is no problem that photon teleportation realizes itself along fully degenerated world-trajectories ($g = 0$) outside the basic pseudo-Riemannian space ($g < 0$), while teleportation trajectories of substantial particles are strictly non-degenerated ($g < 0$) so the trajectories are located in the pseudo-Riemannian space*. It is no problem, because in any point of the pseudo-Riemannian space we can place a tangential space of $g \leq 0$ consisting of the regular pseudo-Riemannian space ($g < 0$) and zero-space ($g = 0$) as two different areas of the same manifold. Such space of $g \leq 0$ will be a natural generalization of the basic space-time of the General Theory of Relativity, permitting teleportation of both substantial particles (outside experiment yet) and photons that has been realized in experiments.

The only difference is that from viewpoint of a regular observer the square of any parallel transferred vector remains unchanged. It is an "observable truth" for also vectors in zero-space, because the observer reasons standards of his pseudo-Riemannian space anyway. So that eikonal equation in zero-space, expressed in his observable world-coordinates, is $K_\alpha K^\alpha = 0$. But being taken in ideal world-coordinates t and x^i the metric inside zero-space $ds^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik} dx^i dx^k = 0$, degenerates into a three-dimensional $d\mu^2$ which, depending on gravitational potential w uncompensated by something other, is not invariant $d\mu^2 = g_{ik} dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 \neq \mathrm{inv}$. As a result, within zero-space the square of a transferred vector, a four-dimensional coordinate velocity vector U^α for instance, being degenerated into a three-dimensional U^i, does not remain unchanged

$$U_i U^k = g_{ik} U^i U^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 \neq \mathrm{const},$$

so that looking Riemannian geometry for a regular observer, the real geometry of zero-space within the space itself is non-Riemannian one.

3. Conclusions

Finishing this brief study, we conclude that instant displacements of particles are naturally permitted in the space-time of the General Theory of Relativity. As it was shown, teleportation of substantial particles and photons realizes itself in different space-time areas. But it would be a mistake to think that teleportation requires to accelerate a substantial particle to super-light speeds (the tachyons area), while a photon needs to be accelerated to infinite speed. No — as it is easy to see from the teleportation condition $\mathrm{w} + v_i u^i = c^2$, if gravitational potential is essential and the space rotates at a speed close to the light velocity, substantial particles may be teleported at regular sub-light speeds. Photons can reach the teleportation condition easier, because they move at the light velocity. From viewpoint of a regular observer, as soon as the teleportation condition realize itself in the neighbourhood around a moving particle, such particle "disappears" although it continues its motion at a sub-light coordinate velocity u^i (or at the velocity of light) in another

* Any space of Riemannian geometry has the strictly non-degenerated metric by definition of such metric spaces. Pseudo-Riemannian spaces are a particular case of Riemannian spaces, where the metric is sign-alternating. So a four-dimensional pseudo-Riemannian space Einstein put the base of the General Theory of Relativity is as well of strictly non-degenerated metric.

space-time area invisible for us. Then, having its velocity lowered or something other that breaks the teleportation condition (lowering gravitational potential or the space rotation speed), it "appears" in the same observable moment in another point of our observable space at that distance and the direction which it has got at u^i there.

In connection with the results, it would be good to remember the "Infinity Relativity Principle", introduced by Abraham Zelmanov (1913–1987), a prominent cosmologist. Having his cosmological studies [1] a base, he had arrived to that "…in homogeneous isotropic cosmological models spatial infinity of the Universe depends on our choice of that reference frame from which we observe the Universe (the observer's reference frame). If the three-dimensional space of the Universe, being observed in one reference frame, is infinite, it may be finite in another reference frame. The same is as well true for the time during which the Universe evolves."

We have arrived to the "finiteness relativity" here. As it was shown, because of a difference between physical observable world-coordinates and ideal ones, the same space-time areas may be very different, being defined in each of the frames. So that, being taken in observable world-coordinates zero-space is a point ($d\tau = 0$, $d\sigma = 0$), while $d\tau = 0$ and $d\sigma = 0$ taken in ideal world-coordinates become $-\left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik} dx^i dx^k = 0$ that is a four-dimensional cone equation like the light cone. Actually here is the "finiteness relativity" for observed objects — an observed point is the whole space taken in ideal coordinates.

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The architecture of triatomic and polyatomic molecules

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We have previously shown that the *classical vibrational period* T_0 of a diatomic molecule can generally be expressed in terms of its *nuclei reduced mass* M_0 and its *internuclear distance* r_0, as

$$T_0 = \frac{4\pi^2}{h\sqrt{n_i n_j}}\sqrt{gM_0 m_e}\, r_0^2 ;$$

n_i and n_j, are the respective *principal quantum numbers of the electrons making up the bond;* m_e is the *electron mass;* h is the *Planck Constant;* we call g the *"bond looseness factor"* of the molecule; it is a Lorentz invariant constant.
g depends only on the *electronic properties of the molecule of concern;* thus it is expected to remain the same for *chemically alike molecules.* This makes that T_0 versus $(1/\sqrt{n_i n_j})\sqrt{M_0}\, r_0^2$ for such molecules, should exhibit a *linear behavior,* the *slope* of which, i.e. $(4\pi^2/h)\sqrt{gm_e}$ fixes g.
For *electronic states configured similarly,* previously we have determined $n_1 n_2$ to be r_0/r_{00}, where r_0 is the internuclear distance of the molecule in hand, and r_{00} the internuclear distance of the molecule of the chemical family we visualize, bearing the lowest classical vibrational period, thus the shortest internuclear distance.
A *triatomic molecule* possesses *two bonds.* One can anticipate that both of the bonds practically act like a diatomic molecule's bond, made of just one atom *on the one hand,* and the rest of the molecule *on the other hand.* We thus expect the above relationship, to hold separately for the bonds of a *triatomic molecule,* and even a *polyatomic molecule.*
Accordingly, out of the available data, we compose *four chemically alike families* of *triatomic or polyatomic molecules,* and we draw T_0, related to the bond in consideration, versus $(1/\sqrt{n_i n_j})\sqrt{M_0}\, r_0^2$, for each of these; here M_0 for a triatomic molecule, is the *reduced mass* of *on the one hand,* the *"mass of the nucleus of the atom"* and, *on the other hand,* the *"total mass of the remaining nuclei of the molecule",* bound by the bond of concern; r_0, is the *length of the bond;* n_i and n_j, are again the respective *principle quantum numbers* of the electrons making up the bond.
The plots T_0, versus $(1/\sqrt{n_i n_j})\sqrt{M_0}\, r_0^2$ indeed satisfactorily turn out to be *straight lines,* passing by the origin, as expected.
We found nothing in the literature, similar to the line we pursue herein.

1. Introduction

In our previous work we established that the classical vibrational period T_0 of a diatomic molecule can be expressed by the *comprehensive relationship*

$$T_0 = \frac{4\pi^2}{h\sqrt{n_i n_j}}\sqrt{gM_0 m_e}\, r_0^2 \quad ; \tag{1}$$

here, M_0 is the *nuclei reduced mass* of the diatomic molecule in hand, r_0 its *internuclear distance,* m_e the *electron mass,* g a Lorentz invariant coefficient which we called the *"bond looseness factor" (given that the inverse of it is roughly proportional to the dissociation energy of the molecule),* h the *Planck Constant,* and n_i and n_j are the respective *principal quantum numbers of the electrons, making up the bond.*
g depends only on the *electronic structure of the molecule;* thus, it is expected to be a *constant* for diatomic molecules belonging to a given *chemical family.*
For *electronic states configured similarly,* previously we have determined $n_i n_j$ to be

$$n_i n_j = \frac{r_0}{r_{00}} \ , \qquad (2)$$

where r_0 is the internuclear distance of the molecule in hand, and r_{00} the internuclear distance of the molecule of the chemical family we visualize, bearing the lowest classical vibrational period, thus the shortest internuclear distance.

Thence, T_0 versus $(1/\sqrt{n_i n_j})\sqrt{M_0 m_e}\, r_0^2$ is expected to behave as a *straight line,* the slope of which is $(4\pi^2/h)\sqrt{g m_e}$, for diatomic molecules belonging to a *given chemical family.* We checked this very satisfactorily, regarding all of the chemical families of diatomic molecules one can compose.[1, 2, 3,4,5] We are embarrassed to cite merely our work, but the fact is, we found nothing similar in the literature.

2. Approach extended to triatomic molecules

A *triatomic molecule* possesses *two bonds.* One can anticipate that both of the bonds practically act like a diatomic molecule bond, made of an atom, on the one hand, and the rest of the molecule, on the other hand.

Henceforth, regarding the *triatomic molecules* belonging to a given chemical family, the classical period of vibration T_0, of *either one* of the two bonds, is expected to behave as a straight line, versus $(1/\sqrt{n_i n_j})\sqrt{M_0 m_e}\, r_0^2$, the slope of which is $(4\pi^2/h)\sqrt{g m_e}$.

Here though, M_0 should be redefined. Thus let us suppose that the triatomic molecule of concern is composed of the atoms A_1, A_2, and A_3, of respective mases m_1, m_2 and m_3, so that the molecule can be represented as A_1 - A_2 - A_3. The bonds in question may be single, double or triple.

Thus one should consider two different nuclei reduced mass, one for the bond A_1 - A_2, and the other for the bond A_2 - A_3.

Let us write the nuclei reduced mass for the latter bond:

$$M_0 = \frac{(m_1 + m_2)m_3}{(m_1 + m_2) + m_3} \quad \text{(for the bond } A_2 - A_3\text{)} \ . \qquad (3)$$

In this case, r_0 is the internuclear distance between the nuclei A_2 and A_3. Further, the *quantum numbers* n_i and n_j are again the respective principal quantum numbers of the electrons making up the bond, in between A_2 and A_3.

Along our approach, we can deal with not only *triatomic molecules,* but also *polyatomic molecules,* in general.

3. Results

Out of the available data, we could assemble just *four families* of *chemically alike triatomic,* or *polyatomic* molecules.[6]

These families are shown in Table 1, together with the related *"bond looseness factor"* [calculated out of the slope, as induced by Eq.(1)], and the standard deviation on this.[7]

Accordingly, we draw the classical vibration period T_0, of the bond of the polyatomic molecule, we focus on, versus $(1/\sqrt{n_i n_j})\sqrt{M_0 m_e}\, r_0^2$, in Figures 1, 2, 3 and 4. These, indeed turn out to be *nearly straight lines.*

Eq.(1) at the same time, can be considered for *a given molecule's excited electronic states,* next to its *ground state.* Thus, we should expect the square of the vibrational period T_0 , of a given state of a given bond in a given *polyatomic molecule,* to behave proportionally to the cube of the internuclear distance r_0, to be associated with this bond's electronic state in consideration; i.e.[5]

$$T_0^2 \sim r_0^3 \ , \qquad (4)$$

the *proportionality constant* being $(4\pi^2/\sqrt{n_i n_j})\sqrt{M_0 m_e}\, r_0^2$, along the definitions given above.

This is successfully checked for all of the polyatomic molecules, we reviewed.

Table 1. The Molecular Bond Strength Factors of the Chemical Families and the Related Relative Errors

The Chemical Families	The Bond Looseness Factor (g)
CO_2, CS_2	0.07
NH_3, PH_3	0.22
H_2O, H_2S, H_2Se, H_2Te	0.16
F_2C, F_2Si	0.03

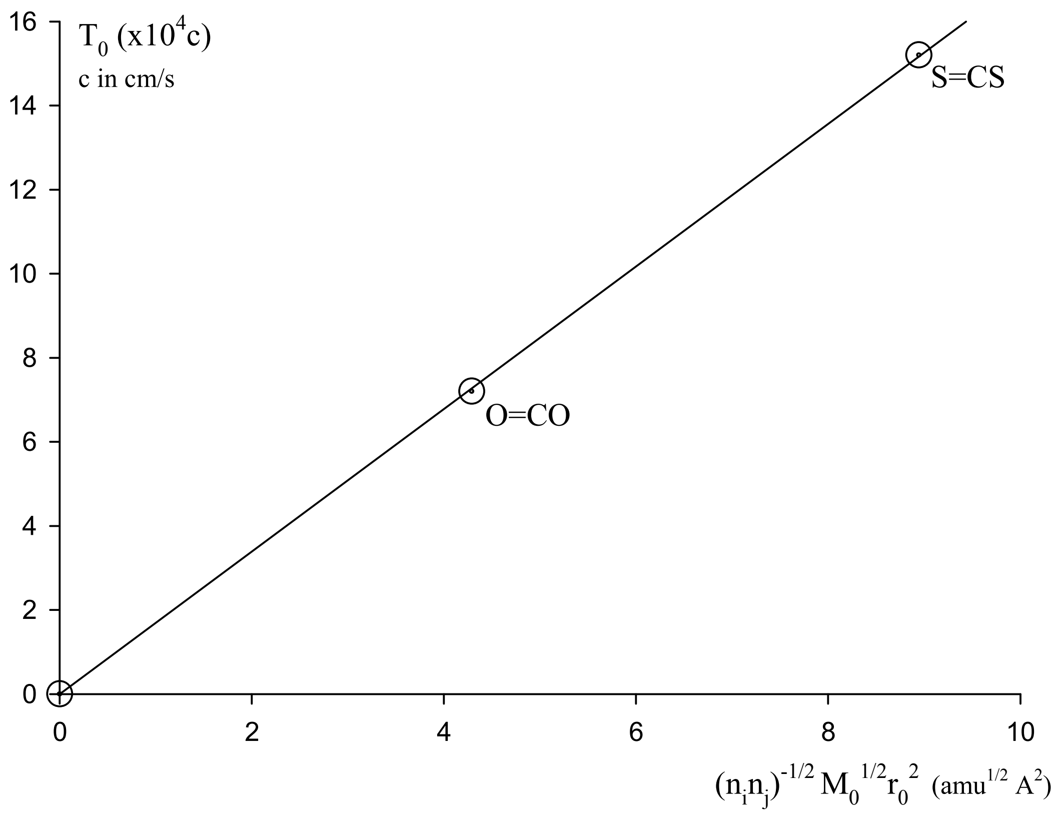

Figure 1 Period of CO_2 and CS_2, versus $(n_i n_j)^{-1/2} M_0^{1/2} r_0^2$

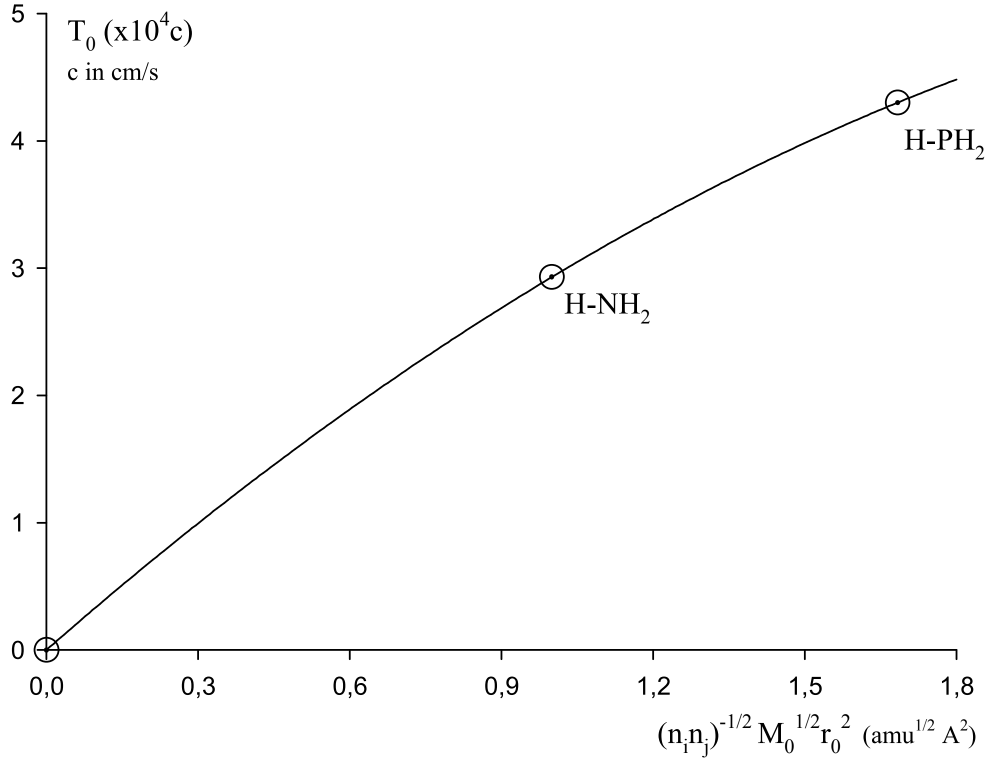

Figure 2 Period of NH_3 and PH_3, versus $(n_i n_j)^{-1/2}\, M_0^{\,1/2}\, r_0^{\,2}$

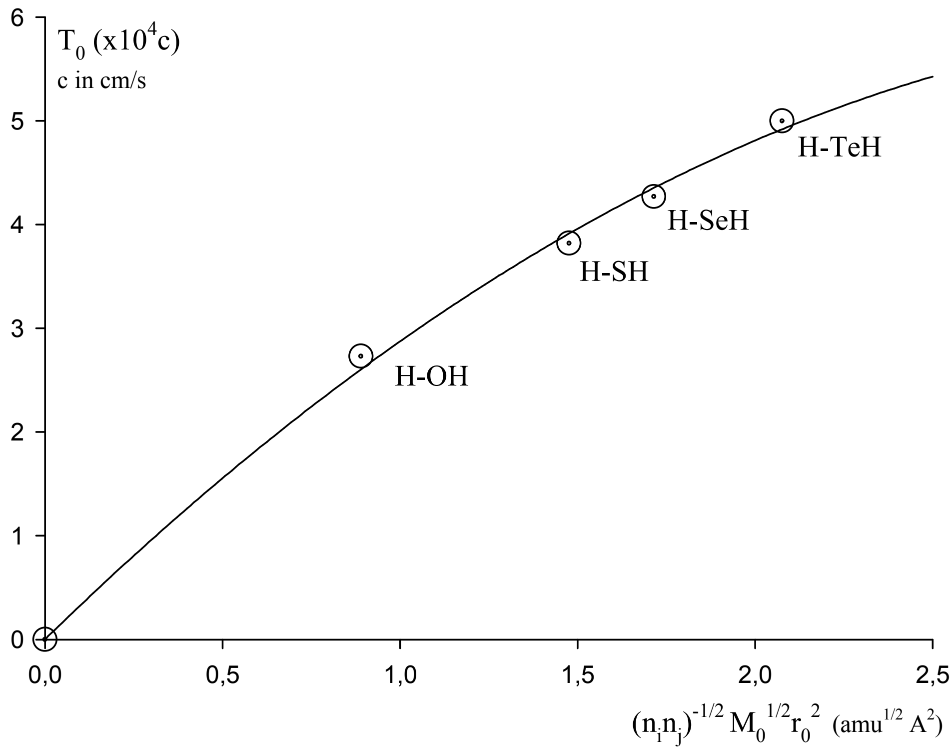

Figure 3 Period of (H_2O, H_2S, H_2Se, H_2Te), versus $(n_i n_j)^{-1/2}\, M_0^{\,1/2}\, r_0^{\,2}$

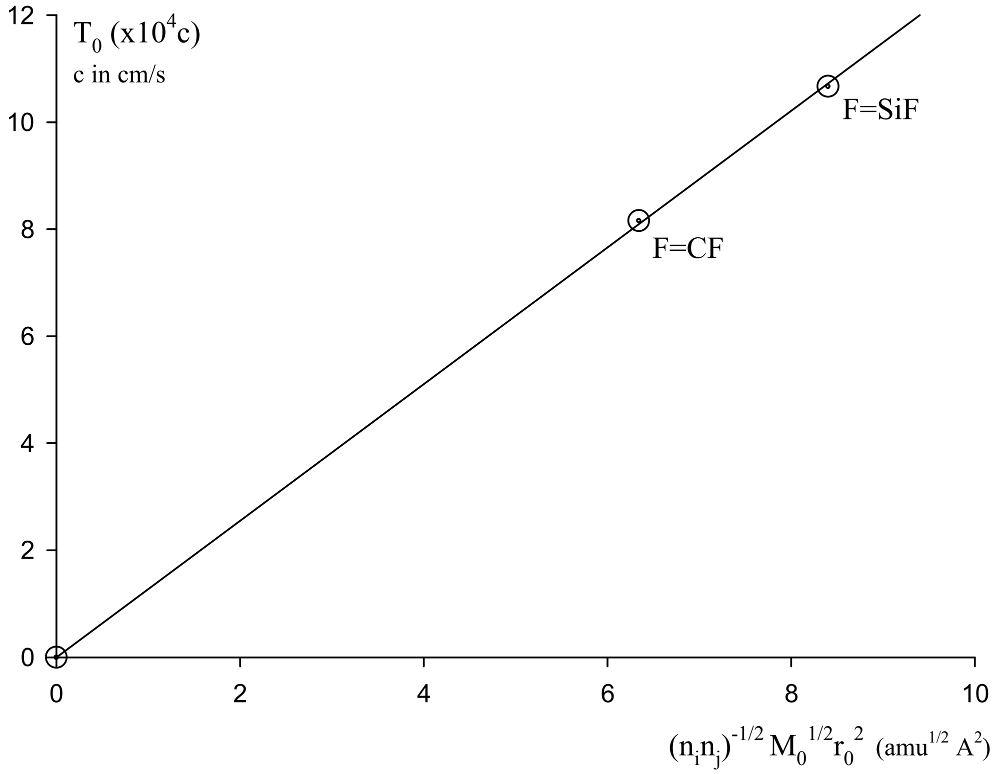

Figure 4 Period of (F_2C, F_2Si), versus $(n_i n_j)^{-1/2} M_0^{1/2} r_0^2$

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A novel approach to the bound muon decay rate retardation: metric change nearby the nucleus

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We show that, just like the gravitational field, the electric field too slows down the internal mechanism of a clock, entering into interaction with the field. This approach explains substantially, the retardation of the decay of the muon bound to a nucleus.

Introduction

For a *"real"* atomistic or molecular wave-like object, i.e. a wave-like object existing in nature, we have shown elsewhere[1] the following theorem, *first,* on the basis of the Schrodinger Equation, as complex as this may be, *then* on the basis of the Dirac Equation, whichever may be appropriate, in relation to the object in hand. A *"real"* atomistic and molecular wave-like object, for instance, involves a *potential energy* made of only "*Coulomb Potential energies"*. Thence even a *relativistic Dirac description* embodying potential energies made of potential energies other than Coulomb Potentials energies, may not represent a *"real"* description, for such an object.

Theorem 1 : In a *"real wave-like description" (thus, not embodying artificial potential energies),* composed of J particles, if all of the different masses m_{j0} (j = 1,…, J) of concern, are overall multiplied by the *arbitrary number* γ, then *concurrently, a)* the *total energy* E_0 associated with the given internal motion of the object, is increased as much, *b) the period of time* T_0 *associated with the motion in consideration, is reduced as much, and c)* the *size* R_0 to be associated with this motion contracts as much; in *mathematical words* this is

$$\{(m_{j0}, j = 1,\ldots, J) \rightarrow (\gamma m_{j0}, j = 1,\ldots, J)\} \Rightarrow \{[E_0 \rightarrow \gamma E_0],\ [T_0 \rightarrow \frac{T_0}{\gamma}],\ [\mathsf{R}_0 \rightarrow \frac{\mathsf{R}_0}{\gamma}]\}\ . \quad (1)$$

What this theorem fundamentally says, is that, if an object ever experiences, for instance an *overall mass decrease,* then its total energy *weakens* as much, yielding a *stretching* of the period of its internal motion framed by the total energy in question, which should be considered quite understandable.

Next we define a quantity called the *"clock mass"* M_0; it is a *compound mass* carrying the internal dynamics of the object; it is manufactured based on different masses embodied by the object in hand; thus multiplying these masses by γ, alters M_0 just as much.

Eq.(1) immediately yields the invariance of the quantity $E_0 M_0 \mathsf{R}_0^2$. This is *remarkable,* since this quantity, is as well, Lorentz invariant *(were the object brought into a uniform translational motion).*

We further show that, the quantity $E_0 M_0 \mathsf{R}_0^2$ is necessarily *"strapped"* to the square of the Planck Constant, h^2 *(being proportional to it, through a rather complex, dimensionless, and relativistically invariant quantity, which is somewhat a characteristic of the bond structure of the wave-like object in hand).*[1]

We call this occurrence, the UMA (Universal Matter Architecture) Cast, disclosing already many *structural properties,* otherwise left obscure since several decades.[2,3,4]

Note that *primarily* what we do is not a "*dimension analysis*"; $E_0 M_0 \mathsf{R}_0^2$ would anyway not be invariant in regards to a mass change, if the wave-like object in question were not *"real*", though of course, *dimension-wise* there would still be no problem.

Our finding further holds for *nuclear wave-like objects* embodying a potential term made of *"real potentials"*.[1]

Anyhow it ought to, since as we just pointed out, the quantity $E_0M_0\mathsf{R}_0^2$ happens to be *Lorentz invariant,* which makes that the *special theory of relativity,* stringently imposes an *interrelation* in between E_0, M_0 and R_0 *(and this, already at rest),* which is precisely the proportionality of $E_0M_0\mathsf{R}_0^2$, to a Lorentz invariant universal constant, i.e. h^2.

The mass increase we introduced above, may very well not be all the way arbitrary, and this is indeed what one experiences for instance, when a clock is removed out of a gravitational field; its *rest mass,* following our claim, as required by the *special theory of relativity,*[5] should be increased as much as the *binding energy* the object displays vis-à-vis the host celestial body of concern *(just like the mass of the hydrogen atom is increased, as the electron is removed away from its orbit around the proton).* The *unit time* displayed by the internal dynamics of the object in hand, were this a wave-like clock, according to our Theorem 1, should then be altered as much.[6] This is exactly what happens in the scope of the general theory of relativity.[5]

According to our approach, the same phenomenon would occur, in exactly the same way, for ionized wave-like clocks in an electric field, or for wave-like clocks bearing an electric dipole, still in an electric field, or for wave-like clocks bearing a magnetic dipole in a magnetic field.[7]

Similarly, if a muon is bound to a proton, its half life should quantum mechanically stretch, as much as its binding energy. This happens, to our knowledge, something totally overlooked.

Calculation of the muon disintegration half life

Keeping *temporarily* aside the *relativistic effect* due to *(had we assumed so)* the motion of the bound muon around the nucleus, and assuming that such a muon preserves its *original identity (besides, its internal dynamics' frequency weakens);* for the bound muon, based on Theorem 1, we can write

$$T = \frac{T_0}{\left(1 - \dfrac{E_B}{m_0c^2}\right)} ; \qquad (2)$$

in this relationship T_0 and T represent the decay half lives of respectively the *free muon* and that of the *bound muon;* E_B is the binding energy of the muon to the nucleus of concern.

Here m_0, should be the *mass of the free muon,* supposing that, the negative electric charge of the muon is distributed uniformly to its entire mass, and that the muon internal dynamics is altered accordingly, when bound to a nucleus.

However this may not be true. Indeed what is bound to the positively charged nucleus, should most likely be the *"muon's electron",* and not the *"muon"* as a whole. This *muonic electron* should then pull, the *neutrino* and the *antineutrino,* together with itself, to the binding state.

Hence, m_0 should be considered as the *highly energetic electron's mass inside the muon*.

Note that there seems to be six different channels of decay of the muon.[8] So the *constituents* of the muon *(supposing that these, acquire their identities inside the muon, at least, prior to the decay),* should really depend on these channels. The one we just considered, is the main decay channel.

We do not know beforehand how, the *energy* subtracted from the muon's electron *(through the binding process),* shall ultimately be accounted by various constituents of the muon.

However, if we were allowed to reason based on the decay data regarding the *main decay channel;* the *mass of the electron* in the free muon, can be guessed to be [0.5 x the mass of the free muon].[8]

It should be this electron's mass alone *(and not the muon as a whole),* which exhibits a *mass deficiency* through the *binding process* of the free muon, to the nucleus in consideration. In other words, we come to expect that the electron's mass, inside the bound muon will decrease as much as the muon's binding energy.

One may check this guess by comparing the *binding energy of the muon* to the nucleus, with the *measured energy shift of the electron thrown from the bound muon,* as referenced to the *energy of the electron thrown from the free muon.*[8] The match is indeed very satisfactory, chiefly for heavy nuclei *(binding the muon).*

Thus we can conlude that, basically the weakened dynamics of the electron inside of the muon, slows down the disintegration of the muon in accordance with Eq.(2).

Now, we can express E_B *(the binding energy of the muon)* for the ground state, based on the Bohr-Sommerfeld, or here the same, the general Dirac Model, with the familiar notation;

$$E_B \cong \frac{2\pi^2 m_{0\mu} Z_0^2 e^4}{h^2}\left(1+\frac{1}{4}\alpha^2 Z_0^2\right) \cong \frac{m_{0\mu} c^2 Z^2 \alpha^2}{2}\ ; \tag{3}$$

$m_{0\mu}$ is the muon's rest mass, Z_0 the atomic number of the nucleus of the *hydrogen-like muoatom,* binding the muon, e the electron's charge; α is the *fine structure constant; it is supposed that the atom is in its ground state.*

Note that Eq.(3) is obtained by expending the rigorous result in power of $Z^2\alpha^2$, but the difference in question remains negligible for the region $1 \le Z < 85$, within which the experimental data is collected.

The electron's mass in the free muon can be expressed as [$f m_{0\mu} c^2$], f following our claim, being 0.5. (Thus $0.5\, m_{0\mu}$ is the *effective mass of the electron,* responsible of the binding of the muon.) α is

$$\alpha = \frac{2\pi e^2}{ch} = \frac{1}{137}\ . \tag{4}$$

The denominator γ, of Eq.(2), thus becomes

$$\gamma = 1 - \frac{E_B}{f m_{0\mu} c^2} = 1 - \frac{1}{2f}\alpha^2 Z_0^2\left(1+\frac{1}{4}\alpha^2 Z_0^2\right)\ ,\ f = 0.5\ . \tag{5}$$

Next, we have to take into account the *time dilation* due to the rotation of the muon around the nucleus *(had we presumed so);* this is

$$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cong \frac{1}{\sqrt{1-\frac{4\pi^2 Z_0^2 e^4}{h^2 c^2}}}\ ; \tag{6}$$

here v the rotational speed of the muon in consideration; it is evaluated through the Bohr-Sommerfeld approximation, which should be expected to be quite satisfactory for light nuclei; for heavy nuclei, quantum effects must be expected to come into play, and it is pointed out that, Eq. (6) is generally an approximation.[9]

Anyway, the overall decay half life T of the bound muon, through Eqs. (2), (3), (4) and (5), quite satisfactorily, becomes

$$T = \frac{T_0}{\left[1-\frac{1}{2f}\alpha^2 Z_0^2(1+\frac{1}{4}\alpha^2 Z_0^2)\right]\sqrt{1-\alpha^2 Z_0^2}}\ . \tag{7}$$

(for the muon bound to the ground state).

It is interesting to note that this expression does not depend on the muon's mass.

Thus, if the electron bears any *internal mechanism,* the above expression would well tell us how this mechanism would slow down, when the electron is in a bound state. (f, though in this case, should be taken as unity.)

Check against experimental and previous theoretical results

We were totally uninformed, in regards to preexisting experimental results, and we are more than happy to discover that our prediction about the bound muon decay, matches quite well with the experimental results.[8,9] Moreover our prediction at a first strike, appears to be much better than previous predictions made so far, no matter how sophisticated, also inevitably cumbersome these may be.

The predictions in question, handle the retardation of the decay process through i) a semiclassical approach, which embodies the *"phase space effect" (consisting in the reduction of the volume of phase space of the muon decay products, because of the binding),* the classical *"relativistic time dilation effect",* and *"the electron Coulomb effect" (consisting in the attraction exerted by the binding nucleus, on the muonic electron)* , and ii) sophisticated quantum mechanical approaches.

It would be interesting to compare quickly our prediction ($_{\text{Author}}$) [cf. Eq.(5)], with the semiclassical (SC) results, exempt of *time dilation effect:*

$$\gamma_{\text{SC}} \cong 1 - \frac{11}{2}\alpha^2 Z_0^2 \qquad \text{(for light } Z_0\text{)}, \tag{8}$$

$$\gamma_{\text{SC}} \cong 0.58\left(1 - \alpha^2 Z_0^2\right)^{5/2} \qquad \text{(for heavy } Z_0\text{)}, \tag{9}$$

$$\gamma_{\text{Author}} \cong 1 - \alpha^2 Z_0^2 \qquad \text{(for all } Z_0\text{)}. \tag{10}$$

Other predictions are so complicated that, they bear no easy series expansions.

In Figure 1 we present the experimental data, and the results of previous calculations *(decay rate normalized to the decay rate of the free muon, versus the atomic number),* achieved to clarify these data. Curve A is a semiclassical calculation including the time dilation effect. Curve B is the same for a Gaussian muon wave function. Curve C is a semiclassical calculation of the time dilation effect alone. Curve D is an interpolation from an anterior calculation achieved by Gilinsky and Mathews.[10] Curve E is interpolated from the calculations achieved by Huff.[9] The experimental results are achieved by Yovanovitch, Barrett, Holmstrom, Keufel, Lederman and Weinrich.[11,12,13]

In Figure 2 we present our prediction, as the denominator of the RHS of Eq.(7), versus the atomic number, together with the corresponding data in hand. We also sketch *separately,* γ of Eq. (5), versus the atomic number, since this constitutes the basis of our claim.

The match of our prediction with data, indeed seems successful.

Analyzing the validity of various proposed contributions, up against that we developed herein, though, constitutes the topic of a subsequent article.

Conclusion

On the whole, clearly our prediction's match with data, is much better than that of other predictions, and constitutes a fundamental explanation to bound muon decay rate retardation.

Our approach however embodies a totally different philosophy than that of others. It is surprizingly simple, whereas other predictions are quite complex.

It is also amazing to note that we came to predict the retardation of the decay of bound muons, through our Theorem 1, which as well yields the end results of the general theory of relativity *(and this, without having to assume the authentic "principle of equivalence").*[6,7]

Thus excitingly enough we come to state that just like *"mass", "electric charge"* too, slows down clocks, interacting with the electric field in question.

This fact induces the *metric change nearby a nucleus,* just like the *metric change nearby a gravitational source.*

Note that the data embody a peak near iron. Our approach did not predict it. Yet neither could the previous attempts. It is suspected that this may be due to the *large background of low energy gamma rays* associated with accompanying *inelastic muon capture* events.

It is worth to emphasize the following interesting *piece of information.* It is that the bound muon's mass is reduced *(as much as the muon's binding energy),* as compared to the free muon's mass. The *mass-energy* equivalence drawn by the special theory of relativity, or the same, the energy conservation law in the broader sense, indeed imposes such an occurrence [cf. Eq.(2)].

This means that just likewise, the bound electron's mass should be smaller than the free electron's mass, and this as much as binding energy, coming into play.

This seems quite trivial, but very much against the general wisdom, since neither Dirac nor anyone else after him, dared to alter the *mass of the bound electron.* Taking it into account, strikingly induces the *change of the metric nearby the nucleus.*[14]

Acknowledgement

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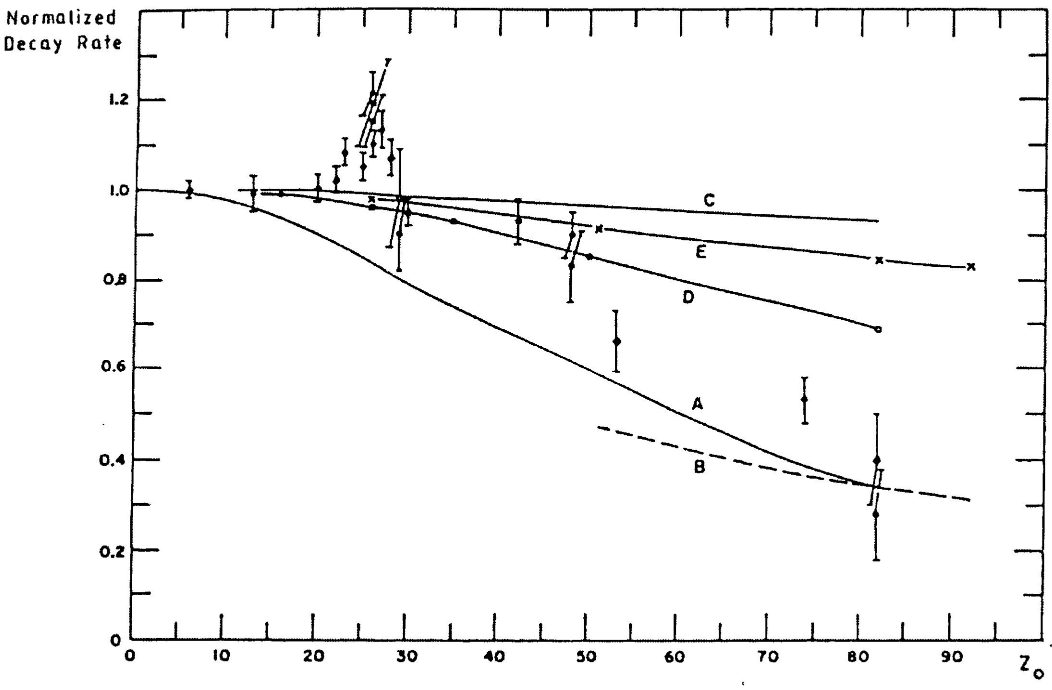

Figure 1 Preexisting experimental results and theoretical predictions [36]

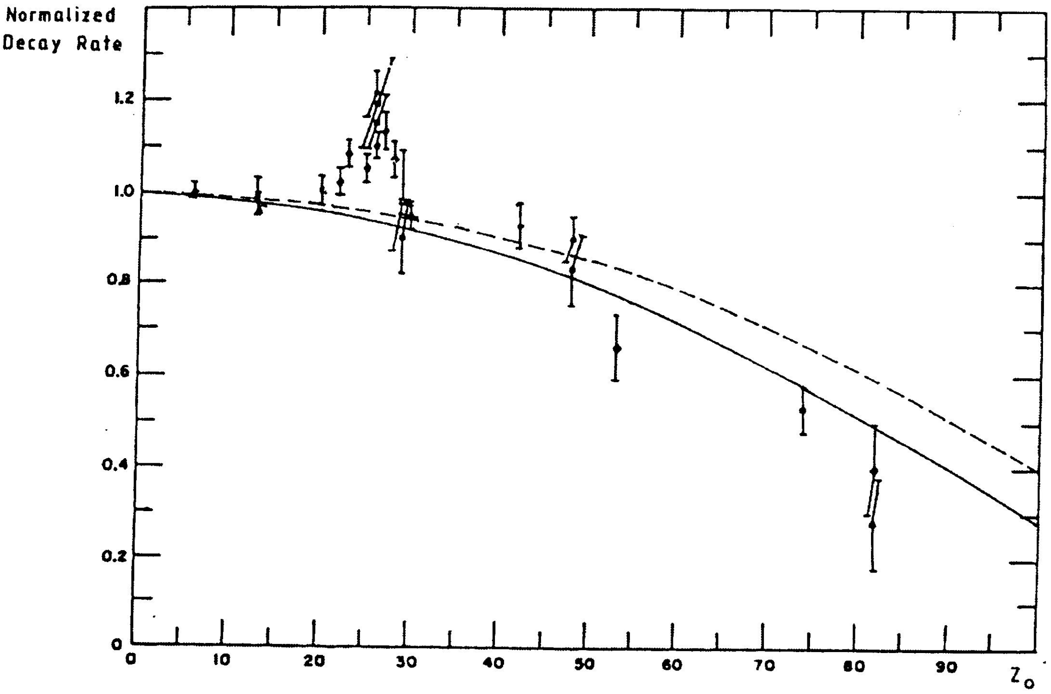

Figure 2 Preexisting experimental results and our predictions:

——— : the overall decay rate

- - - - : the decay rate without the relativistic time dilation effect

General relativity with vacuum corrections

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A new conformal-invariant generalization of the Einstein gravitational theory is proposed which contains a vacuum vectorial field of the Weyl type. Differential equations of the second order for the vacuum field potentials are found and vacuum corrections to the classical dynamic equations for dust-like matter are determined. Cosmological applications of the proposed gravitational theory are considered.

1. Introduction

In recent years a number of difficult problems facing the modern cosmology have arisen.

These problems are caused by observational data obtained from the space telescope "Hubble' and to resolve them cosmologists apply various generalizations of the Einstein gravitational theory containing additional scalar fields and cosmological terms.

For this purpose in our work [1] a new attempt to generalize the Einstein theory based on an additional vectorial field and Weyl's geometry was undertaken. This generalization of the Einstein equations containing four Weyl's potentials λ_i and Weyl's connection Γ^i_{jk} has the form

$$\begin{aligned}
&R_{ik} + R_{ki} - g_{ik}R = (16\pi f / c^4)T_{ik}, \\
&R_{ik} = \partial_l \Gamma^l_{ik} - \partial_k \Gamma^l_{il} + \Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{km}, \quad R = g^{ik}R_{ik}, \\
&\Gamma^i_{jk} = \tfrac{1}{2} g^{im}(\partial_j g_{mk} + \partial_k g_{jm} - \partial_m g_{jk}) + \tfrac{1}{2}(\lambda^i g_{jk} - \lambda_j \delta^i_k - \lambda_k \delta^i_j),
\end{aligned} \tag{1}$$

where f is the gravitational constant, g_{ik} is the metric tensor, R_{ik} is the Ricci tensor, T_{ik} is the energy-momentum tensor of matter and δ^i_k is the Kroneker symbol.

Weyl's connection and hence the gravitational field equations (1) are invariant under the gauge transformations

$$g_{ik} \to \exp(\phi) g_{ik}, \quad \lambda_i \to \lambda_i + \partial_i \phi, \tag{2}$$

where ϕ is an arbitrary differentiable function. This property of Weyl's geometry allows one to give the following simple definition to the four-dimensional interval $ds = (g_{ik}dx^i dx^k)^{1/2}$ which, in contrast with the Einstein theory, does not rest on the principle of least action. According to Weyl's idea, ds^2 is defined as a quadratic differential form describing the kinematics $ds^2 = g_{ik}dx^i dx^k = 0$ of beams of light in a gravitational field [2]. Such a definition just requires that a gravitational theory should be invariant under the conformal transformation (2) of the metric g_{ik} , since it does not change the kinematic equation for beams of light.

As is known, Weyl interpreted the components λ_i as electromagnetic potentials with the aim to unify the gravitational and electromagnetic theories. In order to realize this aim, he relied on a gauge-invariant Lagrangian of second order in the curvature. However, this way led to serious difficulties and, in particular, to gravitational equations of fourth order in the derivatives of the metric [2], in contrast with the Einstein equations of the second order.

That is why we give another interpretation of Weyl's potentials λ_i. Namely, let us regard them as small quantities characterizing a state of the physical vacuum. Then the proposed gravitational equations (1) contain only small corrections to the Einstein equations describing the influence of the vacuum potentials λ_i on physical processes. As will be seen later on, when the standard gauge for the metric is chosen, the small vacuum potentials have the order of $1/A$, where A is the radius of the Universe.

In section 2 we will give main formulae of Weyl's geometry and apply them to find a generalized differential correlation for the energy-momentum tensor T_{ik} in the presence of the vacuum potentials λ_i.

In section 3 we will obtain generalized kinematic equations for dust-like matter and differential equations for the vacuum potentials.

In section 4 we will describe the geometry of the physical vacuum.

In section 5 we will consider the influence of the physical vacuum on moving particles.

In section 6 we will give cosmological applications of the proposed gravitational theory.

2. Differential correlations of Weyl's geometry

Let us note the following properties of the covariant derivative ∇_i based on Weyl's connection and the curvature tensor R^i_{jkn} of Weyl's geometry [1,2]:

$$\nabla_i g_{jk} = \lambda_i g_{jk}, \quad \nabla_i g^{jk} = -\lambda_i g^{jk}, \tag{3}$$

$$\begin{aligned} &R^i_{jkn} = \partial_k \Gamma^i_{jn} - \partial_n \Gamma^i_{jk} + \Gamma^p_{jn}\Gamma^i_{pk} - \Gamma^p_{jk}\Gamma^i_{pn}, \quad R^m_{jmk} = R_{jk}, \\ &R^m_{jkn} = -R^m_{jnk}, \quad R_{ijkn} \equiv g_{im} R^m_{jkn} = -R_{ijnk}, \quad R_{ijkn} = -R_{jikn} + g_{ij}(\partial_n \lambda_k - \partial_k \lambda_n). \end{aligned} \tag{4}$$

Since $\Gamma^i_{jk} = \Gamma^i_{kj}$, we have the Bianchi identities [3]

$$\nabla_m R^i_{jkn} + \nabla_n R^i_{jmk} + \nabla_k R^i_{jnm} = 0. \tag{5}$$

Using formulae (3)-(5), we come to the differential correlations for the Ricci tensor R^{ik}:

$$(\nabla_m + 2\lambda_m)[R^{mk} - \tfrac{1}{2} g^{mk} R + \tfrac{1}{2}(\nabla^m \lambda^k - \nabla^k \lambda^m)] = 0. \tag{6}$$

From (1) and (6) we derive the following generalized differential correlation for the energy-momentum tensor T^{ik} containing the vacuum potentials λ_i:

$$(\nabla_m + 2\lambda_m)[(16\pi f / c^4) T^{mk} + \nabla^k \lambda^m - \nabla^m \lambda^k] = 0. \tag{7}$$

3. Generalized kinematic equations for dust-like matter and differential equations for the vacuum potentials λ_i

Let us choose the standard gauge for the metric tensor and consider the energy-momentum tensor for dust-like matter. Then it takes the following form [3]:

$$T^{ik} = c^2 \rho_0 dx^i / ds \; dx^k / ds, \quad ds^2 = g_{ik} dx^i dx^k = (d\overline{x}^0)^2 - (d\overline{x}^1)^2 - (d\overline{x}^2)^2 - (d\overline{x}^3)^2, \tag{8}$$

where x^i are arbitrary coordinates, $\overline{x}^i$ are rectangular coordinates in a local inertial reference frame and ρ_0 is the rest mass density of the considered dust-like matter.

First, let us write down the differential equation of rest mass conservation in a local inertial reference frame:

$$\partial_m (\rho_0 d\overline{x}^m / ds) = 0. \tag{9}$$

In an arbitrary coordinate system this equation acquires the form

$$(\nabla_m + 2\lambda_m)(\rho_0 dx^m / ds) = 0. \tag{10}$$

Substituting formula (8) for the energy-momentum tensor T^{ik} into correlation (7) and using (10), we find

$$\rho_0 (d^2 x^k / ds^2 + \Gamma^k_{mn} dx^m / ds \; dx^n / ds) + (c^2 / 16\pi f)(\nabla_m + 2\lambda_m)(\nabla^k \lambda^m - \nabla^m \lambda^k). \tag{11}$$

Let us use the following covariant identity [4] which can be easily proved by choosing a local inertial frame:

$$dx_k / ds (d^2 x^k / ds^2 + \overline{\Gamma}^k_{mn} dx^m / ds \; dx^n / ds) = 0, \quad \overline{\Gamma}^k_{mn} = \tfrac{1}{2} g^{ks} (\partial_m g_{sn} - \partial_n g_{ms} - \partial_s g_{mn}), \tag{12}$$

where $\overline{\Gamma}^k_{mn}$ are the classical Christoffel symbols.

Multiplying now Eq. (11) by dx_k / ds and using identity (12), we obtain the equality

$$dx_k / ds[(\nabla_m + 2\lambda_m)(\nabla^k \lambda^m - \nabla^m \lambda^k) - (8\pi f / c^2)\rho_0 \lambda^k] = 0. \quad (13)$$

Since dx^k is arbitrary, from (13) we derive

$$(\nabla_m + 2\lambda_m)(\nabla^k \lambda^m - \nabla^m \lambda^k) = (8\pi f / c^2)\rho_0 \lambda^k. \quad (14)$$

These four equations are the sought differential equations of the second order for the vacuum potentials λ^k. They present the conditions of consistency of the four equations (11).

From Eqs. (11) and (14) we get the sought kinematic equations for dust-like matter containing the vacuum potentials λ^k:

$$d^2 x^k / ds^2 + \Gamma^k_{mn} dx^m / ds \; dx^n / ds + \tfrac{1}{2}\lambda^k = 0. \quad (15)$$

4. Geometry of the physical vacuum

Consider a big region of the physical vacuum in a coordinate system in which the vacuum is homogeneous and isotropic and apply the well-known Robertson-Walker metric [5] for its description in the following form:

$$ds^2 = A^2[d\eta^2 - dr^2/(1-Kr^2) - r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)], \;\; A = A(\eta), \quad (16)$$

where K can take the values -1, 1 or 0, η is a time coordinate and r, θ, φ are spatial spherical coordinates.

In the considered coordinate system the vacuum potentials λ_i are as follows:

$$\lambda_0 = \lambda_0(\eta), \;\; \lambda_1 = \lambda_2 = \lambda_3 = 0. \quad (17)$$

Let us represent the components $\overline{T}^i_k$ of the energy-momentum tensor of the physical vacuum in the following form, using formula (8), since the vacuum is very rarefied, where $\overline{\rho}_0$ is its rest mass density:

$$\overline{T}^i_k = c^2 \overline{\rho}_0 dx^i / ds \; dx_k / ds. \quad (18)$$

Then, substituting metric (16) in the gravitational field equations (1), we obtain

$$\begin{aligned} &3A^{-2} d\alpha / d\eta = -(4\pi f / c^4)(\overline{T}^0_0 - 3\overline{T}^1_1), \;\; \alpha = A^{-1} dA / d\eta - \lambda_0 / 2, \\ &A^{-2}(2K + d\alpha / d\eta + 2\alpha^2) = (4\pi f / c^4)(\overline{T}^0_0 + \overline{T}^1_1), \;\; \overline{T}^1_1 = \overline{T}^2_2 = \overline{T}^3_3. \end{aligned} \quad (19)$$

From Eq. (14) and expressions (17) for λ_i we find

$$\nabla^i \lambda^k - \nabla^k \lambda^i = 0, \;\; \overline{\rho}_0 = 0. \quad (20)$$

Therefore, the rest mass of vacuum particles is zero, they move at the speed of light, and from (18)-(20) we get

$$\overline{T}^m_m = c^2 \overline{\rho}_0 = 0, \;\; \overline{T}^1_1 = \overline{T}^2_2 = \overline{T}^3_3, \;\; \overline{T}^1_1 = -\overline{T}^0_0 / 3. \quad (21)$$

Multiplying the second equation in (19) by 3, adding the first equation in (19) to it and using (21), we find

$$d\alpha / d\eta + \alpha^2 + K = 0, \quad K = \pm 1, 0. \quad (22)$$

This equation has the following solutions, where $\eta_0 = \mathrm{const}$:

$$\begin{aligned} &1)\; K = -1, \;\; \alpha = \mathrm{th}(\eta + \eta_0); \quad 2)\; K = -1, \;\; \alpha = \mathrm{cth}(\eta + \eta_0); \\ &3)\; K = 1, \;\; \alpha = -\mathrm{tg}(\eta + \eta_0); \;\; 4)\; K = 0, \;\; \alpha = 1/(\eta + \eta_0). \end{aligned} \quad (23)$$

From (23) we get that the only nonsingular solution to Eq. (22) is $\alpha = \mathrm{th}(\eta + \eta_0)$ when $K = -1$. In this case from (16), (19) and (23) we have

$$\begin{aligned} &ds^2 = (A_0)^2 \exp(\chi)[d\eta^2 - dr^2/(1+r^2) - r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)], \\ &\lambda_0 = -2\mathrm{th}(\eta + \eta_0) + d\chi / d\eta, \;\; \chi = \chi(\eta), \;\; K = -1, \;\; \alpha = \mathrm{th}(\eta + \eta_0), \\ &\lambda_1 = \lambda_2 = \lambda_3 = 0, \;\; \overline{T}^0_0 = -(3c^4 / 8\pi f) A_0^{-2} \exp(-\chi) \mathrm{ch}^{-2}(\eta + \eta_0), \end{aligned} \quad (24)$$

where $\chi(\eta)$ is an arbitrary differentiable function and $A_0 = \mathrm{const} > 0$.

Let us now choose the standard gauge $\chi = 0$ for this metric. Then from (24) we derive

$$\begin{aligned} &ds^2=(dx^0)^2-d\bar{r}^2/(1+\bar{r}^2/A_0^2)-\bar{r}^2(d\theta^2+\sin^2\theta\, d\varphi^2),\ \ \bar{r}=A_0 r,\\ &\lambda_0=-(2/A_0)\mathrm{th}[(x^0+\beta_0)/A_0],\ \ x^0=A_0\eta,\ \ \beta_0=A_0\eta_0,\ \ \lambda_1=\lambda_2=\lambda_3=0, \end{aligned} \tag{25}$$

where the vacuum potentials λ_i are taken in the coordinate system (x^0,r,θ,φ). As is seen from metric (25), its spatial part presents the Lobachevsky metric with the radius A_0.

Thus, we have come to a nonsingular cosmological solution in which the physical vacuum is described by the Lobachevsky geometry with the constant radius A_0 and has time-dependent negative energy density $\bar{T}_0^0$ and potential λ_0.

5. Influence of the physical vacuum on moving particles

First consider a free movement of a material point in the physical vacuum with the potential λ_0 along the axis x^1 of a rectangular coordinate system (x^1,x^2,x^3). Then from the kinematic equations (15) and formulae (25) we get

$$\begin{aligned} &d^2x^1/ds^2-\lambda_0 dx^0/ds\ dx^1/ds=0,\ \ x^2=x^3=0,\\ &ds^2=(dx^0)^2-(dx^1)^2,\ \ \lambda_0=-(2/A_0)\mathrm{th}[(x^0+\beta_0)/A_0]. \end{aligned} \tag{26}$$

Solving this equation, we find

$$\begin{aligned} &E^2=(m_0c^2)^2+d_0\mathrm{ch}^{-4}(c\tau/A_0),\ \ E=m_0c^2(1-V^2/c^2)^{-1/2},\\ &V=dx^1/d\tau,\ \ \tau=(x^0+\beta_0)/c,\ \ d_0=\mathrm{const}\ge 0, \end{aligned} \tag{27}$$

where E and m_0 are the energy and rest mass of the considered material point, respectively, and τ is a cosmological time. As follows from (24) and (25), the zero time $\tau=0$ corresponds to the maximum absolute value of the vacuum energy density.

Let us apply formula (27) to a photon. Then we get

$$E=h\nu=d_0^{1/2}\mathrm{ch}^{-2}(c\tau/A_0),\ \ m_0=0, \tag{28}$$

where ν is the photon frequency.

It should be noted that formula (28) leads to the Hubble's law giving the relation between the redshift z of the frequency $\nu=\nu(l)$ of the radiation of a galaxy and the distance l gone by its photons:

$$z=[\nu(0)-\nu(l)]/\nu(l)=(H/c)l,\ \ H=2c/A_0,\ \ l\ll A_0,\ \ \tau\ll A_0/c, \tag{29}$$

where H is the Hubble's constant and A_0 is the space curvature radius of the Universe.

Thus, we come to a new interpretation of the Hubble's law. Namely, this law can be interpreted as the result of the influence of the vacuum potential λ_0 on the radiation of galaxies.

Consider the influence of the vacuum potential λ_0 on a non-relativistic material point having the mass m_0 and vector $\mathbf{V}$ of its velocity relative to the physical vacuum. From Eq. (26) we derive that the small force $\mathbf{F}_{\mathrm{vac}}$ which acts on the material point from the physical vacuum is as follows:

$$\mathbf{F}_{\mathrm{vac}}=\lambda_0 c m_0\mathbf{V},\ \ \lambda_0=-(2/A_0)\mathrm{th}(c\tau/A_0),\ \ (\mathbf{V}/c)^2\ll 1. \tag{30}$$

For large positive values of the cosmological time τ from (30) we get the asymptotic formula for the vacuum force $\mathbf{F}_{\mathrm{vac}}$:

$$\mathbf{F}_{\mathrm{vac}}=-(2c/A_0)m_0\mathbf{V},\ \ \tau\to+\infty,\ \ A_0=\mathrm{const}>0. \tag{31}$$

6. Cosmological applications

1) As follows from formula (27), the kinetic energy of free particles exponentially decreases when the cosmological time $\tau\gg A_0/c$. That is why we come to the conclusion that the early Universe, for which the cosmological time τ is near zero, was very hot, similar to the standard cosmology. Hence, the well-known theoretical results of the standard cosmology for the

early Universe concerning the primordial nucleosynthesis and cosmic microwave background radiation [5] could be valid in our conception.

2) As follows from formula (29), the proposed theory, as well as the standard cosmology, describes the well-known Hubble's law, but explains it as the result of a vacuum field influence on the frequency of photons.

3) In contrast with the standard cosmology, the proposed generalization of the Einstein gravitational theory describes a nonsingular Universe.

4) As follows from formula (30), when the cosmological time $\tau > 0$, small decelerating forces act on stars of galaxies. Therefore, stars of a galaxy should move in spiral orbits slowly approaching the galaxy centre. This conclusion allows one to explain the observed spiral structure of many galaxies.

5) Old stars approaching the galaxy centre for a sufficiently long time could be near the centre. Hence, the proposed theory allows one to explain the well-known fact that the galaxy central condensation is mostly composed of old stars, whereas the galaxy spiral arms contain a large number of young stars [6].

6) Since in the proposed theory the stars of the spiral arms of a galaxy approach its centre, the earlier the spiral arms of a galaxy were formed in the past, the closed they are at present to the galaxy centre. This conclusion just corresponds with observational data. Namely, in passing from subclass **c** of the spiral galaxies to subclass **b** and then to **a**, we observe an increasing percentage of old stars and, at the same time, a decreasing spreading of the spiral arms [6].

7) In a similar manner the satellites of the Sun planets should slowly approach them. This allows one to explain the formation of the rings of Saturn, Jupiter, Uranus and Neptune. Such a formation can be possible after the moment when a satellite reaches the Rosh radius of its planet and tidal forces begin to disintegrate the satellite surface.

8) Since the Mars satellite Phobos moves about Mars at the Rosh distance from the planet [7], a very rarefied ring around Mars could form. It is worth noting that the action of particles of this hypothetical ring could explain unexpected failures to communicate with a number of spacecrafts and probes approaching the Martian surface. For example, such failures happened to the Russian "Phobos-2" in 1989, to the American "Mars Observer" in 1993 and "Mars Polar Lander" in 1999 and to the British "Beagle-2" in 2003.

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Einstein Equations in the Higher Order Differential Geometry

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I. INTRODUCTION

In the paper [6], R. Miron and Gh. Atanasiu wrote the Einstein equations and the energy conservation law in the case of the k-jet (or k-osculator, k-tangent) bundle T^kM of a Riemannian manifold in the particular case when the metric structure on T^kM is the Sasaki one $\underset{(0)}{g}_{ij} = \underset{(1)}{g}_{ij} = ... = \underset{(k)}{g}_{ij} =: g_{ij}$ and the metrical linear connection is the one given by (12), which not only preserves by parallelism the distributions generated by the nonlinear connection, but is also absolutely parallel with respect to the k-tangent structure J in (8).

In this paper, we deduce the form of Einstein equations and of the energy conservation law for T^kM endowed with a more general metric structure, namely, as in (11), and also, more general nonlinear connection and metrical linear connection.

II. THE K-TANGENT BUNDLE T^kM

Let M be a real n- dimensional manifold of class $\mathcal{C}^\infty$, $\left(T^kM, \pi^k, M\right)$ its jet bundle of order k, which will be called in the following, the k- tangent bundle (as in [3]). For a point $u \in T^kM$, let $(x^i, y^{(1)i}, ..., y^{(k)i})$ be its coordinates in a local chart.

If N is a nonlinear connection, [5], [7]-[11], with the coefficients $(\underset{1}{N}{}^i_j, \underset{2}{N}{}^i_j, ..., \underset{k}{N}{}^i_j)$, $(i,\ j = 1, ..., n)$, then N determines the direct decomposition

$$T_uT^kM = N_0(u) \oplus N_1(u) \oplus ... \oplus N_{k-1}(u) \oplus V_k(u),\ \forall u \in T^kM. \tag{1}$$

The adapted basis $(\delta_i, \delta_{1i}, ..., \delta_{ki})$ to the decomposition (1) is

$$\begin{cases} \delta_i = \dfrac{\delta}{\delta x^i} = \dfrac{\partial}{\partial x^i} - \underset{1}{N}{}^l_i \dfrac{\partial}{\partial y^{(1)k}} - ... - \underset{k}{N}{}^l_i \dfrac{\partial}{\partial y^{(k)l}} \\ \delta_{1i} = \dfrac{\delta}{\delta y^{(1)i}} = \dfrac{\partial}{\partial y^{(1)i}} - \underset{1}{N}{}^l_i \dfrac{\partial}{\partial y^{(2)l}} - ... - \underset{k-1}{N}{}^l_i \dfrac{\partial}{\partial y^{(k)l}} \\ ... \\ \delta_{ki} = \dfrac{\partial}{\partial y^{(k)i}}, \end{cases} \tag{2}$$

and its dual basis $(\delta x^i, \delta y^{(1)i}, ..., \delta y^{(k)i})$ is

$$\begin{cases} \delta x^i = dx^i \\ \delta y^{(1)i} = dy^{(1)i} + \underset{1}{M}{}^i_l dx^l \\ \delta y^{(2)i} = dy^{(2)i} + \underset{1}{M}{}^i_l dy^{(1)l} + \underset{2}{M}{}^i_l dx^l, \\ ... \\ \delta y^{(k)i} = dy^{(k)i} + \underset{1}{M}{}^i_l dy^{(k-1)l} + ... + \underset{k}{M}{}^i_l dx^l, \end{cases} \tag{3}$$

where $\underset{1}{M}{}^i_l, ..., \underset{2}{M}{}^i_l$ are the dual coefficients of the nonlinear connection N, [5], [7]-[11].

A vector field $X \in \mathcal{X}\left(T^kM\right)$ can be represented in the local adapted basis as

$$X = X^{(0)i}\delta_i + X^{(1)i}\delta_{1i} + ... + X^{(k)i}\delta_{ki}, \tag{4}$$

with the terms

$$hX = X^H = X^{(0)i}\delta_i, \;\; v_1X = X^{V_1} = X^{(1)i}\delta_{1i}, ..., v_kX = X^{V_k} = X^{(k)i}\delta_{ki} \tag{5}$$

(called **d-vector fields**) belonging to the distributions N, $N_1, ..., V_k$ respectively.

A 1-form $\omega \in \mathcal{X}^*\left(T^kM\right)$ will be written as

$$\omega = \omega_i^{(0)}dx^i + \omega_i^{(1)}\delta y^{(1)i} + ... + \omega_i^{(k)}\delta y^{(k)i}; \tag{6}$$

the terms

$$\omega^H = \omega_i^{(0)}dx^i, \omega^{V_1} = \omega_i^{(1)}\delta y^{(1)i}, ..., \omega^{V_k} = \omega_i^{(k)}\delta y^{(k)i} \tag{7}$$

are called **d-covector fields**.

A **distinguished tensor field,** or a **d-tensor field** of type (r, s) on T^kM is a tensor field which acts on r d-covector fields and s d-vector fields:

$$T(\underset{1}{\omega}, ..., \underset{r}{\omega}, \overset{1}{X}, ..., \overset{s}{X}) = T(\underset{1}{\omega}{}^H, ..., \underset{r}{\omega}{}^{V_k}, \overset{1}{X}{}^H, ..., \overset{s}{X}{}^{V_k}).$$

Any tensor field of type (r, s) on T^kM can be decomposed with respect to (1) into a sum of d-tensor fields.

The $\mathcal{F}\left(T^kM\right)$-linear mapping $J : \mathcal{X}\left(T^kM\right) \to \mathcal{X}\left(T^kM\right)$,

$$J\left(\delta_i\right) = \delta_{1i}, \;\; J\left(\delta_{1i}\right) = \delta_{2i}, ..., \;\; J\left(\delta_{(k-1)i}\right) = \delta_{ki}, \;\; J\left(\delta_{ki}\right) = 0 \tag{8}$$

is called the k**-tangent structure on** T^kM,[7]-[11].

III. N-LINEAR CONNECTIONS

By an N-**linear connection** on T^kM, [3], we shall understand a linear connection D on T^kM, which preserves by parallelism the distributions $N_0, N_1, ..., N_{k-1}, V_k$ generated by N. An N-linear connection which is also compatible to the k-tangent structure J ($DJ = 0$) is called, [3], a JN-**linear connection** (let us notice that in the papers [5], [6]-[11], by N-linear connection the authors mean what we here call a JN-linear connection).

An N-linear connection has as coefficients

$$D\Gamma(N) = (\underset{(00)}{L}{}^i{}_{jl}, \underset{(10)}{L}{}^i{}_{jl}, ..., \underset{(k0)}{L}{}^i{}_{jl}, \underset{(01)}{C}{}^i{}_{jl}, \underset{(11)}{C}{}^i{}_{jl}, ..., \underset{(k1)}{C}{}^i{}_{jl}, ..., \underset{(kk)}{C}{}^i{}_{jl}), \tag{9}$$

with

$$\begin{cases} D_{\delta_l}\delta_j = \underset{(00)}{L}{}^i{}_{jl}\delta_i,\ D_{\delta_l}\delta_{1j} = \underset{(10)}{L}{}^i{}_{jl}\delta_{1i}, ..., D_{\delta_l}\delta_{kj} = \underset{(k0)}{L}{}^i{}_{jl}\delta_{ki}, \\ D_{\delta_{1l}}\delta_j = \underset{(01)}{C}{}^i{}_{jl}\delta_i,\ D_{\delta_{1l}}\delta_{1j} = \underset{(11)}{C}{}^i{}_{jl}\delta_{1i}, ..., D_{\delta_{1l}}\delta_{kj} = \underset{(k1)}{C}{}^i{}_{jl}\delta_{ki}, \\ ..., \\ D_{\delta_{kl}}\delta_j = \underset{(0k)}{C}{}^i{}_{jl}\delta_i,\ D_{\delta_{2l}}\delta_{1j} = \underset{(1k)}{C}{}^i{}_{jl}\delta_{1i}, ..., D_{\delta_{2l}}\delta_{2j} = \underset{(kk)}{C}{}^i{}_{jl}\delta_{2i}. \end{cases} \tag{10}$$

In particular, if D is a JN-linear connection, we have

$$\begin{aligned} &\underset{(00)}{L}{}^i{}_{jl} = \underset{(10)}{L}{}^i{}_{jl} = ..., \underset{(k0)}{L}{}^i{}_{jl} =: L^i{}_{jl}, \\ &\underset{(01)}{C}{}^i{}_{jl} = \underset{(11)}{C}{}^i{}_{jl} = ... = \underset{(k1)}{C}{}^i{}_{jl} = \underset{(1)}{C}{}^i{}_{jl}, \\ &... \\ &\underset{(0k)}{C}{}^i{}_{jl} = \underset{(1k)}{C}{}^i{}_{jl} = ... = \underset{(kk)}{C}{}^i{}_{jl} = \underset{(k)}{C}{}^i{}_{jl}. \end{aligned}$$

Let

$$T = T^{i_1...i_r}_{j_1...j_s}\left(x, y^{(1)}, ..., y^{(k)}\right)\delta_{i_1} \otimes ... \otimes \delta_{ki_r} \otimes dx^{j_1} \otimes ... \otimes \delta y^{(k)j_s}$$

be a d-tensor field of type (r, s) and $X \in \mathcal{X}(T^kM)$ is as in (5). Its covariant derivative is

$$D_XT = D^H_XT + D^{V_1}_XT + ... + D^{V_k}_XT,$$

where the h-, v_1-,...,v_k- **covariant derivatives** $D^H_XT = D_{X^H}T$, $D^{V_1}_XT = D_{X^{V_1}}T, ..., D^{V_k}_XT = D_{X^{V_k}}T$ are given by:

$$(D^H_XT)(\underset{1}{\omega}^H, ..., \underset{r}{\omega}^{V_k}, \overset{1}{X}{}^H, ..., \overset{s}{X}{}^{V_k}) = X^H(T(\underset{1}{\omega}^H, ..., \underset{r}{\omega}^{V_k}, \overset{1}{X}{}^H, ..., \overset{s}{X}{}^{V_k})-$$

$$-T(D^H_X\underset{1}{\omega}^H, ..., \underset{r}{\omega}^{V_k}, \overset{1}{X}{}^H, ..., \overset{s}{X}{}^{V_k}) - ... - T(\underset{1}{\omega}^H, ..., \underset{r}{\omega}^{V_k}, \overset{1}{X}{}^H, ..., D^H_X\overset{s}{X}{}^{V_k}),$$

$$(D_X^{V_\beta}T)(\underset{1}{\omega}^H,...,\underset{r}{\omega}^{V_k},\overset{1}{X}^H,...,\overset{s}{X}^{V_k}) = X^{V_\beta}(T(\underset{1}{\omega}^H,...,\underset{r}{\omega}^{V_k},\overset{1}{X}^H,...,\overset{s}{X}^{V_k})-$$

$$-T(D_X^{V_\beta}\underset{1}{\omega}^H,...,\underset{r}{\omega}^{V_k},\overset{1}{X}^H,...,\overset{s}{X}^{V_k}) - ... - T(\underset{1}{\omega}^H,...,\underset{r}{\omega}^{V_k},\overset{1}{X}^H,...,D_X^{V_\beta}\overset{s}{X}),$$

$$(\beta = 1,...,k).$$

In local writing, we obtain:

$$D_X^H T = X^{(0)m}\, T^{i_1...i_r}_{j_1...j_s|m}\delta_{i_1}\otimes ...\otimes \delta_{ki_r}\otimes dx^{j_1}\otimes ...\otimes \delta y^{(k)j_s},$$

where

$$T^{i_1...i_r}_{j_1...j_s|m} = \delta_m T^{i_1...i_r}_{j_1...j_s} + \underset{(00)}{L}{}^{i_1}_{hm}T^{hi_2...i_r}_{j_1...j_s} + ... + \underset{(k0)}{L}{}^{i_r}_{hm}T^{i_1...i_{r-1}h}_{j_1...j_s} -$$

$$-\underset{(00)}{L}{}^{h}_{j_1m}T^{i_1...i_r}_{hj_2...j_s} - ... - \underset{(k0)}{L}{}^{h}_{j_sm}T^{i_1...i_r}_{j_1...j_{s-1}h},$$

respectively,

$$D_X^{V_\beta} T = X^{(\beta)m}\, T^{i_1...i_r}_{j_1...j_s}\overset{(\beta)}{|}_m\ \delta_{i_1}\otimes ...\otimes \delta_{ki_r}\otimes dx^{j_1}\otimes ...\otimes \delta y^{(k)j_s},$$

where

$$T^{i_1...i_r}_{j_1...j_s}\overset{(\beta)}{|}_m = \delta_{\beta m} T^{i_1...i_r}_{j_1...j_s} + \underset{(0\beta)}{C}{}^{i_1}_{hm}T^{hi_2...i_r}_{j_1...j_s} + ... + \underset{(k\beta)}{C}{}^{i_r}_{hm}T^{i_1...i_{r-1}h}_{j_1...j_s} -$$

$$-\underset{(0\beta)}{C}{}^{h}_{j_1m}T^{i_1...i_r}_{hj_2...j_s} - ... - \underset{(k\beta)}{C}{}^{h}_{j_sm}T^{i_1...i_r}_{j_1...j_{s-1}h},\quad (\beta = 1,...,k).$$

IV. D-TENSORS OF TORSION AND CURVATURE

The torsion $T(X,Y) = D_XY - D_YX - [X,Y]$ of the N-linear connection D is well determined by its components which are d-tensors of $(1,2)$-type ([6], [7]):

$$v_\gamma T(\delta_{\beta k},\delta_{\alpha j}) = \underset{(\alpha\beta)}{\overset{(\gamma)}{T}}{}^i_{jk}\delta_{\gamma i},\quad (\alpha,\beta,\gamma = 0,...,k).$$

In the above cited papers, the **d-tensors of torsion** $\underset{(\alpha\beta)}{\overset{(\gamma)}{T}}{}^i_{jk}$ are denoted by:

$$hT(\delta_k,\delta_j) = \underset{(00)}{\overset{(0)}{T}}{}^i_{jk}\delta_i = \underset{(00)}{T}{}^i_{jk}\delta_i,\qquad v_\gamma T(\delta_k,\delta_j) = \underset{(00)}{\overset{(\gamma)}{T}}{}^i_{jk}\delta_{\gamma i} = \underset{(0\gamma)}{T}{}^i_{jk}\delta_{\gamma i},$$

$$hT(\delta_{\beta k},\delta_j) = \underset{(0\beta)}{\overset{(0)}{T}}{}^i_{jk}\delta_i = \underset{(\beta 0)}{P}{}^i_{jk}\delta_i,\qquad v_\gamma T(\delta_{\beta k},\delta_j) = \underset{(0\beta)}{\overset{(\gamma)}{T}}{}^i_{jk}\delta_{\gamma i} = \underset{(\beta\gamma)}{P}{}^i_{jk}\delta_{\gamma i},$$

$$v_\gamma T(\delta_{2k},\delta_{1j}) = \underset{(12)}{\overset{(\gamma)}{T}}{}^i_{jk}\delta_{\gamma i} = \underset{(2\gamma)}{Q}{}^i_{jk}\delta_{\gamma i},$$

$$v_\gamma T(\delta_{\beta k},\delta_{\beta j}) = \underset{(\beta\beta)}{\overset{(\gamma)}{T}}{}^i_{jk}\delta_{\gamma i} = \underset{(\beta\gamma)}{S}{}^i_{jk}\delta_{\gamma i},$$

$(\beta, \gamma = 1, ..., k)$.

The curvature of the N-linear connection D, $R(X,Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]}Z$, is completely determined by its components

$$R(\delta_{\gamma l}, \delta_{\beta k})\delta_{\alpha j} = \underset{(\alpha\beta\gamma)}{\mathcal{R}}{}^{i}_{j\,kl}\delta_{\alpha i},\ (\alpha, \beta, \gamma = 0, 1, ..., k),$$

where the coefficients $\underset{(\alpha\beta\gamma)}{\mathcal{R}}{}^{i}_{j\,kl}$ are d-tensors, named the **d-tensors of curvature** of the N-linear connection D. For a JN- linear connection, there holds

$$\underset{(0\beta\gamma)}{\mathcal{R}}{}^{i}_{j\,kl} = \underset{(1\beta\gamma)}{\mathcal{R}}{}^{i}_{j\,kl} = ... \underset{(k\beta\gamma)}{\mathcal{R}}{}^{i}_{j\,kl}, \quad (\beta, \gamma = 0, 1, ..., k).$$

V. METRIC STRUCTURES ON T^kM

A **Riemannian metric** on T^kM is a tensor field G of type $(0, 2)$, which is nondegenerate in each $u \in T^kM$ and is positively defined on T^kM.

In this paper, we shall consider only metrics in the form

$$G = \underset{(0)}{g}{}_{ij}dx^i \otimes dx^j + \underset{(1)}{g}{}_{ij}\delta y^{(1)i} \otimes \delta y^{(1)j} + ... + \underset{(k)}{g}{}_{ij}\delta y^{(2)i} \otimes \delta y^{(2)j}, \tag{11}$$

where $\underset{(\alpha)}{g}{}_{ij} = \underset{(\alpha)}{g}{}_{ij}(x, y^{(1)}, ..., y^{(k)})$; this is, so that the distributions N, $N_1, ..., V_k$ generated by the nonlinear connection N be orthogonal with respect to G.

An N-linear connection D is called a **metrical** N-linear connection if $D_X G = 0$, $\forall X \in \mathcal{X}(T^kM)$,or, in local writing,

$$\underset{(\alpha)}{g}{}_{ij|l} = \underset{(\alpha)}{g}{}_{ij} \overset{\beta}{|}_l = 0,\ \alpha = 0, 1, ..., k,\ \beta = 1, ..., k.$$

In the particular case $\underset{(0)}{g}{}_{ij} = \underset{(1)}{g}{}_{ij} = ... = \underset{(k)}{g}{}_{ij} =: g_{ij}$, we obtain the metrical JN-linear connection used in [6], namely $C\Gamma(N) = (L^i{}_{jl}, \underset{(1)}{C}{}^i{}_{jl}, ..., \underset{(k)}{C}{}^i{}_{jl})$, given by

$$\begin{aligned} L^i{}_{jl} &= \frac{1}{2}g^{ih}(\frac{\delta g_{jh}}{\delta x^l} + \frac{\delta g_{hl}}{\delta x^j} - \frac{\delta g_{jl}}{\delta x^h}), \\ \underset{(\beta)}{C}{}^i{}_{jl} &= \frac{1}{2}g^{ih}(\frac{\delta g_{jh}}{\delta y^{(\beta)l}} + \frac{\delta g_{hl}}{\delta y^{(\beta)j}} - \frac{\delta g_{jl}}{\delta y^{(\beta)h}}), \quad (\beta = 1, ..., k). \end{aligned} \tag{12}$$

The existence of metrical N-linear connections which are not JN-linear connections is proved in [3] (for $k = 2$, see [1], [2]).

VI. THE RICCI TENSOR $Ric(D)$

Let us notice that, if D is not J- compatible, we could expect that the components of the Ricci tensor should look in a more complicated way that the ones in the Miron-Atanasiu theory, [6]-[11].

Indeed, if we consider the Ricci tensor $Ric\,(D)$,[4], as the trace of the linear operator

$$V \mapsto R\,(V,X)\,Y,\ \forall V = V^{(0)a}\delta_a + V^{(1)a}\delta_{1a} + ... + V^{(k)a}\delta_{ka} \in \mathcal{X}\left(T^2M\right), \tag{13}$$

then we have:

$$\begin{aligned} Ric\,(D)\,(X,Y) = trace(V \mapsto R\left(V^H, X\right)Y + R\left(V^{V_1}, X\right)Y + \\ +... + R\left(V^{V_k}, X\right)Y). \end{aligned} \tag{14}$$

By a straightforward calculus, one can see that

$$Ric(D)(\delta_{\beta j}, \delta_{\alpha i}) = \underset{(\alpha\beta\alpha)}{\mathcal{R}}{}_{i\ jm}^{\ m}, \quad (\alpha, \beta = 0, 1, ..., k).$$

By expressing the above relation on each distribution, we obtain:

$$\begin{aligned}
Ric\,(D)\left(\frac{\delta}{\delta x^j}, \frac{\delta}{\delta x^i}\right) &= \underset{(000)}{\mathcal{R}}{}_{i\ jm}^{\ m} =: R_{ij}, \\
Ric\,(D)\left(\frac{\delta}{\delta y^{(\beta)j}}, \frac{\delta}{\delta x^i}\right) &= \underset{(0\beta0)}{\mathcal{R}}{}_{i\ jm}^{\ m} =: -\underset{(\beta)}{\overset{2}{P}}{}_{ij}, \\
Ric\,(D)\left(\frac{\delta}{\delta x^j}, \frac{\delta}{\delta y^{(\beta)i}}\right) &= \underset{(\beta0\beta)}{\mathcal{R}}{}_{i\ jm}^{\ m} =: \underset{(\beta)}{\overset{1}{P}}{}_{ij}, \\
Ric\,(D)\left(\frac{\delta}{\delta y^{(\gamma)j}}, \frac{\delta}{\delta y^{(\beta)i}}\right) &= \underset{(\beta\gamma\beta)}{\mathcal{R}}{}_{i\ jm}^{\ m} =: \underset{(\beta\gamma)}{S}{}_{ij},
\end{aligned}$$

$(\beta, \gamma = 1, ..., k)$.

Consequently, the Ricci scalar $Sc\,(D)$ writes as

$$Sc\,(D) = \underset{(0)}{g}{}^{ij} R_{ij} + \underset{(1)}{g}{}^{ij} \underset{(11)}{S}{}_{ij} + ... + \underset{(k)}{g}{}^{ij} \underset{(kk)}{S}{}_{ij}, \tag{15}$$

where $\underset{(0)}{g}{}^{ij}, \underset{(1)}{g}{}^{ij}, ..., \underset{(k)}{g}{}^{ij}$ are the components of the inverse matrix of G.

VII. EINSTEIN EQUATIONS

The Einstein equations associated to the metrical N-linear connection D are

$$Ric\,(D) - \frac{1}{2} Sc\,(D)\,G = \kappa\mathcal{T}, \tag{16}$$

where κ is a constant and $\mathcal{T}$ is the energy-momentum tensor, given by its components

$$\underset{(\alpha\beta)}{\mathcal{T}}{}_{ij} = \mathcal{T}(\delta_{\beta j}, \delta_{\alpha i}).$$

Expressing the above relation in the adapted frame (2), we obtain

Theorem 1 *The Einstein equations associated to the metrical N- linear connection D are*

$$
\begin{aligned}
& R_{ij} - \tfrac{1}{2} Sc(D) \underset{(0)}{g}_{ij} = \kappa \underset{(00)}{\mathcal{T}}_{ij}, \\
& \underset{(\beta)}{\overset{1}{P}}_{ij} = \kappa \underset{(\beta 0)}{\mathcal{T}}_{ij}, \\
& \underset{(\beta)}{\overset{2}{P}}_{ij} = -\kappa \underset{(0\beta)}{\mathcal{T}}_{ij}, \\
& \underset{(\beta\beta)}{S}_{ij} - \tfrac{1}{2} Sc(D) \underset{(\beta)}{g}_{ij} = \kappa \underset{(\beta\beta)}{\mathcal{T}}_{ij}, \\
& \underset{(\beta\gamma)}{S}_{ij} = \kappa \underset{(\beta\gamma)}{\mathcal{T}}_{ij}, \quad (\beta, \gamma = 1, ..., k, \quad \gamma \neq \beta).
\end{aligned} \tag{17}
$$

In particular, if D is the JN-linear connection used in [6], the result above coincides with the one in the cited paper.

Now, if we impose the condition that the divergence of the energy- momentum tensor should vanish, in the adapted frame we get

Theorem 2 *The law of conservation on T^kM endowed with the metrical N-linear connection D is given by*

$$
\left(R^i_j - \tfrac{1}{2} Sc(D)\, \delta^i_j\right)_{|i} + \underset{(1)}{\overset{1}{P}}{}^i_j \overset{(1)}{|}_i + ... + \underset{(k)}{\overset{1}{P}}{}^i_j \overset{(k)}{|}_i = 0,
$$

$$
-\underset{(\beta)}{\overset{2}{P}}{}^i_{j|i} + \underset{(1\beta)}{S}{}^i_j \overset{(1)}{|}_i + \underset{(2\beta)}{S}{}^i_j \overset{(2)}{|}_i + ... + \left(\underset{(\beta\beta)}{S}{}^i_j - \tfrac{1}{2} Sc(D)\, \delta^i_j\right) \overset{(\beta)}{|}_i +
$$

$$
+ \underset{(k\beta)}{S}{}^i_j \overset{(k)}{|}_i = 0, \qquad \beta = 1, ..., k,
$$

where

$$
R^i_j = \underset{(0)}{g}{}^{im} R_{mj}, \quad -\underset{(\beta)}{\overset{2}{P}}{}^i_j = -\underset{(0)}{g}{}^{im} \underset{(\beta)}{\overset{2}{P}}_{mj}, \quad \underset{(\beta)}{\overset{1}{P}}{}^i_j = \underset{(\beta)}{g}{}^{im} \underset{(\beta)}{\overset{1}{P}}_{mj}, \quad \underset{(\beta\gamma)}{S}{}^i_j = \underset{(\beta)}{g}{}^{im} \underset{(\beta\gamma)}{S}_{mj},
$$

$(\beta, \gamma = 1, ..., k)$.

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Разностное уравнение, описывающее эмпирический закон Тициуса-Боде

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Характеризующий Солнечную систему закон Тициуса-Боде представлен разностным уравнением, с помощью которого установлена математическая связь планетных расстояний с итерациями метода Ньютона. Высказана гипотеза о применимости этого уравнения к планетам в других звёздных системах.

1. Введение

Известны различные попытки найти дискретный закон пространственного распределения планет в Солнечной системе. Например, Иоганн Кеплер (1571-1630) в «Космографической тайне» сопоставил орбитам планет последовательность концентрических сфер, вписанных или описанных вокруг вложенных друг в друга правильных многогранников, называемых телами Платона. Солнечная система была представлена последовательностью геометрических фигур возрастающего объёма: сфера Меркурия, октаэдр, сфера Венеры, икосаэдр, сфера Земли, додекаэдр, сфера Марса, тетраэдр, сфера Юпитера, куб, сфера Сатурна. Георг Вильгельм Фридрих Гегель (1770 – 1831) в своей философской диссертации «Об орбитах планет» связал планетные расстояния с числовой последовательностью

1; 2; 3; 4; 9; 16; 27,

содержащей степени двойки и тройки. Наиболее удачным оказался так называемый закон Тициуса-Боде.

2. Эмпирический закон Тициуса-Боде.

Немецкие астрономы Иоганн Даниель Тициус (1729-1796) и Иоганн Элерт Боде (1747-1826) во второй половине 18 века предложили эмпирическую формулу для средних расстояний планет от Солнца, измеренных в астрономических единицах:

$$r_n = a + b \cdot 2^n \qquad (n \in \mathbf{Z}). \tag{1}$$

Здесь $a = 0{,}4$; $b = 0{,}3$. $\mathbf{Z}$ – множество целых чисел. Каждая планета имеет свой номер n: $n = -\infty$ (Меркурий), $n = 0$ (Венера), $n = 1$ (Земля), $n = 2$ (Марс), $n = 3$ (пояс астероидов или гипотетическая планета Фаэтон), $n = 4$ (Юпитер), $n = 5$ (Сатурн), $n = 6$ (Уран). Нептун из этой зависимости выпадает, а Плутону приписывается номер $n = 7$. Допустимыми являются такие значения n , которым соответствуют целочисленные значения степени двойки.

Недавно в астероидном поясе Койпера (обширной зоне, лежащей за орбитой Нептуна) были открыто несколько планетообразных тел: 2003 UB313 (Ксена – десятая планета), 2003 EL61 (Санта), 2005 FY9 (Истербанни), 2003 VB12 (Седна), 2004 DW (Оркус), 2002 LM60 (Квавар) и так далее. Ещё только предстоит систематизировать эти тела и осмыслить их роль в рамках закона Тициуса-Боде. Трудно также понять имеет ли какое-либо отношение к закону Тициуса-Боде кометное облако Оорта, важный объект Солнечной системы.

Данная последовательность $\{r_n\}$ планетных расстояний удовлетворяет аддитивному линейному разностному уравнению

$$2\,r_{n-1} + r_{n+1} = 3\,r_n \qquad (n \in \mathbf{Z}). \tag{2}$$

С другой стороны, общее решение уравнения (2) представляется формулой (1) с произвольными константами a и b . С последним уравнением связано ещё одно разностное уравнение

$$r_{-\infty} + r_{n+1} = 2\,r_n \qquad (r_{-\infty} = a), \tag{3}$$

являющееся первым интегралом уравнения (2).

3. Асимптотическое разностное уравнение метода Ньютона.

Последовательные приближения x_n к корню x_* уравнения $f(x)=0$ могут быть найдены при помощи нелинейного разностного уравнения

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad . \tag{4}$$

При $n \to +\infty$ последовательность $\{x_n\}$ сходится к x_*. Формула (4) лежит в основе метода Ньютона, или метода касательных, широко применяемого для решения уравнений. Метод Ньютона позволяет решать алгебраические и трансцендентные уравнения, системы уравнений. Более того, этим методом решаются дифференциальные уравнения и функциональные уравнения (метод Ньютона-Канторовича).

При достаточно больших значениях номера n последовательность $\{x_n\}$ подчиняется асимптотическому мультипликативному линейному разностному уравнению

$$y_{n-1}^2 \cdot y_{n+1} = y_n^3 \qquad (n \in \mathbf{Z}), \tag{5}$$

в котором последовательность $\{ y_n = x_n - x_* \}$ задаёт погрешности, или ошибки, для приближённых значений x_n корня x_*. Если $n \to +\infty$, то $x_n \to x_*$, $y_n \to 0$. Уравнение (5) имеет общее решение

$$y_n = A \cdot B^{2^n} \qquad (n \in \mathbf{Z}) \tag{6}$$

и первый интеграл

$$y_{-\infty} \cdot y_{n+1} = y_n^2 \qquad (y_{-\infty} = A). \tag{7}$$

Здесь A и B – произвольные константы.

Вместо погрешности $y_n = x_n - x_*$ можно также рассматривать точность

$$d_n = -\lg | y_n |$$

для приближённых значений x_n корня x_*. Если $n \to +\infty$, то $y_n \to 0$, $d_n \to +\infty$. Используемый здесь десятичный логарифм связан с выбором десятичной системы счисления. Число правильных (верных) десятичных знаков, взятых после запятой, для приближённого значения x_n корня x_* определяется как целая часть точности: $D_n = [\, d_n \,]$.

В случае выбора системы счисления с основанием p, отличным от десяти, точность приближённых значений x_n корня x_* будет задаваться аналогичной формулой

$$d_n = -\log_p | y_n | \ .$$

Уравнения (5), (6) и (7), характеризующие погрешность, могут быть также переписаны для точности:

$$2\, d_{n-1} + d_{n+1} = 3\, d_n \qquad (n \in \mathbf{Z}), \tag{8}$$

$$d_n = \alpha + \beta \cdot 2^n \qquad (n \in \mathbf{Z}), \tag{9}$$

$$d_{-\infty} + d_{n+1} = 2\, d_n \qquad (d_{-\infty} = \alpha). \tag{10}$$

Здесь α и β – произвольные константы.

4. Математическая эквивалентность уравнений.

Уравнения (1), (2), (3) для планетных расстояний и уравнения (6), (5), (7) для погрешностей в методе Ньютона математически эквивалентны друг другу, но отличаются формой записи: в первом случае имеет место аддитивная форма записи, во втором – мультипликативная. Рассмотрение точности вместо погрешности позволяет уйти от мультипликативной формы записи к аддитивной. В результате приходим к следующему выводу: планетные расстояния и точности в методе Ньютона описываются одинаково, так как уравнения (1), (2), (3) идентичны уравнениям (9), (8), (10).

5. Заключение.

В настоящей работе даны разностное уравнение (2) и его первый интеграл (3), описывающие планетные расстояния в соответствии с законом Тициуса-Боде. Кроме того, при помощи этих уравнений установлена математическая связь последовательности планетных расстояний с последовательностями погрешностей и точностей в итерационном методе Ньютона. Асимптотическая универсальность метода Ньютона (уравнение (5) для $\{x_n\}$ не зависит от конкретного вида функции *f*) и аналогия между последовательностями $\{y_n\}$, $\{d_n\}$, с одной стороны, и $\{r_n\}$, с другой стороны, приводят к гипотезе о возможной универсальности разностного уравнения (2), представляющего закон Тициуса-Боде: уравнение (2) определяет средние расстояния планет в других звёздных системах (а не только в Солнечной системе). Каждой звёздной системе соответствуют свои значения констант a и b.

Generalized theories of gravity and conformal continuations

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Many theories of gravity admit formulations in different, conformally related manifolds, known as the Jordan and Einstein conformal frames. Among them are various scalar-tensor theories of gravity and high-order theories with the Lagrangian $f(R)$ where R is the scalar curvature and f is an arbitrary function. It may happen that a singularity in the Einstein frame corresponds to a regular surface $\mathbb{S}_{\rm trans}$ in the Jordan frame, and the solution is then continued beyond this surface. This phenomenon is called a conformal continuation (CC). We discuss the properties of static, spherically symmetric configurations of arbitrary dimension $D \geq 3$ in scalar-tensor and $f(R)$ theories of gravity and indicate necessary and sufficient conditions for the existence of solutions admitting a conformal continuation. Two cases are distinguished, when $\mathbb{S}_{\rm trans}$ is an ordinary regular sphere and when it is a Killing horizon. Explicit examples of continuations are discussed.

Dispersion of flat wave on dot charged center in approximation geometrical optics

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Trajectory of light rays in vacuum calculates by Ferma principle, which formula of rays in static gravity field appear as [1]

$$\delta\left(\int \frac{dl}{\sqrt{g_{00}}}\right) = 0\,, \tag{1}$$

g_{00} - is time component of metric tensor, which for charged spherical body might be written the following way [2]:

$$g_{00} = c^2\left(1 - \frac{a}{r} + \frac{b}{r^2}\right), \tag{2}$$

where $a = \frac{2GM}{c^2}$, $b = \frac{Gk^2q^2}{c^4}$, r - is distance from center of body, M - is mass of body, q - is volume of charge, $c=3\cdot10^8$ m/s - is speed of light in vacuum, $k = \frac{1}{4\pi\varepsilon_0} \approx 9\cdot10^9$ m/F, $G=6{,}67\cdot10^{-11}$ $H\cdot m^2/kg^2$ - is gravity number.

Taking into account formula of element of length in polar system of coordinates $dl = \sqrt{r^2 + r_\varphi^2}d\varphi$ equation (1) might be written as follows:

$$\delta\left(\int \frac{\sqrt{r^2 + r_\varphi^2}d\varphi}{\sqrt{c^2\left(1 - \frac{a}{r} + \frac{b}{r^2}\right)}}\right) = 0\,.$$

In order to find solution using methods of calculus of variations let us write down the Lagrange equation

$$\frac{dL}{dr} - \frac{d}{d\varphi}\left(\frac{dL}{dr_\varphi}\right) = 0\,,$$

where

$$L = \frac{\sqrt{r^2 + r_\varphi^2}}{\sqrt{c^2\left(1 - \frac{a}{r} + \frac{b}{r^2}\right)}}\,.$$

Finally we have the equation below

$$\frac{2r^4 + 4r_\varphi^2\cdot r^2 - 3ar^3 - 5ar_\varphi^2\cdot r + 4br^2 + 6br_\varphi^2}{2\left(r^2 - ra + b\right)} = r\cdot r_{\varphi\varphi} \tag{3}$$

Bringing in characteristic length r_0 and replacing variable $r = r_0 r_1$ in the equation (3), we receive (further index of new variable is dropped out):

$$\frac{2r^4 + 4r_\varphi^2 \cdot r^2 - \frac{a}{r_0}\left(3r^3 + 5r_\varphi^2 \cdot r\right) + \frac{b}{r_0^2}\left(4r^2 + 6r_\varphi^2\right)}{2\left(r^2 - r\frac{a}{r_0} + \frac{b}{r_0^2}\right)} = r \cdot r_{\varphi\varphi} \tag{4}$$

To model solving we select parameters of dot charged center as for the electron $q = 1,6 \cdot 10^{-19}$ kl, $M = 9,1 \cdot 10^{-31}$ kg. Then: $a \approx 1,35 \cdot 10^{-58}$ m, $b \approx 1,7 \cdot 10^{-62}$ m^2.

Let us consider size of characteristic length as an assessment as $r_0 = 10^{-30}$ m and, therefore, $\frac{a}{r_0} \approx 1,35 \cdot 10^{-28}$, $\frac{b}{r_0^2} \approx 1,7 \cdot 10^{-2}$.

Consequently, having characteristic lengths $r_0 \sim 10^{-30}$ number $\frac{a}{r_0}$ in equation (4) we can ignore in comparing to $\frac{b}{r_0^2}$. Taking this into account, equation (4) might be written as follows:

$$\frac{r^4 + 2r_\varphi^2 \cdot r^2 + \frac{b}{r_0^2}\left(2r^2 + 3r_\varphi^2\right)}{r^2 + \frac{b}{r_0^2}} = r \cdot r_{\varphi\varphi}. \tag{5}$$

This equation might be integrated one time in distinct form:

$$\frac{dr}{d\varphi} = \pm r \sqrt{\frac{Cr^4 - r^2 - \frac{b}{r_0^2}}{r^2 + \frac{b}{r_0^2}}}. \tag{6}$$

Number of integration C might be found through value of target parameter of ray $C = \frac{1}{L^2}$ (target parameter L – is the distance from initial direction of ray dispersion to the center of charge), this permits to write minimal distance, that ray is approaching to the center:

$$r^2_{MIN} = \frac{L^2 + \sqrt{L^4 + 4L^2 \frac{b}{r_0^2}}}{2}. \tag{7}$$

According to (7) when L is increasing, size of minimal distance r_{MIN} aims to L - i.e. dispersion do not observed.

Equation (5) was calculated using the Eller method. Example of calculation of rays trajectories near charged dot center is shown on Fig.1. The results of calculations show that, near charged dot center ray trajectory twists rays disperse into different directions depending on L. In this case even reflection of rays is possible.

There is an area behind the center, where rays cannot enter. In accordance with (7) further from the center smaller distortion of trajectory is.

Using calculated ray trajectories it is possible to model phase surface of flat wave movement when it falls on charged dot center.

The results of calculation are presented on Fig. 2. As one can see there is dispersion of flat wave. At the same time dispersed wave is in fact spherical one.

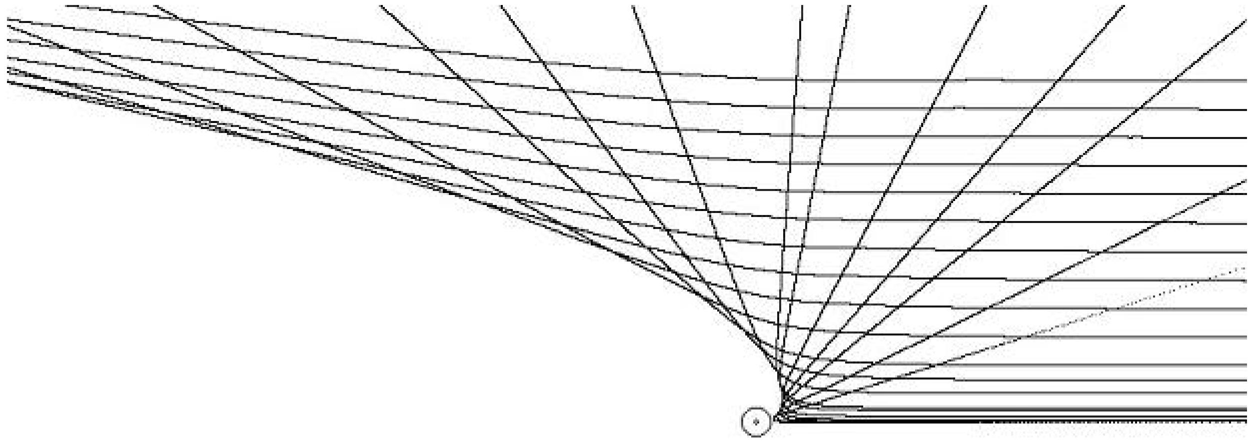

Fig. 1. Rays trajectories in electric field of dot charged center. Characteristic length 10^{-31} m. (Circle radius is half of characteristic length). Rays go from right to left.

As follows from (2) while approaching the center the time component of metric tensor grows up. This leads, finally, to deformation of flat phase surface of wave near the center (Fig. 3).

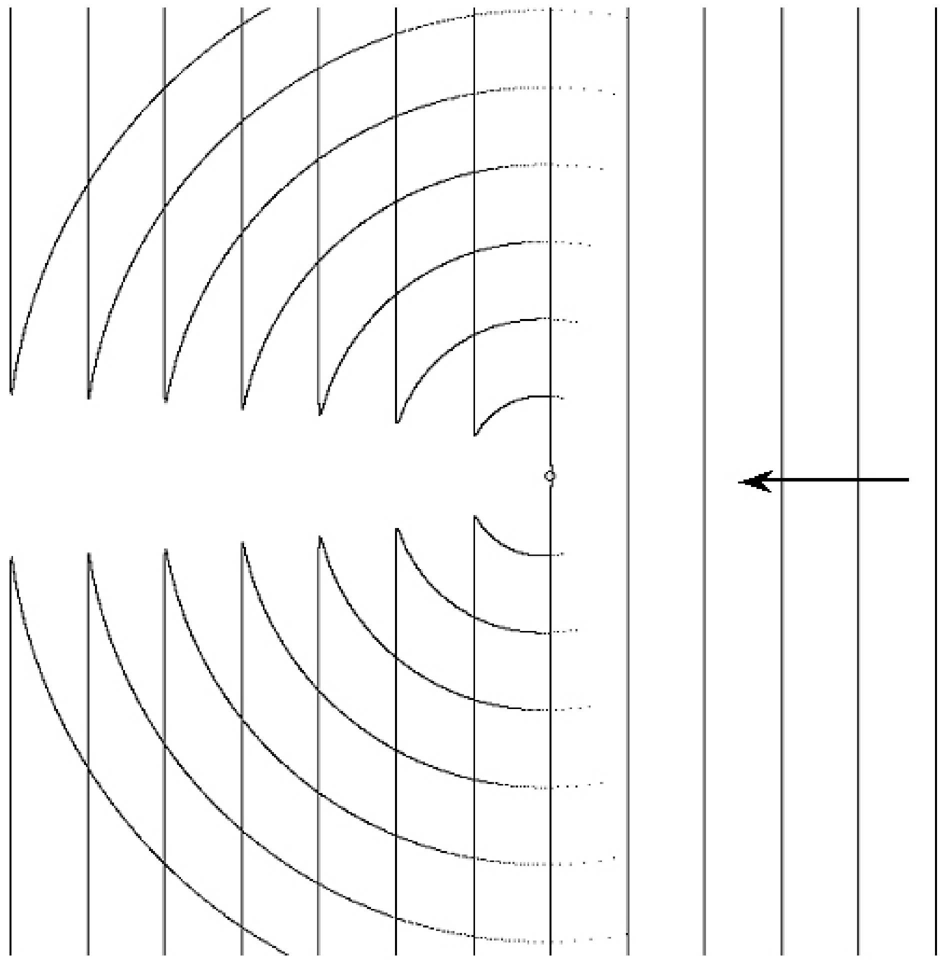

Fig. 2. Dispersion of flat wave on dot center. Arrow shows direction of waves.

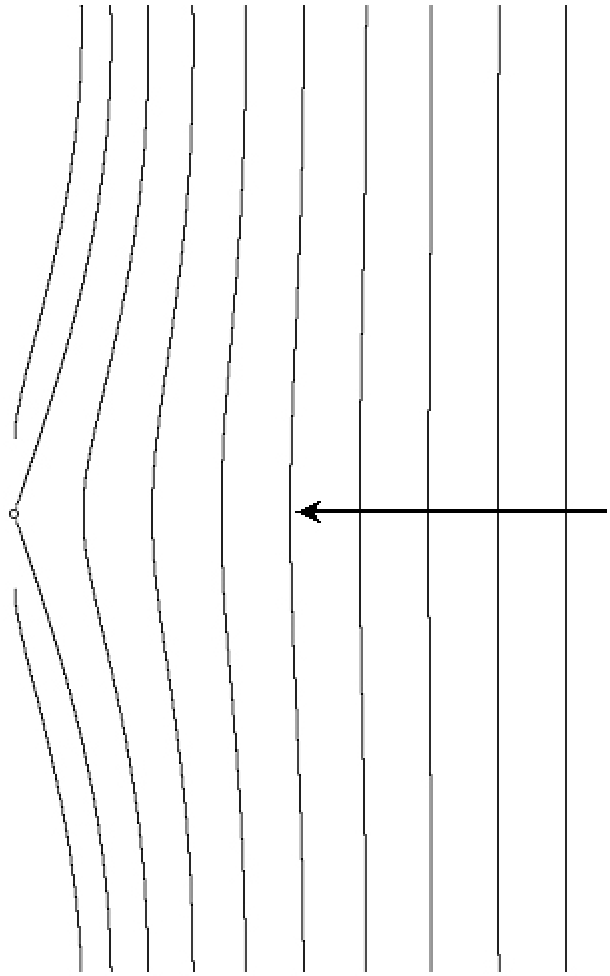

Fig. 3. Distortion of phase surface of flat wave in approaching to charged center. Wave direction is shown by arrow.

In conclusion if should be noticed that “taking into consideration” effects predicted by Gravity Theory permits to model dispersion of rays on charged centers without knowing their inner structure and any hypothesis regarding voluminous distribution of charge. Static charged center must be a source of spherical waves when falling on outside electromagnetic radiation it is dispersing.

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The influence of spin-spin interaction on the energy levels of the Moon- Earth system

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The influence of spin-spin interaction in quantum relativistic weak gravitation Moon-Earth problem was considered. The levels energy and levels width of two bodies system such as Moon and Earth were calculated. The influence of real form of Earth was considered also. It is shown that the end light velocity influence is important in this case. The evolution of Moon orbit is discussed.

1. Introduction

The simplest two bodies problem of quantum gravitation theory which is very impotent is problem of the Moon-Earth system. For example, it's very interesting to compare quantum levels of these system with quantum levels in solid state on the Earth and the Moon including widths.

2. Analysis of problem

We have considered the more complete model of quantum gravitation theory for Moon-Earth system. Our relativistic model has included spin-spin interaction and non spherical form of Earth also. We have supposed that the Moon and the Earth are the point particles.

The normalized whole mechanical energy of particle in gravitation field had in our case the following form:

$$\frac{E}{m_1c^2} = g_{00}{}^{1/2}\left(\frac{E_1 + E_{ss}}{m_1c^2}\right) - 1, \tag{1}$$

where m_1 is a mass of small particle (the Moon), c is light velocity, g_{00} is metric component, E_1, E_{ss} are free part energy, spin-spin interaction energy. For the gravitation field with central symmetry g_{00} is known [1] and $g_{00} = (1 + \frac{2\varphi}{c^2})$ where φ is gravitation potential. It contained contribution from non spherical form of the Earth and ratio $\frac{2\varphi}{c^2}$ was equal $\frac{1}{\eta}(1 + \frac{f}{\eta^2})$ where $\eta = \frac{r}{r_g}$, r is orbit radius, r_g is gravitation radius, $r_g = \frac{2Gm_2}{c^2}$, G is the Newton's gravitation constant, m_2 is a mass of big particle (the Earth), f is non spherical form parameter, $f = \mu(\frac{R}{r_g})^2$, μ is dimensionless small parameter ($\mu \approx 3*10^{-3}$ [2]), R is a radius of the Earth. We have used for E_1 the following expression: $E_1 = (1+\xi^2)^{1/2} m_1c^2$ where $\xi = \frac{p}{m_1c}$. We have supposed that the Moon and the Earth move in one plane. Then we have the following formula for E_{ss}: $-\frac{f_1}{\eta^3} m_1c^2$ where $f_1 = K_1K_2(\frac{m_{pl}{}^4}{m_1m_2{}^3})$, K_1, K_2 are integer and equal $\frac{L_{1z}}{\hbar}, \frac{L_{2z}}{\hbar}$ where L_{1z}, L_{2z} are z-projections of angular momentum, $\hbar$ is the Dirac constant. So we have formula

$$\frac{E}{m_1c^2} = (1 - \frac{1}{\eta}(1 + \frac{f}{\eta^2}))^{1/2}((1+\xi^2)^{1/2} - \frac{f_1}{\eta^3}) - 1, \tag{2}$$

As in work [3] we have used the Bohr's rule of quantization

$$\xi\eta = N\Lambda_g,\ N = 1,2,3,... \tag{3}$$

Using approximation $(1+\xi^2)^{1/2} \cong 1+\frac{\xi^2}{2}$ and condition (3) and conserving terms of order not upper $\frac{1}{\eta^3}$ we have produced new form of formula (2)

$$\frac{E}{m_1c^2} = -\frac{1}{2\eta} + \frac{(N\Lambda_g)^2}{2\eta^2} - \frac{1}{\eta^3}\left(\frac{f}{2} + f_1 + \frac{(N\Lambda_g)^2}{4}\right), \tag{4}$$

We have found that minimum of energy take place at value of η determined by formula

$$\eta = (\Lambda_g N)^2 \left(1 \pm \left(1 - \frac{3}{2(\Lambda_g N)^2} - \frac{3(f+2f_1)}{(\Lambda_g N)^4}\right)^{1/2}\right). \tag{5}$$

So integer N must be positive, it is determined by formula

$$N \ge N_c = \frac{1}{\Lambda_g}\left(\frac{3}{4} + \left(\frac{9}{16} + 3(f+2f_1)\right)^{1/2}\right)^{1/2}. \tag{6}$$

Parameter f is more than unity ($2*10^{13}$) as ratio $\frac{R}{r_g}>10^7$. Another parameter f_1 more than unity also ($f_1 \approx 100$). Evaluation of parameter Λ_g was equal $9*10^{-63}$ and $N_c \cong 10^{62}$. So we have $r > r_g$.

Evaluate τ - time of transition between levels with numbers N and $N+1$. Radiation power of accelerated charge q is determined by formula

$$\frac{dW}{dt} = -\frac{2}{3}\frac{(qa)^2}{c^3}, \tag{7}$$

where a is acceleration. We have supposed that $q = G^{1/2}m$. Our substitution is step which is analog of some assumption in [4]. Using dependence $W = m_1c^2\left(-\frac{1}{2\eta} + \frac{C_N{}^2}{\eta^2} - \frac{C_{fN}}{\eta^3}\right)$ in form $W = m_1c^2\left(-\frac{1}{2\eta}\right)$, we have produced formula

$$\frac{dt}{d\eta} = 6\left(\frac{Gm_2{}^2}{c^3m_1}\right)\eta^2. \tag{8}$$

Integrating formula (8) we have deduced formula for τ

$$\tau = 12\left(\frac{m_{pl}{}^{12}G}{m_1{}^7m_2{}^4c^3}\right)N^5. \tag{9}$$

Using formulas for τ and W, we have found formula for $\Delta W = W(N+1) - W(N)$ and formula for evaluation recovering of energy levels

$$\frac{\hbar}{\tau\Delta W} = \frac{m_1{}^4m_2{}^2}{12m_{pl}{}^6N^2}. \tag{10}$$

Conclusions

1. Relativistic effects change quadratic dependences the energy and orbit radius of number.
2. Relativistic effects determine the range of shut orbits.
3. Relativistic effects determine crossovers of energy levels .

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PHYSICAL NATURE OF LOBACHEVSKY PARALLEL LINES AND A NEW INERTIAL FRAME TRANSFORMATION

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The synchronous process of particle motion and light beams propagation has been found to reveal a new physical equivalent for Lobachevsky geometrical axiom on parallel lines in the velocity space. The process revealed also its fruitfulness in solving in a new way the main problem in relativity - the problem of time synchronization for different space points [1]. The first obvious consequences of the new solution - such as simultaneity, proper time, inertial frame coordinate transformation and relativistic velocity summation law - are also presented in this paper.

1 Introduction

The Lobachevsky velocity space being adequate to the relativistic mechanics is widely used to study particle interaction processes in modern high energy physics [2]. The main Lobachevsky axiom, violaiting the Euclidean V-th postulate, is known as the geometrical equivalent of the experimental fact of the two photons pion decay $\pi^o \rightarrow \gamma\gamma$ [3]. Due to the requirement of the constant light velocity principle its kinematics gives arise the Lobachevsky parallel lines (LPL) in the velocity space. But the dynamics of this decay mode is still not known and its kinematics does not explain the LPL intersection point absence in the velocity space. LPL have only a geometrical interpretation - either as LPL plane curvature or as hordes on the Euclidean circle (Beltrami model) [2]. As it has turned out there exists a new more fruitful physical equivalent for the LPL.

Further developments of the approach published earlier in [4] have been described in this paper. We consider light propagation according to the Huygens principle and the independency of the light beams. So, the phenomena of light diffraction and interference are not considered. It is assumed that the time counting for a space point starts when a light front comes to that point. This is also the moment of a secondary light hemisphere emission, according to the Huygens principle. We accept the constant light velocity principle and we use the same plane light fronts as widely used to explain the light reflection and refraction phenomena. The basic knowledge of Lobachevsky geometry [2, 3, 5] is assuming.

2 Physical nature of Lobachevsky parallel lines

Let us consider two inertial frames K and K_s. Each of the frames may be associated with a particle. The space axises of both frames are parallel and K_s is moving with constant velocity V along the X-axis of frame K. It is assumed that their origins, O and O_s, coincide when the plane light front directed at the parallel angle θ_L reaches the point O

(a lateral beam is moving from bottom to top in XY-plane as shown in Fig.1a). At this initial moment a light sphere (hemisphere to the falling front) starts to spread out from O. The parallel angle θ_L is defined as

$$\cos\theta_L \equiv \cos\Pi(\rho/k) = th(\rho/k) = V/c \equiv \beta, \qquad (k=c) \tag{2.1}$$

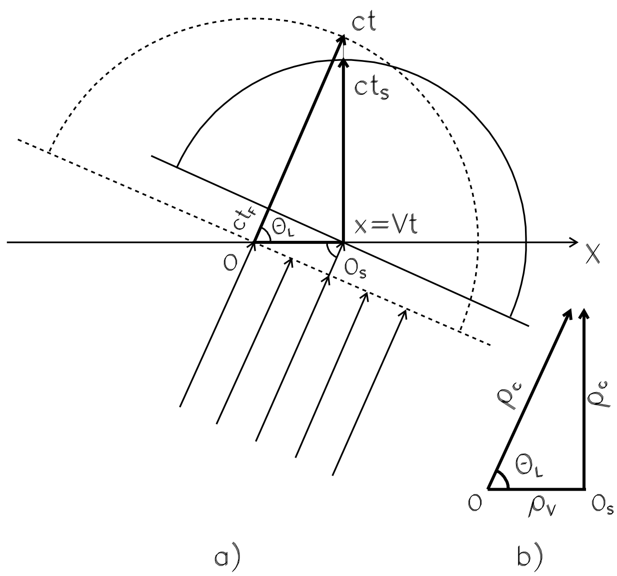

Figure 1: *a) Synchronization of the K_s-motion (Vt) and the light rays (ct and ct_s) propagation by the side light beam. b) Lobachevsky parallel lines in the velocity space plane corresponding to synchronous motions of ct, ct_s and Vt in Euclidean plane ($c = 1$ is used for rapidities).*

here β is the velocity V in units of c, ρ/k is a value of rapidity ρ in units of $k = c$, $\Pi(\rho/k) \equiv \theta_L$ is a parallel angle, k is the Lobachevsky constant, c is the velocity of light. The second equality $\beta = th(\rho/c)$ in (2.1) is known from the Beltarami model [2] and used to define a particle rapidity:

$$\rho/c = 1/2\ln\left((1+\beta)/(1-\beta)\right). \tag{2.2}$$

The first equality in (2.1) can be rewritten as

$$\theta_L \equiv \Pi(\rho/k) = 2arctg\, e^{-\rho/c}, \tag{2.3}$$

known as the Lobachevsky function. It is seen from (2.1) that for any rapidity (and its velocity) there is a definite angle θ_L. For the negative argument of the Lobachevsky function the parallel angle θ_L changes to $\pi - \theta_L$ [2], which corresponds to the same velocity but for the opposite direction.

Let us consider a space-time point $(x = Vt, t)$ in frame K. The light ray from the origin O will get to this point in time x/c (Einstein's signal) but the lateral beam's ray will come there first with some delay (relatively to O) in the moment of time t_F as

$$ct_F = x\cos\theta_L = Vt\cos\theta_L = ct\cos^2\theta_L\,, \tag{2.4}$$

and then a new light sphere starts to spread out from the x-point. By the given moment of time t a new sphere will spread out to the radius

$$ct_s = ct - ct_F = ct - x\cos\theta_L = ct - xV/c, \qquad t_s = t - xV/c^2\,, \tag{2.5}$$

and for $x = Vt$:

$$ct_s = ct - ct\cos^2\theta_L = ct\sin^2\theta_L = ct\,(1 - V^2/c^2)\,, \tag{2.6}$$

where ct is the light sphere radius from origin O, so that $ct_s < ct$.

Let us choose two light rays from these two spheres: one, ct, emitted from O under the angle θ_L to the X-axis in some plane, and the other, ct_s, emitted from O_s (located at x) perpendicular to the X-axis in the same plane (see Fig.1a). Three segments ct, Vt and ct_s form a rectangular triangle. But two sides of triangle, ct and ct_s, have no common (intersection) point at no moment of time t, so they are parallel in any chosen Euclidean plane. As rapidity (2.2) for the light velocity is the infinity, then the obtained triangle transforms into the LPL or, more precisely, into the parallel lines in one side on the Lobachevsky plane in the velocity space as it is illustrated in Fig.1b.

Thus, the LPL in a velocity space corresponds to the light rays ct and ct_s emitted (according to the Huygens principle) from different points and different times and synchronized with particle motion Vt by the side light beam. The physical reason for the lack of intersection point in Eucledean space is the time delay t_F (see (2.4)). As the value of time delay t_F for given x and V is defined by c (with changing V the θ_L changes but not the c) then one can conclude that the basic reason for the V-th postulate violation in the velocity space is the constant light velocity principle.

To find out light rays corresponding to LPL in another side, one can consider a lateral beam to another direction (from top to bottom) in the same plane (as shown in Fig.2a and Fig.2b).

For light rays corresponding to the LPL (in both sides) for negative argument of Lobachevsky function (for $V < 0$), one should use a pair of lateral beams directed opposite to X-axis, i.e. from right to left (for $V > 0$ the beams were directed from left to right), as shown in Fig.2c and Fig.2d.

Thus, the moving reference frame (for $V > 0$ and/or $V < 0$) can be associated with the definite lateral light beams. The rest frame ($V = 0$) is associated with the direct beams at $\theta_L = \pi/2$ (as shown in Fig.2). Lobachevsky function has the same form for the rest frame and for the moving ones, i.e. it follows the principle of relativity. So, Lobachevsky function expresses the constant light velocity principle at $k = c$.

The synchronization method used to reveal the new physical nature of Lobachevsky parallel lines is also fruitful in solving the main problem of relativity - the problem of time synchronization for different space points.

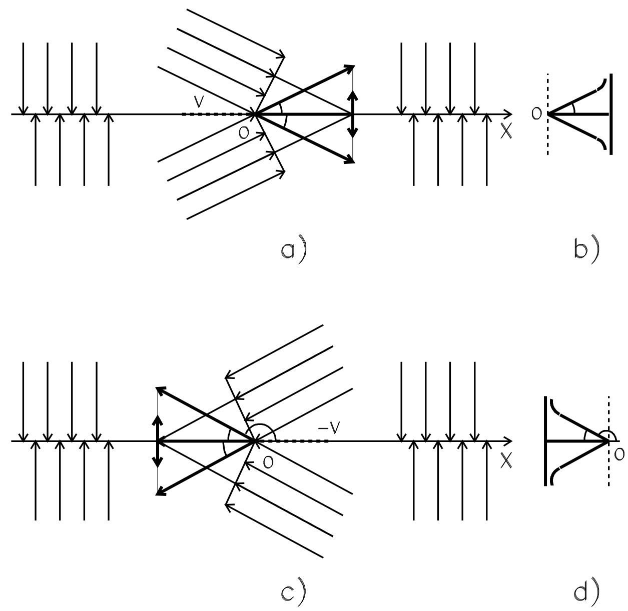

Figure 2: *a) Two lateral light beams (for $V > 0$) give two pairs of light rays ct and ct_s for both sides of the plane (top and bottom), synchronous with K_s-motion Vt. b) Parallel lines in both sides on Lobachevsky plane, corresponding to synchronous motions in a). The plots for $V < 0$ are shown in c) and d).*

3 x and t- coordinate transformation and light ether concept

Let us continue with the inertial frames K and K_s for $V > 0$. One can assume that a pair of direct beams (from top and bottom) reaches X-axis at the same moment of time as a pair of lateral beams (from left to right) reaches the point where both origins coincide. All x-points (including O) are "exited" simultaneously, and this moment of time is usually chosen as the initial one for K frame (the same for all coordinates). The initial moment of time for any x-point is delayed by t_F relative to the lateral beams (see (2.4)) so that time t_s at a given moment of time t (in K) is defined by (2.5). Thus, due to the synchronization of K and K_s frames (by the corresponding pairs of direct and lateral fronts) two moments of time, t and t_s, can be defined at any x point. For the chosen event (x, t) time t_s depends only on the velocity of the moving frame K_s.

Let us define the time t in the fixed frame via the distance ct passed by the light ray emitted from the point O at the parallel angle θ_L to X-axis in some plane. It is seen from Fig.1-Fig.3 that for any event (x, t) the delay time ct_F is just a projection of the given x-point on the chosen light ray ct.

Obviously, the displacement of K_s origin $Vt = ct\ cos\theta_L$ is just a projection of the light ray ct on the X-axis. So, for any given coordinate x at a given time t a value x_s relative

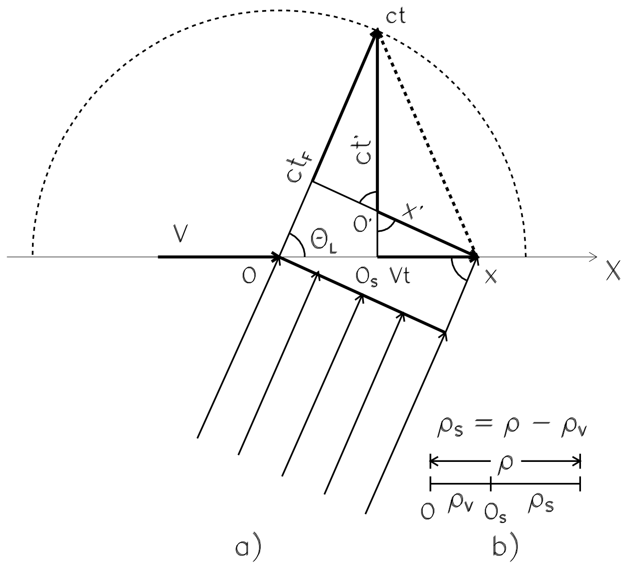

Figure 3: *a) An illustration of the inertial frame x and t coordinate transformation (including Lorentz transformation). b) A velocity space diagram corresponding to x and t shifts. The x-coordinate is the x-position of a particle, moving with a velocity of $v = x/t$ in K frame by the moment of time t.*

to the origin O_s is

$$x_s = x - Vt = x - ct\cos\theta_L\,. \tag{3.1}$$

For any event $(x = Vt,\ t)$ a relative coordinate is $x_s = 0$. It means that time t_s (see (2.5) and (2.6)) is the proper time of K_s, i.e. the time "measured" by means of a "moving clock", when one spectator observes the light sphere with the radius ct in K and in the same time t a moving spectator observes another light sphere with the radius ct_s (both spheres are triggered off by the lateral light beams). For the event (x, t) the corresponding moment of time t_s is the time "measured" by means of the "moving clock" located at the point x_s of K_s. Unlike of t in K, the time t_s defined for O_s is not all the same for the points on X_s-axis.

Indeed, from (2.4) one can see that the initial moment of time (provoked by the lateral light front) propagates along X-axis with the velocity v_F:

$$v_F \equiv \Delta x/\Delta t_F = x/t_F = c/\cos\theta_L = c^2/V = c/\beta > c. \tag{3.2}$$

So, for $0 < V < c$ any two events (x_1, t) and (x_2, t) have different time t_s in K_s. For $V \to 0$ ($\theta_L \to \pi/2$ for side beams) the velocity $v_F \to \infty$ and one comes to the Newton time $t_s \to t$, and for $V = c$ ($\theta_L = 0$) the proper time $t_s = 0$.

Thus, for any event (x, t) in K the corresponding coordinates in K_s are simple shifts (see (2.5) and (3.1)). To obtain the values of shifts, one should make symmetrical projections as described above.

We have used this symmetry to find out the Lorentz coordinates x' and t' for a moving frame. To get them, one has to find the crossing point O' of two perpendiculars producing the projections for any (x,t) event (see Fig.3). Then the length of the interval from O' to x corresponds to x':

$$x' = (x - ct\cos\theta_L)/\sin\theta_L = (x - Vt)/\sqrt{1 - V^2/c^2}\,, \qquad x_s = x'\sin\theta_L\,, \tag{3.3}$$

and the distance from O' to the ct corresponds to ct':

$$ct' = (ct - x\cos\theta_L)/\sin\theta_L = (ct - xV/c)/\sqrt{1 - V^2/c^2}\,, \qquad ct_s = ct'\sin\theta_L\,. \tag{3.4}$$

It is seen from (3.3) and (3.4) that primed and shifted coordinates are related as the corresponding projections. But the point O', which is always considered as the origin of the moving frame, does not coincide in space with O_s. It is also seen that the line $O'x'$ is not parallel to the X-axis. So, it seems obvious that the primed values can not be regarded as the coordinates in a moving frame.

The distance between the given points x and ct (dashed line in Fig.3) can be defined via the primed and unprimed values:

$$l^2 \equiv c^2t^2 + x^2 - 2ctx\cos\theta_L = c^2t'^2 + x'^2 + 2ct'x'\cos\theta_L \equiv l'^2\,, \tag{3.5}$$

or as a sum of two terms, either as $l^2 = s_1^2 + s_2^2$ (to get it one should add $\pm x^2$ to the left part of (3.5) and $\pm x'^2$ to its right part), or as $l^2 = -s_1^2 + s_3^2$ (add $\pm c^2t^2$ to the left part of (3.5) and $\pm c^2t'^2$ to the right part), where:

$$s_1^2 = c^2t^2 - x^2 = c^2t'^2 - x'^2 = \gamma^2(c^2t_s^2 - x_s^2)\,, \qquad \gamma = 1/sin\theta_L = 1/\sqrt{1 - V^2/c^2}\,, \tag{3.6}$$

$$s_2^2 = 2x(x - ct\cos\theta_L) = 2x'(x' \pm ct'\cos\theta_L)\,, \qquad s_3^2 = 2ct(ct - x\cos\theta_L) = 2ct'(ct' \pm x'\cos\theta_L)\,. \tag{3.7}$$

Term s_1^2 is known as an invariant interval. Obviously, it is only a part of the full distance l^2 and is a result of cancelling of two equal values, either s_2^2, or s_3^2 in the expressions for $l^2 = l'^2$. Terms s_2^2 and s_3^2 may differ by sign: (+)/(-) corresponds to the point O' located inside/outside the cone defined by the angle θ_L. For an event $(x = Vt, t)$ term s_2^2 is equal to zero (as $x_s = x' = 0$) and $s_3^2 = 2s_1^2$, so $l^2 \equiv s_1^2 \equiv l'^2$. The Lorentz coordinate transformations for this particular case have being usually presented in the manuals (e.g. [6]).

From (3.7) one can find (using the second formulae in (3.3, 3.4))

$$x = (x_s + ct_s\cos\theta_L)/\sin^2\theta_L = (x_s + Vt_s)/(1 - V^2/c^2)\,, \tag{3.8}$$

and

$$ct = (ct_s + x_s\cos\theta_L)/\sin^2\theta_L = (ct_s + Vx_s/c)/(1 - V^2/c^2)\,, \tag{3.9}$$

which are the reverse transformation from the moving frame to the rest frame. To check that, one can solve (2.5) and (3.1) for x and ct (once the factor $1/sin\theta_L$ is inserted into the brackets then the terms in brackets became the lengths of perpendiculars corresponding to the mentioned projection symmetry).

It is seen from (2.5),(3.1) and (3.8-3.9) that the direct and reverse transformations are different: the latter could not be obtained by changing V to $-V$. This means that one already knows that the frame either moves, or not. When changing V on $-V$ one should also choose an appropriate lateral light beam direction for a moving frame. So, if K_s moves backward to X ($V < 0$) one should change the sign in (2.5), (3.1) and in nominators of the reverse formulae (3.8-3.9). Thus, for any two frames one frame can be regarded as a moving frame and other one as the rest frame and vise versa by choosing the corresponding direct and lateral light beams (according to the known parallel angles).

A possible way to realize these opportunities is to make an assumption about the presence of many light streams of any directions. One may assume an ether, not a restful one, but the moving light ether. The absence of the absolute frame testifies upon the absence restful ether and does not contradict the presence of the moving light ether. Thus, the relation between space and time coordinates expresses through the parallel angle or through the corresponding velocities. So, this relation is generated by the presence of the corresponding light streams and particles.

4 y, z- coordinate transformation and invariants

Let us consider event $(x, y, z = 0, t)$ in K frame. The lateral light beam is reaching X-axis in XY-plane as shown in Fig.4, i.e. it spreads from bottom to top, first enters the plane point (x, y) and then the point $(x, y = 0)$ at the X-axis (if y-coordinate has an opposite sign, then one can choose another lateral beam heading from top to bottom).

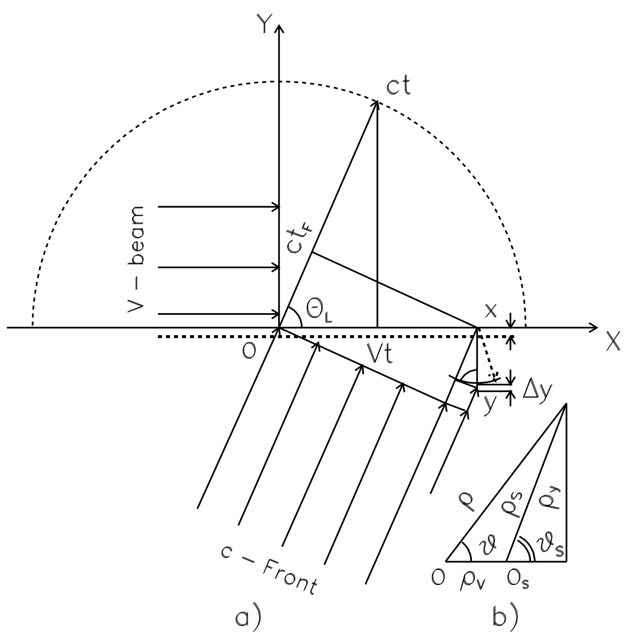

Figure 4: *a) An illustration of the Δy-shift origin due to the light way difference, and b) a corresponding velocity space diagram (see note in Fig.3b).*

The secondary light sphere spreads out from the first point to the point $(x, y = 0)$ at the X-axis in a time of y/c. The lateral beam ray reaches this point in a moment of time

$y \, sin\theta_L/c$ (since the secondary sphere starts to spread out from the first point). So, the light way difference is

$$c\Delta t \equiv \Delta y = y - y \sin\theta_L \,. \tag{4.1}$$

To compensate for this difference and make the initial moment of time counting caused by the lateral beam to be the same for x_s and y_s, the origin of K_s frame should be shifted along the Y-axis by the value of Δy (4.1). Then the y-coordinate in K_s frame is

$$y_s = y - \Delta y = y \sin\theta_L = y\sqrt{1 - V^2/c^2} \tag{4.2}$$

and the transverse coordinate

$$z_s = z - \Delta z = z \sin\theta_L = z\sqrt{1 - V^2/c^2}. \tag{4.3}$$

The reverse transformation is also obvious:

$$y = y_s/\sin\theta_L = y_s/\sqrt{1 - V^2/c^2}, \qquad z = z_s/\sin\theta_L = z_s/\sqrt{1 - V^2/c^2}. \tag{4.4}$$

Then for the non-invariant interval (see(3.6)) one can get

$$c^2t^2 - x^2 - y^2 - z^2 = \gamma^2(c^2t_s^2 - x_s^2 - y_s^2 - z_s^2). \tag{4.5}$$

So, for any event (x, y, z, t) in K there is the "parallel" event (x_s, y_s, z_s, t_s) corresponding to the moving K_s frame shifted in space and time in an appropriate way. These two sets of coordinates are related by the equation (4.5).

The obtained coordinate transformation leads to the contracted interval but this does not contradict to the relativistic velocity summation law. Since the energy-momentum transformation is a direct consequence of the velocity summation law, then the Lorentz energy-momentum transformation is valid in this approach [7]. Also in [7] relativistic effects considered in detail, and the four elements complex fraction invariant and a new wave equation in framework of this approach were proposed.

5 Conclusions

• A complete correspondence has been established between Lobachevsky parallel lines in the velocity space and the synchronous process of particle and light beams propagation in the Euclidean space.

• The constant light velocity principle and a time delay in the emission of two light rays has been found as the physical reason for absence of their intersection point in the Eucledean space and for the violation of the V-th postulate in the Lobachevsky velocity space.

• Lobachevsky function has been found as a tool to express the constant light velocity principle.

• A new method of time synchronization for different space points have been found and a new contents of the simultaneity conception, common time and proper time, have been formulated.

• A new inertial frame coordinate transformation, as the simple shifts, has been found. It leads to the known relativistic velocity summation law and requires the existence of the light (moving) "ether".

• It has been shown, that the initial moment of time counting for the moving frame propagates in space in the same direction with a finite velocity greater than the velocity of light.
• The relativistic effects have been shown to take place due to the coordinate and time shifts of the origin point. One can find the values of space or time intervals to be the same in the moving and the rest frames by changing the measurement way.
• It has been shown, that Lorentz energy-momentum transformation is a straightforward consequence of the relativistic velocity summation law.
• The four elements complex fraction invariant and a possible wave equation have been presented.

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Geometry of spacetime and Berwald spaces

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Abstract

A key question regarding the universe is the nature of the spacetime geometry. In general relativity this is assumed to be a pseudo-Riemannian manifold. This is usually justified by invoking the motion of idealised test particles and light rays which specify the projective and conformal structures of the underlying geometry respectively. This view was futher strengthened by showing that staring from a Weylian geometrical framework, the above assumptions together with the additional assumption regarding the norm of vectors under parallel displacement, reduce the geometry of the spacetime to be Riemmanian [1]. This was at times taken to suggest that these motions indeed fix the spacetime geometry of the spacetime to be Riemannian.

On the other hand differential geometers and mathematical physicists have long studied other candidates as the geometry of the spacetime, including a natural metrical generalisation of he Riemannian geometry, the Finsler geometry.

Here I briefly summarise some earlier results which demonstrate that the answer to the above question crucially depends on the general geometrical framework assumed. In particular, it turns out that starting from a Finslerian geometrical framework and demanding Riemannian conformal and projective structures, together with the additional assumption regarding the norm of vectors under parallel displacement, do not reduce the geometry of the spacetime to be Riemannian but rather Berwaldian geometry [2, 3, 4].

There are also generalisations of this result to Generalized Lagrange Spaces [5, 6].

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Raman scattering of light in photon traps

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The spectroscopy of Raman scattering of light now is a perspective direction of research of ultradisperse environments (powders, heterostructures, suspensions, biological environments, etc.). Feature of such mediums is the big relation of the area of a surface of a disperse phase to volume of substance. The transfer of light in the environment thus has features.

- Presence of the big number of submicronic particles increases intensity of scattering of light, and the scattering albedo increases.
- Dense packing of particles in the medium creates conditions for occurrence of a coherent component of scattering of light by particles of medium that creates local heterogeneity of radiation.
- Presence of the developed surface can influence spectral structure of the secondary radiation, one of which component is Raman scattering.

Feature of Raman scattering in homogeneous mediums is small value of signal in comparison with intensity of exciting radiation. In usually applied technique of registration of spectra of secondary radiation in the condensed medium exciting laser radiation focus near to a surface of medium. At enough high intensity of exciting radiation it leads to change of characteristics of substance: photodestruction, to the local warming up, the photoinduced phase transformations. In case of the disperse medium probably also sintering of particles of substance .

For ultradisperse medium it is possible to expect increase of signal strength Raman scattering owing to diffusive character of carry of radiation in the medium. Thus focusing of radiation is not necessary. The technique of work developed by authors was based on use of the pulse-periodic laser, cavity a dish of teflon and optical waveguides.

The scheme of experimental installation is presented on fig. 1. For excitation of a Raman scattering the laser on pairs of copper 1, generating radiation in visible area of a spectrum with lengths of waves 510,6 nanometers and 578,2 nanometers is used. Lasing was carried out in the form of short impulses by duration of 20 nanoseconds, the following with frequency of recurrence 16 kHz with average power more than 1 W. Generation was carried out in a monochromatic mode (□ = 510,6 nanometers) for what the yellow line has been suppressed by filters. Duration of an impulse - 15 nanoseconds. Pulse power - 15 kW. Exciting radiation of the laser 1 by means of an optical waveguides 2 went inside ditches with the sample 3. Secondary radiation was included into other optical waveguides 2', directing it to an entrance crack of monochromator 6 by means of a lens 4. After the photo multiplier the impulse of a current acts in the block of processing of a signal 11. The block of management 7 carries out discrete scanning on a spectrum with the set step of scanning and time of accumulation in each point. The computer 13 accumulates the digital information on a spectrum of secondary radiation and operates the step-by-step engine of monochromator, carrying out discrete turn diffraction lattices of this device. The target crack of monochromator has a photomultiplier 8 types. The Power unit 9 of photomultipliers provided the stabilized voltage up to U = 2,3 kV, necessary for amplification the electric impulses arising in the photomultiplier as a result of hit on the photocathode of photons, caused with secondary radiation in the researched sample. Sensitivity of device - 10^{-15} W.

Special ditches have been developed for carrying out of researches (fig. 2). The principles stated in the patent application [1] are put in a basis of a design. In the case dish 3 the cylindrical cavity 7, being by working volume and filled by researched substance is cut out. From above and from below to the case dish fasten washers 2 and 5 into which 1 optical waveguide having diameter of a fiber 50 - 100 microns are inserted entrance 6 and the day off. The case and washers are executed from Teflon. Between the case and the bottom washer the lining 4 of teflon a film by thickness 10 - 20 microns is installed. This lining well passing exciting radiation, separates an end face of an entrance optical path from researched substance and pressurize to a ditch at removal of an entrance optical waveguide.

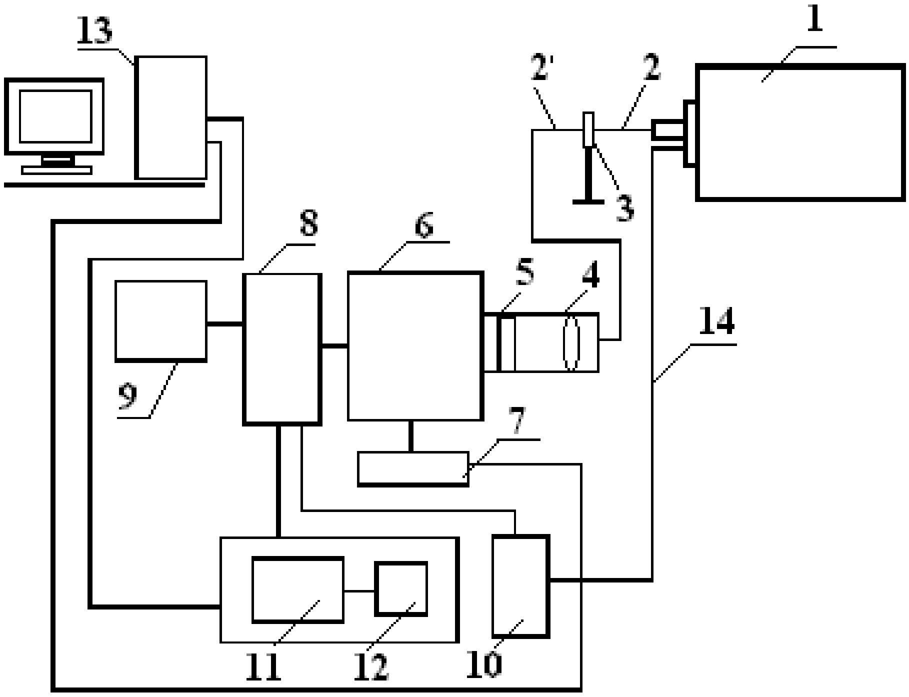


Fig. 1. The block diagram of installation for research of volumetric secondary radiation in the condensed environments at импульсно-periodic laser excitation (the scheme " on a gleam "): 1 - the laser; 2, 2' - opticals waveguide; 3 - a dish with the analyzed sample; 4 - a lens; 5 - the filter; 6 - a monochromator; 7 - the block of management of a monochromator; 8 - the photomultiplier; 9 - a power unit of the photomultiplier; 10 - a strobe-shaper; 11 - the block of processing of a signal; 12 - a line of a delay; 13 - a computer; 14 - an optical fibre

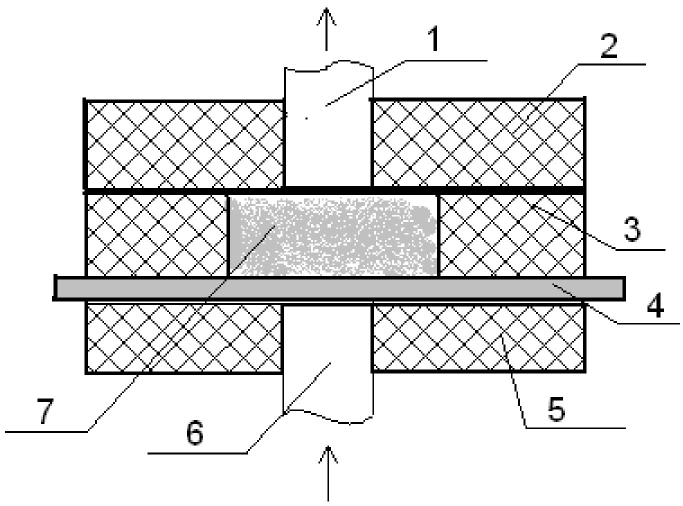


Fig. 2. The scheme dish: 1, 6 - entrance and target opticals waveguides; 2, 5 - washers; 3 - the case dish; 4 - a film lining; 7 - working volume of dish

In experimental installation as the case at once a several ditch was used fluoroplastic a sheet in which cylindrical apertures have been drilled. It has allowed to prepare for research at once a multiples of substances, and replacement a dish during measurements was made only by rearrangement of optical waveguide that has essentially simplified researches.

The design ditches enables to research samples as in the ultradisperse form, so in liquid and firm forms.

In the work have been researched a line of organic and inorganic substances in the ultradisperse form.

Results of measurements are presented on fig. 3 - 7.

Application developed a ditch has allowed to receive a signal of secondary radiation, comparable on intensity with exciting radiation, especially for organic substances.

From spectra of a Raman scattering of salts potassium on fig. 3, 4 it is possible to determine spectral shift of a line of a Raman scattering: 740 cm^{-1} and 846 cm^{-1} for potassium iodate and potassium bichromate accordingly.

The common component for the researched organic substances: stilbene, PPO, and POPOP - the six-nuclear cyclic group is. Stilbene and PPO incorporate on two groups, POPOP - three groups. Cyclic groups in PPO and POPOP are connected accordingly through one and two C_3NO groups, and in stilbene - through two CH groups connected by dangling bond.

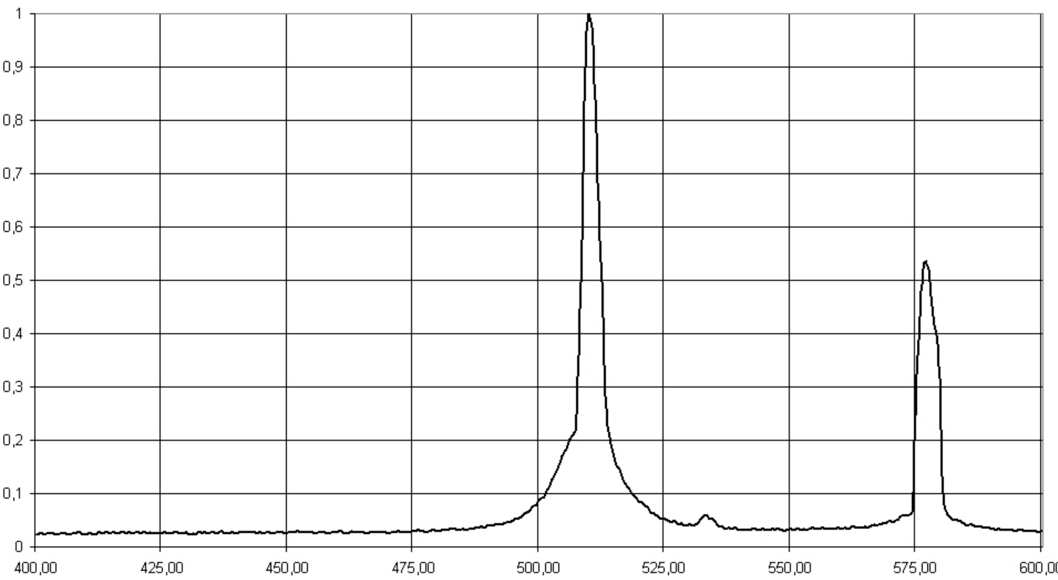


Fig. 3. A spectrum of secondary radiation K_2CrO_4

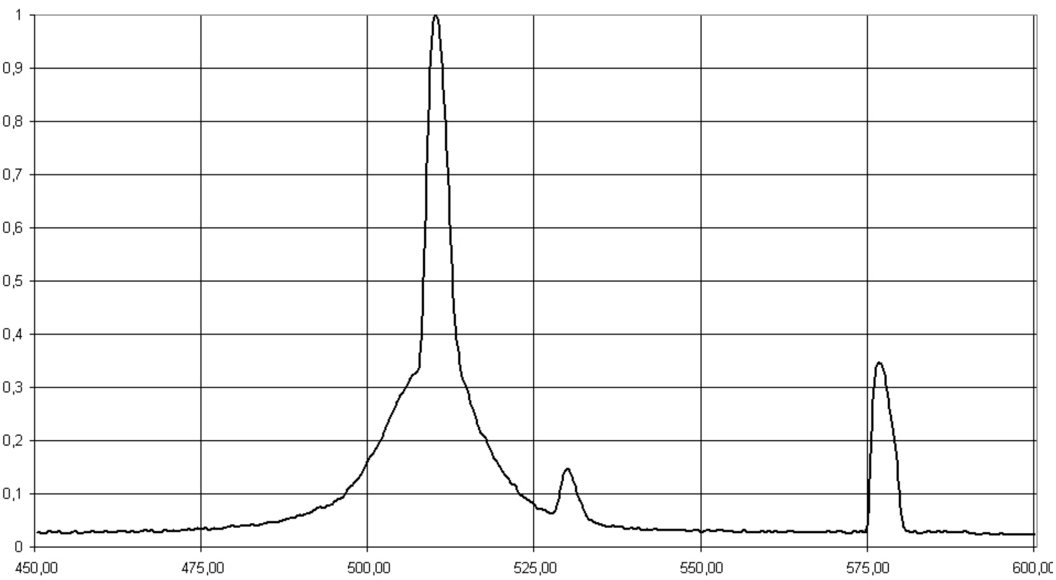


Fig. 4. A spectrum of secondary radiation KIO_3

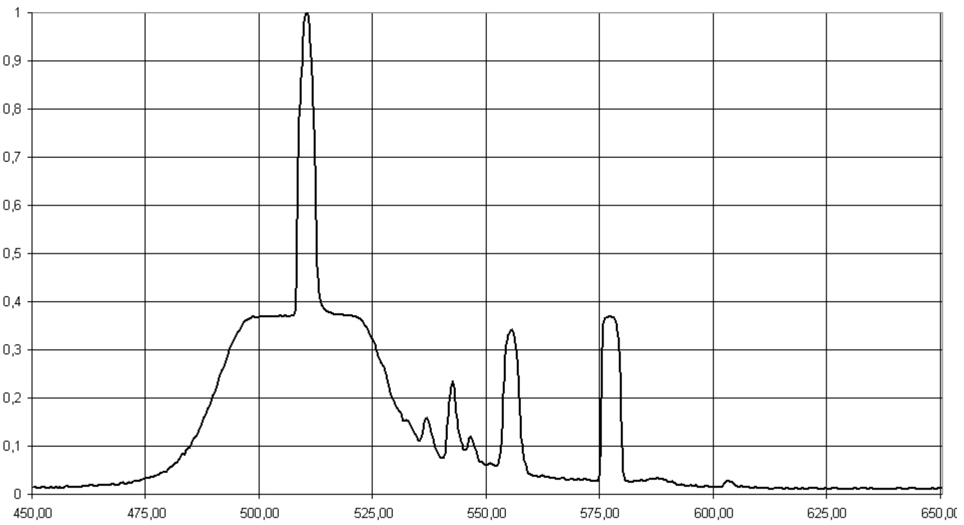


Fig. 5. A spectrum of secondary radiation of a stilbene (1,2-diphenylethylene $C_6H_5CH = CHC_6H_5$)

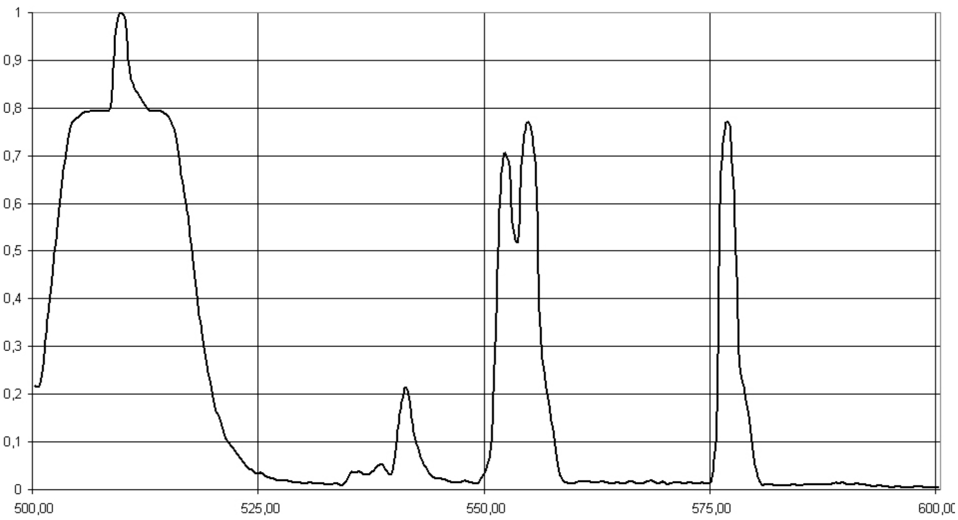


Fig. 6. A spectrum of secondary radiation POPOP

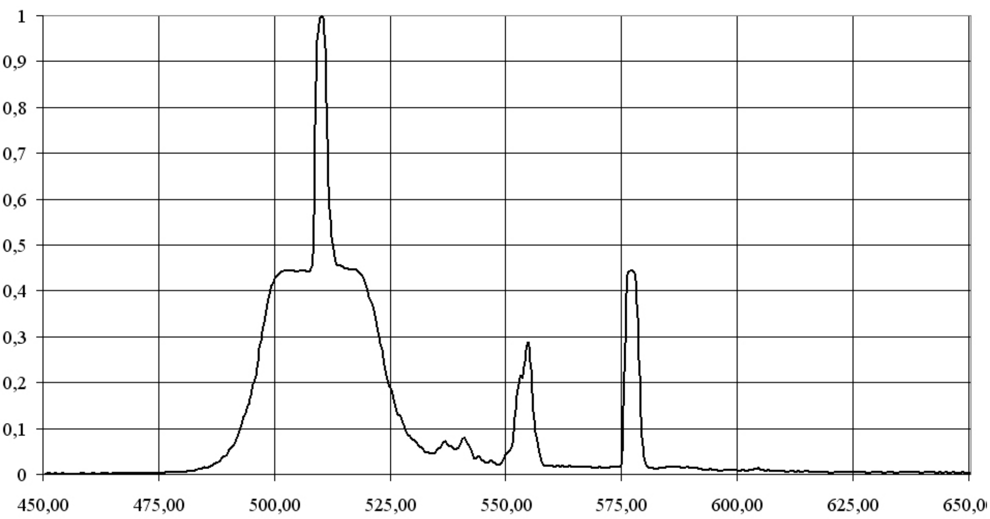


Fig. 7. A spectrum of secondary radiation PPO

On fig. 8 the site of a spectrum of a Raman scattering in a range 900 - 1650 cm^{-1} for all three substances. Position of the "main" maximum shifted from an exciting line on the greatest distance is well visible, that, in process of increase in number of cyclic groups moves in area of smaller frequencies, and its form out of one-topmost becomes two-topmost.

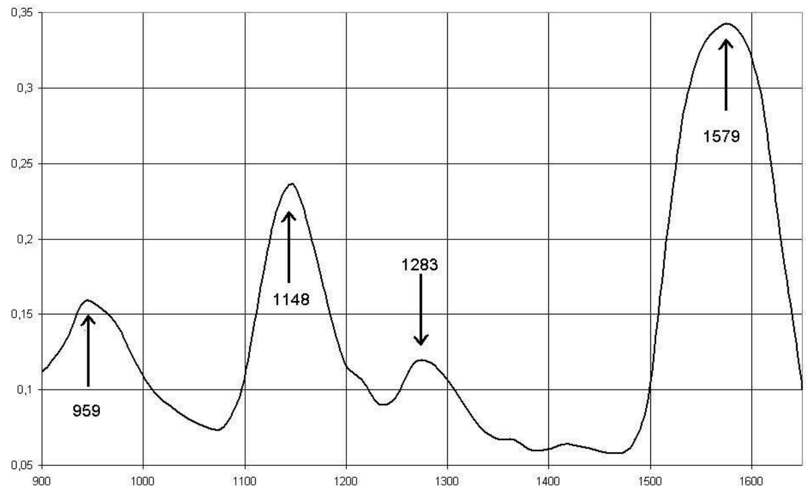
Poorly expressed at a stilbene and well appreciable at PPO the line of 1416 cm^{-1} is not visible in spectrum POPOP.

Shift in area of low frequencies and other lines is observed: 1148 cm^{-1} at a stilbene, 1097 cm^{-1} at PPO, 1011 cm^{-1} at POPOP.

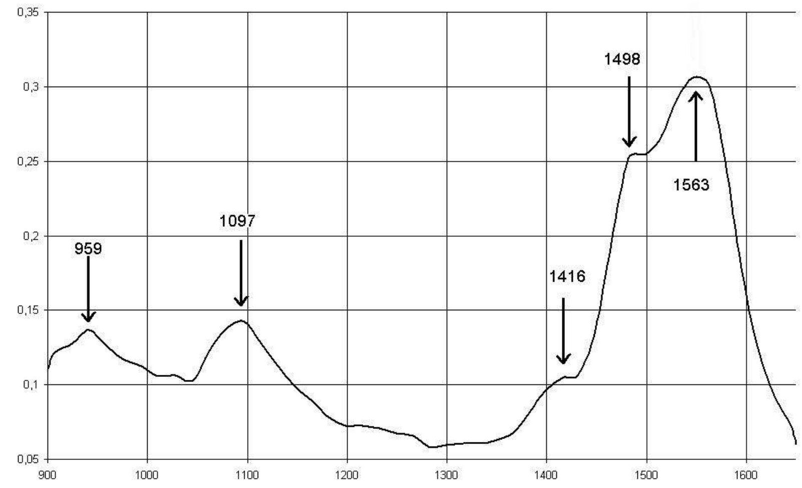
The line of 959 cm^{-1} well appreciable at a stilbene and PPO, disappears in spectrum POPOP (from fig. 8c it is visible, that there a minimum), but at POPOP there is a line of 924 cm^{-1}.

In experiment it has been noticed, that the degree of change of the form of an exciting line depends on a design dish and from quantity of substance in a dish. Whether is this phenomenon lack of a design or display of features of the ultradisperse form of substance it is necessary to specify in the further researches.

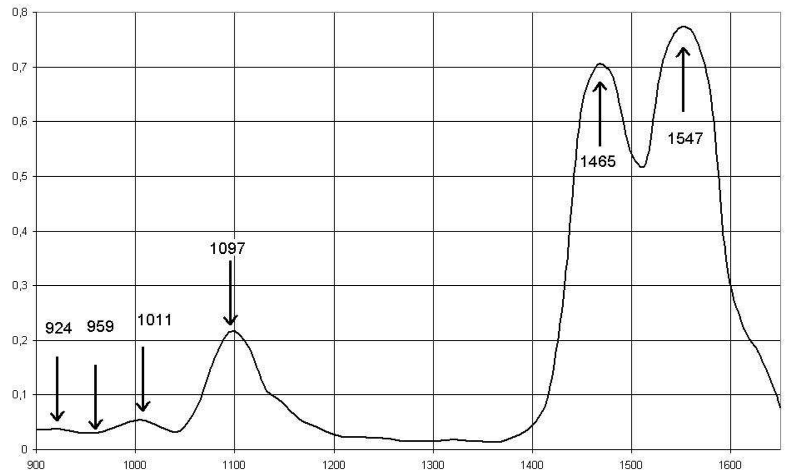
Carried out researches have shown, use special resonator a dish allows to receive a high level of intensity of secondary radiation in the substances which are being the ultradisperse form. It allows to increase volume and reliability of the information on researched substances.



a)



b)



c)

Fig. 8. A site of a spectrum 900 - 1650 cm^{-1} for a stilbene (a), PPO (b) and POPOP (c). Numbers near arrows - spectral shift by the noted arrow of a line.

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1. Устройство для возбуждения вторичного излучения в молекулярных соединениях. – Заявка № 2005108927 от 30.03.2005 на пат. РФ.

Positive and negative values of an index of refraction and effective mass of a quantum in a globular photon crystal

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Introduction

The opportunity of existence of negative masses is object of a discussion in the relativistic theory. In the article the requirements of origin of negative mass of quantum in globular photon crystal is investigated. The globular photon crystal represent new unique objects interesting as to basic researches, and for the practical appendices [1,2]. In the given article as objects of investigations the simulated opals representing one of types of globular photon crystal were utilized. The simulated opals are built from identical spherical particles (globule) silica packed by the way of a cubic lattice. The size globule can receive values in a range 200 - 400 nm. In the given article the optical behavior of globular photon crystal is analyzed on the basis of usage of analytical expressions for the law of a dispersion of photon bands.

1. Laws of a dispersion of photon bands in globular crystal.

As displays the theoretical analysis, reference property of globular photon crystal is the availability in them of several dispersion branches щ$_j$(k) (j=1,2, …) in a spectrum of electromagnetic waves. The precise calculation of the laws of a dispersion щ$_j$(k) of photon bands in a globular crystal is very difficult. For approximation of a view of dispersion curves it is possible to utilize that fact, that far from critical points of a Brillouin zone the group velocity dщ/dk of electromagnetic waves should be comparable to speed of light in vacuum, and near to critical points the derivative dщ/dk should aim at null. In this connection we selected following approximation for the first three photon branches:

$$\omega_1(k) = \frac{2c}{a}\sin\frac{ka}{2} \tag{1}$$

$$\omega_2(k) = \sqrt{\omega_0^2 - 4\frac{c^2}{a^2}\sin^2\frac{ka}{2}} \tag{2}$$

$$\omega_3(k) = \sqrt{\omega_0^2 + 4\frac{c^2}{a^2}\sin^2\frac{ka}{2}} \tag{3}$$

Here with *c* - constant, close to speed of light in vacuum, and *a* - diameter globule (lattice constant), *щ$_0$* – value of a circular frequency at center of a Brillouin zone for the second and third branches.

In a fig. 1 the obtained theoretical dependence of frequency on a wave vector for following values of parameters is showed: *щ$_0$* = 5,39*10^{15} 1/c, and *a* = 1,68x10^{-7} m. The lowermost diagram starting from null, corresponds to the first branch of oscillations. The diagrams of the second and third branch start with value *щ=щ$_0$*. To the second branch there corresponds a curve, directional downwards; the upper curve corresponds to the third branch.

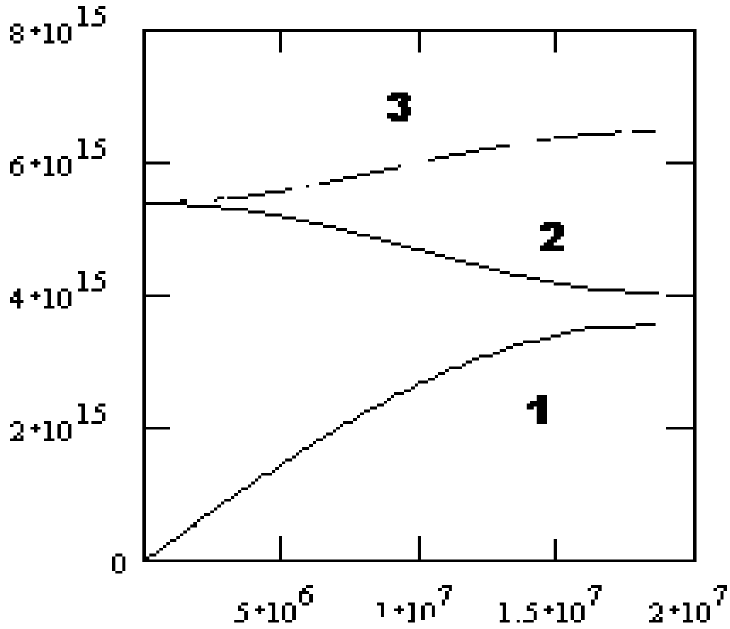

Fig. 1. A theoretical view of the law of a dispersion of a globular photon crystal for three lower branches (1-3); on an axis of ordinates the circular frequency щ in rad /s with is postponed.; on an abscissa axis - wave vector k in м$^{-1}$.

2. Dispersion of a reflectivity.

The reflectivity R (щ) from a surface of a photon crystal was calculated under the known formula:

$$R = \left| \frac{k - k_0}{k + k_0} \right|^2 . \tag{4}$$

Here k_0= щ/c - wave vector in vacuum, and k - a wave vector in medium calculated from relations (1-3).

Accordingly, the transmittance T (щ) was from a relation:

$$T = 1 - \left| \frac{k - k_0}{k + k_0} \right|^2 . \tag{5}$$

We explored angular dependencies of spectrums of reflection for two types of simulated opals experimentally.

The Fig. 2 and 3 is illustrated by the obtained spectral dependencies of intensity of reflection at normal incidence and at incidence 45 degrees on a surface of an opal. As it is visible from these figures, there is an essential distinction of standings of a maximum of intensity in spectrums of reflection at normal incidence of a beam on a surface and at incidence 45 degrees and subsequent mirroring. On the basis of the formulas for a reflectivity the relevant dependencies of a reflectivity under the formula (4) by matching of reference parameters were calculated so that to supply conformity to theoretical and experimental dependence. In a fig. 4 the obtained theoretical dependencies for following parameters are given: щ$_0$ = 5,39x10^{15} 1/sec, and a= 1,68x10^{-7} m. As it is visible from comparing experimental and theoretical dependencies, the qualitative consent of the theory with experiment is watched; difference in the shape of apparent curves at the same time takes place.

3. Dispersion of a group velocity and effective mass of a quantum in a globular photon crystal

The group velocity depends on a wave vector as follows:

$$v = \frac{d\omega}{dk}, \tag{6}$$

Where щ(k) - dispersion dependence of one of branches. For the first branch, allowing (1) and (6), we gain:

$$v = c\sin(ka/2). \tag{7}$$

For the second branch, allowing (2) and (6), we discover:

$$v = -\frac{2c^2\sin(ka/2)\cos(ka/2)}{\sqrt{\omega_0^2 a^2 - 4c^2 + 4c^2(\cos(ka/2))^2}} \tag{8}$$

For the third branch, allowing (3) and (6), we gain:

$$v = \frac{2c^2\sin(ka/2)\cos(ka/2)}{\sqrt{\omega_0^2 a^2 + 4c^2 - 4c^2(\cos(ka/2))^2}} \tag{9}$$

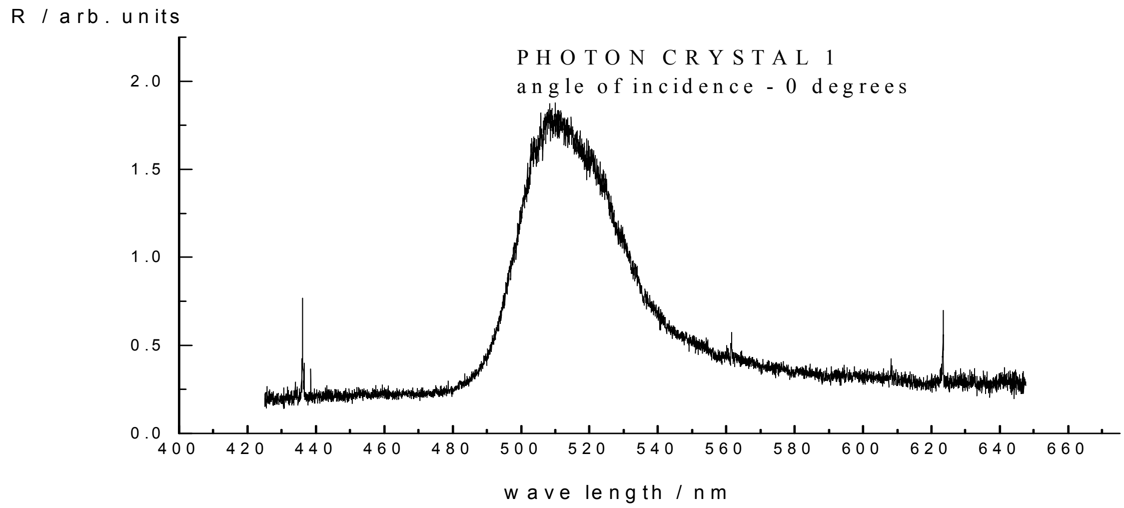

Fig. 2. Specter of reflection of an opal at normal incidence.

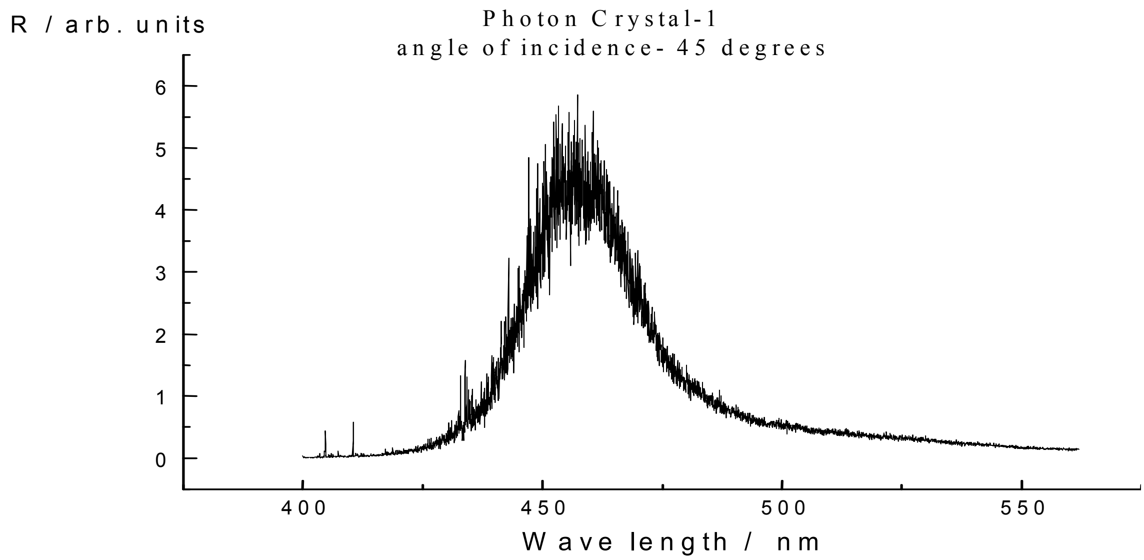

Fig. 3. Specter of reflection at incidence and reflection of a beam bevel way 45 degrees.

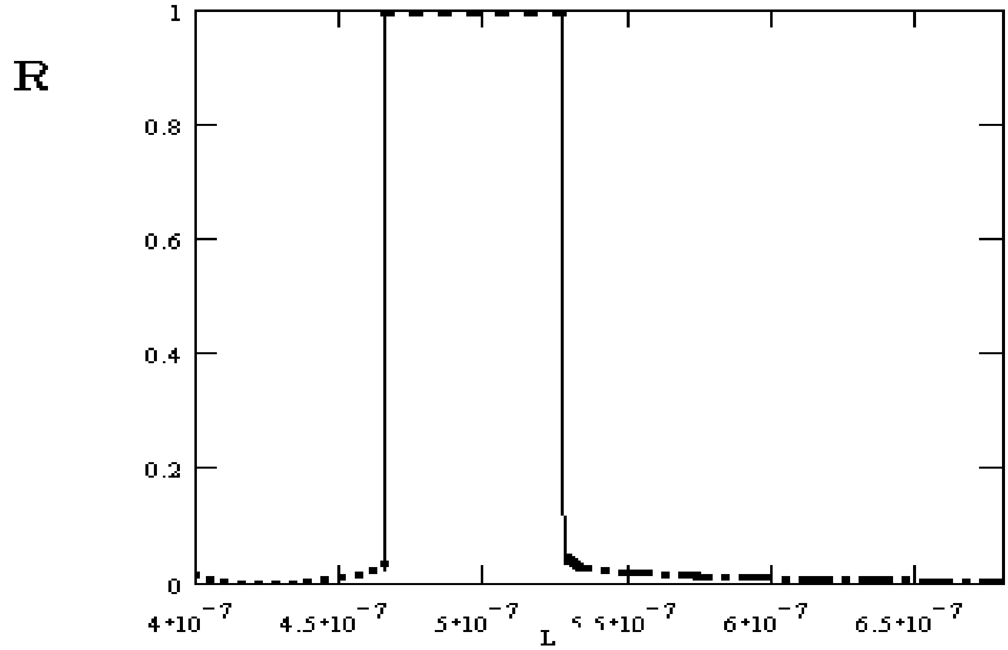

Fig. 4. Theoretical dependence of a reflectivity R from a wave length L (м).

In a fig. 5 the calculated dependence of a group velocity on a wave vector is showed at following parameters $\omega_0 = 5{,}39\text{x}10^{15}$ 1/sec, and $a = 1{,}68\text{x}10^{-7}$ m. The upper diagram (full curve) corresponds to the first branch. From this figure it is visible, that the sign of a group velocity $v = \frac{d\omega}{dk}$ everywhere is plus. The dashed line corresponds to the third branch; for it the velocity also is plus. The dot line corresponds to the second branch; to it there corresponds a negative sign of a group velocity.

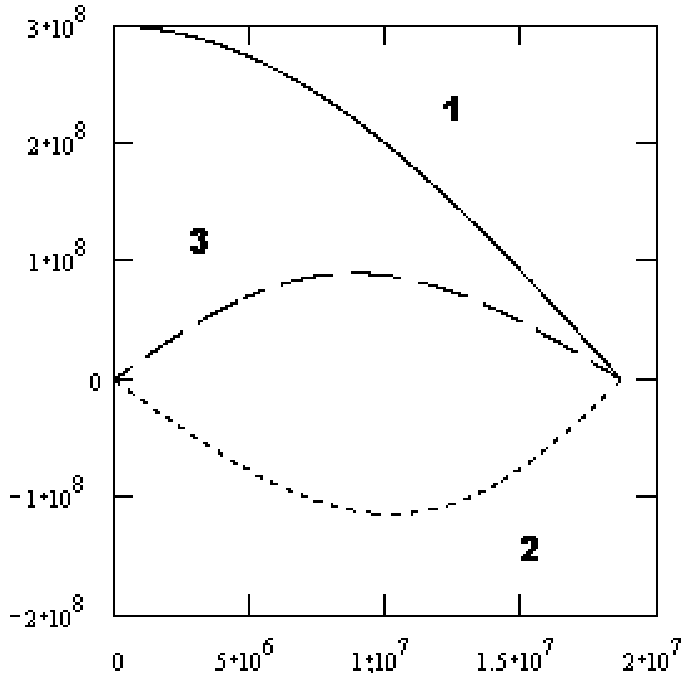

Fig. 5. Dependence of a group velocity v (m/sec) from a wave vector k (м$^{-1}$).

The calculation of dependence of effective mass from a wave vector was carried out on the basis of the known formula [4]:

$$m = \frac{h}{2\pi \frac{d^2\omega}{dk^2}}, \tag{10}$$

Where h - Plank constant. For the first branch, allowing relations (1) and (10) is gained the following formula:

$$m = -\frac{h}{\pi c a \sin(ka/2)}. \tag{11}$$

The relevant dependence is showed in a fig. 6a in terms of h.

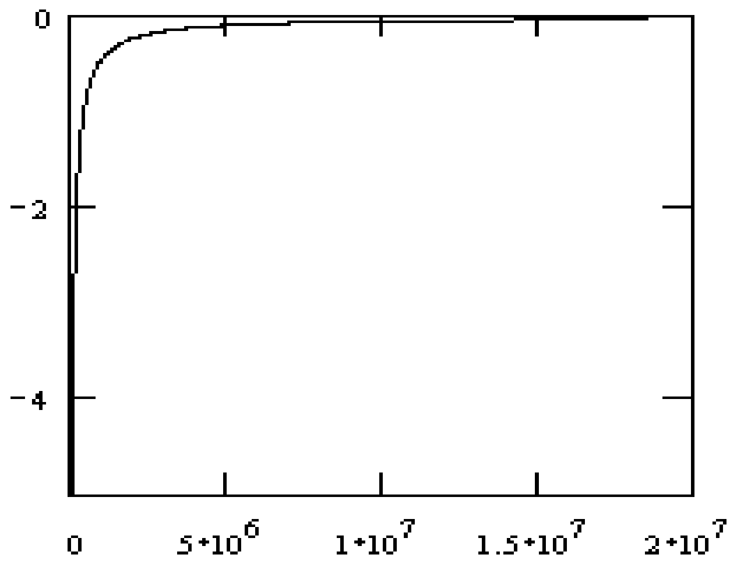

Fig. 6a. Dependence of effective mass m/ h (kg / joule sec) from a wave vector k ($м^{-1}$) for the first dispersion branch (see fig. 2).

Accordingly, for the second branch is gained:

$$m = -\frac{ha\sqrt{\omega_0^2 a^2 - 4c^2(\sin(ka/2))^2}}{2\pi(4c^2\cos^2(ka/2)\sin^2(ka/2) - c^2a^2\cos^2(ka/2) + c^2a^2\sin^2(ka/2)}. \tag{12}$$

The obtained dependence is illustrated a fig. 6b.

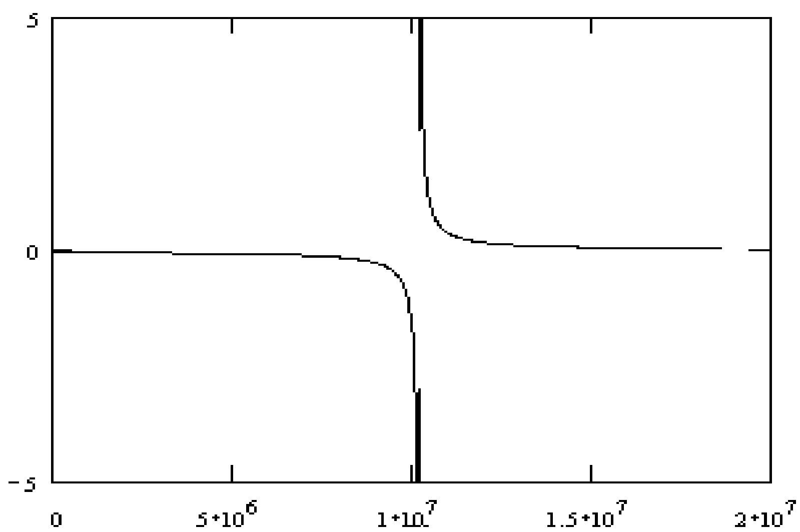

Fig. 6b. Dependence of effective mass m/ h (kg / joule sec) from a wave vector k ($м^{-1}$) for the second dispersion branch (see fig. 2).

For the third branch takes place:

$$m=-\frac{ha\sqrt{\omega_0^2a^2+4c^2(\sin(ka/2))^2}}{2\pi(4c^2\cos^2(ka/2)\sin^2(ka/2)+c^2a^2\cos^2(ka/2)-c^2a^2\sin^2(ka/2)}. \qquad (13)$$

The obtained dependence is showed in a fig. 6c.

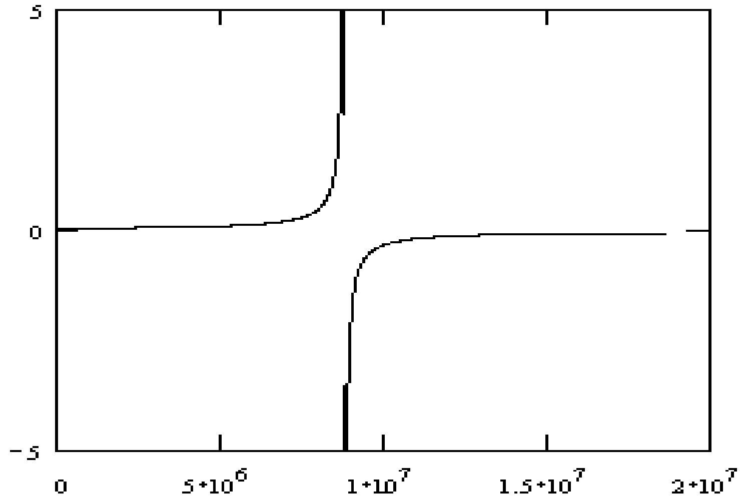

Fig. 6c. Dependence of effective mass m/ h (kg / joule sec) from a wave vector k (м$^{-1}$) for the third dispersion branch (see fig. 2).

4. Dispersion dependence of an effective index of refraction of a crystal

For a flat simple harmonic wave the direction of a phase velocity $\mathbf{v_{ph}}$= (щ/k)**k**/k coincides a direction of a wave vector **k**. When the group velocity is negative, the directions of vector of a group velocity **v** and wave vector **k** are antiparallel. On the other hand, the directions of vectors **v** and with (with **c** - velocity vector of a wave in vacuum) for normal incidence are identical. It means, that group and phase velocity thus are antiparallel. For an index of refraction in case of normal incidence takes place:

$\mathbf{v_{ph}}$= (щ/k)**k**/k=**c**/n. (14)

Thus, if the sign of a group velocity is negative, the effective index of refraction too becomes negative.

In a fig. 7a is showed calculated according to (14) dependencies of an index of refraction on frequency for the first branch. It is visible, that in this case at small щ and *k* the index of refraction is plus and is close to unity.

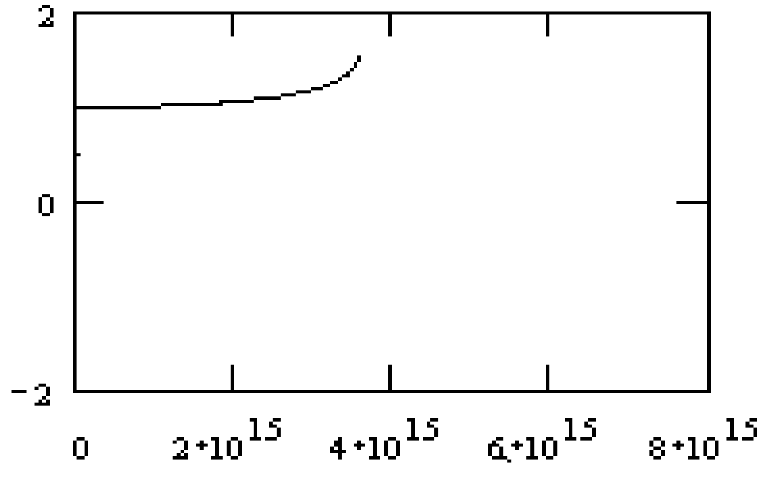

Fig. 7a. Dependence of an index of refraction on frequency for the first branch; an abscissa axis is the circular frequency щ *(rad / sec).*

In a fig. 7b the dependence of an effective index of refraction on frequency for the second branch is showed. It is visible, that the index of refraction for all branch is negative.

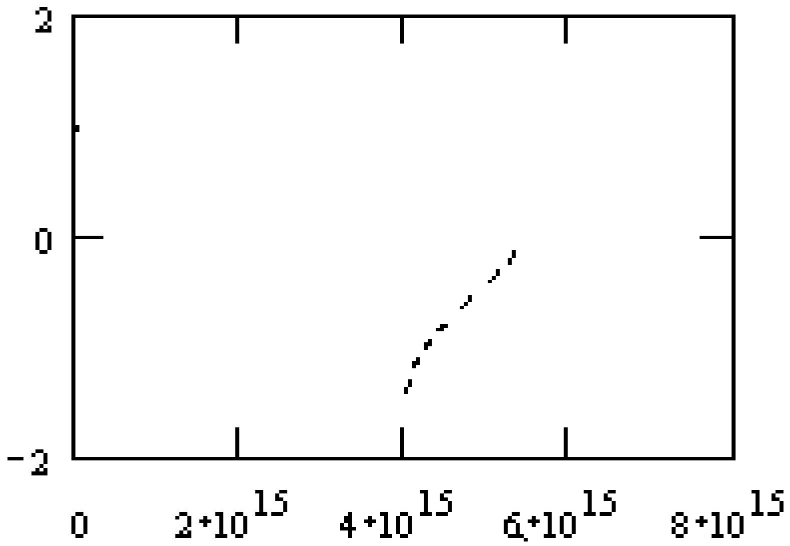

Fig. 7b. Dependence of an effective index of refraction on frequency for the first branch; an abscissa axis is the circular frequency щ*(rad / sec).*

In a fig. 7c the dependence of an index of refraction on frequency for the third branch is showed. From this figure it is visible, that the index of refraction for all this branch is to positive and smaller unity.

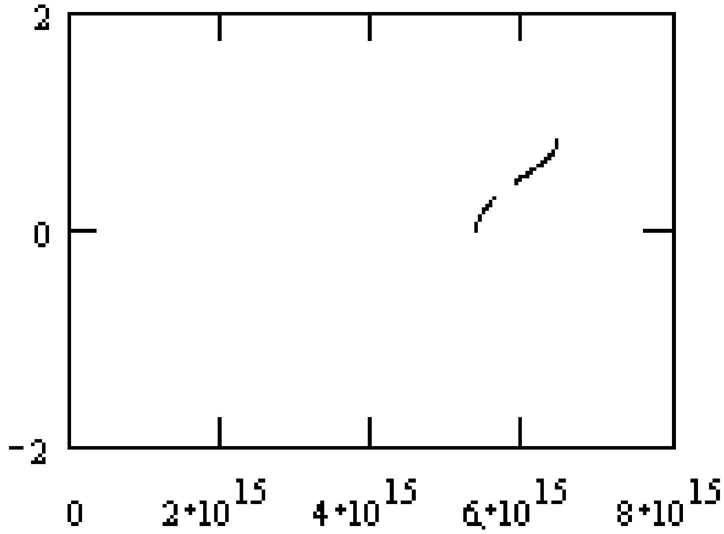

Fig. 7c. Dependence of an index of refraction on frequency for the third branch; an abscissa axis is the circular frequency щ *(rad / sec).*

In a fig. 7d the dependence of an index of refraction on frequency for all three branches is showed. It is visible, that the graph of an index of refraction from frequency on the second branch transfers without a disrupter in the diagram for the third branch. Between the graphs for the first and second branch the discontinuity is seen.

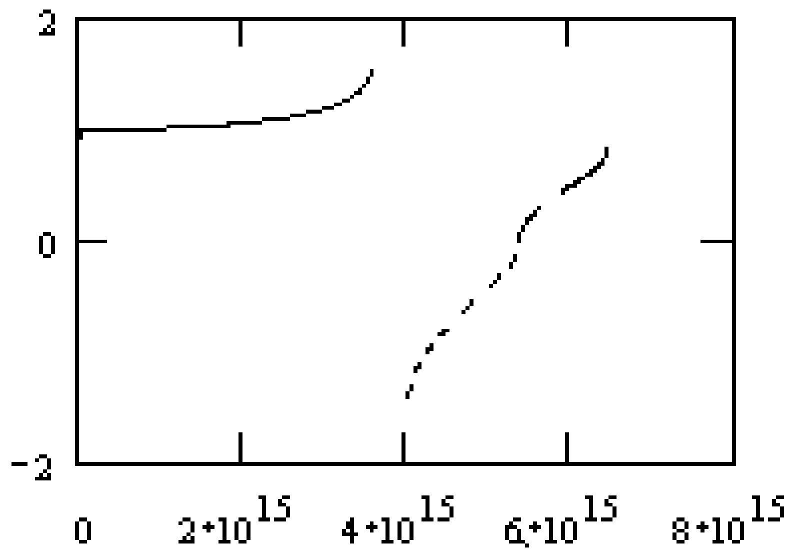

Fig. 7d. Dependence of an index of refraction on frequency for all three branches; an abscissa axis is the circular frequency щ*(rad / sec).*

5. Conclusion.

The opportunity of existence of negative masses is object of a discussion in the relativistic theory. In the article the requirements of origin of negative mass of quantum in globular photon crystal is investigated.

Thus, in the present article on the basis of a prime model of the laws of a dispersion of photon bands of a photon crystal such as an opal the dispersion dependencies of a reflectivity, group velocity, effective mass and index of refraction are calculated. The theoretical dependence of a reflectivity is as a whole close to experimental, though has other shape. For the first and third branches the group velocity is positive, and for second - negative; and effective mass of quantum for the first branch - is negative. For the second and third branches effective mass of quantum can be both positive, and negative and has a discontinuity at variation of a sign. The effective index of refraction for the first branch is to positive and major unity. For the second branch the effective index of refraction is negative. For the third branch he is to positive and smaller unity.

The article opens opportunities for installation of a microstructure of globular photon crystal on the basis of a view of their spectrums of reflection and transmission in visible range of a spectrum.

As a whole it is possible to draw a conclusion that usage of dispersion curves by the way (1-3) allows satisfactorily to describe dispersion dependencies of an index of refraction on frequency and spectral dependence of a reflectivity on a surface of a photon crystal.

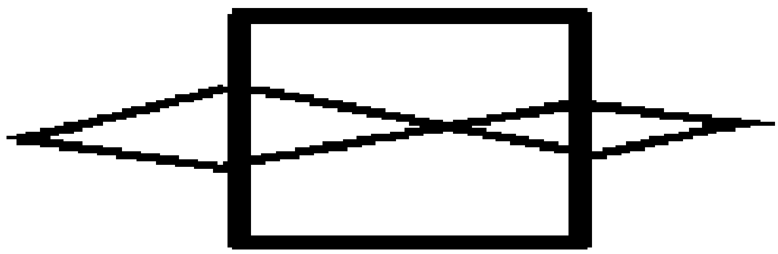

Fig. 8. A diagrammatical view « lenses Veselago» on the basis of a photon crystal.

In area, where index of refraction is negative, on the basis of photon crystal is possible making known « lens Veselago » in visible ranges of a spectrum (see fig. 8). In areas, where the index of refraction modulus is less than unity (for the second and third branches), the effect of complete exterior reflection from a surface of a photon crystal (see fig. 9) is possible.

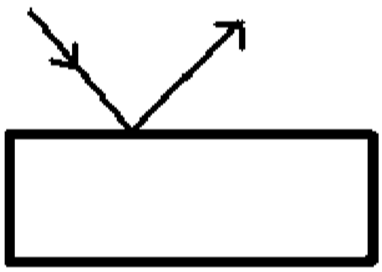

Fig. 9. Effect of complete exterior reflection from a surface of a photon crystal.

Fig. 10. An optical waveguide on the basis of a photon crystal.

On the basis of such effect the optical waveguide such as tubes can be generated, on an interior surface which one marks a material by the way globular photon crystal (for example, opal). Thus light owes completely is reflected from an interior surface of a tube, as the relevant index of refraction is less than unity (see fig. 10).

For area of a spectrum, where dn/dω (aims at infinity, i.e. near to a discontinuity, there is an opportunity to produce so-called «superprism» described anomalously by a high dispersion.

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On the sizes of the moving rigid bodies determined from conditions of equilibrium of the ions of the crystalline lattice.

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In this article, we consider changes of the sizes of rigid bodies uniformly moving along some direction. The macroscopic change of the sizes of the body is caused by displacement of the points of equilibrium of the ions of the crystalline lattice due to change of the convection potential. Some applications of the results, we found, are considered in an interpretation of the Michelson-Morley experiment. The effect of the contraction due to the convection potentential and relativistic Doppler shift can be used, in principle, to detect a motion of some space apparatus with respect to the cosmic background radiation frame.

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I. INTRODUCTION

When FitzGerald and then Lorentz suggested a hypothesis on contraction of the moving bodies to explain null results of the Michelson-Morley (MM) experiment, they could not give any argument in favor of their suggestion. Later in attempts to substantiate this hypothesis, Lorentz argued [1], that "if it is assumed that the molecular forces holding Michelsons interferometer together are affected by the Earths motion through the ether in the same way as Coulomb-forces are affected, the interferometer will experience a contraction of the kind needed to explain the negative result of the Michelson-Morley experiment". On level of scientific knowledge of the end of XX century, it was impossible to explain an origin of these forces but now we know the laws which the atoms forming the solid state obey. So it would be worthwhile to verify Lorentz's hypothesis having known what factors are responsible for forming the structure of the material of which the arms of the Michelson interferometer are made.

Our approach to this problem will be linked neither with the special relativity nor Lorentz's absolute ether theory because it is based only on the Maxwell equations and some results of the solid state physics. In frame of this approach, we will show that while considering the relativistic contraction of the moving bodies, for example in the MM experiment, one factor is omitted. This factor is the influence of the convection potential created by the ions in the lattice of uniformly moving crystal [11] to the surrounding ions which comes to re-distribution of the ions of the lattice to other points of electrostatic equilibrium. So total contraction of the moving body must be stronger than it is predicted by both the special relativity [2] and Lorentz's concept of the ether [3, 4].

This paper is arranged as follows. In Sec. II, we consider how the conditions of electrostatic equilibrium of the ions in the lattice of some crystal body change if this body begins to move uniformly. Also we calculate the distances between new points of equilibrium where the ions are located and how it results to changing the sizes of the whole body. In Sec. III we consider an application of the results of Sec. II to the interpretation of the measurement data of the MM experiment. In Sec. IV we consider the numerical calculation of the potential of an ionic crystal lattice. Sec. V contains some conclusions and description of

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a possibility to detect a motion of some space apparatus with respect to the cosmic background radiation frame.

II. CONTRACTION OF THE MOVING BODIES DUE TO THE CONVECTION POTENTIAL

Let us consider a motion of some rigid body made of ionic crystal (for example, it can be a NaCl crystal). Our choice of the material of the body is caused by the intention to reduce analysis of behavior of the solid state to analysis of the electrostatic forces providing equilibrium of the crystalline lattice. Actually, one can calculate the change of the points of equilibrium of ions of the lattice of some perfect metal. In this case, one should consider a distribution of the electrons of conductivity in the lattice. The dominant factor determining the points of location of the ions in the sites of the lattice is the electrostatic repulsion force between the ions. This force is shielded by the spatial negative charge due to electrons of conductivity. So even in this case, the task can be reduced to an electrostatic one. It can be established from consideration of the Hamiltonian of the crystal (Ch. 1.3 of [5], Eq. (1.3.1))

$$H = \frac{1}{2}\sum_l \frac{\boldsymbol{p}_l^2}{m} + U(\boldsymbol{R}_1, \boldsymbol{R}_2, ...\boldsymbol{R}_l, ...) \ , \tag{1}$$

where the potential energy of the crystal is a function of distances $\boldsymbol{R}_1, \boldsymbol{R}_2, ...\boldsymbol{R}_l, ...$ between the atoms of the lattic only ((Eq. 1.3.3) of [5]). So in the equilibrium configuration, when the atoms are located exactly in the sites of the lattice, we have

$$\frac{\partial U}{\partial \boldsymbol{r}_l} = 0 \quad ; \quad \boldsymbol{r}_1 = \boldsymbol{r}_2 = ... = \boldsymbol{r}_N = 0 \tag{2}$$

for all $\boldsymbol{r}_l = \boldsymbol{R}_l - l\boldsymbol{a}$, $\boldsymbol{a}$ is the vector of elementary cell of the lattice. In the solid state problems, while studying the dynamical properties of the crystal, the potential energy is represented as

$$U(\boldsymbol{r}_1, \boldsymbol{r}_2, ...\boldsymbol{r}_l, ...) = U_0 + \sum_{l,l'} \boldsymbol{r}_l \boldsymbol{r}_{l'} \frac{\partial^2 U}{\partial \boldsymbol{r}_l \partial \boldsymbol{r}_{l'}} \ . \tag{3}$$

Additive constant in the energy U_0 is not essential in studying the dynamical properties of the crystal so it is omitted in further consideration of the solid state problems. But we will be interested just in this additive constant, i.e. how it changes if the forces acting on the atoms change too. Obviously, U_0 obeys Eq. (2) and we shall analyze this equation below.

When each ion moves concordantly with the whole lattice we are able to consider the fields of one elementary cell. For NaCl, the lattice is of the cubic type and it is sufficient to consider how the points of equilibrium change in longitudinal and transversal direction to the motion of the body (we suggest that one axis of symmetry of the lattice is oriented in direction of motion so two other axes are directed transversally). For uniformly moving charges, the EM fields created by these charges are stationary, which means that they become to be static in the co-moving frame. We find the magnitude of the interaction force between the ions [6],

$$\boldsymbol{F} = e\left(\boldsymbol{E} + [\boldsymbol{v} \times \boldsymbol{B}]\right) \ , \tag{4}$$

where $\boldsymbol{v}$ is the velocity of uniform motion of the lattice, $\boldsymbol{E}$ and $\boldsymbol{B}$ the electric and magnetic fields created by one ion. The values of the EM fields can be found from the expressions for the Liennard-Wiechert potentials written in the 'present time' coordinates. The force is given by

$$\begin{aligned} \boldsymbol{F} &= e^2 \left[-\boldsymbol{\nabla}\left(\frac{1}{s}\right) + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\frac{\boldsymbol{v}}{c^2 s} + \frac{\boldsymbol{v}}{c^2} \times \left(\boldsymbol{\nabla} \times \frac{\boldsymbol{v}}{s}\right)\right] \ , \\ s &= \sqrt{(x - x'^2) + (1 - v^2/c^2)\left[(y - y')^2 + (z - z')^2\right]} \ . \end{aligned} \tag{5}$$

Here, x, y, z and x', y', z' are the coordinates of the interacting ions, and x-axis is assumed to be parallel to $\boldsymbol{v}$, without restriction of generality. Further, we assume that charges do not change with the velocity v. Neglecting the sign, Eq. (5) can be presented as

$$\boldsymbol{F} = e^2 \boldsymbol{\nabla} \left(\frac{1 - v^2/c^2}{s} \right) = e^2 \boldsymbol{\nabla} \Psi \ ,$$

$$\Psi = \frac{1 - v^2/c^2}{\sqrt{(x - x'^2) + (1 - v^2/c^2)\left[(y - y')^2 + (z - z')^2\right]}} \ , \tag{6}$$

where the scalar function Ψ is called the *convection potential.* So we see from Eq. (6) that for the system of the moving charges, the electrostatic potential is changed to the convection potential. Because the electrostatic potential determines the points of location of the ions where they are being in equilibrium when the lattice is at rest, we can suggest that if the lattice moves, just the convection potential must determine the points of equilibrium of ions of the moving lattice. It follows from Eq. (6) that if the velocity of motion of the lattice changes, the magnitude of the convection potential changes too. Therefore, the points of locations of the ions must change too. Below we find how new points of equilibrium depend on the velocity.

It is reasonable to assume that if the magnitude of the potential providing the equilibrium of the lattice changes, the ions tend to displace in such a way that changing of the potential is compensate for the total energy conserves.

Let us assume that when the lattice is being at rest, the distance between two neighbor ions is d. The energy of the lattice is the potential electrostatic energy of the ions

$$W_{rest} = - \sum_{l\, m,\, n=0}^{\infty} \frac{(-1)^{l+m+n} e^2}{\sqrt{(ld)^2 + (md)^2 + (nd)^2}} = -\frac{e^2}{d} \sum_{l,\, m,\, n=0}^{\infty} \frac{(-1)^{l+m+n}}{\sqrt{l^2 + m^2 + n^2}} \ , \tag{7}$$

where summation over l corresponds to summation of the ions along the x axis, summation over m does along the y axis and summation over n does along the z axis; the term $l = m = n = 0$ is excluded.

When the lattice moves, the electrostatic energy of the ions changes because of changing the convection potential with velocity as

$$W_{mov} = - \sum_{l,\, m,\, n=0}^{\infty} \frac{(-1)^{l+m+n}(1 - v^2/c^2) e^2}{\sqrt{(ld)^2 + (1 - v^2/c^2)\left[(md)^2 + (nd)^2\right]}} \ , \tag{8}$$

The terms $1 - v^2/c^2$ in the numerator and in the denominator, respectively, of Eq. (8) arise from the finite speed c of propagation of electromagnetic forces. The case of two moving charges (charge 1 and charge 2) with their distance perpendicular to speed v is easier to explain (see also Fig. 1b). If md is such a perpendicular distance, then the traveling distance of the electromagnetic force in the resting frame is enlarged in anology to the hypotenuse of an right angle triangle with adjacent side vt and opposite side ct by the factor $\sqrt{1 + v^2/c^2}$, or $1/\sqrt{1 + v^2/c^2}$ if we are considering both terms, in the denominator, and in the numerator, respectively. Traveling time for the distance md is t.

To explain the parallel case one needs the assumption that action and reaction of the electromagnetic force need twice the distance - from charge 1 to charge 2 and back. If ld is the parallel distance we find as traveling time t_1 of the action from charge 1 to charge 2 the amount $t_1 = (ld + vt_1)/c$ or $t_1 = ld/(c - v)$. The traveling time backwards, t_2, is $t_2 = (ld - vt_2)/c$ or $t_2 = ld/(c + v)$. The averaged simple traveling distance is then $c(t_1 + t_2)/2 = ld/(1 - v^2/c^2)$, i.e., exactly the factor of the distance ld in Eq. (8).

Since there is no internal motion of the ions the total internal energy is equal to the electrostatic energy. Comparing Eq. (7) to Eq. (8) shows that the total internal energy of the lattice increases with the velocity. It means that the ions are not located at the points of equilibrium and it is necessary to find new points of location of the ions in such a way that the magnitude of W_{mov} will be equal to the value of W_{rest}. To do it, we consider partial sums over m, n and l separately. We have for Eq. (7) and (8) at $m = n = 0$

$$W_{rest}^{\parallel} = -\frac{e^2}{d} \sum_{l=0}^{\infty} \frac{(-1)^l}{l} \quad ; \quad W_{mov}^{\parallel} = -\frac{e^2}{d} \sum_{l=0}^{\infty} \frac{(-1)^l (1 - v^2/c^2)}{l} \ . \tag{9}$$

The total energy does not change if the magnitudes of $W^{||}_{rest}$ and $W^{||}_{mov}$ are equal. Because the only parameter, which can change in Eq. (9), is the interatomic distance d, the equivalence of these quantities is provided by changing the interatomic distance (in x direction, length contraction due to von Weber [7] as

$$d^{\parallel}_{rest} = \frac{d^{\parallel}_{mov}}{1 - v^2/c^2} \quad \Longrightarrow \quad d^{\parallel}_{mov} = \left(1 - v^2/c^2\right) d^{\parallel}_{rest} \ . \tag{10}$$

Respectively, analysis of partial sums at $l = 0$ gives

$$W^{\perp}_{rest} = -\frac{e^2}{d} \sum_{m,\,n=0}^{\infty} \frac{(-1)^{m+n}}{\sqrt{m^2+n^2}} \quad ; \quad W^{\perp}_{mov} = -\frac{e^2}{d} \sum_{m,\,n,=0}^{\infty} \frac{(-1)^{m+n}\sqrt{1-v^2/c^2}}{\sqrt{m^2+n^2}} \ . \tag{11}$$

and the interatomic distances which are transversal to the motion direction for the lattice at rest and the moving lattice are connected (due to cross contraction) as

$$d^{\perp}_{rest} = \frac{d^{\perp}_{mov}}{\sqrt{1 - v^2/c^2}} \quad \Longrightarrow \quad d^{\perp}_{mov} = \sqrt{1 - v^2/c^2}\, d^{\perp}_{rest} \, . \tag{12}$$

Using conditions (10) and (12), one can evaluate changing the distance between two arbitrary ions separated by l, m and n sites ($d^{\parallel}_{rest} = d^{\perp}_{rest} = d$)

$$\begin{aligned} d_{mov}(l,m,n) &= \sqrt{l^2\left(d^{\parallel}_{mov}\right)^2 + [m^2+n^2]\,(d^{\perp}_{mov})^2} = \\ &= \sqrt{(1-v^2/c^2)^2\,(ld)^2 + (1-v^2/c^2)\,[(md)^2+(nd)^2]} \, . \end{aligned} \tag{13}$$

Using Eq. (13), we calculate the electrostatic energy of the moving lattice taking into account that the transversal component of the interatomic distance should enter into the formula with the factor $(1-v^2/c^2)$

$$\begin{aligned} W_{mov} &= -\sum_{l,\,m,\,n=0}^{\infty} \frac{(-1)^{l+m+n}(1-v^2/c^2)e^2}{\sqrt{\left[d^{\parallel}_{mov}(l,m,n)\right]^2 + (1-v^2/c^2)\,[d^{\perp}_{mov}(l,m,n)]^2}} = \\ &- \sum_{l,\,m,\,n=0}^{\infty} \frac{(-1)^{l+m+n}(1-v^2/c^2)e^2}{\sqrt{(1-v^2/c^2)^2\,(ld)^2 + (1-v^2/c^2)(1-v^2/c^2)\,[(md)^2+(nd)^2]}} = \\ &- \frac{e^2}{d} \sum_{l,\,m,\,n=0}^{\infty} \frac{(-1)^{l+m+n}}{\sqrt{l^2+m^2+n^2}} = W_{rest} \ . \end{aligned} \tag{14}$$

So we find that if the conditions (10) and (12) are fulfilled, the total electrostatic energy of the lattice does not change with velocity (we consider the steady-state regimes of motion only).

This is an important result for the analysis of the electrodynamics of the moving bodies. It is easily to find that any other kinds of changing the interatomic distances of the lattice do not provide the minimum of the electrostatic energy when the body is being in motion. For example, if we assume that only relativistic contraction of the bodies occurs, *i.e.* the interatomic distance changes only in direction of motion,

$$d^{\parallel}_{mov} = \sqrt{1 - v^2/c^2}\, d^{\parallel}_{rest} \quad ; \quad d^{\perp}_{mov} = d^{\perp}_{rest} \ , \tag{15}$$

we have for W_{mov} taking into account Eqs. (8) and (15)

$$\begin{aligned} W_{mov} &= -\frac{e^2}{d^{rel}_{rest}} \sum_{l,\,m,\,n=0}^{\infty} \frac{(-1)^{l+m+n}(1-v^2/c^2)}{\sqrt{(1-v^2/c^2)\,(l)^2 + (1-v^2/c^2)\,[m^2+n^2]}} = \\ &- \frac{\sqrt{1-v^2/c^2}\,e^2}{d^{rel}_{rest}} \sum_{l,\,m,\,n=0}^{\infty} \frac{(-1)^{l+m+n}}{\sqrt{l^2+m^2+n^2}} = \sqrt{1-v^2/c^2}\, W_{rest} \ , \end{aligned} \tag{16}$$

and the electrostatic energy of the lattice does not reach its minimum.

If the interatomic distances change in accordance to Eqs. (10) and (12), the whole sizes of the moving body must change as

$$L^{\parallel}_{mov} = \left(1 - v^2/c^2\right) L^{\parallel}_{rest} \quad ; \quad L^{\perp}_{mov} = \sqrt{1 - v^2/c^2} L^{\perp}_{rest} \ . \tag{17}$$

We should note that the relativistic transformations of the fields are included in our calculations (the expression for the convection potential) but it follows from more sophisticated consideration of the change of the electrostatic energy, and, therefore, the conditions for equilibrium of the ions forming lattice demand that some additional contraction of the moving bodies must occur.

III. APPLICATION OF THE ABOVE RESULTS TO INTERPRETATION OF THE MICHELSON-MORLEY EXPERIMENT

Now we show that the contraction of the moving body described by Eq. (17) can be used for explanation of the *null results* of the Michelson-Morley (MM) experiments [8, 9] (the scheme of this experiment is give in Fig. 1).

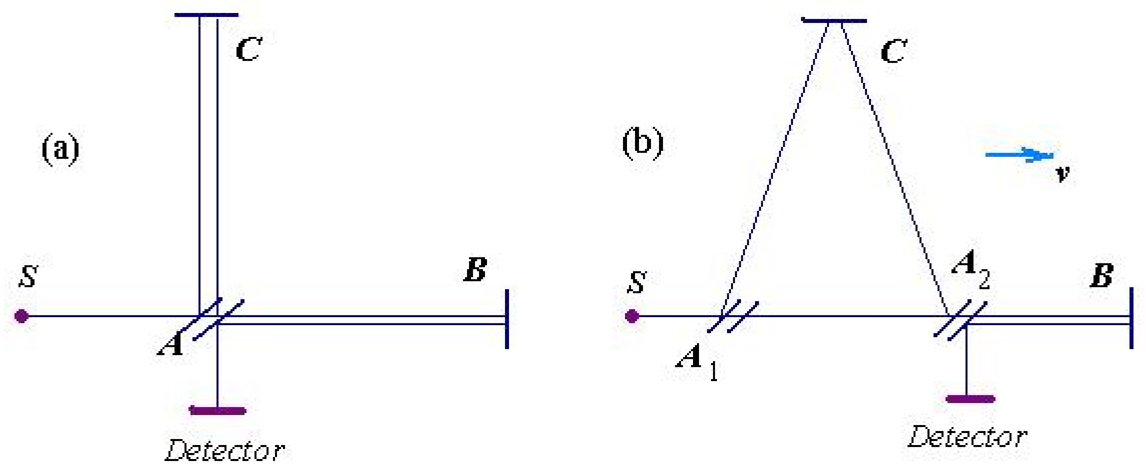

FIG. 1: The scheme of the MM experiment. (a) the interferometer is being at rest with respect to the cosmic background radiation (CBR) frame, and (b) the interferometer moves with the velocity v with respect to this frame. AB and AC are the arms of the interferometer, A, B and C the mirrors, s the source of the light.

First, we note that the optical paths of the light beam in the Michelson interferometer are determined by the lengths of the arms $\boldsymbol{AB}$ and $\boldsymbol{AC}$ supporting the mirrors. The arms are made of the solid state material and, therefore, they should contract, while the frame with the interferometer moves, in accordance to Eq. (17).

We denote the inertial frame in which the Michelson interferometer moves (with the Earth) as the CBR frame and the inertial frame where the interferometer is being at rest as the laboratory frame. Let us assume that the length of the path $\boldsymbol{AB}$ for the Michelson interferometer being at rest is $L^{\parallel}_{rest}$ [12] and the length of the path $\boldsymbol{AC}$ is $L^{\perp}_{rest}$. Then the optical paths are

$$P^{\|}_{rest} = 2L^{\|}_{rest} \ ; \ P^{\perp}_{rest} = 2L^{\perp}_{rest} \ , \tag{18}$$

and the diference in the optical paths is

$$\Delta P_{rest} = P^{\|}_{rest} - P^{\perp}_{rest} = 2[L^{\|}_{rest} - L^{\perp}_{rest}] \ . \tag{19}$$

so a difference in the traveling times of the light beam in this frame is

$$\Delta t_{rest} = \frac{2[L^{\|}_{rest} - L^{\perp}_{rest}]}{c} \tag{20}$$

Now we go to the CBR frame where the interferometer moves. When the arm $L^{\|}$ is oriented in parallel to the light beam $\boldsymbol{A} \to \boldsymbol{B} \to \boldsymbol{A}$, the traveling time of the light is

$$t_{ABA} = t_{AB} + t_{BA} = \frac{L^{\|}_{mov}}{c-v} + \frac{L^{\|}_{mov}}{c+v} = \frac{2cL^{\|}_{mov}}{c^2 - v^2} = \frac{2c(1 - v^2/c^2)L^{\|}_{rest}}{c^2 - v^2} = \frac{2L^{\|}_{rest}}{c} \, . \tag{21}$$

When the arm $L^{\perp}$ is oriented transversally to direction of motion, the traveling time of the light is

$$t_{ACA} = t_{AC} + t_{CA} = \frac{2L^{\perp}_{mov}}{\sqrt{c^2 - v^2}} = \frac{2\sqrt{1 - v^2/c^2}L^{\perp}_{rest}}{\sqrt{c^2 - v^2}} = \frac{2L^{\perp}_{rest}}{c} \, . \tag{22}$$

so the difference in the traveling times of the light beam in the laboratory frame is

$$\Delta t_{mov} = t_{ABA} - t_{ACA} = \frac{2[L^{\|}_{rest} - L^{\perp}_{rest}]}{c} \tag{23}$$

Because the difference in the optical paths is

$$\Delta P_{mov} = c\Delta t_{mov} = [L^{\|}_{rest} - L^{\perp}_{rest}] \ , \tag{24}$$

we see from Eqs. (19) and (24) that the type of contraction of the moving body (17) provides conservation of the optical paths of the light beam in the Michelson interferometer independently of the frame. Because just the difference in the optical paths determines the interference pattern distribution, we conclude that this type of contraction explains the *null result* of the MM experiment.

IV. THE NUMERICAL CALCULATION OF THE POTENTIAL OF AN IONIC CRYSTAL LATTICE

When the lattice moves, the electrostatic energy of the ions changes because of changing the convection potential with arising the velocity as

$$W_{mov} = - \sum_{l,\,m,\,n=0}^{\infty} \frac{(1 - v^2/c^2)e^2(-1)^{l+m+n}}{\sqrt{(ld)^2 + (1 - v^2/c^2)\left[(md)^2 + (nd)^2\right]}} \ . \tag{25}$$

Since there is no internal motion of the ions the total internal energy is equal to the electrostatic energy. Comparing Eq. (7) to Eq. (25) shows that the total internal energy of the lattice arises with the velocity. It means that the ions are not located at the points of equilibrium and it is necessary to find new points of location of the ions in such a way that the magnitude of W_{mov} will be equal to the value of W_{rest}.

The volume of the two possible space cells of our NaCl type crystal is $(2d)^3$. The nearest distance of two ions of the same kind is $2d$. One space cell consists of 8 ions of the first kind positioned in the corners of a cube, positive e.g., and one central ion of the other kind, negative then. The second possible space cell is built similarly, but the ions changed.

β	P_1/P_0	P_2/P_0	P_3/P_0
0.000	1.0000000	1.0000000	1.0000000
0.001	0.9999993	0.9999995	1.0000000
0.002	0.9999973	0.9999980	1.0000000
0.003	0.9999940	0.9999955	1.0000000
0.004	0.9999893	0.9999920	1.0000000
0.005	0.9999833	0.9999875	1.0000000
0.006	0.9999760	0.9999820	1.0000000

TABLE I: Relative potentials of the central ion in an ionic crystal lattice for different $\beta = (v/c)$ and different contraction formulas.

We consider spherical shells surrounding a central ion. The number $N(r)$ of ions in a shell with thickness dr and radius r is proportional to $4\pi r^2 \mathrm{d}r$. With greater r we can consider the number $N(r)$ as statistically defined. With Bernoulli we assume a statistical error of $N^{-1/2}$. Since positive and negative ions are mixed up in a shell the number of positive acting ions is $[N_+ - N_-]$ with a statistical error of about $(2N_+)^{-1/2}$ or $N_0 r$. The constant factor N_0 contains the number of ions per spatial unit, and additionally contains the constants $4\pi \mathrm{d}r$. The acting potential of the $[N_+ - N_-]$ ions on the central ion is then statistically averaged $P_0 r/r = P_0$. That means the potential of each shell is zero with a statistically error of constant variance over all radii. Integrating the contribution of the shells we get a divergent integral similarly to the integral of sine or cosine.

The Laplace transform shows a way to handle such divergent integrals. We introduce the damping factor $\exp(-\delta r)$ for the integrand with $r = \sqrt{(ld)^2 + (md)^2 + (nd)^2}$ and calculate the sum according to Eq. (8) using this damping factor. The practical computation written in C-language uses a spherical body of ions with radius $r_{max} = 500$ simple ion distances d. The damping factor δ was chosen so that for r= rmax we got $\exp(-\delta r) = 10^{-5}$. The thickness of a shell was $\mathrm{d}r = d/150$. Table 1 shows the results of the numerical calculation.

Here P_0 is the potential for $\beta = 0$. P_1 is the potential calculated without any contraction of the crystal. P_2 is the potential calculated with Lorentz contraction, *i.e.*, contraction only in x-direction here. P_3 is the potential calculated with length- and cross-contraction introduced above. Since the absolute value is not of interest, table 1 shows the relative change of the calculated potentials with increasing β.

The relative potential P_1/P_0 changes as $(1-\beta^2)^{2/3}$. The relative potential P_2/P_0 changes as $(1-\beta^2)^{1/2}$. Relative potential P_3/P_0 does not change its value with changing β. Any decrease of the potential energy of the crystal lattice is physically not explanable. So, case 3 with cross and length contraction is the only case here with non decreasing potential.

Based on the above numerical result, we consider the process of displacement of the ions in the lattice while the velocity of the crystal increases. Because the absolute values of relativistic length contraction for the lattice cell are too small we conclude that the shift of the ions in each cell of the lattice occurs in both directions due to the above defined cross and length contraction. Because the quotient of length and cross contraction $(1-\beta^2)/(1-\beta^2)^{1/2} = (1-\beta^2)^{1/2}$ is the same as the SR or Lorentzian length contraction $(1-\beta^2)^{1/2}$, we get the same null result for the MM experiment as the SR or the Lorentzian ether theory.

V. CONCLUSON

The authors showed starting from basic considerations how the sizes of the moving rigid bodies are determined from conditions of equilibrium of the ions of the lattice. It seems, however, that the other type of contraction of relativistically moving bodies yields the same effects as the SR predicts. Actually, if we calculate the difference in the optical paths in the moving interferometer, we obtain the same value as Eq. (24) gives. The SR states that, while the interferometer moves, the arms contract as

$$L^{\parallel}_{mov} = \sqrt{1 - v^2/c^2}\, L^{\parallel}_{rest} \quad ; \quad L^{\perp}_{mov} = L^{\perp}_{rest} \; . \tag{26}$$

Then, instead of Eqs. (21) and (22) we have

$$t'_{ABA} = t'_{AB} + t'_{BA} = \frac{L^{\parallel}_{mov}}{c-v} + \frac{L^{\parallel}_{mov}}{c+v} = \frac{2cL^{\parallel}_{mov}}{c^2-v^2} = \frac{2c\sqrt{1-v^2/c^2}L^{\parallel}_{rest}}{c^2-v^2} = \frac{2L^{\parallel}_{rest}}{c\sqrt{1-v^2/c^2}} \ , \qquad (27)$$

and

$$t'_{ACA} = t'_{AC} + t'_{CA} = \frac{2L^{\perp}_{mov}}{\sqrt{c^2-v^2}} = \frac{2cL^{\perp}_{rest}}{\sqrt{c^2-v^2}} \ . \qquad (28)$$

One obtains the difference in the traveling times from Eqs. (27) and (28)

$$\Delta t'_{mov} = \frac{2L^{\parallel}_{rest} - 2L^{\perp}_{rest}}{c\sqrt{1-v^2/c^2}} \ . \qquad (29)$$

In the SR the factor $1/\sqrt{1-v^2/c^2}$ is eliminated by the time dilation

$$\Delta t'_{mov} \rightarrow \frac{\Delta t'_{mov}}{\sqrt{1-v^2/c^2}}$$

so Eq. (29) should be transformed to

$$\frac{\Delta t'_{mov}}{\sqrt{1-v^2/c^2}} = \frac{2L^{\parallel}_{rest} - 2L^{\perp}_{rest}}{c\sqrt{1-v^2/c^2}} \ . \qquad (30)$$

hence the difference in the optical paths conserves

$$\Delta P^{SR}_{mov} = c\Delta t'_{mov} = 2[L^{\parallel}_{rest} - 2L^{\perp}_{rest}] = \Delta P_{rest} \qquad (31)$$

which yields a correct null result. We find the same time dilation in the Lorentz-FitzGerald theory yielding the null result too.

However, despite both types of the contraction of the bodies, *i.e.* relativistic and considered above, predict no change of the interference picture when the velocity of the interferometer, with respect to the cosmic background radiation frame, changes there is one difference between these types of contraction. Below we analyze it in more detail.

1. The magnitude of the contraction we consider above (WO contraction) is stronger than the magnitude of the SR contraction. Because we derive the WO contraction from the Maxwel equations which are primary with respect to the SR [13], we should conclude that the rigid bodies, contract, while they move, in accordance to the WO contraction. The optical path $\boldsymbol{A} \rightarrow \boldsymbol{B} \rightarrow \boldsymbol{A}$ of the light beam in the interferometer can be found, in the CBR frame, either from Eq. (21) or from geometric considerations of Fig. 1

$$P^{\parallel}_{mov} = \frac{L^{\parallel}_{mov}}{1-v/c} + \frac{L^{\parallel}_{mov}}{1+v/c} = \frac{2L^{\parallel}_{mov}}{1-\dfrac{v^2}{c^2}} = 2L^{\parallel}_{rest} \qquad (32)$$

where $L^{\parallel}_{mov}$ contracts in accordance to Eq. (17).
2. We will measure this optical path in number of the wavelengths λ', *i.e* in the number of the crests of the EM field distribution in the resonance cavity of the Michelson interferometer. Because distribution of the EM field in the resonanse cavity is of stationary type, the factor $\exp(i\omega t)$ in the term describing the EM wave can be neglected.
3. Because the light source is rigidly linked to the laboratory frame, we should transform λ' to the CBR frame. It is made in accordance to Eq. (11.19) or Eq. (11.22) of [10]

$$\lambda = \frac{\lambda'}{\sqrt{1-v^2/c^2}} \ . \qquad (33)$$

So the number of the crests in the resonance cavity is

$$N = \frac{P_{mov}^{\|}}{\lambda} = \frac{2\sqrt{1 - v^2/c^2}L_{rest}^{\|}}{\lambda'} \ . \tag{34}$$

4. Now we use the fact that the quantity N is *invariant* because the invariant quantity is the phase of the EM wave (Sec. 11.4 of [10]). It means that if we count the number of crests of the EM wave passing some distance in two frames, we must obtain the same value of these crests. So if we count this quantity from the above equation in the laboratory frame, we should obtain the correct result. All quantities in the *rhs* of Eq. (34) are now defined in the laboratory frame so this formula can serve to count the crests of the EM field distribution in the resonance cavity of the interferometer.
5. To find the quantity N, it is very difficult but in principle a solvable experimental task. Actually, one needs to solve a simplier technical task. One can see from Eq. (34) that the number of crests depends on the velocity v of the laboratory frame with respect to the CBR frame and evaluation of Eq. (34) for the values $L_{rest}^{\|} \approx 1$ meter, $\lambda' \approx 1\ \mu m$ and if the velocity v changes from 300 km/sec (when the main axis of the resonator is directed along the vector $\boldsymbol{v}$) to 0 km/sec (when the main axis is directed transversally to $\boldsymbol{v}$) gives

$$\Delta N = N(v) - N(0) \approx 1 \ . \tag{35}$$

We note that a possibility to detect a quantity ΔN, depending on a velocity of the device with respect to the CBR frame, is a result of two effects, *namely* contraction of the crystalline lattice of the material of the device due to the convection potential and the relativistic Doppler shift. Actually, a technical tast of detection of ΔN equal to the task of detection of the phase of coherent light incoming to some detector; if the arm, at which ends the source and the detector are rigidly fixed, turns, according to Eq. (35) the phase of the light ocsillations should change. We suggest that this effect can be used for orientation of the space apparatuses in cosmos because a device working with this effect does not need in exchanging information with the surrounding space; the arm with the source and the detector fixed at its ends is completely closed system, however, if our calcualtions are correct, it allows to determine magnitude of the velocity and direction of the vector of the velocity with respect to the CBR frame.

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[11] The type of the material of the rigid body is insignificant. It can be either crystalline or amorphous material; what is significant here is that the ions of the material are located at the points of equilibrium and that these points are determined by electrostatic forces. The only difference in the types of the material of the rigid body is that due to ordering of the crystalline lattice, the calculation of the points of equilibrium of the ions is made much easier than for the amorphous material.
[12] The arm $\boldsymbol{AB}$ is directed along the vector of the velocity of the laboratory frame with respect to the CBR frame.
[13] The SR was derived from the symmetries of the Maxwell equations.

Обобщенно-конформные преобразования

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1 Введение

Замечательные результаты, которые получены в теории функций комплексной переменной и теории конформных преобразований евклидовой плоскости, заставляют многих исследователей снова и снова пытаться проделать подобные построения в четырехмерном пространстве-времени. Для этого каждому событию должно соответствовать некое гиперкомплексное число. До сих пор не сложилось достаточно определенное мнение, какая конкретно система гиперкомплексных чисел может осуществить такую "алгебраизацию" четырехмерного пространства-времени. И конечно, от такой "алгебраизации"ожидается не просто описание уже существующих фактов, а дальнейшее нетривиальное развитие релятивистской теории.

Какая бы систему гиперкомплексных чисел ни выбирали, точного воспроизведения ситуации, имеющей место на комплексной плоскости не происходит. Так обязательно (по разным причинам для разных систем гиперкомплексных чисел) приходится обобщать понятие аналитичности функций соответствующей гиперкомплексной переменной, и при этом имеет место некая неоднозначность такого обобщения.

Соглашаясь с Павловым [5] в том, что в качестве чисел, которые ставятся в соответствие точкам четырехмерного пространства-времени, следует выбирать поличисла, то есть ассоциативно-коммутативные гиперкомплексные числа, например, H_4 - поличисла, изоморфные алгебре квадратных диагональных действительных матриц 4x4, конкретизируем задачу, решение которой представлено в данном докладе.

Поличисловые пространства размерности больше двух являются метрическими финслеровыми пространствами, и их метрические функции выражаются через форму n-го порядка. Для евклидовых и псевдоевклидовых пространств это всегда форма второго порядка. Между аналитическими функциями поличисловой переменной и конформными преобразованиями соответствующего координатного финслерова пространства существует точно такая же связь как между аналитическими функциями комплексной переменной и соответствующими конформными преобразованиями евклидовой плоскости. В связи с этим возникла идея, сохраняя такую связь, вначале обобщить понятие конформных преобразований, а уже потом вернуться к обобщению понятия аналитичности.

Если V_n некое риманово или псевдориманового пространство с координатами x^i и метрическим тензором $g_{ij}(x)$, то коэффициенты связности Γ^i_{kl} в этом пространстве, как известно, определяются формулой

$$\Gamma^i_{kl}(g) = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right). \quad (1)$$

Пространства с метрическими тензорами g_{ij} и G_{ij},

$$G_{ij}(x) = \Lambda(x) \cdot g_{ij}(x), \tag{2}$$

где $\Lambda(x) > 0$ - некая скалярная функция координат, называются конформно связными [2]. Объекты связности этих пространств выражаются друг через друга по формуле

$$\Gamma^i_{kl}(G) = \Gamma^i_{kl}(g) + \frac{1}{2\Lambda}\left(\frac{\partial\Lambda}{\partial x^l}\delta^i_k + \frac{\partial\Lambda}{\partial x^k}\delta^i_l - g^{im}\frac{\partial\Lambda}{\partial x^m}g_{kl}\right). \tag{3}$$

Каждому взаимно однозначному отображению некоторой области пространства в некоторую, вообще говоря, другую область этого же самого пространства можно сопоставить переход от одних координат к другим, то есть сопоставить преобразование координат. Так как коэффициенты связности преобразуются при переходе от одной системы координат к другой по формулам:

$$\frac{\partial x^{i'}}{\partial x^i}\Gamma^i_{kl} = \Gamma^{i'}_{n'p'}\frac{\partial x^{n'}}{\partial x^k}\frac{\partial x^{p'}}{\partial x^l} + \frac{\partial^2 x^{i'}}{\partial x^k \partial x^l} \tag{4}$$

- конформные преобразования координат, осуществляемые функциями f^i для метрического тензора g_{ij}, не зависящего от точки пространства, должны удовлетворять следующей системе уравнений:

$$\frac{\partial^2 f^i}{\partial x^k \partial x^l} = \frac{1}{2\Lambda}\left(\frac{\partial\Lambda}{\partial x^l}\delta^m_k + \frac{\partial\Lambda}{\partial x^k}\delta^m_l - g^{mp}\frac{\partial\Lambda}{\partial x^p}g_{kl}\right)\frac{\partial f^i}{\partial x^m}. \tag{5}$$

Свернем левую и правую части уравнений (5) с тензором g^{kl} сразу по двум индексам, получим

$$g^{kl}\frac{\partial^2 f^i}{\partial x^k \partial x^l} = \frac{2-n}{2\Lambda}g^{kl}\frac{\partial\Lambda}{\partial x^k}\frac{\partial f^i}{\partial x^l}. \tag{6}$$

Таким образом, функции, которые осуществляют конформное преобразование в евклидовых или псевдоевклидовых пространствах, являются решениями дифференциального уравнения (6).

2 Обобщение конформных преобразований евклидовых и псевдоевклидовых пространств

Если в некоторой геометрии аффинной связности к коэффициентам связности аддитивно добавить тензор

$$T^i_{kl} = \frac{1}{2}(p_k\delta^i_l + p_l\delta^i_k), \tag{7}$$

где p_i - произвольное ковариантное поле, то геодезические останутся теми же кривыми [2]. Такое преобразование объектов связности принято называть геодезическим преобразованием [2].

Пусть функции f^i осуществляют взаимно однозначное отображение некоторой области евклидового или псевдоевклидового пространства с метрическим тензором g_{ij} на некоторую другую область того же пространства, и при этом эти функции удовлетворяют системе уравнений

$$\frac{\partial^2 f^i}{\partial x^k \partial x^l} = \left[\frac{1}{2}(p_l\delta^m_k + p_k\delta^m_l) - g^{mp}\frac{\partial L}{\partial x^p}g_{kl}\right]\frac{\partial f^i}{\partial x^m}, \tag{8}$$

где p_i - некое ковариантное векторное поле, а L - некое скалярное поле, тогда такое отображение (преобразование координат) будем называть элементарным обобщенно-конформным. Иными словами, элементарное обобщенно-конформное преобразование - это конформное преобразование с точностью до геодезического преобразования соответствующего объекта аффинной связности.

Таким образом, каждая функция (компонента) элементарного обобщенно-конформного преобразования удовлетворяют следующему уравнению:

$$g^{kl}\frac{\partial^2 f^i}{\partial x^k \partial x^l} = g^{kl}\left(p_k - \frac{n}{2}\frac{\partial L}{\partial x^k}\right)\frac{\partial f^i}{\partial x^l}. \tag{9}$$

Для того чтобы показать нетривиальность такого обобщения понятия конформных преобразований приведем одно из решений системы уравнений (8):

$$f^i = \frac{x^i}{a + b\cdot g_{kl}x^k x^l}, \tag{10}$$

где a и b - некоторые действительные числа, причем

$$\Lambda = \frac{d}{(a - b\cdot g_{kl}x^k x^l)^2}, \tag{11}$$

где d - некоторое действительное число.

В случае евклидовой (комплексной) плоскости (x, y)

$$z = x + iy, \qquad F(z) = f^1 + if^2, \tag{12}$$

формула (10) дает функцию комплексной переменной

$$F(z) = \frac{z}{a + bz\bar{z}}, \tag{13}$$

которая при $a \neq 0$ и $b \neq 0$ не является ни аналитической, ни комплексно сопряженной аналитической, но осуществляет элементарное обобщенно-конформное преобразование евклидовой плоскости. При $a = 0$ эта функция становится комплексно сопряженной аналитической

$$F(z) = \frac{1}{b\bar{z}}, \tag{14}$$

что соответствует конформному отображению второго рода. При $b = 0$ функция $F(z)$ является аналитической,

$$F(z) = \frac{1}{a}z, \tag{15}$$

что соответствует конформному отображению первого рода.

3 Поличисла H_4

Система гиперкомплексных чисел H_4 - метрическое плоское финслерово пространство, для которого в ψ - базисе ($H_4 \ni X = \xi^i\psi_i$) четвертая степень элемента длины имеет вид

$$(ds)^4 = d\xi^1 d\xi^2 d\xi^3 d\xi^4. \tag{16}$$

Это частный случай класса финслеровых пространств, которые изучались Богословским и Геннером [3], [4]. Конформно связная финслерова геометрия будет иметь четвертую степень элемента длины вида

$$(ds)^4 = \Xi d\xi^1 d\xi^2 d\xi^3 d\xi^4, \tag{17}$$

где $\Xi > 0$ - некоторое скалярное поле. Такая геометрия аналогична геометрии аффинной связности с коэффициентами связности [?]

$$\Gamma^i_{kj} = \frac{1}{2}(p_k\delta^i_j + p_j\delta^i_k) - p^i_{kj}\frac{1}{\Xi}\frac{\partial\Xi}{\partial\xi^{j_-}}, \tag{18}$$

где

$$\psi_k\psi_j = p^i_{kj}\psi_i, \qquad p^i_{kj} = \begin{cases} 1, \text{ если } i = j = k, \\ \\ 0, \text{ во всех остальных случаях}, \end{cases} \tag{19}$$

p_k - произвольное тензорное поле, индекс $j_- = j$, но по ним не ведется суммирование. Аналогична в том смысле, что геодезические (точнее, экстремали) финслерова пространства (17) и геодезические пространства аффинной связности с объектами связности (18) совпадают. Важно отметить, что сопоставить финслеровой геометрии, конформно связной с геометрией поличисла P_n для $n > 2$, геометрию аффинной связности можно только с точностью до геодезического преобразования объектов связности. Именно это, в какой-то мере, является обоснованием предлагаемого обобщения понятия конформного преобразования.

Таким образом, система уравнений для функций f^i, которые осуществляют элементарное обобщенно-конформные преобразования в координатном пространстве поличисел H_4, имеет следующий вид:

$$\frac{\partial^2 f^i}{\partial\xi^k\partial\xi^l} = \left[\frac{1}{2}(p_l\delta^m_k + p_k\delta^m_l) - p^m_{kl}\frac{\partial L}{\partial\xi^{l_-}}\right]\frac{\partial f^i}{\partial\xi^m}, \tag{20}$$

где L - некоторое скалярное поле.

Общий вид аналитической функции H_4-переменной в ψ-базисе определяется формулой

$$F(X) = f^1(\xi^1)\psi_1 + f^2(\xi^2)\psi_2 + f^3(\xi^3)\psi_3 + f^4(\xi^4)\psi_4, \tag{21}$$

где $f^i(\xi)$ - произвольные функции одной действительной переменной.

Любая аналитическая функция переменной H_4 которая осуществляет взаимно однозначное отображение некоторой области координатного пространства H_4 на некоторую другую область того же пространства осуществляет конформное отображение и удовлетворяет системе уравнений (20), причем

$$p_i = 0, \qquad \Xi = \dot{f}^1\dot{f}^2\dot{f}^3\dot{f}^4, \qquad L = \ln|\Xi/\Xi_0|. \tag{22}$$

Множество решений системы уравнений (20) не сводится только к аналитическим функциям переменной H_4. Так решением этой системы уравнений являются функции

$$f^i = \frac{f^i_0 \ln\left|\dfrac{\xi^{i_-}}{\xi_0^{i_-}}\right|}{a + b\ln\left|\dfrac{\xi^1\xi^2\xi^3\xi^4}{\xi^1_0\xi^2_0\xi^3_0\xi^4_0}\right|}, \tag{23}$$

которые только при $b = 0$ становятся компонентами аналитической функции переменной H_4. В формуле (23) a, b, ξ^i_0, f^i_0 - постоянные, но, конечно, не все они независимы. Для функций (23)

$$\Xi = \frac{const}{\xi^1\xi^2\xi^3\xi^4}. \tag{24}$$

Так как в пространстве H_4 можно образовать тензор

$$q_{ij} = p^m_{ik}p^k_{mj}, \qquad (q_{ij}) = diag(1, 1, 1, 1), \tag{25}$$

то существует и дважды контравариантный тензор q^{ij}, причем

$$(q^{ij}) = diag(1,1,1,1), \tag{26}$$

поэтому каждая компонента элементарного обобщенно-конформного преобразования в пространстве H_4, должна удовлетворять следующему уравнению:

$$q^{kl}\frac{\partial^2 f^i}{\partial\xi^k\partial\xi^l} = q^{kl}\left(p_k - \frac{\partial L}{\partial\xi^k}\right)\frac{\partial f^i}{\partial\xi^l}. \tag{27}$$

Сравнивая уравнения (9) и (27), видим, что уравнение (9), решениями которого являются функции, осуществляющие обобщенно-конформные преобразования в 4-мерном евклидовом пространстве, и уравнение (27), которому подчиняются функции, осуществляющие обобщенно-конформные преобразования в пространстве H_4, имеют совпадающие левые части и правые части одинаковой структуры. Этот интересный факт еще требует своего осознания.

4 Обобщенно-конформные преобразования

Предыдущие построения позволяют предположить, что наиболее общий вид системы уравнений, которые определяют элементарные обобщенно-конформные преобразования в пространстве поличисловой переменной, таков:

$$\frac{\partial^2 f^i}{\partial x^k\partial x^l} = \left[\frac{1}{2}(p_l\delta_k^m + p_k\delta_l^m) - \Delta_{kl}^{pm}\frac{\partial L}{\partial x^p}\right]\frac{\partial f^i}{\partial x^m}, \tag{28}$$

где Δ_{kl}^{pm} - симметрический по нижним индексам числовой тензор в аффинной системе координат исходной метрической геометрии, L и p_k - скалярное и одноковариантное поля.

Из формул (28) следует, что любое линейное невырожденное преобразование является элементарным обобщенно-конформным с

$$p_i = 0, \qquad L = const. \tag{29}$$

Особый интерес представляют числовые метрические пространства поличисел $P_n \ni X = x^i e_i$, где x^i - координаты в базисе e_i, $e_i e_j = p_{ij}^k e_k$, для которых тензор $q_{ij} = p_{ik}^m p_{mj}^k$ имеет невырожденную матрицу, то есть $det(q_{ij}) \neq 0$, поэтому для таких пространств всегда можно построить тензор q^{ij}, а значит для таких пространств функции f^i, осуществляющие элементарные обобщенно-конформные преобразования, удовлетворяют следующему уравнению:

$$q^{kl}\frac{\partial^2 f^i}{\partial x^k\partial x^l} = \left(p_k q^{km} - q^{kl}\Delta_{kl}^{pm}\frac{\partial L}{\partial x^p}\right)\frac{\partial f^i}{\partial x^m}. \tag{30}$$

Элементарные обобщенно-конформные преобразования не образуют группу, но всевозможные произведения (то есть последовательное выполнение) таких преобразований вместе с обратными элементарными преобразованиями образуют группу, которую будем обозначать $G_n(\Delta_{kl}^{pm})$ и называть группой обобщенно-конформных преобразований. Элементы этой группы являются решения системы уравнений

$$\begin{aligned}\frac{\partial^2 f^i}{\partial x^k\partial x^l} &= \left[\frac{1}{2}(p_l\delta_k^m + p_k\delta_l^m) - \Delta_{kl}^{pm}\frac{\partial L}{\partial x^p}\right]\frac{\partial f^i}{\partial x^m} - \\ &- \left[\frac{1}{2}(p'_r\delta_s^i + p'_s\delta_r^i) - \Delta_{sr}^{pi}\frac{\partial L'}{\partial f^p}\right]\frac{\partial f^s}{\partial x^k}\frac{\partial f^r}{\partial x^l},\end{aligned} \tag{31}$$

где p_l, p'_k, L, L' - некоторые поля, Δ^{pi}_{sr} - тот же самый скалярный тензор, что и в системе уравнений (28); причем подразумевается, что производные $\dfrac{\partial L}{\partial f^p}$ явно выражены через частные производные по x^i.

Нам не удалось строго доказать, что среди решений системы уравнений (31), осуществляющих взаимно однозначное отображение одной области метрического пространства на некоторую, вообще говоря, другую область, нет отличных от тех, которые присутствуют в группе $G_n(\Delta^{pm}_{kl})$. Если такие имеются, то понятие обобщенно-конформных преобразований можно ещё несколько обобщить.

5 Заключение

Хотелось бы надеяться, что изложенный в докладе несколько более общий взгляд на конформные преобразования позволит выбрать правильное обобщение понятия аналитичности и построить в четырехмерном пространстве-времени теорию столь же красивую, какой является теория функций комплексной переменной и конформных преобразований евклидовой плоскости.

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The generalized Finslerian metric tensors

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The generalized Finslerian metric tensors are proposed. These tensors can have different number of indeces dependent on space dimension as well as space properties. The relationship of these tensors with the Finsler spaces associated with commutative associative algebras is analyzed. Nearest perspectives to research of the tensors of this type are discussed. The generalized differential equations of Finsler geodesics are derived and discussed.

1. Introduction

The notion of metric tensor of Riemann and Finsler geometry is the central notion that determines the metric properties of space considered. The metric tensor is the well known notion and tensor analysis of metric space is hardly possible without it. It is usual to consider the metric tensor as a tensor of the second rank. Let us ask whether it is possible to generalize this fundamental notion of Finsler space not to restrict ourselves by the second rank's type of tensor. If this approach is possible mathematically it will permit to look for some applications in modern relativity and quantum physics. The more the rank of the metric tensor the more components it has and it gives possibility to look for, for example, correspondence between these components and fundamental physical interactions. This article is the first attempt to consider the generalized metric tensor as a mathematical notion.

The Finslerian metric tensor is well known historically to be found by Berwald L, Synge G.L. and Taylor J. H. at 1925 by analogy with .the Riemannian metric tensor [1]. Although this analogy has helped to develop the Finsler space analysis it has its own boundary. The Riemannian metric tensor has fundamental role but it is not right for Finsler geometry because the Finslerian metric tensor of the second rank has special properties unlike its the Riemannian predecessor. The further consideration gives possibility to doubt universal role of the Finslerian metric tensor of the second rank and therefore gives some background of its generalization.

2. Difference between the Finslerian metric tensor and the Riemannian metric tensor

The components of the Riemannian metric tensor appeared initially as the coefficients of the second order's expansion of the distance between near points, that is, we have:

$$ds^2 = g_{ij}dx^i dx^j \tag{1}$$

Therefore the components in the fixed system of coordinates depend only on the point of Riemann space:

$$g_{ij} = g_{ij}(x)$$

Unlike Riemann space Finsler manifold is determined by set of axioms one of which represents the property of homogeneity of the Finslerian metric function. Owing to this important axiom the metric function has the next form analogous to (1):

$$F^2 = g_{ij}(x, y)y^i y^j \tag{2}$$

Similarity between (1) and (2) is limited because the components in (2) depend not only on the point of base manifold x, but also on the contravariant vector of tangent manifold y. This imparts new character to (2): this expansion is multiple and hence it has not universal nature.

There is the fundamental formula of the Finsler metric tensor components in the books of this geometry [2-4]:

$$g_{ij} = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^j \partial y^i} \tag{3}$$

However it should be noted that the expansion (1) with the aid of (3) is not unique.

To illustrate it we consider Finsler space associated with the commutative associative algebra H_3. This algebra is the product of three real number's algebras: $H_3 = R \times R \times R$. The metric function of it is [5]:

$$F^3 = y^1 y^2 y^3 \tag{4}$$

It isn't difficult to check up that the square of this very metric function can be expanded as the following (2) using not only the classical metric tensor but also some other matrix (6):

$$F^2 = g_{ij} y^i y^j = \widetilde{y}_{ij} y^i y^j$$

$$g_{ij} = \begin{Vmatrix} -\frac{1}{9}\frac{(y^2y^3)^{2/3}}{(y^1)^{4/3}} & \frac{2}{9}\frac{(y^3)^{2/3}}{(y^1y^2)^{1/3}} & \frac{2}{9}\frac{(y^2)^{2/3}}{(y^1y^3)^{1/3}} \\ \frac{2}{9}\frac{(y^3)^{2/3}}{(y^1y^2)^{1/3}} & -\frac{1}{9}\frac{(y^1y^3)^{2/3}}{(y^2)^{4/3}} & \frac{2}{9}\frac{(y^1)^{2/3}}{(y^2y^3)^{1/3}} \\ \frac{2}{9}\frac{(y^2)^{2/3}}{(y^1y^3)^{1/3}} & \frac{2}{9}\frac{(y^1)^{2/3}}{(y^2y^3)^{1/3}} & -\frac{1}{9}\frac{(y^1y^2)^{2/3}}{(y^3)^{4/3}} \end{Vmatrix} \quad (5), \qquad \widetilde{y}_{ij} = \begin{Vmatrix} 0 & \frac{3}{4}g_{12} & \frac{3}{4}g_{13} \\ \frac{3}{4}g_{12} & 0 & \frac{3}{4}g_{23} \\ \frac{3}{4}g_{13} & \frac{3}{4}g_{23} & 0 \end{Vmatrix} \quad (6)$$

Besides the components of the Finslerian metric tensor unlike Riemann space has another property. These components may have a singularity at the point $y = 0$ if $y^i \to 0$ by some special way. For example in the Berwald-Moore space of the third order (4) a component will tend to infinity if the denominator tends to zero more fast the numerator does.

The possibility of such singularities may be considered as another disadvantage of two rank tensors. But the generalized metric tensor may has not this disadvantage. For example the generalized three rank metric tensor of the Berwald-Moore space associated with the algebra H_3 has constant components and consequently the notion of three rank metric tensor is more appropriate for this space.

3. The generalized three rank Finslerian metric tensors

Owing to the key property of homogeneity of metric tensor in the form

$$F(x, ky) = kF(x, y)$$

we can determine a generalized metric tensor.

The Euler's theorem of homogenous function gives the next identities:

$$F^2 = \underbrace{\frac{1}{2}\frac{\partial F^2}{\partial y^i}}_{y_i} y^i = \underbrace{\frac{1}{2}\frac{\partial^2 F^2}{\partial y^j \partial y^i}}_{g_{ij}} y^i y^j \tag{7}$$

It is usual way to determine the covariant components of tangent vector y_i and metric tensor g_{ij}. Their connection with the contravariant components y^i is expressed by a formula:

$$y_{ij} = g_{ij} y^i$$

Due to the Euler theorem for homogenous functions it is possible by analogy with (7) to expand not only the second one but also the higher powers of the Finslerian function to the sum of products of the contravariant components of the vector. Further expanding of the 3-d and 4-th power of this function is going over and as a consequence of which the generalized metric tensors are defined.

$$F^3 = \underbrace{\frac{1}{3}\frac{\partial F^3}{\partial y^i}}_{y_i^* = y_i F} y^i = \underbrace{\frac{1}{6}\frac{\partial^2 F^3}{\partial y^j \partial y^i}}_{y_{ij}^{(3)}} y^i y^j = \underbrace{\frac{1}{6}\frac{\partial^3 F^3}{\partial y^k \partial y^j \partial y^i}}_{G_{ijk}} y^i y^j y^k \tag{8}$$

The components of the covariant vector y_i^* are appeared to be at the first step of (8), but they differ from the components of the covariant vector y_i just by the factor of F and that is why are out of any interest. The second step of this expansion gives the doubly covariant metric tensor $y_{ij}^{(3)}$. The tensor $\widetilde{y}_{ij}$ is a result of this tensor division by the Finslerian function F, that is, we have:

$$\widetilde{y}_{ij} = y_{ij}^{(3)} \big/ F . \tag{9}$$

The tensor (9) is the tensor that takes part in the alternative expansion of the square of the Finslerian function and that is why can be considered as a partial analogue of the fundamental metric tensor g_{ij} (3). The relationship of these two tensors is expressed by the following formula:

$$\widetilde{y}_{ij} = \left(g_{ij} + y_i y_j \big/ F^2\right) \big/ 2 . \tag{10}$$

Easy to see that the tensor (10) has resemblance to the angular Finslerian metric tensor h_{ij} [4]:

$$h_{ij} = g_{ij} - y_i y_j \big/ F^2 .$$

Have a look at the tensor $\widetilde{y}_{ij}$ properties.

1. As the fundamental metric tensor g_{ij}, the tensor $\widetilde{y}_{ij}$ is homogeneous function of zero degree, that is:

$$\widetilde{y}^{ij}(x, ky) = \widetilde{y}^{ij}(x, y) .$$

2. As the fundamental metric tensor g_{ij}, the tensor $\widetilde{y}_{ij}$ can be used for raising and lowering index of arbitrary tangent vector, that is:

$$y^i = \widetilde{y}^{ij} y_j , \quad y_i = \widetilde{y}_{ij} y^j .$$

3. Unlike the fundamental metric tensor g_{ij}, the tensor $\widetilde{y}_{ij}$ does not allow to raise and lower indeces of tensors of the second rank and the highest one. For example the result of lowering index of an arbitrary tensor of two rank T_{ij} by $\widetilde{y}_{ij}$ if it had been raised by g_{ij} is expressed by the formula:

$$(\widetilde{y}_{ij} g^{jk}) T_{kl} = \frac{1}{2}\left[T_{il} + \frac{y_l}{F}\left(\frac{y^j}{F} \cdot T_{jl} \right) \right] .$$

4. The internal product of $\widetilde{y}_{ij}$ by the tensor g_{ij} is equal to 1:

$$\widetilde{y}_{ij} g^{ij} = \widetilde{y}^{ij} g_{ij} = 1 .$$

5. If we construct the Cristoffel symbols on the base of $\widetilde{y}_{ij}$ by usual way, that is, we have:

$$\widetilde{\gamma}_{ijk} = \frac{1}{2}\left[\frac{\partial \widetilde{y}_{ij}}{\partial x^k} + \frac{\partial \widetilde{y}_{jk}}{\partial x^i} - \frac{\partial \widetilde{y}_{ik}}{\partial x^j}\right],$$

this geometrical object obeys the usual equations of the Finslerian geodesics:

$$\frac{d^2 x^i}{ds^2} + \widetilde{\gamma}^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0,\ \widetilde{\gamma}^i_{jk} = g^{il}\widetilde{\gamma}_{ljk} = \widetilde{y}^{il}\widetilde{\gamma}_{ljk} \tag{11}$$

The proof of this property is analogous to the proof of assertion (20) (see further).

At the last, third step of the expansion (8) we determine the third rank metric tensor G_{ijk}:

$$G_{ijk} = \frac{1}{6}\frac{\partial^3 F^3}{\partial y^k \partial y^j \partial y^i} \tag{12}$$

It should be noted that the generalized metric tensors (9) and (12) are symmetrical by all their indices and so are all the metric tensors.

4. The generalized four rank Finslerian metric tensors

It is possible to expand the fourth degree of the Finslerian function by analogy with (8) to give the next set of identities:

$$F^4 = \underbrace{\frac{1}{4}\frac{\partial F^4}{\partial y^i}}_{y_i^* = y_i F^2} y^i = \underbrace{\frac{1}{12}\frac{\partial^2 F^4}{\partial y^j \partial y^i}}_{y_{ij}^{(4)}} y^i y^j = \underbrace{\frac{1}{24}\frac{\partial^3 F^4}{\partial y^k \partial y^j \partial y^i}}_{y_{ijk}} y^i y^j y^k = \underbrace{\frac{1}{24}\frac{\partial^4 F^4}{\partial y^l \partial y^k \partial y^j \partial y^i}}_{G_{ijkl}} y^i y^j y^k y^l \tag{13}$$

At the first step of (13) we have the covariant vector y^* the components of which differ from the components y_i by the factor of F^2 while we have the doubly covariant tensor $y_{ij}^{(4)}$ at the second step. The trebly covariant tensor y_{ijk} and four times covariant tensor G_{ijkl} are appeared to be at the third and the last, fourth step accordingly.

Review the properties of the tensor G_{ijkl}.

First, on considering an indicatrix of Finsler space G_{ijkl} gives possibility to write down not only the equation of tangent plane to a indicatrix's point (14) but also the equations of tangent surfaces of two and three order (15)-(16):

$$G_{ijkl}\left(x^m, y^m_{(0)}\right)\cdot y^i_{(0)} y^j_{(0)} y^k_{(0)} y^l = 1 \tag{14}$$

$$G_{ijkl}\left(x^m, y^m_{(0)}\right)\cdot y^i_{(0)} y^j_{(0)} y^k y^l = 1 \tag{15}$$

$$G_{ijkl}\left(x^m, y^m_{(0)}\right)\cdot y^i_{(0)} y^j y^k y^l = 1 \tag{16}$$

Consequently the known classifications of surfaces of the second and third order permit us to classify the indicatrix's points with the aid of G_{ijkl}.

Secondly, the tensor G_{ijkl} allows to set the five rank geometrical object the components of which may be called the generalized Christoffel symbols. We define the components of this object as the following:

$$\gamma_{i_1 i_2 i_3 i_4 i_5} = \frac{1}{12}\left\{\frac{\partial G_{i_1 i_2 i_3 i_4}}{\partial x^{i_1}} - \frac{\partial G_{i_1 i_3 i_4 i_5}}{\partial x^{i_2}} + \frac{\partial G_{i_1 i_2 i_4 i_5}}{\partial x^{i_3}} - \frac{\partial G_{i_1 i_2 i_3 i_5}}{\partial x^{i_4}} + \frac{\partial G_{i_1 i_2 i_3 i_4}}{\partial x^{i_5}}\right\} \quad (17)$$

The generalized 5-rank Christoffel symbols of the first kind have properties analogous to the properties of the symmetry of classic 3-rank symbols of Christoffel:

a) a symmetry property by 1, 3 and 5, and also by 2 and 4 indices:

$$\gamma_{i_1 i_2 i_3 i_4 i_5} = \gamma_{i_5 i_2 i_3 i_4 i_1} = \gamma_{i_3 i_2 i_1 i_4 i_5} = \gamma_{i_1 i_4 i_3 i_2 i_5} \quad (18)$$

b) a property connected with the permutation of 1 and 2,4 and 5 indices:

$$\gamma_{i_1 i_2 i_3 i_4 i_5} + \gamma_{i_2 i_1 i_3 i_5 i_4} = \frac{1}{6}\partial G_{i_1 i_2 i_4 i_5} \Big/ \partial x^{i_3}$$

c) a property connected with a shift $dx^i (x'^i = dx^i / ds)$ along curve with the natural parameter s:

$$\gamma_{i_1 i_2 i_3 i_4 i_5} x'^{i_1}\, x'^{i_2}\, x'^{i_4}\, x'^{i_5} = \frac{1}{12} \cdot \frac{\partial G_{i_1 i_2 i_4 i_5}}{\partial x^{i_3}} x'^{i_1} x'^{i_2} x'^{i_4} x'^{i_5} \quad (19)$$

With the help of the generalized Christoffel symbols the following assertion can be formulated:

Assertion. The following generalized form of equations for the Finslerian geodesics is fair**:**

$$\frac{d^2 x^i}{ds^2} + \gamma^i_{jklm} \frac{dx^j}{ds}\frac{dx^k}{ds}\frac{dx^l}{ds}\frac{dx^m}{ds} = 0, \quad (20)$$

where $\gamma^i_{jklm} = \widetilde{y}^{(4)in} \gamma_{jknlm}, \widetilde{y}^{(4)}_{in} = y_i y_n - y^{(4)}_{in}$.

On the proving of this assertion we shall predicate upon the equation of Euler-Lagrang where the length along the curve s as natural parameter is used:

$$\frac{d}{ds}\left(\frac{\partial F}{\partial x'^i}\right) - \frac{\partial F}{\partial x^i} = 0 \quad (21)$$

Transform the first item into (21):

$$\frac{d}{ds}\left(\frac{1}{4F^3}\frac{\partial F^4}{\partial x'^i}\right) = \frac{1}{F^6}\left(\frac{d}{ds}\left[\frac{1}{4}\frac{\partial F^4}{\partial x'^i}\right]F^3 - \frac{3}{4}F^2 \frac{\partial F}{\partial s}\cdot\frac{\partial F^4}{\partial x'^i}\right) \quad (22)$$

Now transform the incoming into (22) derivatives:

$$\frac{d}{ds}\left(\frac{1}{4}\frac{\partial F^4}{\partial x'^i}\right) = \frac{dy^*_i}{ds} = \frac{d}{ds}\left(y^{(4)}_{ij} x'^j\right) = \frac{dy^{(4)}_{ij}}{ds} x'^j + y^{(4)}_{ij} x''^j,$$

where $\dfrac{dy^{(4)}_{ij}}{ds} = \dfrac{\partial y^{(4)}_{ij}}{\partial x'^k} \cdot \dfrac{dx'^k}{ds} = 2y_{ijk} ds \cdot x''^k$, while $\dfrac{dF}{ds} = \dfrac{\partial F}{\partial x'^k} \cdot \dfrac{\partial x'^k}{\partial s} = \dfrac{\partial F}{\partial x'^k} x''^k$.

Substituting into (22) we get the following expression:

$$\frac{d}{ds}\left(\frac{\partial F}{\partial x'^i}\right) = \frac{1}{F^6}\left\{F^3\left[2y_{ijk} ds \cdot x'^j x''^k + y^{(4)}_{ij} x''^j\right] - 3F^5 \frac{\partial F}{\partial x'^k}\frac{\partial F}{\partial x'^i} x''^k\right\}$$

Note that $F(x, x') = 1$ due to our choice of the length along the curve as a parameter. Besides it is evident from (13) that $y_{ijk} x'^j ds = y^{(4)}_{ik}$. As a result the first item in the Euler-Lagrang equation looks like the following simple form:

$$\frac{d}{ds}\left(\frac{\partial F}{\partial x'^i}\right) = 3\left(y^{(4)}_{ij} - y_i y_j\right) \cdot x''^j$$

Transforming the second item of the Euler-Lagrang equation is possible as well:

$$\frac{\partial F}{\partial x^i} = \frac{1}{4F^{3/4}} \cdot \frac{\partial G_{jklm}}{\partial x^i} \cdot x'^j x'^k x'^l x'^l$$

Taking into account the property c) of the generalized Christoffel symbols (19) the equations of geodesics look like the following form:

$$\widetilde{y}^{(4)}_{ij} x''^j + \gamma_{jkilm} x'^j x'^k x'^l x'^m = 0, \quad \text{where } \widetilde{y}^{(4)}_{ij} = y_i y_j - y^{(4)}_{ij}$$

Introducing matrix $\widetilde{y}^{(4)ij}$ inversed to the matrix. $\widetilde{y}^{(4)}_{ij}$ and marking $\gamma^i_{jklm} = \widetilde{y}^{(4)in} \gamma_{jknlm}$ we get the very equation (20).

5. Classification of the generalized Finslerian metric tensors

In conclusion to systematize the available concepts of generalized metric tensors we shall classify them.

Definition. We'll say that the generalized metric tensor belongs to the class (m, n), if its rank is equal to, and its components are the coefficients in expanding of n-power of the Finslerian function, i.e. equality is correct

$$F^n = \sum_{i_1,..,i_m=1}^{n} G^{(n)}_{i_1..i_m} \cdot y^{i_1} \cdot \ldots \cdot y^{i_m}$$

According to this definition the components of the metric tensor of class (m, n) is determined by the formula:

$$G^{(n)}_{i_1..i_m} = \frac{m!}{n!} \frac{\partial^m F^n}{\partial y^{i_1} \ldots \partial y^{i_m}}, \quad (n \ge m > 2)$$

Note that the fundamental metric tensor belongs to the class (2,2).

6. Concluisions

The generalized Finslerian metric tensor is determined in this paper, some their properties are investigated and their classification is proposed. Besides the generalized five rank Christoffel symbols are proposed; it gives possibility to generalize the differential equations of the Finslerian geodesics.

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It should be noted that the idea to generalize the metric tensor was originated from Dr. ºº Pavlov, and this article is the first effort to realize this idea. The author is also grateful to Dr. Asanov for helpful discussion.

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Six-Dimensional Treatment of CPT-symmetry

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In distinct of standard formulation of the CPT-theorem, in which the properties of particles and antiparticles, respectively, under direct and reverse flow of time are collated, in the six-dimensional treatment of CPT-symmetry the properties of the same elementary particle are collated under direct and reverse flow of time. In this treatment the charges of particles and antiparticles are the same but the signs of the corresponding electrical and magnetic fields are defined by the sense of revolution of the particle or antiparticle in the extra dimensions space (in a circle of Compton radius). Under change of the flow of time, the sense of this revolution is changed on reverse one, that leads to the change of signs of the fields on opposite ones. By this the corresponding trajectories in the whole space occurs to be as reflected from a mirror. The motion of a particle along the helical line (of Compton radius) with revolution to the left (right), viewing in the direction of travel, is changed onto the motion along the mirror-reflected helical line with revolution to the right (left). The corresponding formulation of the theorem is following: If the flow of time is reversed, the particle moves in the whole space backward along the same trajectory as under direct flow of time. By this automatically the signs of the fields change on opposite ones, and the trajectory, viewing in the direction of travel, in the whole space occurs to be as reflected from a mirror, so that this particle acquires all properties of the antiparticle. The sign of charge may be regarded as nothing but a mark corresponding to positive or negative sense of revolution in the space of extra dimensions. The six-dimensional treatment of the Coulomb force of interaction between two charges is given. The electric force is due to motion of charges in the extra-dimensional subspace and is equal to correspondent Lorentz force.

The equation of dispersion is the same for acoustic waveguide, electromagnetic one, and de Broglie waves: $v_{ph}v_g = c^2$, where v_{ph} is the phase velocity, v_g group velocity, c speed of waves in a free medium (speed of sound in the first case and of light in two other cases). The main characteristic of any waveguide is that it has finite transverse dimensions. The dispersion of waves is due to just these dimensions. It indicates that the space with which we deal is three-dimensional only approximately, but has a small (Compton) extra-dimension thickness.

The proposed treatment is based on the principle of simplicity [1] giving preference to that among competing hypotheses which is based on smaller number of postulates, that is, more simple. It rises from Einstein's statement "the nature saves on principles" and idea of F. Klein [2-4] on movement of particles with speed of light in a multi-dimensional space. These ideas entered in that principley.

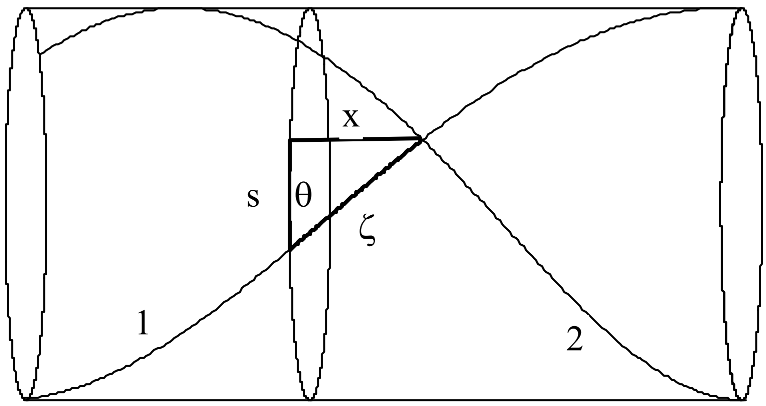
It is well known that the light and as well particles of substance have corpuscular as well wave properties of which examples are diffraction of electrons, when they represent as a wave, and photoeffect, when photon represents as a particle. On this reason, following to the principle of simplicity, it is naturally to suppose that several basic properties of light and particles are similar. The basic property of light is its propagation with the same speed in any system of reference. Then as well elementary particles of substance must move with the same speed. It is impossible in three-dimensional space but possible in multi-dimensional one if positions of particles are recording by an observer in projection on three-dimensional space x_1, x_2, x_3 (X) which we shell consider as homogeneous and isotropic. By this, Newtonian insight extended on six-dimensional Euclidian space (R_6) with projection on three-dimensional space X give known relativistic results.

The whole space is supposed to be six-dimensional one, as only for it a simple interpretation of spin and isospin of electron and other elementary particles is possible. The first substantiation of six-dimensionality of space was given in [5], where fundamental physical constants are calculated.

Assume that for moving with speed of light in six-dimensional space R_6 elementary particles considered as material points, formulas of the Newtonian mechanics are applicable with appropriate chose of time (specifying below). The particles should be acting by a force (of cosmological nature), which is orthogonal to subspace X and keeping them in small vicinity of X. Without such force, withstanding centrifugal force, existence of macroscopic three-dimensional bodies in the Universe would be impossible. The positions of particles are fixing by an observer in the projection on subspace X. (More precisely, we use cosmological small site of X tangential to three-dimensional Universe as three-dimensional sphere in six-dimensional space with neglecting the curvature near this site).

The particle, which is at rest in a projection on X in an inertial frame of reference, moves with the speed of light c, in the simplest case, in a circumference in three-dimensional subspace Y adding up X until R_6, with the center of the circumference in X (by $y_1 = y_2 = y_3 = 0$). In any other inertial frame of reference this particle is moving in a helical line located on a cylindrical surface (a motion pipe) in R_6 with an axis in X.

By natural measure of the proper time of a particle is the number of its revolutions in additional subspace Y around the axis of pipe. Accordingly, we assume that the proper time of a particle is proportional to the number of such revolutions in Y or to the path length traveling in Y.



1 – helical trajectory of a particle moving in six-dimensional space with speed of light c along the cylinder surface of Compton radius $a = \hbar/(mc)$ with axis in subspace X and directrix in subspace Y

2 – helical line of equal proper time of this particle. It passes through the particle perpendicular to that helical trajectory. It moves along the same cylinder surface with velocity of de Broglie wave. Its pitch is equal to the de Broglie wavelength

Generally, the number of revolutions of a particle is proportional to $|\cos\theta|$, where θ is the angle of an inclination of a helical line, as shown in figure. Therefore, if a particle makes one revolution per a proper time τ by clock of the observer “at rest”, relatively of which the particle moves along the tube with a speed $v = c\sin\theta$, where c is speed of light, then it will take place per time $t = \tau/|\cos\theta|$. It is obvious that

$$\sin\theta = v/c,\ \cos\theta = \pm\sqrt{1-(v/c)^2}\,, \tag{1}$$

where upper sign is referred to the particle revolving about the axis of tube in positive sense and lower sign is referred to the antiparticle revolving in negative sense. Such a chose of sign corresponds to the following relation between lapses of proper time of a particle (or antiparticle) $d\tau$ and time of the observer at rest dt:

$$dt = \pm d\tau/\cos\theta = d\tau/\sqrt{1-(v/c)^2}\,. \tag{2}$$

In the frame of reference at rest (K), a particle moving with speed of light c on the motion tube under angle θ to the direcrix of the tube has a component of speed along the direcrix equal to $v\cos\theta$. According to (2) the proper time of a particle from the point of view of the observer at rest is proportional to $\cos\theta$ as well, so that the particle in proper frame of reference (K') moves with speed of light c as well.

A particle at rest in K, a particle moving with speed of light c along the direcrix, displaces per proper time $d\tau$ in an interval ds equal to

$$ds = \pm c d\tau . \tag{3}$$

The momentum of this particle is a vector directed along the tangent to the direcrix at a point where this particle is placed at given time. The magnitude of this vector is mc being the product of mass m of the particle by its speed c. This is the momentum at rest in relativistic mechanics. The energy at rest E_0, according to definition, is the product of momentum and speed of a particle: $E_0 = mc^2$. In the general case, the total momentum of a particle is the vector directed along the tangent to the helical trajectory. Its value p is the product of mass m of a particle by the relation of its path

$$d\varsigma = cdt \tag{4}$$

in R_6 to a proper time $d\tau$ expended for this path:

$$p = m\frac{d\varsigma}{d\tau} = \frac{mc}{|\cos\theta|} = mc\Big/\sqrt{1-(v/c)^2} \tag{5}$$

This is relativistic formula for total momentum of a particle [6].

Projections p_x and p_y of a total momentum on the generatrix and directrix of a tube are equal to co-ordinate and temporal components of 4- momentum of a particle, respectively [6]:

$$p_x = \pm mc\tan\boldsymbol{\theta} = mv\Big/\sqrt{1-(v/c)^2} \ , \quad p_y = \pm mc. \tag{6}$$

In the general case, $\theta \neq 0$ and the total energy of a particle E is the product of the total momentum p by the speed of movement c along a helical line:

$$E = pc = \frac{mc^2}{|\cos\theta|} = mc^2\Big/\sqrt{1-(v/c)^2} \tag{7}$$

This value is the total relativistic energy of a particle. Note that the relation of the total energy to the total momentum of a particle occurs to be the same as for a photon. It is yet another common property of light and substance.

Let us assume that particles having charges of opposite signs revolve about the axis of motion tube in opposite senses. Particles and antiparticles have charges of opposite signs and revolve in opposite senses. For time undergoes a reversal, a particle would go back along its helical trajectory and hence revolve in opposite sense. This signifies that its charge has to change its sign, so that this particle has to transform to its antiparticle. In this case, the motion of such a particle will be as reflected in mirror. The sum of above properties is CPT-symmetry.

The displacement of a particle in an interval ds along a directrix of a motion tube and respective turn through a central angle $d\phi = ds/a$ about the axis of the tube, where a is radius of the tube, is identical in any frame of reference, is invariant. It is because an angle ϕ of a turn of a particle about the axis of the tube is independent on a velocity of an observer in X relative to this particle.

Denoting through dx in system of reference K a projection of a displacement $d\varsigma$ of a particle on the surface of the tube on its generatrix and applying the Pitagorian theorem to the rectangular triangle shown in figure, one obtains the expression for the interval: $(ds)^2 = (cdt)^2 - (dx)^2$. The projection of sides of that triangle on the trajectory of the particle gives

$$s\cos\theta + x\sin\theta = \varsigma . \tag{8}$$

Put initial conditions in the form $t = \tau = 0$ by $x = s = 0$. Then referring to (3) and (4) it follows:

$$s = \pm c\tau \ , \ \ \varsigma = ct . \tag{9}$$

Substituting (1) and (9) into (8) gives the Lorentz transform for time:

$$\tau = \pm\left[t - (x/c)\sin\theta\right]/\cos\theta = \left[t - \left(xv/c^2\right)\right]\Big/\sqrt{1-(v/c)^2} \ .$$

Similar consideration applied to the system of reference K' with account for that the system K moves relatively to considered particle with velocity $-v$ leads to the reversed transform: $t = \pm\left[\tau + (x'/c)\sin\theta\right]/\cos\theta = \left[\tau + \left(x'v/c^2\right)\right]/\sqrt{1-(v/c)^2}$, where x' is the co-ordinate along the generatrix in K. To the transition from the system K to K' corresponds a turn throw an angle $-\theta$ about the origin $x = s = 0$ of co-ordinate net x,s on the surface of the motion tube, together with trajectories of particles on it. This turn transfer a helical trajectory in the directrix of the tube.

For a geometrical interpretation of rest Lorentz transforms let us consider a trajectory of a particle moving along the tube with the same velocity v and intersecting at a time $t = 0$ the helical line $s\cos\theta + x\sin\theta = 0$ at its arbitrary point. In the system of reference K, trajectories inclined under the angle θ to the directrix are the lines of constant co-ordinate x' of the system K'. The co-ordinate x' is measured along the helical line describing by equation (8). The measuring is taken from the normal section of tube $x = vt = \varsigma\sin\theta$ until a section of which the particle achieves at time t. Projecting segments x', x, ς, and s on the generatrix and directrix, the trajectory of particle, and the helical line (along x') perpendicular to the trajectory one obtains by $\cos\theta > 0$:
$x'\cos\theta + \varsigma\sin\theta = x$, $s\cos\theta + x\sin\theta = \varsigma$, $\varsigma\cos\theta - x'\sin\theta = s$, $x\cos\theta - s\sin\theta = x'$.
Dividing these equalities throw by $\cos\theta$ and eliminating s,ς and θ by means of (1) and (9) according to which, in considered case, $s = c\tau$, $\varsigma = ct$, $\sin\theta = v/c$, $\cos\theta = \sqrt{1-(v/c)^2}$, one may easy obtain the Lorentz transforms in the standard form.

The proper length of moving rigid scale is the difference of co-ordinates x' of its ends. In the system K, it is equal to the length of a segment of the helical line perpendicular to the trajectories of particles moving with this segment between normal sections of the motion tube corresponding to those ends. It is a segment of the line of equal time in the system K'. The length of the same scale in the system at rest K is the difference of co-ordinates x of its ends. It is equal to the distance along the generatrix between those normal sections that is in $1/\cos\theta$ less then the proper length.

Thus, the Lorentz contraction of moving scales is a result of projection of lengths in multi-dimensional space on three- dimensional space. Non-simultaneity of spatially spaced events in one system of reference with simultaneity in another is explained by non-parallelism of helical lines of equal time in system of reference moving one relatively another.

Above interpretation of the formula (2) holds as well for curve axis of a motion tube because in any case all normal sections of such tube are perpendicular to any directions in the subspace X to which belongs the axis of a tube.

The energy of a photon is equal to $h\nu$, where ν is frequency of light, h the Plank's constant. By virtue of a principle of similarity of the basic properties of substance and light concretizing the principle of simplicity, the rest energy of a particle may be represented as a quantum of energy $h\nu$, so that

$$mc^2 = h\nu . \tag{10}$$

Unique and natural frequency ν for a particle of substance is the frequency of its revolutions in extra-dimensional subspace Y. On the other hand, the particle moves with speed of light along the directrix of the motion tube, whence $2\pi a = c/\nu$, where is radius of tube. Eliminating ν from this equality and (10), one finds $2\pi a = h/mc$, $a = \hbar/mc$, that is the length of directrix is equal to the length of Compton wave.

Another helical line placed on the same tube perpendicular to helical trajectory of a particle and passes through the particle, is the line of equal proper time of the system K'. This helical line moves along the tube with velocity of de Broglie wave $V_\phi = c/\sin\theta = c^2/v$, where v is velocity of the particle in the subspace X. The pitch ℓ of the helical line is equal to the de Broglie wavelength

$\ell = \frac{2\pi a}{|\tan\theta|} = \frac{h}{mc|\tan\theta|} = \frac{h}{p_x} = \frac{h}{|mv|}\sqrt{1-(v/c)^2}$, as it is seen from (6) and above figure. The angle co-ordinate s/a of the helical line describing by (8) and (9) is equal to $\frac{s}{a} = \frac{\varsigma}{a\cos\theta} - \frac{x}{a}\tan\theta =$ $= \left(t\frac{c}{\cos\theta} - x\tan\theta\right)\frac{mc}{\hbar}$. Whence and from (6) and (7) is seen that s/a is equal to the phase of de Broglie wave $\pm\left[Et - p_x(x/\hbar)\right]$. In the place of position of the particle $x = vt$ this phase is an angle of turn of itself particle on the motion tube. Tthe function $\exp(i\,s/a)$ satisfies the Klein-Gordon equation.

The proper moment of momentum $\boldsymbol{S}$ of a particle is a vector product of the proper momentum and radius vector of this particle. The component of the radius vector and the component of velocity of the particle on the axis of the motion tube are perpendicular to the plane of revolving in Y and therefore do not give any contribution in $\boldsymbol{S}$. Hence for a particle moving in six-dimensional space along a helical line but consequently in a straight line in a projection on X, $\boldsymbol{S}$ is a vector product of projection of momentum and radius vector of this particle on Y. In this case, the magnitude of momentum $\boldsymbol{S}$ becomes $S = |\boldsymbol{S}| = |p_y a| = mc\hbar/mc = \hbar$. This formula remains some arbitrariness in the orientation of vector $\boldsymbol{S}$ in six-dimensional space: it may be oriented in any direction in four-dimensional subspace perpendicular to the plane of revolving in Y. In the general case, vector $\boldsymbol{S}$ has four non-zero components along directions perpendicular each to other and plane of revolving of the particle in Y. In the case of revolving in the plane y_2, y_3, such components are S_1, S_2, S_3, S_4 along the axes x_1, x_2, x_3, y_1, respectively, and $S = \left(S_1^2 + S_2^2 + S_3^2 + S_4^2\right)^{1/2} = \hbar$. Components S_1, S_2, S_3 are components of spin of the particle, S_4 is a projection of isospin of the particle. Thus, spin and isospin are the projections on X and Y, respectively. By (6), p_y is independent on velocity v. Hence spin and isospin are independent on velocity v also and do not subjected to the Lorentz transforms.

Vector $\boldsymbol{S}$ remaining perpendicular to the plane of revolving of the particle has three degree of freedom and may be oriented in arbitrary manner relative to those co-ordinate axes. To particles with spin one half corresponds uniform distribution of components of the vector over above four axes perpendicular each to other and plane of revolving of the particle in Y. Then these components are equal to $+\hbar/2$ or $-\hbar/2$, and the sum of squares of these components in X is equal to $(3/4)\hbar^2$. In quantum mechanics it is “total” (in three-dimensional space) square of the proper momentum of a particle.

To last case orientations of vector $\boldsymbol{S}$ obtained from previous orientations through allowable turns retaining one or two given components invariable are referred as well. So, if one of components of the vector in X and one component in Y have a fixed value $+\hbar/2$ or $-\hbar/2$, then the vector retain a possibility to turn about two correspondent axes. In this case, two non-fixed components will not have of specific values (it is ordinary situations in quantum mechanics, where absence of fixation of quantities is rather the exclusiveness then a rule). For equal allowed probabilities of orientations of that vector, means-square components mentioned above are equal to $\hbar/2$. Change of a direction of revolving of a particle about the axis of the motion tube on the opposite sense as well changes the signs of the components on opposite and corresponds to the transition to antiparticle.

The relations of Heisenberg uncertainty are due to uncertainty of co-ordinates and momenta of a particle in Y. In fact, let the directix of a motion tube of a particle is displaced in the plane y_1,

y_2. Then projections of the momentum of a particle on axes y_1 and y_2, and coordinates of the particle along this axes are equal to $p_{y1} = -mc\sin\phi$, $p_{y2} = mc\cos\phi$, $y_1 = \frac{\hbar}{mc}\cos\phi$, $y_2 = \frac{\hbar}{mc}\sin\phi$, where ϕ is the angle of a turn of the particle about the axis of tube reckoned from the axis y_1. Average over ϕ values of coordinates and projections of the momentum are equal to zero but their mean-square values are equal to $\left\langle y_1^2 \right\rangle = \left\langle y_2^2 \right\rangle = \frac{1}{2}\left(\frac{\hbar}{mc}\right)^2$, $\left\langle p_{y1}^2 \right\rangle = \left\langle p_{y2}^2 \right\rangle = \frac{1}{2}(mc)^2$, whence one finds seeking relations $\left\langle p_{y1}^2 \right\rangle \cdot \left\langle y_1^2 \right\rangle = \left\langle p_{y2}^2 \right\rangle \cdot \left\langle y_2^2 \right\rangle = \hbar^2/4$.

It is of interest, why the values of the proper momentum and its components in X and Y, that is spin and isospin, are independent on mass of an elementary particle? In six-dimensional treatment the answer is obvious: the momentum is proportional to this mass but the radius of the Compton orbit in Y for this particle is inversely proportional to this mass, and therefore the product of momentum and radius of the Compton orbit is independent on this mass.

The proper magnetic moment μ of a charged elementary particle is defined similarly to the proper moment of momentum $\boldsymbol{S}$ accordingly to the known formula of electrodynamics [7]: $\boldsymbol{\mu} = \frac{e}{2c}[\boldsymbol{Rc}]$, where $\boldsymbol{R}$ is six-dimensional radius vector of the particle, $\boldsymbol{c}$ vector of its velocity in Y. Since a contribution in this vector product gives only the projection $\boldsymbol{a}$ of the radius vector $\boldsymbol{R}$ on subspace Y, one finds $\boldsymbol{\mu} = \frac{e}{2c}[\boldsymbol{ac}]$. Whence, accounting for mutual perpendicularity of vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ as well equalities $|\boldsymbol{a}| = a$ and $|\boldsymbol{c}| = c$, one finds the magnitude μ of the proper moment $\boldsymbol{\mu}$ of the particle which occurs to be equal to the Bohr magneton:

$$\mu = \frac{|e|a}{2} = \frac{|e|\hbar}{2mc} = \mu_B. \tag{11}$$

In the simplest case, when the vector $\boldsymbol{\mu}$ has not components in subspace Y, the components of $\boldsymbol{\mu}$ in X defines a three-dimensional vector, of which magnitude is equal to the Bohr magneton.

A projection of the magnetic moment onto arbitrary chosen direction (called the axis of quantization) in subspace X may have a fixed value only in the case when the projection of the proper moment of momentum has a fixed value as well. In this case, according to (11) $\mu_x = \pm\mu_B$. At uniform distribution of components of the proper moment of momentum over four axes which perpendicular each to other and a plane of revolving of a particle in Y, in considered case $S_x = \pm\, mca/2$, that equals $+1/2$ or $-1/2$ (in units of $\hbar$). Whence $\mu_x / S_x = e/mc$ in accordance with the experiment of Stern and Gerlach.

The six-dimensional treatment of considered above and other physical values and phenomena stated in [8-12].

In the general case, the moment of momentum has four nonzero components along directions perpendicular each to other and a plate of revolving of a particle. Therefore the theory of spin and isospin must use explicitly or implicitly four co-ordinates and four projections of vectors on the axes of that co-ordinates. The total moment of momentum $\boldsymbol{M}$ in R_6 is the vector product of the total momentum $\boldsymbol{p_x} + m\boldsymbol{c}$ and radius-vector $\boldsymbol{r} + \boldsymbol{a}$ of a particle in R_6, where $\boldsymbol{p_x}$ and $\boldsymbol{r}$ denote momentum and radius vector in X, $m\boldsymbol{c}$ and $\boldsymbol{a}$ momentum and radius vector in Y. Moment $\boldsymbol{M}$ is four-dimensional vector perpendicular to the plane of revolving of a particle (in Y). On average over a period of revolution about the axis of the tube, the cross terms disappear and then $\boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$, where $\boldsymbol{L}$ is the orbital moment in X, and $\boldsymbol{S} = [\boldsymbol{a}\ m\boldsymbol{c}]$ the spin-isospin moment of revolving in Y. Three components of $\boldsymbol{S}$ represent the spin projections S_1, S_2, S_3 on X, and the component on Y represents isospin S_4. Hence, on account of the mutual perpendicularity of vectors $\boldsymbol{a}$ and $\boldsymbol{c}$, and equalities $|\boldsymbol{a}| = a$, $|\boldsymbol{c}| = c$, one obtains $S = \hbar$, $S_1^2 + S_2^2 + S_3^2 + S_4^2 = \hbar^2$.

At uniform distribution of components on four axes of co-ordinates, which are perpendicular to the plane of revolving in Y, one finds $\left|S_j\right| = \hbar/2$, $j = 1,2,3,4$; $S_1^2 + S_2^2 + S_3^2 = 3\hbar^2/4$.

The disposition of two electrons on the opposite sides of the same tube of motion has energetic advantage. By this the distance between them in the whole space is equal to $R = \sqrt{r^2 + 4a^2}$ where r is the distance between projections of the particles onto X, a is the distance from the axis of their revolution in Y. The tube radius is depended on r and tending asymptotically to $a_\infty = \hbar/(mc)$ with increasing of r, m and c being mass of particle and speed of light at infinity, respectively. By such a revolution with the shift in phase π between two particles the Coulomb force of their repulsion in the whole space is equal to e^2/R^2 where e is the charge of electron. Projections of this force onto subspaces X and Y are $F_{\|} = \left(e^2/R^2\right)\sin\chi$ and $F_\perp = \left(e^2/R^2\right)\cos\chi$, respectively, where $\sin\chi = r/R$, $\cos\chi = 2a/R$, so that $F_{\|} = e^2 r/R^3$, $F_\perp = 2e^2 a/R^3$. The force $F_\perp$ reacts against centripetal force $F_0 = mc^2/a_\infty$. On this cause the radius of revolution a is a little in excess of the tube radius a_∞ at infinity.

The energy at rest and centrifugal force in Y, as in the theory of gravitation [10, 11], are equal to $E_0 = p_y c_\varsigma = mc^2\sqrt{\gamma}$ and $F_c = p_y c_\varsigma/a = E_0/a$, respectively, where c_ς is speed of the particle on the motion tube, $p_y = \hbar/a$ momentum at rest, $\sqrt{\gamma} = c_\varsigma\, a_\infty/ca$, so that $c_\varsigma = ca\sqrt{\gamma}/a_\infty$.

The balance of forces in Y is $F_0 = F_\perp + F_c$. Referring to the relation $e^2/mc^2 = \alpha a_\infty$ (this is the classical radius of electron, α is the constant of fine structure) and introducing $z = a/a_\infty$, this balance of forces may be represented as

$$\sqrt{\gamma} = z - 2\frac{\alpha}{\rho^3}z^2, \tag{1(12)}$$

where $\rho = \sqrt{(r/a_\infty)^2 + 4z^2}$, $r = a_\infty\sqrt{\rho^2 - 4z^2}$, r is the three-dimensional distance. Under the condition $c_\varsigma a = ca_\infty$ of conservation of angular moment, one finds

$$c_\varsigma = c/z, \qquad \sqrt{\gamma} = 1/z^2. \tag{13}$$

If1 $r = 0$, then $\rho = 2z$ and by (12) and (13) one obtains the equation $z^3 - \dfrac{\alpha}{4}z - 1 = 0$, whence

$$z = \sqrt[3]{\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{1}{2}\left(\frac{\alpha}{6}\right)^3}} + \sqrt[3]{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{1}{2}\left(\frac{\alpha}{6}\right)^3}} = 1 + \frac{\alpha}{12} - \frac{1}{3}\left(\frac{\alpha}{12}\right)^3 + \frac{1}{3}\left(\frac{\alpha}{12}\right)^4 + \cdots \tag{14}$$

If electrons are ejected on head one to other with the same speed v in X, the principle of energy yields

$$\frac{e^2}{R}\left(1-\frac{v}{c_\varsigma}\right)^{-1}+E_0\frac{1}{\beta}+\frac{mc^2}{a_\infty}\left(a-a_\infty\right)=mc^2\frac{1}{\beta_\infty}, \tag{15}$$

where $\beta=\sqrt{1-\left(v/c_\varsigma\right)^2}$, $\beta_\infty=\sqrt{1-\left(v_\infty/c\right)^2}$, v_∞ is the speed at infinity. The first term in the left side of equation (15) is an electric potential, created by the electron coming nearer, in the position point of other electron. The second term is the total energy of the electron under consideration, the third term is the work done by this electron against the cosmological force $F_0=mc^2/a_\infty$ at increasing of the tube radius from a_∞ to a. The right side of (15) is the total energy of electron at infinity. By (12), (13), the equation (15) may be represented in the form

$$z-1+\frac{\alpha}{\rho}\left(1-\frac{v}{c}z\right)^{-1}+\frac{1}{\beta z^2}=\frac{1}{\beta_\infty}. \tag{16}$$

Whence by $v=0$ from (16) one has $z-1+\dfrac{\alpha}{\rho}+\dfrac{1}{z^2}=\dfrac{1}{\beta_\infty}$. If as well $r=0$, then

$$\frac{1}{\beta_\infty}=z-1+\frac{\alpha}{2z}+\frac{1}{z^2}=2z-1+\frac{\alpha}{4z}, \tag{17}$$

and according to (14) the kinetic energy at infinity is equal to $mc^2\left(\dfrac{1}{\beta_\infty}-1\right)=mc^2 3.03945468\times10^{-3}$ that for electron is equal to 1553. 146 eV.

Applying the Bio – Savar formula to six-dimensional space, the total magnetic field of the charge at rest in X is defined at the distance R from the charge e as $\boldsymbol{H}_{\text{tot}}=\dfrac{e}{cR^2}[\boldsymbol{c}\ \boldsymbol{R}_0]$ where $\boldsymbol{R}_0$ is the unit vector directed from the charge to the point of observation, $\boldsymbol{c}$ the velocity of the charge. For R being the distance between two electrons

$\boldsymbol{R}_0=\boldsymbol{r}_0\sin\chi+\boldsymbol{a}_0\cos\chi=\boldsymbol{r}_0\left(r/R\right)+\boldsymbol{a}_0\left(2a/R\right)$,

$$\boldsymbol{H}_{\text{tot}}=\frac{e}{R^2}[\boldsymbol{c}_0\boldsymbol{R}_0]=\frac{e}{R^2}\{[\boldsymbol{c}_0\ \boldsymbol{r}_0]\frac{r}{R}+[\boldsymbol{c}_0\ \boldsymbol{a}_0]\frac{2a}{R}\}, \tag{18}$$

where $\boldsymbol{r}_0$ is unit vector along radius vector $\boldsymbol{r}$ in X, $\boldsymbol{a}_0$ unit vector along radius vector of the charge e in the plane of revolution in Y, and $\boldsymbol{c}_0$ is unit vector along velocity $\boldsymbol{c}$.

Let us show that the Coulomb force of interaction between the two charges (e and e') is the Lorentz force acting on this charges as moving in Y. Referring to (18) this force is equal to $\boldsymbol{f}=\dfrac{e'}{c}[\boldsymbol{c}'\boldsymbol{H}_{\text{tot}}]=\dfrac{e'e}{cR^2}\{[\boldsymbol{c}'[\boldsymbol{c}_0\ \boldsymbol{r}_0]]\dfrac{r}{R}+[\boldsymbol{c}'[\boldsymbol{c}_0\ \boldsymbol{a}_0]]\dfrac{2a}{R}\}$. Whence, with account of that for two interacting electrons $\boldsymbol{c}'=-\boldsymbol{c}$, $\boldsymbol{f}=-\dfrac{e'e}{R^2}\{[\boldsymbol{c}_0[\boldsymbol{c}_0\boldsymbol{r}_0]]\dfrac{r}{R}+[\boldsymbol{c}_0[\boldsymbol{c}_0\boldsymbol{a}_0]]\dfrac{2a}{R}\}$. Revealing the triple vector products and taking into account mutual perpendicularity of involved vectors and that in the case under consideration $e'=e$, one obtains $\boldsymbol{c}_0\ [\boldsymbol{c}_0\ \boldsymbol{r}_0]]=-\boldsymbol{r}_0$, $[\boldsymbol{c}_0[\boldsymbol{c}_0\boldsymbol{a}_0]=-\boldsymbol{a}_0$, $\boldsymbol{f}=\dfrac{e^2}{R^3}r\ \boldsymbol{r}_0+\dfrac{e^2}{R^3}2a\ \boldsymbol{a}_0$. In the last formula the first term represents the projection of the Coulomb force onto X, the second term is its projection onto Y. Their magnitudes are equal to $F_{\|}$ and $F_{\perp}$, respectively. From this is seen that electric forces are due to the moving of charges in subspace Y, in distinct of that usual magnetic forces are caused by moving of charges in the same subspace X. The force $F_{\|}$ equals zero at $r = 0$. This is the point of indifferent equilibrium, near which electrons may be slow moving comparatively long time if they were ejected on head one to other with original energy 1553.146 eV [12].

Under change of the flow of time, the sense of revolution of particles in Y is changed on reverse one, that leads to the change of signs of the fields on opposite ones. By this the corresponding trajectories in the whole space occurs to be as reflected from a mirror. The motion of a particle along the helical line (of Compton radius in Y) with revolution to the left (right), viewing in the direction of travel, is changed onto the motion along the mirror-reflected helical line with revolution to the right (left). The sign of charge may be regarded as nothing but a mark corresponding to that or other (positive or negative) sense of revolution of a particle in the space of extra dimensions. In distinct of standard formulation of the CPT-theorem, in which the properties of particles and antiparticles, respectively, under direct and reverse flow of time are collated, in the six-dimensional treatment of CPT-symmetry the properties of the same elementary particle are collated under direct and reverse flow of time. In this treatment the charges of particles and antiparticles are the same but the signs of the corresponding electrical and magnetic fields are defined by the sense of revolution in the extra dimensions space. The corresponding formulation of the theorem is following: If the flow of time is reversed, the particle moves in the whole space backward along the same trajectory as under direct flow of time. By this automatically the signs of the fields change on opposite ones, and the trajectory, viewing in the direction of travel, in the whole space occurs to be as reflected from a mirror, so that this particle acquires all properties of the antiparticle.

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The theory of relativity within the framework of Galilean electrodynamics and Newtonian mechanics

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1. Introduction

The null result of the Michelson-Morley experiment [1] led to the development of the special relativity. The essence of the special relativity is the Lorentz transformation for the space and time. This allows to explain the null result of the Michelson-Morley experiment. The special relativity is logically controversial because operates with two inconsistent coordinate systems, the local system following the Galilei transformation, and the global system following the Lorentz transformation. This means that an experimenter extracts two inconsistent conclusions from one and the same experiment. This gives rise to unobservable phenomena within the framework of the special relativity. The Lorentz local time, $t' = t - vx/c^2$, and the Fitzgerald-Lorentz contraction of length, $l' = l(1 - v^2/c^2)^{1/2}$, are examples of unobservable phenomena. Such phenomena are non-physical by definition. Therefore the physical theory cannot contain such phenomena. It is necessary to search for an explanation of the null result of the Michelson-Morley experiment other than that of the special relativity.

The above ambiguousness of the special relativity means the double standards for the scales of length, time and mass. Einstein extended this ambiguousness to the general relativity. According to Einstein, an observer measures invariant scales of length, time and mass in his own frame but the scales shifted by the factor $(1 - v^2/c^2)^{1/2}(1 - v/c)^{-1}$ in the other frame (special relativity) and by the factor $(1 - 2\Phi/c^2)^{1/2}$ (general relativity). Hence invariance of the scales of length, time and mass holds true only for an observer in his own frame and cannot be verified by an observer in the other frame. Then the theory of relativity, both special and general, is non-verifiable.

Illustrate the situation in the general relativity. Consider a particle orbiting around a gravitating body. The general relativistic corrections for a particle orbiting around a gravitating body can be incorporated in the Newtonian framework via an effective potential of the form

$$\Phi_{eff} = -\frac{Gm}{r} + \frac{v_c^2}{2} - \frac{Gmv_c^2}{rc^2} \qquad (1)$$

where G is the Newton constant, m is the mass of the gravitating body, v_c is the circular velocity, c is the velocity of light. The last term in eq. (1) is the general relativistic correction to the Newtonian potential. This term causes the general relativistic shift (advance) of the particle's perihelion. According to the Einstein general relativity, an observer in the frame of the particle measures the Newtonian potential, $\Phi_N = -Gm/r$, while an observer in the background Euclidean space measures the modified Newtonian potential, $\Phi' = \Phi_N(1 + v_c^2/c^2)$. Hence an observer in the background Euclidean space views the violation of the principle of equivalence. The principle of equivalence holds true for an observer in the frame of the particle. However such an observer cannot measure the general relativistic shift of the particle's perihelion. Then, within the framework of the Einstein general relativity, we cannot reveal an observer in the frame of the particle. Thus the principle of equivalence in the general relativity cannot be verified.

2. Relativity within the framework of galilean electrodynamics

Consider electromagnetic field as a wave with the vector potential $\vec{A}$ in the Euclidean space and absolute time of a preferred reference frame. The Maxwell-Lorentz equations are given by

$$\frac{\partial^2 \vec{A}}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = 0. \tag{2}$$

One can represent the solution of eq. (2) as a plane wave [2]

$$\vec{A} = A_0 \vec{b} e^{-i\phi} \tag{3}$$

where $\vec{b}$ is the polarization vector, $\phi = \omega t - kr$ is the phase, ω is the frequency, $\vec{k}$ is the wave vector.

Suppose that electromagnetic wave propagates with the velocity c with respect to a preferred reference frame independently of the velocity of the source. Let the emitter and receiver of the electromagnetic wave be situated in the frame moving with the velocity v with respect to a preferred reference frame. The travel time of the electromagnetic wave is a function of the velocity of the frame. Then the phase of the electromagnetic wave should be a function of the velocity of the frame. The Michelson-Morley experiment [1] is thought of to determine the phase shift due to the velocity of the frame with respect to a preferred reference frame. However the Michelson-Morley experiment [1] does not give the above phase shift. It was introduced the Lorentz transformation for the space and time to explain the null result of the Michelson-Morley experiment.

Within the quantum mechanics framework one can conceive electromagnetic wave as a bunch of photons. The energy of a bunch of photons is given by

$$\varepsilon_{em} = N\hbar\omega V |\psi|^2 \tag{4}$$

where N is the number of photons, $\hbar$ is the Planck constant, V is the volume, ψ is the wave function of photon given by

$$\psi = \frac{\vec{b}}{\sqrt{V}} e^{-i\phi}. \tag{5}$$

The momentum and energy of photon are given by correspondingly

$$p_{ph} = \hbar k \qquad \varepsilon_{ph} = \hbar\omega. \tag{6}$$

In quantum mechanics, the wave function describes the wave of probability. The square of the wave function yields the density of probability. One cannot determine the wave function of photon but can determine the square of the wave function of photon. The density of probability to register the photon, the square of the wave function of photon, does not depend on the phase of photon

$$|\psi|^2 = \frac{b^2}{V} e^{-i\phi} e^{i\phi} = \frac{b^2}{V}. \tag{7}$$

This means that one cannot determine the phase by measuring the energy (frequency) of photons. The discreteness of the electromagnetic wave means that one always determines zero phase.

In quantum mechanics, the Heisenberg uncertainty principle binds coordinate and dynamical parameters as

$$pr \geq \frac{\hbar}{2} \qquad \varepsilon t \geq \frac{\hbar}{2}. \tag{8}$$

The Heisenberg uncertainty principle forbids to determine at once coordinate and dynamical parameters of the quantum wave. When treating electromagnetic wave as a quantum object, the time coordinate and the frequency as well as the space coordinate and the wave vector are bound with the Heisenberg uncertainty principle as follows from eqs. (6), (8). When detecting the electromagnetic wave with the wave vector k and the frequency ω one can determine the space and time coordinates as

$$r = \frac{1}{2k} \qquad t = \frac{1}{2\omega}. \tag{9}$$

The intervals of length and time given by eq. (9) yield the limits of photon as a wave. When detecting the photon, the wave properties are restricted by these limits. From this it follows that one

cannot determine the travel time and the corresponding phase of photon. It is forbidden by the Heisenberg uncertainty principle.
So when treating electromagnetic field as a quantum wave one cannot determine the travel time and the corresponding phase of the electromagnetic field. Then one cannot determine the phase shift in the Michelson-Morley experiment. The null result of the Michelson-Morley experiment is trivial from the viewpoint of quantum mechanics. So the null result of the Michelson-Morley experiment does not favour for the invariance of the electromagnetic field and cannot serve as an evidence for the special relativity. Therefore there is no necessity in the Lorentz transformation for the space and time to explain the null result of the Michelson-Morley experiment. Then it is reasonable to consider electrodynamics in the Euclidean space and absolute time of a preferred reference frame. One should use the Galilei transformation for the space and time. The Maxwell-Lorentz equations are not invariant under the Galilei transformation. That is electromagnetic field is non-invariant. Such an approach was developed in [3-6].
Note that Marinov [7] determined the velocity of the Earth with respect to a preferred reference frame in the one-way experiment utilizing the rotating toothed wheel ahead of the receiver. The result of the Marinov experiment supports non-invariance of the electromagnetic field. Note that Kholmetskii [8] investigated the problem of invariance for the stationary electromagnetic field. He showed theoretically and experimentally that the Faradey induction law is non-invariant.
Non-invariance of the Maxwell-Lorentz equations under the Galilei transformation means that the size of photon and correspondingly the time characterizing photon are shifted in the moving frame as

$$r_{ph} \propto \frac{(1-v/c)}{(1-v^2/c^2)^{1/2}} \qquad t_{ph} \propto \frac{(1-v/c)}{(1-v^2/c^2)^{1/2}}. \tag{10}$$

In view of eqs. (6,9), the momentum and energy of photon are shifted in the moving frame as

$$p_{ph} \propto \frac{(1-v^2/c^2)^{1/2}}{(1-v/c)} \qquad \varepsilon_{ph} \propto \frac{(1-v^2/c^2)^{1/2}}{(1-v/c)}. \tag{11}$$

From eqs. (11) one can deduce relativistic effects for the electromagnetic wave such as Doppler effect, Sagnac effect.
Consider the dynamics of the electron with the charge e and the mass m in the electromagnetic field with the strength $\vec{E}$ [4]. In the frame moving with the velocity v, the acceleration due to the electromagnetic force experienced by the electron is given by

$$w' = \frac{eE}{m}(1-v^2/c^2)^{1/2}. \tag{12}$$

The following interpretation may be given that the mass of the electron is invariant and the electromagnetic field decreases as $E \propto (1-v^2/c^2)^{1/2}$. Thus the mass of the electron is invariant that provides invariant scale of mass for massive bodies.
We consider electrodynamics in the Euclidean space and absolute time that provides invariant scales of length and time. As follows from eq. (4) the energy of a single photon and the number of photons are bound with the Heisenberg uncertainty principle. Depending on the experiment one can measure the variation of the energy of the electromagnetic field as a variation of the energy of a single photon or as a variation of the number of photons. In the former case one measures the Doppler shift for the frequency of the electromagnetic field. In the latter case one measures the decrease of the flux of photons, $N \propto (1-v^2/c^2)^{1/2}(1-v/c)^{-1}$, with the energy of a single photon being constant. Thus, when measuring the flux of photons, the energy of a single photon is constant that provides invariant scale of mass (energy) for the electromagnetic field.
So we consider electromagnetic field in the Euclidean space and absolute time of a preferred reference frame. That is we consider electromagnetic field within the framework of Galilean electrodynamics. The scales of length, time and mass are invariant while the electromagnetic field is not. Stress once more that we can deal with invariant scales of length, time and mass and non-invariant electromagnetic field due to the quantum behaviour of the electromagnetic field.

When considering electromagnetic field as a quantum wave within the framework of Galilean electrodynamics, there is no need in the double standards for the scales of length, time and mass as well as for the electromagnetic energy. In this way we make the relativity a verifiable theory.
It is reasonable to think that relativistic effects pertain only to the energy and momentum of the electromagnetic field while the scales of length, time and mass are not relativistic. Then it is reasonable to think that electromagnetism is a relativistic phenomenon while gravitation is not. Then gravitation should be described within the framework of Newtonian mechanics. Relativistic effects in the gravitational potential pertain only to the energy and momentum of the electromagnetic field while the scales of length, time and mass do not depend on the gravitational potential.

3. Relativistic effects within the framework of newtonian gravity

So we consider electromagnetic filed in the gravitational potential within the framework of Newtonian mechanics. Suppose that electromagnetic wave propagates with the velocity c with respect to a preferred reference frame independently of the gravitational potential. Put the frame with gravitational potential Φ into correspondence with the frame with the velocity v

$$v^2 = 2\Phi . \tag{13}$$

In view of eq. (11), electromagnetic energy is a function of the gravitational potential

$$\varepsilon_{em} \propto (1 - 2\Phi / c^2)^{1/2} \tag{14}$$

that yields the gravitational redshift of the electromagnetic field.
Describe the bending of light in the gravitational potential [9]. Put the gravitational mass into correspondence with the electromagnetic energy. Consider the relation between the gravitational mass and the electromagnetic energy within the Newtonian mechanics. Put the particle of the mass m moving with the speed of light into correspondence with the electromagnetic field of the energy ε_{em}. Then the kinetic energy of the particle is equivalent to the electromagnetic energy

$$\varepsilon_{em} = \frac{1}{2} mc^2. \tag{15}$$

This means that the electromagnetic field possesses the gravitational mass and can take part in the gravitational interaction. The electromagnetic field as a particle of the mass m suffers the attraction of a gravitating body. Let the electromagnetic field move transversely to a gravitating body. At a length Δl, the electromagnetic field acquires the radial momentum due to the gravitational potential Φ

$$\Delta p_r = \frac{m\Phi\Delta t}{r} = \frac{2\varepsilon_{em}\Phi\Delta l}{rc^3}. \tag{16}$$

The deflection of the electromagnetic field at a length Δl is given by

$$\Delta\theta = \frac{\Delta p_r}{p} = \frac{2\Phi\Delta l}{rc^2} \tag{17}$$

wherein we use the expression for the momentum of the electromagnetic field

$$p = \frac{\varepsilon_{em}}{c}. \tag{18}$$

We obtain the same result as in the general relativity. Remind [1] that the experimental data support this result. Thus the true description of the bending of light by a gravitating body is possible in the Newtonian mechanics.
It is worth to stress that we use the relation between the electromagnetic energy and gravitational mass other than that in the general relativity. The well known Einstein relation $\varepsilon = mc^2$ follows from the relativistic dynamics [1] based on the interpretation of eq. (12) as an increase of the mass of electron. This is just the opposite to the interpretation under consideration. When dealing with the ratio of the mass to the electromagnetic field, both interpretations are the same. The Einstein relation makes sense only for the ratio of the mass to the electromagnetic field. Then, under the considered approach, one can use the relativistic dynamics in the electromagnetic interaction and

cannot in the gravitational interaction. When considering the gravitational interaction we should use the relation given by eq. (15) instead of the Einstein one.

4. Effective gravity: footprints in the solar system

So we should describe the motion of the massive bodies in the gravitational potential within the Newtonian mechanics. Hence explanation of the anomalous shift of the perihelion of Mercury remains open. In [10] the effective gravity is considered which includes the Newtonian potential and the fixed potential

$$\Phi = -\frac{Gm}{r} + \Psi \tag{19}$$

with the fixed potential of a gravitating body being

$$\Psi = \frac{4\pi}{3} G\rho r_{NS}^2 \tag{20}$$

where ρ is the density of the body, r_{NS} is the radius of neutron star for the body. At the radius of neutron star the fixed potential balances the Newtonian potential. It is worth to note that the fixed potential does not modify the Newtonian gravity. The fixed potential produces the outward inertial acceleration

$$w_{in} = \frac{\Psi}{r}. \tag{21}$$

So introduction of the fixed potential Ψ means the presence of the inertial repelling forces.

Footprints of the fixed potential of the Sun for a particle orbiting around the Sun may be revealed as an anomalous shift of the perihelion of the Keplerian orbit of a particle or as an anomalous shift of the frequency of light or as a polarization of the particle's satellite orbit. Although we have the rough estimation of the fixed potential of the Sun we can test it by comparing the results of different observations. Following [10] we shall consider three consequences of the fixed potential of the Sun: the shift (advance) of the perihelion of Mercury, the shift of the frequency of light at the Earth seen in ranging of distant spacecraft, and the polarization of the Moon's orbit.

The shift (advance) of the perihelion of Mercury due to the fixed potential of the Sun per revolution is given by

$$\delta\varphi \approx \frac{6\pi a(1-e^2)\Psi_S}{Gm_S} \tag{22}$$

where a is the semi-major axis, e is the eccentricity, Ψ_S is the fixed potential of the Sun, m_S is the mass of the Sun.

The inertial acceleration due to the fixed potential gives contribution into the first order relativistic effect. The inertial acceleration of the Earth w_E due to the fixed potential of the Sun can be seen in ranging of distant spacecraft as a blue shift of the frequency of the electromagnetic field

$$\frac{\Delta\omega}{\omega} \approx \frac{w_E t}{c} = \frac{\Psi_S t}{c r_{SE}} \tag{23}$$

where r_{SE} is the distance between the Earth and Sun. In ranging of distant spacecraft, the acceleration of the Earth outward the Sun looks like the acceleration of the spacecraft inward the Sun, $w_{sc} = 2w_E$. The factor 2 takes into account that the acceleration of the Earth gives contribution into the shift of the reference frequency during the time of two-leg light travel while the acceleration of the spacecraft gives contribution into the shift of the observed re-transmitted frequency during the time of one-leg light travel.

The fixed potential of the Sun yields the polarization of the Moon's orbit in the direction of the Sun. While adopting the sum $\Phi_S + \Psi_S$ as a gravitational potential of the Sun at the radius r_{SE}, we reveal an additional acceleration of the Moon outward the Earth average for the period of revolution of the Moon

$$w_M = \frac{\Psi_S}{\sqrt{2} r_{SE}}. \qquad (24)$$

This can be seen in lunar laser ranging as a first order relativistic effect.
Compare the above three effects. Determine the fixed potential of the Sun from the data on the anomalous shift of the perihelion of Mercury, 43 arcseconds [11]. Then we obtain the value, $\Psi_S = 6.4 \times 10^5$ cm^2/s^2. This value yields the effective inertial outward acceleration of the Earth, $w_E = 4.25 \times 10^{-8}$ cm/s^2. This may be interpreted as the inward acceleration of the distant spacecraft, $w_{sc} = 2w_E = 8.5 \times 10^{-8}$ cm/s^2, which is consistent with the observed anomalous inward acceleration acting on Pioneer 10 and 11, $w_P = (8.74 \pm 1.25) \times 10^{-8}$ cm/s^2 [12]. The acceleration of the Moon outward the Earth due to the fixed potential of the Sun is equal to $w_M = \Psi_S / \sqrt{2} r_{ES} = 3.0 \times 10^{-8}$ cm/s^2. This can be seen in lunar laser ranging as a velocity, $v_M = \Psi_S r_{EM} / \sqrt{2} c r_{ES} = 3.8 \times 10^{-8}$ cm/s, where r_{EM} is the distance between the Earth and Moon. There is a difference in the rate of the lunar semi-major axis increases obtained from telescopic observations and from lunar laser ranging, $\dot{a}_{LLR} - \dot{a}_{tel} = 1.29$ cm/yr$= 4.1 \times 10^{-8}$ cm/s [13]. This anomalous increase in the lunar semi-major axis is consistent with the acceleration of the Moon outward the Earth due to the fixed potential of the Sun. Thus the polarization of the Moon's orbit due to the fixed potential of the Sun may explain the anomalous increase in the lunar semi-major axis. So the fixed potential of the Sun allows to explain three anomalous phenomena, the anomalous shift of the perihelion of Mercury, the anomalous acceleration acting on Pioneer 10, 11, the anomalous increase in the lunar semi-major axis.

5. Summary

It is considered electromagnetic field in the Euclidean space and absolute time of a preferred reference frame. Electromagnetic wave propagates with the velocity of light independently of the velocity of the source. Within the quantum mechanics framework, the Heisenberg uncertainty principle restricts the wave properties of the electromagnetic field by the size of photon. This means that one cannot determine the travel time and the corresponding phase by measuring the energy (frequency) of photons. Within the quantum mechanics framework, one cannot determine the phase shift in the Michelson-Morley experiment. Then the null result of the Michelson-Morley experiment cannot serve as an evidence for the special relativity. Therefore it is reasonable to refuse from the Lorentz transformation and to consider the theory of relativity within the framework of Galilean electrodynamics, with the space and time following the Galilei transformation. So we refuse from the invariance of the Maxwell-Lorentz equations under the Lorentz transformation then electromagnetic field is non-invariant.
Due to non-invariance of the Maxwell-Lorentz equations under the Galilei transformation the size of photon in the moving frame is shifted that results in the corresponding shift of the energy of photon. This shift can explain relativistic effects for the electromagnetic wave such as Doppler effect, Sagnac effect.
We consider electrodynamics in the Euclidean space and absolute time that provides invariant scales of length and time. When measuring the flux of photons, the energy of a single photon is constant while the number of photons is a function of the velocity of the frame. In this case the energy of a single photon provides invariant scale of mass. Then one can provide invariant scales of length, time and mass in the electrodynamics.
In the interpretation of relativity under consideration, relativistic effects pertain only to the energy and momentum of the electromagnetic field while the scales of length, time and mass are not relativistic. We can infer that electromagnetism is a relativistic phenomenon while gravitation is not. Gravitation should be described within the framework of Newtonian mechanics. One can extend relativistic effects in the moving frame to the frame with the gravitational potential by means

of the principle of equivalence. Hence electromagnetic energy is a function of the gravitational potential that yields the gravitational redshift of the electromagnetic field.
Relativistic effects in the gravitational potential pertain only to the energy and momentum of the electromagnetic field while the scales of length, time and mass do not depend on the gravitational potential. Then the motion of light and massive bodies should be described within the framework of Newtonian mechanics. It is given the description of the bending of light in the gravitational potential by putting the gravitational mass into correspondence with the electromagnetic energy. It is proposed an explanation of the anomalous shift of the perihelion of Mercury with the hypothetical fixed potential of the Sun. It is shown that the hypothetical fixed potential of the Sun can also explain the anomalous acceleration acting on Pioneer 10, 11 and the anomalous increase in the lunar semi-major axis.

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Properties of the “field ether”

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”...Neither will a space in absolute rest endowed with special properties be introduced nor will a velocity vector be associated with a point of empty space in which electromagnetic processes take place”. (A. Einstein)

Physicists of X1X century successfully used ether to describe various phenomenons. Synthesis of different branches followed their development. A yield idea here was using a field concept. A field notation means to join force vector with every point. Two adjacent volumes act one at another. But at what substance the force acts?
Before the Relativity Theory appeared the Ether was such a substance. It was thought absolute as space. Later the Relativity Theory rejected Absolute Ether.
The “ether” concept regularly recovers, like the Phoenix bird from the ashes, despite of a great number of the destructive attacks performed by the relativistic theory. This paper presents the analysis of the Maxwell ether properties, which are complemented by the statement of its density dependence from the field. It was shown that the “field ether” was pulled into the area of higher field strain increasing its own density and hence increasing it dielectric permeability ε. Field ether of electromagnetic wave exists as ensemble of jets. So it is not absolute and it can not be used as a frame of references.

1. Introduction

J. C. Maxwell based his electromagnetic theory on the coincidence of the equations for motion of fluids under hydrodynamic pressure, and equations describing charges and currents behavior in the field of magnetic or electric forces. He stated that the ether has certain mechanical and electrical properties [1– 4]. Later, The Relativity Theory washed out these electrodynamics foundations. Nevertheless physicists working in various areas keep trying to involve some ether concepts for fine effects analysis. Modern electrodynamics rejects the ether for it is yet absolute. Than, classical electrodynamics was not capable to analyze micro scale processes due to linear deformations of Maxwell ether. It seems quite natural to revise old ether concept from the modern point of view.

2. Ether density distribution

According to the modern notion electromagnetic field (EMF) is a simple independent substance that “cannot be reduced to anything simpler” [5]. “After spreading far from it source field gains own character” [6]. If dielectric and magnetic permeability $\varepsilon_v = \mu_v = 1$ EMF follows to the linear Maxwell equations:

$$\mathbf{rot\ E} = -\partial \mathbf{B}/\partial t \qquad (1)$$

$$\mathbf{rot\ H} = \partial \mathbf{D}/\partial t. \qquad (2)$$

However the wave process, that is characterized by conversion the magnetic flow changes into electric field vortexes and vise versa, does not occur in the empty space but within small region of the space where certain energy density w are present. The fact that we cannot picture clearly the medium required for the vortexes conversions should not lead to the denial of this medium existence. The region where Maxwell’s transmutations occur is filled in by the substance which could be named by various terms: ether, field substance, or “physical vacuum”. The expressions (1) and (2) are valid only for the regions where $w \neq 0$. Furthermore the wave spreading does not require this substance occurring everywhere outside. It is sufficient enough to have ether bunch traveling synchronously with the spreading of the wave train.

So, how such ether is distributed over the space? Let’s chose for simplicity “attendant” coordinates. That allows us to take account of electric field only. Then following to the Maxwell idea we assume if empty space has energy **DE** is concentrated in the ether. Ether mass density τ is

$$\tau = \mathbf{DE}/c^2. \qquad (3)$$

We can imagine that when some source generates a field, at first there is an extrusion of the jet stream of ether, which occurs to be polarized instantly. Let’s name such substance as “field ether” in order to distinguish it from uniform ether medium of XIX century. Figuratively speaking, every beam of light uses it own jet of ether. This extremely important statement of the ether localization and flow opens new approached to the problem of compatibility the ether concept and the principles of Relativity Theory. It is reasonable to stress out that in the analysis of weighty matter motion the ”field ether” cannot be used any more as a base for the universal frame of references. Introduction of the localized ether concept appears to be a significant base for the rationality in using the simultaneity definition given by Einstein as well as in interpretation of the local time concept.

Suggested concept of the ether demands corresponding corrections to some classical electrodynamics equations. The ether theory and Lorenz electronic theory produce different expressions for the bulk force that follows from the Maxwell tension tensor [7]

$$T_{\alpha\beta} = E_\alpha D_\beta - 0.5\delta_{\alpha\beta}E_\gamma D_\gamma + H_\alpha B_\beta - 0.5\delta_{\alpha\beta}H_\gamma B_\gamma \tag{4}$$

Tension tensor divergence determines some bulk force that could be recorded as

$$\partial T_{\alpha\beta}/\partial x_\beta = [\mathbf{F}_{ev} + \partial[\mathbf{D},\mathbf{B}] /\partial t]_\alpha. \tag{5}$$

$\mathbf{F}_{ev}$ is force acting on non-uniform bodies, charges and currents; the second term that remains in emptiness is pro rata to the derivative of the Poynting vector. In XIX century this was understood as the field effect at the ether. But it caused a perplexity after the ether concept rejection: it was not possible to understand, what this force is acting on. Since the only force that should be acting in electrodynamics in a substance absence is the Lorenz force, the Relativity Theory demands was satisfied by voluntary replacement “+” sign by the “-” sign [7]. In the model with field ether it is acceptable to leave “+” sign in (5).

From equations (1) and (2) could be derived the expression

$$\mathbf{div}[\mathbf{E},\mathbf{H}] = -(\mathbf{H},\partial\mathbf{B}/\partial t) - (\mathbf{E},\partial\mathbf{D}/\partial t) \tag{6}$$

that represents the local energy conservation law [7,9]

$$\mathbf{div}\boldsymbol{\sigma} = - \partial w/\partial t. \tag{7}$$

We rewrite it in order to provide it with new physical sense using for right side

$$\tau = (1/2)(\varepsilon\varepsilon_o E^2 + \mu\mu_o H^2)/c^2. \tag{8}$$

For plane monochromatic wave the mean value of the Poynting vector is equal to the mean value of energy density that is multiplied by speed vector $\mathbf{u}$, [7]

$$\boldsymbol{\sigma}_m = w_m\mathbf{u} = \tau_m c^2\mathbf{u}. \tag{9}$$

If designated region is so small that within its boundaries the τ is constant, we can omit the mean subscript. This all allows us to rewrite the energy conservation law (6) in terms of continuity equation, which wide scale application in hydromechanics is related to the fluids’ continuousness;

$$\mathbf{div}(\tau\ \mathbf{u}) = - \partial\tau/\partial t, \tag{10}$$

The important conclusion follows from the last equation: ether maintains its own continuity. The polarized ether in this respect behaves like a liquid, the particles of which are attracting each other. For the narrow light beam these forces prevent the ether stream disintegration. Furthermore, interaction of the ether streams combines it into a single whole substance spread over the space. Since at least weak field exists at any pint of space, the ether is present everywhere, while its density beyond the stream is extremely small, the adhesive forces are correspondingly insignificant. In this meaning the ether is ubiquitous, while its density is significantly changeable through the space as well as over the time. Given ether interpretation helps us to change our notation of the “physical vacuum” of quantum electrodynamics: vacuum is also moveable and consists of separate dense jets bounded by rare component.

3. Bases for quantitative description of the “field” ether

J. Maxwell [3,§15] pointed out that parts of the ether are polarized, elastically bounded, and field forces can cause their displacement. We assume that ether possesses only dielectric properties and does not dissipate energy. Than according to the Kramers-Kronig expressions $\varepsilon(\omega)$=const. So it is enough to examine only how ether’s dielectric permeability ε depends on static electric field strength $\mathbf{E}$. Let’s use two typical electrostatic problems to illustrate ether density distribution: the field of the point charge $\mathbf{q}$, and the field of the endless straight uniformly charged wire with a linear charge density λ («linear charge»). Studying interaction of the ether fragments, Maxwell [4,§59] concluded that "ether medium should be in the state of mechanical stress" that causes "charged bodies' motion". Since ether fragments are capable to be polarized and to gain the dipole moment depending from E, two adjacent fragments located along the field line are appeared to be polarized in the same direction, hence they attract each other. Mutual attraction of the neighbor fragments is give rise to the pressure that one fragment produces on another. Sequent, compressing forces are acting on the every ether fragment. Bearing in mind the existence of spherical or cylindrical symmetry in the chosen example problems, we can state the existence of pressure gradient in these cases. Due to the charge field action the ether fragments displace toward the center producing the gradient of ether density directed toward the charge. This mechanism of compression is similar to the known processes in material mediums that result in compacting of a liquid material dielectric caused by electrostriction and drawing this dielectric into the places with higher electric field strength.

Then we can notice that displacement of a separate ether fragment towards the center and its compression are followed by the growth of "counter-field" strength as compression results in the increase of field lines density. So ether compression (like material medium compression) produces its polarization growth, hence it reduces external field effect. Usually such response of a medium is interpreted as increase in its dielectric permeability ε. The process in electrostatic field, resulting in the increase of dielectric permeability due to medium compression and growth of its polarization, will be called as "field reduction" effect. Field reduction typical for dielectric materials differs from the shielding caused by free mobile charges and observed in conductors. The field reduction effect is known also in the quantum theory. It is used for example in the analysis of the interaction between an electron and external electrostatic field in the "physical vacuum". Right after the virtual e - p pair springing up, the polarization of this pair by the electron occurs: real electron attracts virtual positrons of "vacuum" and repels virtual electrons. The electron is appeared to be covered by the layer made of the positrons originated from virtual pairs; the displacement of the elements of these pairs is termed as "physical vacuum" polarization. The size of polarization region is comparable by order with the Compton Wavelength ($\lambda_e = 2.43 \cdot 10^{-12}$м). The last case should be rather termed as field reduction, than electron screening and its effective charge reduction.

It does not matter, how to call the substance responsible for the field interactions, ether or "vacuum", it is important that its polarization leads to the field reduction.

Thus the task is simplified to the analysis of electrostatic field forces acting on the ether fragments that have dielectric nature and can be polarized, deformed and displaced by the forces action.

4. Ether dielectric permeability

The external electrostatic field bulk force **f**dv acting on the volume part dv of dielectric substance could be derived from the math expression for the work done on the field energy change

$$\delta U = \int \mathbf{E} \cdot \delta \mathbf{D} dv. \tag{11}$$

Existence of the slow virtual dielectric flows are usually assumed for the forces calculation [5-7], where the work done by the external forces over each fragment is taken to be equal to the scalar product of the bulk force **f** by the slow speed **u** of its flow. The rate of the field free energy change at the virtual ether flow can be express as

$$dU/dt = - \int (\mathbf{u} \cdot \mathbf{f}) dv. \tag{12}$$

Using the calculation procedure [6,7], which is typical for electrostatics problems, we have to convert (11) to appearance (12). Energy changes occur only when dielectric permeability $\boldsymbol{\varepsilon} \neq 1$.

Total energy change could be determined by the integration over the whole volume in the frame of references related to the charge

$$\delta U = \int (\mathbf{E_2 D_2} - \mathbf{E_1 D_1}) dv. \tag{13}$$

Taking in the consideration field symmetry and using for the normal component of induction vector the obvious equation $D_1 = D_2$ (since regardless to the flow, the distance between center of gravity of the element dv and the charge remains constant) we can state that

$$\delta U = \int (\mathbf{E_2 D_1} - \mathbf{E_1 D_2}) dv = - \varepsilon_o \int (\varepsilon_2 - \varepsilon_1) \mathbf{E_2 E_1} dv. \tag{14}$$

As $|\mathbf{E_2} - \mathbf{E_1}| << |\mathbf{E_1}|, |\mathbf{E_2}|$; $|\varepsilon_2-\varepsilon_1| << \varepsilon_1,\varepsilon_2$

$$dU/dt = - \varepsilon_o \int \mathbf{E}^2(\partial\varepsilon/\partial t)dv. \tag{15}$$

If perform several rearrangements (see attachment), switch from ether density to the pressure, and equate the obtained bulk force to the pressure gradient, then we can derive from (15) the differential equation

$$(1/p)\ \partial p/\partial r = \varepsilon_o \partial\ (\mathbf{E}^2(d\varepsilon/dp))/\partial r. \tag{16}$$

Growth of ε leads to the decrease of separate ether fragment energy ($U \sim 1/\varepsilon$). As a result, it becomes energetically more beneficial for a separate fragment to be drawn in the higher strength field region. Such ethers' characteristic was specified by Maxwell particularly [4].

To integrate the equation (16) I assume that

$$\varepsilon = 1 + p/Y. \tag{17}$$

We suppose that Y is the universal constant which characterizes the field reduction degree as a result of ether compression. Let's name the constant Y as baro-reduction ether module. Replacing ε in (16) via (17) results in

$$\partial(\ln p)/\partial r = \varepsilon_o \partial\ (\mathbf{E}^2/Y)/\partial r. \tag{18}$$

We use for (18) well known expressions for the fields of the point charge **q** or (and) of the uniformly distributed along the infinite straight line charge with a linear density **λ**.

$$\mathbf{E_s} = q/4\pi\varepsilon_o\varepsilon_s r^2 \tag{19a}$$

$$\mathbf{E_c} = \lambda/2\pi\varepsilon_o\varepsilon_c r \tag{19b}$$

After integration and substitution p by ε we receive for the spherically or cylindrically symmetrical fields:

$$\ln(\varepsilon_s - 1) - \ln(\varepsilon_{start} - 1) = q^2/\ Y16\pi^2\varepsilon_o\varepsilon_s^2 r^4 \tag{20a}$$

$$\ln(\varepsilon_c - 1) - \ln(\varepsilon_{start} - 1) = \lambda^2/\ Y4\pi^2\varepsilon_o\varepsilon_c^2 r^2. \tag{20b}$$

Here $\ln(\varepsilon_{start}-1)$ is used as a constant of integration which equals to the ε value at the significantly remote point. We can ignore this constant of integration if analysis will be limited by the internal field regions where pressure and dielectric permittivity reach the large values ($\varepsilon>>2$). This approach gives us the formulas

$$\ln(\varepsilon_s - 1) = q^2/Y16\pi^2\varepsilon_o\varepsilon_s^2 r^4. \tag{21a}$$

$$\ln(\varepsilon_c - 1) = \lambda^2/Y4\pi^2\varepsilon_o\varepsilon_c^2 r^2. \tag{21b}$$

Unfortunately formulas (21) could not be recorded in the form of obvious dependency $\varepsilon = f(r)$, but assigning different values to ε (starting from ε=2), we can unambiguously receive corresponding values of r. To carry on calculations we need to know module **Y** and assign q and λ. There are no experimental data for the non-linear interaction of the electrostatics fields in vacuum. Let's take, however, that **Y** is universal ether characteristic independent from the experimental conditions. I estimated Y module by extrapolation summative experimental data [10,11] for depressive non-linearity for several isotropic and crystal optical mediums, that are represented on the Figure 1. There $\tilde{n}_2$ (cm^2/kW) is the field depended non-linear coefficient of increment to the real part of light refraction index, and τ_{nl} is the time for non-linear response establishment.

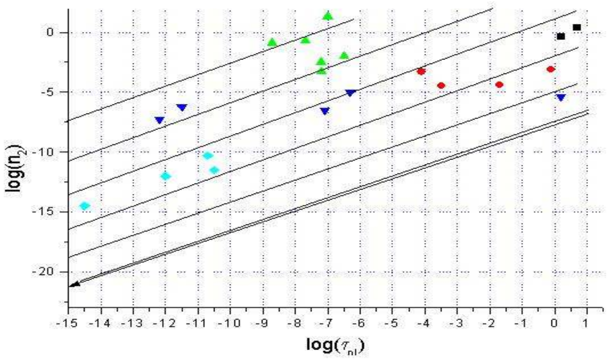

Fig. 1. Extrapolation of data and estimation of module Y.

We can use interdependences from [10] to rewrite the well-known in non-linear optics dependence of the refraction index n from the field strain amplitude module Ã :

$$\varepsilon = n^2 \approx n_o^2 + \tilde{n}_2 |\tilde{A}|^2 (3/4\pi), \tag{22}$$

n_o is the refraction index for small light intensity. The $(3/4\pi)$ factor makes an account on the fact that $\tilde{n}_2$ on the Figure 1 is expressed in cm^2/kW while $\tilde{A}$ is in the CGSE units. For the fields interaction in empty space ($n_o= 1$), using (17), (22), and (A6), we can receive the expression

$$1/Y = [6 /(2\varepsilon-1)]\, \tilde{n}_2. \tag{23}$$

According to the phenomenological theory concepts for a certain refraction index the coefficient of increment $\tilde{n}_2$ grows when the time for non-linear response establishment increases. Correspondingly, the data for non-linear coefficient $\tilde{n}_2$ for number of optically active materials on the Figure 1 are grouped by means of the straight lines family with the same slope with optical density used as a parameter. In order to estimate the coefficient $\tilde{n}_2$ for the non-linear interaction of light with light in empty space we need to extrapolate the encompassing dependencies [10,11] of non-linear response $\tilde{n}_2$ to the time for non-linear response establishment 10^{-15}c. As far as at "zero" field strength the optical density of ether is smaller than for any optical material, the dependence for $\tilde{n}_2$ from τ_{nl} corresponding to the value $n_o=1$ should be used. Therefore the additional straight line with the same slope is plotted on this graph lower on one repeating period of the bordering straight lines. (There is some arbitrariness on this step, but we can hope that the estimation will not be far from the real value). According to these data, the ether non-linear coefficient $\tilde{n}_2$ equals to $3\cdot10^{-21}$ cm^2/kW; and correspondingly for this value

$$Y \approx 10^{19}\ \text{N/м}^2 \tag{24}$$

In order to compute by formulas (21) radius r for a given value of ε we have to set charge value q and linear density λ. We choose the point charge to be equal to the charge of an electron $q = 1.6\cdot10^{-19}$ Q. For the linear distribution we assume that straight infinite line wire has the same charge spread over the length equal to the first Bohr's orbit ($2\pi\cdot0.5\cdot10^{-10}$m), which correspond to the $\lambda = 0.482\cdot10^{-9}$ Q/m. Then we group all constants in the separate co-factors (correspondingly, for the spherical and cylindrical field symmetry):

$$R_s^{\,4} = q^2/Y16\pi^2\varepsilon_o \approx 1.834\cdot10^{-48}\ \text{м}^4 \tag{25a}$$

$$R_c^{\,2} = \lambda^2/Y4\pi^2\varepsilon_o \approx 0.665\cdot10^{-26}\ \text{м}^2. \tag{25b}$$

Now the expressions (21a), (21б) take an appearance

$$r_s^{\,-2} = R_s^{-2}\varepsilon_s \sqrt{\ln(\varepsilon_s - 1)}. \tag{26a}$$

$$r_c^{\,-1} = R_c^{\,-1}\varepsilon_c\sqrt{\ln(\varepsilon_c - 1)}. \tag{26b}$$

It is obvious that R_s and R_c are certain critical lengths that determine areas for the change of the permeability dependence character from radius ε(r). Indeed, each of the dependencies (26a) or (26b) contains the product of two functions that have different rhythm of changes. Correspondingly graphs lg(ε) - lg(1/r) (on the Figures 2,3 distances are shown in cm) have two regions.

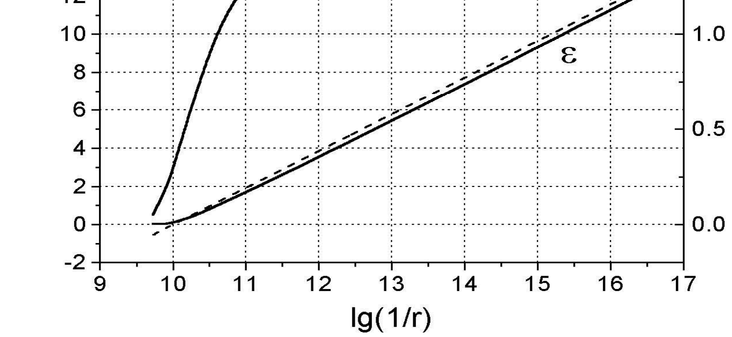

Fig.2. Ether dielectric permeability and potential for point charge.

For r>R the logarithmic component plays the key role in (26), while closer to the center – the law $\varepsilon(r)$ is determined by factor $1/r_s^2$ (sphere) or $1/r_c$ (cylinder). In both cases as ether compression increases, its dielectric permeability grows and reaches the value of $\varepsilon \sim 10^5$ on the distance $r_c \sim 10^{-19}$ m or $r_s \sim 10^{-15}$ m. Let's name the spherical or cylindrical zone of radius R starting from which ε rapidly grows as "reduction zone". Note that for $q = 1.6 \cdot 10^{-19}$ Q the radius of the spherical "reduction zone" is about $R_s \sim 10^{-12}$ м = 1pm, which is close to the Compton Wavelength of electron. For linear distributed charge this distance can be estimated as $R_c \sim 0.8 \cdot 10^{-13}$ м.

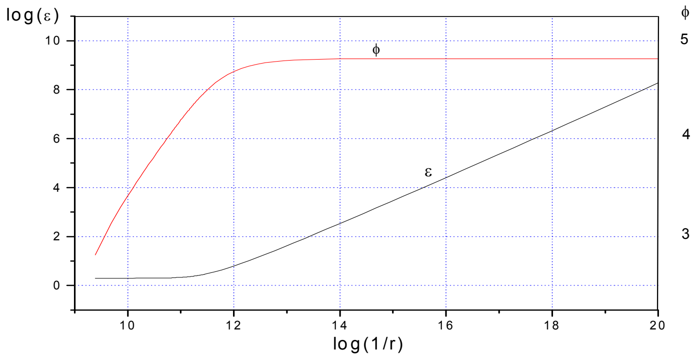

Fig. 3. Ether dielectric permeability and potential for linear distributed charge.

The type of dependence of the field potential from distance is also of great interest. The effect of field "reduction" resulting from the ether thickening allows to get rid of the overwhelming over classical electrodynamics infinite point charge energy phantom. Potential is defied as a work done for displacement of the unit sample charge

$$\varphi = \int E dr. \qquad (27)$$

The result of numerical integration by (27) is presented on the Figures 2, 3. If one have ignored the "reduction" effect and left ε= 1 he would receive the straight line with a slope 1/r that results in $\varphi \to \infty$ at $r \to 0$. But, according to the graph, the potential is growing if distance decreases, while in the range of distances $r_s \sim r_c = 10^{-14} \ldots 10^{-16}$ m the φ value goes to the saturation level. The filed **E** inside the selected spherical or cylindrical region with radius $r_{cr} \sim 10^{-15}$ m is limited, i.e. the charge is blocking itself out. A zone inside which the field becomes constant, i.e. where the complete field reduction occurs, will be called for short as "micro-block". It is interesting to note that in both cases we get the same magnitude of "micro-block" radius $\sim 10^{-15}$ m though at the edge of cylindrical "micro-block" permeability reaches only ε =38 (as we mentioned at the edge of point charge "micro-block" $\varepsilon \sim 10^5$). Please note, that in the φ value computing we did not use any arbitrary value for any physical parameter (for the only one theory parameter Y we took the value resulting from extrapolation of the dependencies originated from non-linear optics). Nevertheless the main dependence for dielectric permeability (21) helped us to determine two distances that are often used in the quantum electrodynamics: "micro-block" radius coinciding by the order of magnitude with "classical electron radius" $2 \cdot 10^{-15}$m. "Reduction zone" radius $R_s = 10^{-12}$ m is close to the specific length of the polarization region for "physical vacuum", i.e. to the order of magnitude of the Compton Wavelength for electron $\lambda_e = 2.43 \cdot 10^{-12}$m. In spite of different symmetry and great slopes $\varepsilon(r)$ distinction both cases shows almost the same magnitudes of "reduction zone" and "micro-block" radii.

5. Conclusion

The space devoid of omnipresent substance is demanded for relativity theory. To adjust this standing with medium necessity for Maxwell's equations it is assumed that ether density is proportional to the field energy density. It is supposed that ether has properties of a dielectric medium that could be polarized, elastically deformed and displaced by electrical forces. It is shown that the ether follows to the continuity equation and behaves like liquid. It is inferred that ether should inevitably compress itself, increasing its density. Dielectric permeability enlarged especially rapid inside so-called "reduction zone" with radius $R \sim 10^{-12}$ m. So inside microscopic bulks ether behavior is to be described by non-linear equations. At the distance $\sim 10^{-15}$m from point static charge dielectric permeability rises to $\varepsilon \sim 10^5$.

Due the "reduction effect" electrostatic field is forming from itself a special region ("micro-block") with radius $\sim 10^{-15}$m, within which the potential is limited and field blocks itself up. For the linearly distributed charge the radius of the "reduction zone" is approximately the same.

"Field ether" of free EMF exists as ensemble of separate jets. The attraction that acts between jets is very slight, but it allows us to say that ether is omnipresent. It is obvious that uniform physical vacuum contradicts the relativity theory, so its density is also altering.

The statement of the ether concentration and flow opens new approached to the problem of the ether concept compatibility with the principles of Relativity Theory. It is reasonable to stress out that in the analysis of weighty matter motion the "field ether" cannot be used as a base for the universal frame of references. Introduction of the ether concentrated jets concept appears to be a significant base for the rationality in accounting for the simultaneity definition given by Einstein as well as in interpretation of the local time notion.

Attachment

To perform several rearrangements in (15) we use ε and τ functional dependences from velocity vector **u**

$$d\varepsilon/dt = \partial\varepsilon/\partial t + (\mathbf{u}\ \mathbf{grad}\varepsilon), \tag{A1}$$

$$d\tau/dt = \partial\tau/\partial t + (\mathbf{u}\cdot\mathbf{grad}\tau), \tag{A2}$$

and

$$d\varepsilon/dt = (d\varepsilon/d\tau)(d\tau/dt). \tag{A3}$$

Using (10) one can get from (A2)

$$d\tau/dt = -\operatorname{div}(\mathbf{u}\tau) + (\mathbf{u}\cdot\mathbf{grad}\tau), \tag{A4}$$

or

$$d\tau/dt = -\tau\operatorname{div}(\mathbf{u}). \tag{A5}$$

To switch from ether density to the pressure one could express p from Maxwell's formula

$$|T| = p = \varepsilon_o\mathbf{E}^2(2\varepsilon-1). \tag{A6}$$

After dividing (A6) by (3) one get

$$p/\tau = (2\varepsilon-1)c^2/2\varepsilon = c^2(1-1/2\varepsilon). \tag{A7}$$

On neglecting by dependence p(ε) one can get

$$d\tau/dt = -\tau\operatorname{div}(\mathbf{u}) = (d\tau/dp)(dp/dt) \approx (dp/dt)/c^2(1-1/2\varepsilon), \text{ and } dp/dt = -p\cdot\operatorname{div}\mathbf{u}. \tag{A8}$$

So one can rewrite (A2) as

$$\partial\varepsilon/\partial t = -(\mathbf{u}\ \mathbf{grad}\varepsilon) + d\varepsilon/dt = -(\mathbf{u}\ \mathbf{grad}\varepsilon) + (d\varepsilon/dp)(dp/dt) = -(\mathbf{u}\ \mathbf{grad}\varepsilon) - p\cdot(d\varepsilon/dp)\operatorname{div}\mathbf{u}. \tag{A9}$$

It is necessary to take **u** out of parenthesis. So we rewrite second term in (A9), multiplied by $\mathbf{E}^2$

$$\mathbf{E}^2p(d\varepsilon/dp)\operatorname{div}\mathbf{u} = \operatorname{div}(\mathbf{E}^2p(d\varepsilon/dp)\mathbf{u}) - \mathbf{u}\cdot\mathbf{grad}(\mathbf{E}^2p(d\varepsilon/dp)). \tag{A10}$$

Than using Gauss's theorem one can see that div is vanishing at infinity. Finally (11) takes an appearance

$$dU/dt = -\varepsilon_o\int\mathbf{u}\{\mathbf{grad}(\mathbf{E}^2p(d\varepsilon/dp)) - \mathbf{E}^2\mathbf{grad}\varepsilon\}dv. \tag{A11}$$

So

$$\mathbf{f} = \varepsilon_o\mathbf{grad}(\mathbf{E}^2p(d\varepsilon/dp)) - \varepsilon_o\mathbf{E}^2\mathbf{grad}\varepsilon. \tag{A12}$$

Now we equate the obtained bulk force to the pressure gradient

$$\mathbf{grad}p = \varepsilon_o\mathbf{grad}(\mathbf{E}^2p(d\varepsilon/dp)) - \varepsilon_o\mathbf{E}^2\mathbf{grad}\varepsilon. \tag{A13}$$

Or

$$(1/p)\mathbf{grad}p = \varepsilon_o\mathbf{grad}(\mathbf{E}^2(d\varepsilon/dp)). \tag{A14}$$

Due to the symmetry of the field one can write

$$(1/p)\,\partial p/\partial r = \varepsilon_o\partial(\mathbf{E}^2(d\varepsilon/dp))/\partial r. \tag{A15}$$

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Macro- and micro worlds from the point of view of perception: different levels of reflecting in observer's consciousness

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In given paper it has been shown that there is a deep connection between inequalities of quantum physics $\Delta p \Delta x \leq h$ and relativity theory $v \leq c$.

The explanation of nature of Heisenberg uncertainty principles is of a great interest. It can be understudied, gone without saying, if we will turn out attention to philosophical conceptions of perception, reflection and observation [1]. The famous physicist Wigner has written [2] that in order to obtain full scientific knowledge it is necessary to understand more deep the observation and perception processes. The perception as "visual thinking" arises in consequence of spontaneous (as it is) influence of objects of real world to our sensual organs. This form of cognition is main and point of departure in cognition process. But some physicist account that scientific cognition is to obtain only objective knowledge existing out of separate individual, for example, some scientific text. One can answer by means of following Wigner's notice:"The separation our perception and law of nature is no more than simplification. Although we are convinced of it is of harmless character but nevertheless we ought not to forget about it". The perception is the whole (integer) reflection of objects, phenomena, events in consequence of spontaneous influence to sensual organs. But is the perception of micro world whole? It is not, of course! Invisible world of micro particles can't spontaneously, directly affect to our organs and therefore it is perceived by means of devices. Of course, the result of it is no whole reflection to consciousness of man. It is important to agree that the observation in macro world, for example, sunrise and the observation in micro world, for example, the changing some numbers on the device aren't the same. The understanding of what, how process is behind of these numbers on the device depends on the level of scientific knowledge of man. Thus the perception of macro world by man doesn't depend on standard of scientific knowledge but that of micro world does. As it is known from philosophy the scientific knowledge is the reflection of objective characteristics of reality to man's consciousness. Therefore the level of scientific knowledge depends on the level of reflection. From philosophy it is known that different forms and levels are presented by various kinds and levels of consciousness. The end therefore the perception of micro world depends on consciousness of man. That is why the consciousness of observer takes place in quantum mechanics. More full knowledge about which has been written by Wigner demands not to consider separately the physical phenomena and phenomena of thinking, consciousness. As Wigner has written the decisive step to such knowledge is to establish the limit of our ability to percept surrounding world. It is clear that this limit is finished by perception of our world – macro world that we see, hear, and fell. In fact the whole perception of macro world results in that there is no uncertainties at determining impulse p and coordinate x of particle, i.e. Δp and Δx equal 0 and therefore $\Delta p\Delta x=0$, more exactly $\Delta p \Delta x \leq h$. The famous philosopher Hegel would say that such being is being as it is [3]. Being as it is because it is perceived directly from our sensual organs. The no whole perception of micro world results in that there is uncertainties at determining impulse p and coordinate x of particle, i.e. Δp and Δx don't equal 0, more than 0 and therefore $\Delta p\Delta x>0$. The more exact quantitative tie between Δp andΔx was established by Heisenberg, i.e. $\Delta p\Delta x \geq h$. This inequality shows us where the perception of micro world began and plank constant h is that limit about that Wigner says above and that corresponding our ability perceive the surrounding world.

Thus if $\Delta p\Delta x \leq h$ then the surrounding world is perceived by us habitually (usually), i.e. simplify and this world is macro world and applicable physics is the classical physics. On the contrary if $\Delta p\Delta x \geq h$ then the surrounding world is perceived by us unhabitually (unusually), i.e. there is a ambiguity and such world is micro world and applicable physics is the quantum physics. The just ambiguous perception of micro world results in that the perception become another. The various perceptions mean different forms and levels of reflection which, as it is mentioned above, are presented by various kinds and levels of consciousness. Therefore in the micro world observer's consciousness is differed from one in the macro world, i.e. from usual consciousness and this difference results in that the consciousness must be accounted and, in reality, it is accounted in the quantum mechanics. Is there another, with the exception of Plank constant h, limit corresponding our ability perceive the surrounding world? Yes, there is and it is a velocity of light c. From the philosophy it is known that space and time are apriori forms of contemplation [4]. The development of the relativity theory results in such conclusions that following from experiments objective properties of space and time are reflected just by Lorentz transformations. Therefore the principal postulates of this theory says us that any physical law must satisfy Lorentz transformations and if $v \geq c$ then Lorentz transformations lose me sense. Therefore always the body motion velocity $v \leq c$. Hegel would say that it is mediocre essence. One is mediocre because we do its such, suitable for us. Thus if $v \geq c$ then the objective properties of space and time are lost and it means loss of our ability to contemplate the world around, i.e. in this case there isn't a experience. Is there a connection between the inequalities of quantum mechanics $\Delta p\Delta x \leq h$ and relativity theory $v \leq c$?

Any physicist will not argue that the source of knowledge is an experience. But is knowledge product only experience? Empiricism accounts that it is true. No knowledge without feelings and experience can arise (R.Bekon). Rationalism considers that it isn't true. Only the intellect (mind) can give knowledge generality and necessity (Dekart). Kant of genius taking none of them side, but between them, understanding that the reason can not contemplate and the sense can't think have said the following. 1) the experience have unfinished character, 2) the mind perfect knowledge [4]. Usually we account that the experience consist of only aposteriori elements. In it Kant sees its incompleteness. Kant has said that if we want to give experience finished character then we must announce that the experience consist of both the aposteriori elements and apriori elements. Aposteriori elements are sensations which we receive after experience. But what will be an aprioristic element of experience? Kant has understood that it is not sensations which as result of influence must be only aposteriori. He has understood that this element is necessarily connected with consciousness of the person, namely with speculation, contemplation. As it is known from philosophy the contemplation is the direct relation consciousness to the object. It seems us that direct relation consciousness to the object takes place in only case when the object appear us. But Kant has said that it is empirical contemplation. In order to understand what contemplation is and to separate from it the apriori contemplation Kant has introduced the following conception: phenomenon, substance, form, space and time. What appeared us is the object of empirical contemplation. In the phenomena Kant has differed substance and form. The matter of phenomenon is the sensations or variety of sensations. This variety is organized and regulated by mean of form of phenomenon, i.e. contemplation. It is appropriate to note once more that Kant has postulated that the reason can not contemplate and therefore, unlike Dekart, he has accounted that the contemplation must be only sensual. Kant has said that contemplation is important moment of the sensuality and as for as apriori contemplation is no sensual sensuality. Kant has accounted that contemplations are both real and ideal. The contemplations are real because they give a chance to be experience. The contemplations are ideal because they exist before experience. In fact, there must be this property (apriori contemplation) in the subject in order to arise the direct notion about object in consequence the influence of object on subject. If this property would absent in subject then the influence will not equal to the notion. But what are forms of apriority contemplation? Kant has accounted that it is space and time. Being in form of "pure contemplation" they already take place in the soul in the ready-made condition. By means of them (space and time) it is organized the first given to us appearance. In other words, it is come, appeared us what is organized by means of space and time. One can say that experience as paste is laid in the ready cake size(mould) or as waves some of which can be received by the receiver. Therefore Kant says that it is cognized what is come, appeared, but they aren't "things-in-themselves". "Things-in-themselves" take place behind the limit (boundary) of the contemplation and therefore it isn't cognized by us. It testifies about boundary of our cognition. Kant of genius has known that knowledge going out experience is possible. In other words, the paste that is not laid in the ready cake size(mould) or the waves some of which can't be received by the receiver. Thus first of all Kant has put the problem of existence knowledge in two forms – the empirical knowledge (into experience) and the theoretical knowledge (out of experience). He has said that the theoretical knowledge – knowledge without contemplation, knowledge about objects comprehended by mind (Kant called them noumens) is possible. But this knowledge never can be original, general. Kant has written:" The contemplation ties sensual impressions and creates from them the phenomenon: the phenomena are product of our contemplation and object of the mind (reason). The mind connects phenomena and creates from them the cognition…" So, the sensual impressions are tied by the contemplation whose apriori forms (space and time) are in our soul, intellect in ready-made condition. But which space and time? This question isn't put by Kant because at past it was known only one geometry – geometry of Evklid. Therefore Kant has accounted that it is euklidean space. He has emphasized once for all that without fail our intellect organize our space sensations in accordance with law of euklidean geometry. Kant was convinced that our mind already owing forms of euklidean space lays them on received sensual impressions which after that are organized, regulated by the ready schemes. These schemes are apriori synthetic knowledge, for example such statement as "straight line is the shortest distance between two points" or "the plane is determined by three points which isn't on the straight line" or famous euklidean axiom about parallel lines are automatically put in our intellect [5]. However from appearance other geometries – geometry of Lobachevski, Riman etc. it is clear that these schemes can be quite another. Therefore the question: "What kind of space and time is in our soul, intellect?" becomes very interesting. It is very interesting what Kant would say if he knew about these geometries consequently another kind of spice and time – other forms of apriority contemplation. Maybe, he would say that form of apriority contemplation with which we deal in our direct experience is the usual for us space and time – evklidean space. But if there are other forms of apriority contemplation – unusual for us space and time, then by means of them can be organized such way of the first given to us appearance that doesn't come directly to us in experience, but come to us through devices (instruments, apparatus). The question, that knowledge is possible in this case, can be answered by modern philosophers words [6] : "… it is also possible real knowledge about such objects which aren't directly given in our human experience. With such objects are dealt both modern micro physics and cosmology". In other words, one can contemplate object that don't come tj us in direct experience and giving of this object will be organized by another way – another space and time. In fact, in the relativity theory another form of apriori contemplation is the space of Minkovski, Finsler, etc. As it was said above, the followed from experiments objective properties of space and time are reflected by Lorentz transformations. In order that these transformations are carried out it is necessary that $v \leq c$. Thus, in spite of space of Minkovski, Finsler are quite another form of apriority contemplation nevertheless they have objective properties, but unlike usual space and time, i.e. euklidean space, they satisfy non Halliley transformations, but Lorentz transformations. It is clear that if $v \geq c$ then objective properties of space and time are lost. But what does mean the objective properties of space and time from more deep point of view. That is the properties of such space and time which are concerned to the usual contemplation. But what is unusual contemplation? The unusual contemplation is the unusual relation of consciousness to the object and therefore unusual way of the first given to us appearance which is already organized no just another bur unusual form of apriority contemplation. In this case the space and time have non objective properties, i.e. properties not following from our experiments. Is the knowledge possible in this case? Kant would answer this question that not, because the knowledge coming out boundary of experience can't be actual. Therefore it will be better if one is limited by experience – directly ore non directly one. However Hegel would say that knowledge not only can but also must go out boundary of experience because in only this case we can understand essence of being. Thus we have analyzed the notions of space and time from philosophical point of view. In the relativity theory the notions of space and time are important. According to this theory the objective properties of space and time take place in case when $v \leq c$. Also from philosophical point of view we have considered the problems connected with perception [7]. We have understood that when we don't deal with direct experience i.e. micro world then reflection is differed from usual reflection. This no whole reflection, perception was tied with Heisenberg uncertainty principles ($\Delta p \Delta x \leq h$). The perception and the contemplation are deeply connected with each other. In fact, as it is spoken above by means of contemplation apriority from of which is space and time it is appear direct notion about object. The notion is connected with perception because the notion (image) is form of early perceived object or phenomena. So, the space and time are kinds of perception. Therefore one can say that inequalities $\Delta p \Delta x \leq h$ and $v \leq c$ are connected with each other and this connection is the evidence of unity of micro and macro words laws. It is very interesting to remember the following. At past Hegel has said that mediocre essence (here $v \leq c$) and direct being ($\Delta p \Delta x \leq h$) separately taken not yet keep real knowledge about object. The essence and the being must be considered in connection with each other, in such one when from essence it is explained its phenomena or the being. Thus let's consider the following connection:

1) $\Delta p \Delta x \leq h$ and $v \leq c$. This case correspond to no relativistic macro world. Here, as Hegel said, the direct being and the mediocre essence
2) $\Delta p \Delta x \geq h$ and $v \leq c$. This case correspond to no relativistic micro world.
3) $\Delta p \Delta x \leq h$ and $v \geq c$. This case isn't possible. Here, the unusual contemplation of perceived macro world take place. This fact is postulated by the relativity theory too.
4) $\Delta p \Delta x \geq h$ and $v \geq c$. This case is possible. Here, the unusual contemplation of no wholly perceived micro world take place.

Thus, in case 3) we see that philosophy confirm the conclusion of relativity theory about body motion velocity. Sometimes the philosophy can draw a conclusion before the natural science do it. For example, in 1846 Kant wrote that three dimensionality of our space follows from character of Newton's law of universal gravitation. It is quite true, but it is proved by physicists no sooner than many years after. Kant has confirmed that from another law of gravitation would follow another structure of space, another number of measurements and if it is really possible then it is probably the God arrange it somewhere. From philosophical point of view it is very interesting the case 4). This case is just that case which, as Kant considered, go out the limit of experience. Here, the knowledge going out the boundary of experience can't be true. Therefore in Kant's philosophy the case 4) doesn't take place. However in Hegel's philosophy this case not only take place but also attract his attention. Here, the knowledge not only can, but must go out the boundary of experience because in just this case we can comprehend the essence of things.

Let's consider the case 1), when $\Delta p \Delta x \leq h$ and $v \leq c$. Let's assume x= vt. However it isn't supposed that movement is uniform, i.e. $\Delta v \neq 0$ and $\Delta x = v\Delta t + t\Delta v$. We are distinguishing from each other observers which is moving and which is rest or one can say that system in which the measurements take place is moving and that is rest. So, $m\Delta v(v\Delta t + t\Delta v) \leq h$. Hence, $v \leq \dfrac{h}{m\Delta v \Delta t} - \dfrac{\Delta v}{\Delta t} t$. On the another hand, $v \leq c$. Therefore $\dfrac{h}{m\Delta v \Delta t} - \dfrac{\Delta v}{\Delta t} t =$ c. Consequently, $t(\Delta v)^2 + c\Delta t \Delta v - \dfrac{h}{m} = 0$. Let's find the solutions Δv of this quadratic equation. So, $(\Delta v)_{1,2} = \dfrac{-c\Delta t \pm \sqrt{D}}{2t}$, where $D = c^2(\Delta t)^2 + \dfrac{4ht}{m}$. It is clear that solution $\Delta v_2 \leq 0$ doesn't our conditions. We take an interest in $\Delta v_1 \geq 0$ solution and therefore $\Delta v = \dfrac{-c\Delta t + \sqrt{D}}{2t} =$

$\frac{-c\Delta t+\sqrt{c^2(\Delta t)^2+\frac{4ht}{m}}}{2t}$ (1). As it is clear, in macro world in determining the velocity Δv there is a uncertainty. But in which phenomenon of macro world we can see this uncertainty, but not perceiving it as uncertainty? It is clear that this phenomenon must be kinematics. The kinematics is the part of mechanics in which the geometrical motion of body is being studied. Therefore in the kinematics the space and time are principal notions. As it was said above, the space and time are various kind of perception. It is known that Dopler's effect is the phenomenon of kinematics. Directly perceived by us this effect is Dopler effect in acoustics. The sound source is considered in two cases, when it is rest and it is moving, for example toward observer with velocity *v*. The velocity of the sound wave is the same in both case (*V*). However the sound frequency *w* which is perceived by observer depends from source motion velocity *v*. The formulae describing this dependence the following: $w=\frac{w_0}{1-\frac{v}{V}}$ (2). This effect can be analyzed by us from point of view of the unity of the macro and micro worlds laws and therefore existing also uncertainty in macro world. This uncertainty can be seen by us in the velocity *v* of the source. This velocity *v* can be considered as Δv . We don't perceive it as uncertainty because there is Dopler's effect by means of which we find *v* exactly. If macro world wasn't wholly perceived by us then Dopler's effect would not be observed. Really does not it look like that, as Hegel said, from the essence (macro world is perceived) it is explained its phenomena (on hand Dopler's effect), the being. From the relativity theory it is known that the time of event isn't the absolute value. It can be understood if we are distinguishing from each other observers which is moving and which is rest or one can say that system in which the measurements take place is moving and that is rest. In the formula $\Delta x = v\Delta t + t\Delta v$ because of $\Delta v \neq 0$, the factor t of Δv must be differed on dependence of measure momentum. Therefore, $t_1 \neq t_2$ and, consequently, $\Delta t \neq 0$. By comparing the formulae obtained by us it can be obtained the formula for Δt. From formula (2) we have: $\Delta v = V\left(1-\frac{w_0}{w}\right)$ (3). Comparing the formula (1) with (3) we obtain:

$V\left(1-\frac{w_0}{w}\right)=\frac{-c\Delta t+\sqrt{c^2(\Delta t)^2+\frac{4ht}{m}}}{2t}$. From this formula we obtain: $\Delta t=\frac{\frac{h}{m}-tV^2\left(1-\frac{w_0}{w}\right)^2}{cV\left(1-\frac{w_0}{w}\right)}$. In the macro world $\frac{h}{m}\to 0$ and therefore: $\Delta t=\frac{tV\left(\frac{w_0}{w}-1\right)}{c}$ (3). In the case of non relativistic macro world when the body motion velocity, in that number the velocity of wave in Dopler's effect, is more less than the velocity of light ($V \ll c$), then $\Delta t \to 0$. Therefore at perceiving non relativistic macro world, world in which velocities body, wave sources and waves themselves are more less than light velocity, the time of events are absolute. However, if in this world it is considered the source of no sound, but light then quite another act, i.e. $\Delta t = t\left(\frac{w_0}{w}-1\right)$. Therefore the contemplation of light by observer, i.e. the reference of consciousness to light, light phenomena is very unusual. Therefore in any inertial system the light velocity is constant. Really, this fact is postulated by the relativity theory. Now, let's consider the Dopler's effect in the relativistic macro world. In this case the effect of relativistic slowing-down of time take into account. It is known the formula $w = \dfrac{w_0}{\left(1-\dfrac{v}{V}\right)\gamma}$, where $\gamma = \left(1-\dfrac{v^2}{c^2}\right)^{-1/2} = \dfrac{c}{\sqrt{c^2 - v^2}}$. As we have spoken above, v is represented as Δv. Thus, for this case in the previous calculations in formula (3) the factor γ before ω take place: $\Delta t = \dfrac{tV\left(\dfrac{w_0}{w\gamma}-1\right)}{c} = \dfrac{tV\left(\dfrac{w_0\sqrt{c^2-\Delta v^2}}{wc}-1\right)}{c}$. Thus, $\Delta t = \dfrac{tV\left(w_0\sqrt{c^2-\Delta v^2} - wc\right)}{wc^2}$. Here $\Delta v \to c$ and therefore $\Delta t = \dfrac{-tV}{c}$. Let's compare this formula with formula (3), i.e. let's compare the relativistic case ($\Delta v \to c$) with the non relativistic case. The transition from non relativistic case to relativistic case is marked by that the factor $\left(\dfrac{w_0}{w}-1\right)$ will be equal to -1. Therefore $\dfrac{w_0}{w}$ =0 and, consequently, $w \to \infty$. Therefore if the source of oscillations (any waves) moves, with the velocity near velocity of light, towards observer (we have analyzed this case) or opposite (at analyzing this case we would such calculations as well) then the frequency of wave w perceived by observer will be greater than frequency of wave w_0, given off by the source itself. Thus, the perception of the relativistic world, i.e. world with velocities near light velocities, by observer is very differed from usual perception. In the relativistic world if the source gives off light then $\Delta t = -t$ (the case of drawing near source) and $\Delta t = t$ (the case of going away source). We satisfy oneself once again that in any case (relativistic or non relativistic) the contemplation of light is unusual.

Now let's consider the second case, when $\Delta p \Delta x \geq h$ and $v \leq c$. this case corresponds to non relativistic world. As in previous case, let's make transformation. So, $v \geq \dfrac{h}{m\Delta v \Delta t} - \dfrac{\Delta v}{\Delta t}t$. Taking into account $v \leq c$ we obtain: $\dfrac{h}{m\Delta v\Delta t} - \dfrac{\Delta v}{\Delta t}t \leq c$. Transforming this inequality we obtain: $\Delta v \Delta t \geq \dfrac{h}{mc} - \dfrac{(\Delta v)^2 t}{c}$ (4). Consequently, $\Delta v \Delta t \geq \dfrac{h}{mc}$, where $\dfrac{h}{mc}$ is the Kompton's wave length of particle λ_k, i.e. it is the wave length before the scattering. The equation(4) can be presented as: $\Delta v \Delta t = \dfrac{h}{mc}\left(1 - \dfrac{m(\Delta v)^2 t}{h}\right)$. Can we say that Δv is v? Most likely, if we remember that above mentioned case we consider v as Δv due to effect of perception. Let's assume that we perceived micro world as whole world. Then Δv will not be presented as uncertainty. During time interval Δt we can find the distance without uncertainty and it is possible due to whole reflection to our sensual organs. Then $v\Delta t = \Delta\lambda$. Let's remember that Kompton's effect is the scattering of electromagnetic wave which is accompanied by decreasing frequency. $\Delta\lambda = \lambda' - \lambda = \dfrac{h}{mc}(1-\cos\alpha)$, where λ and λ' are wave lengths before and after scattering, α is the angle of scattering. Thus, the Kompton's effect is effect of micro world which get rid of uncertainty in our whole perception of macro world. Really, our skill of finding wave lengths before and after scattering testifies about it.

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Observation possibilities of the gravitational waves detection based on the method of the opto-metrical parametric resonance

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Abstract
To apply the method of the direct observation of the gravitational waves (GW) based on the opto-metrical parametric resonance (OMPR) suggested earlier, one needs to analyze the characteristics of the astrophysical system that can be used. In particular, the characteristics of such possible sources of the gravitational radiation as neutron stars and binaries and such sensitive elements of the remote GW detectors as cosmic masers and cosmic lasers. It is also important to account for the distance between the GW source and the cosmic maser or laser. It appears that the variety of the situations suitable for the OMPR based observations of the GW is rather wide. But the OMPR problem for the cosmic laser requires some modifications in comparison to the similar maser problem because in the laser case the lower atomic level may be not the ground one. The amplitude of the non-stationary component of the laser signal in case of the OMPR is obtained. The problem of distinguishing the signal in question out of variety of others is discussed. It is shown that the elements of the GW-map of the sky can be obtained using the OMPR method right now.

Экспериментальные возможности наблюдения гравитационных волн на основе метода оптико-метрического параметрического резонанса

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Для того, чтобы использовать метод прямой регистрации гравитационных волн (ГВ), основанный на эффекте оптико-механического параметрического резонанса (ОМПР), предложенном ранее, необходимо проанализировать свойства необходимой для этого астрофизической системы. В частности, характеристики таких возможных источников ГВ, как нейтронные и двойные звезды, и таких чувствительных элементов удаленных детекторов ГВ, как космические мазеры и лазеры. Важно также выполнить оценки расстояний между источниками ГВ и космическими мазерами или лазерами. Оказывается, что число ситуаций, пригодных для наблюдений ГВ с использованием ОМПР, довольно велико. Однако, например, задача об ОМПР для космического лазера требует некоторых модификаций по сравнению с аналогичной проблемой для мазеров, поскольку в случае лазеров нижний атомный уровень может быть не основным. Получена амплитуда нестационарной составляющей лазерного сигнала для случая ОМПР. Кроме того, обсуждается проблемы выделения интересующего сигнала среди возможных помех. Показано, что элементы ГВ-карты звездного неба могут быть получены с помощью метода ОМПР уже в настоящее время.

Введение

Теоретические основы этой работы были изложены в [1], где явление ОМПР обсуждалось в деталях. Данная работа и ее результаты адресованы экспериментаторам. Для того чтобы сделать ее самодостаточной, напомним вкратце основные идеи.

Усилия по обнаружению ГВ в настоящее время сосредоточены на интерферометрических методах, в которых используется изменение расстояния между детектором и зеркалом, вызванное действием ГВ. Поскольку (безразмерная) амплитуда h ГВ, приходящих на Землю, весьма мала, чувствительность и избирательность инструментов должна быть очень высокой, и это – основная проблема современных исследований ГВ. В стандартной экспериментальной установке источник монохроматического электромагнитного излучения (ЭМИ), например, лазер, размещен на одном блоке с детектором, так что световой луч проходит путь между источником и детектором в оба конца (Рис.1а).

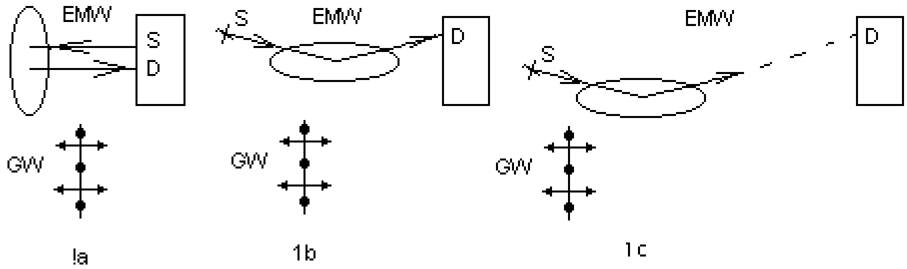

Рис.1

Расположим источник монохроматического ЭМИ с другой стороны зеркала (Рис.1в). Тогда ГВ будет изменять фазу сигнала так же, как и на Рис.1а, и такая экспериментальная установка, оставаясь локальной, также может быть использована для детектирования ГВ.

Превратим теперь локальную установку в удаленную. Увеличим расстояние между блоком «источник ЭМИ – зеркало» и детектором и расположим зеркало достаточно близко к источнику ГВ (Рис.1с). С учетом дальнейшего нас будут интересовать периодические источники ГВ – нейтронные звезды или короткопериодные двойные. В этом случае амплитуда (периодического) изменения фазы сигнала ЭМИ, отраженного от зеркала, станет гораздо больше, чем в локальном эксперименте. В то же время период изменения фазы сигнала будет по-прежнему равен периоду ГВ, и, следовательно, фаза детектируемого сигнала будет также меняться с периодом ГВ. Вместо зеркала в таком удаленном детекторе можно использовать любой объект, способный реагировать с монохроматическим ЭМИ. Например, это может быть атом с переходом, резонансным ЭМИ. Необходимое ЭМИ может доставляться космическим мазером, которому принадлежит этот атом.

Вопросы о влиянии ГВ на атомные уровни, о влиянии ГВ на ЭМИ, взаимодействующее с атомом, о динамике атома в поле ГВ были рассмотрены в [1]. Там также было учтено, что поскольку расстояние между зеркалом и детектором становится очень большим, следует изменить теоретический подход, обычно используемый для описания локальных интерферометрических экспериментов и использовать уравнение геодезической вместо уравнения геодезического отклонения [2]. При этом была обнаружена возможность специфического параметрического резонанса. Она следует из того обстоятельства, что в задаче имеются частотные члены одного порядка величины: частота изменения скорости атома, совпадающая с частотой ГВ *D*, и частота Раби $\alpha = \frac{\mu E}{\hbar}$,

характеризующая силу ЭМИ (μ – индуцированный дипольный момент атома, E – электрическая напряженность поля, $\hbar = 1.05 \cdot 10^{-27}$ *эрг·с)*. Вопросы о достаточности интенсивности мазера для создания эффекта и о возможности гашения эффекта, вызываемого одной частью мазера, действием другой его части были рассмотрены в [3]. Было показано, что интенсивности космических мазеров достаточны для ОМПР, а их размеры, хотя и накладывают некоторые ограничения, не препятствуют возможности наблюдения.

Явление оптико-механического параметрического резонанса («предшественника» оптико-метрического параметрического резонанса в [1]) обсуждалось в [4-6]. Оно состоит в следующем. Если механически колеблющийся атом поместить в спектроскопически сильное резонансное поле и потребовать выполнения некоторых условий на частоту и амплитуду колебаний атома и на частоту Раби системы «атом-поле», то наблюдаемый спектр рассеянного излучения приобретет нестационарную компоненту с большой амплитудой. «Спектроскопически сильное поле» означает, что вынужденные переходы будут доминировать над спонтанными. «Нестационарная компонента с большой амплитудой» появляется, например, на спектре поглощения пробной волны [7] в виде периодической смены поглощения усилением, происходящей с большой амплитудой на некоторой частоте, неподалеку от резонансной, Рис.2. Но вообще говоря, нестационарная компонента с большой амплитудой соответствует периодической смене поглощения усилением потока энергии, когда ЭМИ взаимодействует с атомной средой [8].

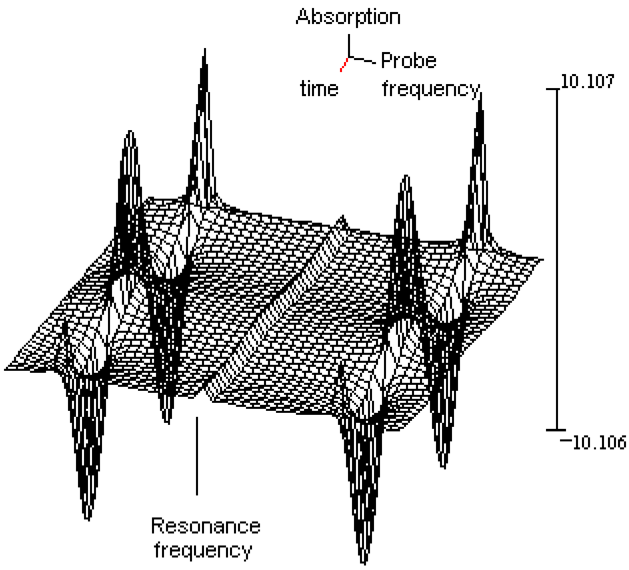

Рис.2

Следует подчеркнуть, что этот вид сигнала существенно отличается от других. Это не просто низкочастотная составляющая спектра. При обычных измерениях никакой дополнительный сигнал не регистрируется из-за усреднения по времени. Для регистрации и измерения обсуждаемой нестационарной компоненты необходимо модифицировать процедуру измерения с учетом периодической смены усиления поглощением – использовать схему детектора с дополнительным устройством, подающим отпирающий импульс определенной длительности.

Период указанной смены равен периоду механических колебаний атома, амплитуда такого переменного сигнала оказывается выше, чем стационарный пик, это – типичное резонансное явление. Следует отметить, что обычный (стационарный) пик (например, коэффициента поглощения) остается без изменений.

Как показано в [1], скорость атома космического мазера в окрестности источника ГВ периодически изменяется, и условия ОМПР могут в принципе быть выполнены. Это означает, что если подобрать подходящий мазер (например, в окрестности источника ГВ) и модифицировать процедуру измерений, то можно зарегистрировать нестационарную компоненту и, таким образом, получить прямое свидетельство существования ГВ.

Условия ОМПР были найдены в [1,3]. Они таковы: поле ЭМИ должно быть спектроскопически сильным

$$\alpha >> \gamma \tag{1}$$

частотное условие

$$\alpha \approx D \tag{2}$$

амплитудное условие

$$h / \omega D \approx \gamma / \alpha \tag{3}$$

Здесь ω – частота ЭМИ (т.е. частота космического мазера), γ – постоянная распада атома, h – безразмерная амплитуда ГВ. Соотношение между расстоянием до пульсара r_S, параметрами пульсара и амплитудой ГВ дается выражением [9]

$$h = \frac{GMR^2 D^2 g_e}{c^4 r_S} \tag{4}$$

где M – масса пульсара, R – радиус пульсара, g_e – гравитационная эллиптичность пульсара, $G = 6.67 \cdot 10^{-8}$ см^3/г·с^2.

Очевидно, вероятность обнаружить стабильный мазер в окрестности такого активного объекта, как нейтронная звезда, невелика. Основная цель данной работы – провести детальный анализ всех наблюдательных возможностей и показать, что метод ОМПР может быть эффективно использован для получения элементов ГВ-карты звездного неба уже сегодня. В последующих разделах мы рассмотрим требования к источникам ГВ, к источникам ЭМИ, их характеристики и требования к расстояниям между этими источниками, которые могут обеспечить возможность прямого наблюдения ГВ с помощью метода ОМПР. Недавнее открытие лазерного эффекта в звездных оболочках [10] требует исследования соответствующей ситуации для космического лазера. Это означает, что необходимо решить задачу для случая, когда нижний атомный уровень не является основным. В заключении приводятся основные результаты.

1. Источники ГВ

В первую очередь обсудим природу источников периодических ГВ, представляющих интерес для нашей задачи. Очевидным кандидатом является нейтронная звезда (пульсар). Типичные значения массы и радиуса пульсара составляют $M = 10^{33}$ г, $R = 10^6$ см. Гравитационная эллиптичность g_e – параметр, значение которого установить довольно трудно. В [9] эта величина равна $g_e = 10^{-3}$, в то время, как в более ранней работе [11] она равна $g_e = 10^{-6}$. Существует теоретический нижний предел периода вращения пульсара [12], равный 0.5 мс и зависящий от выбора уравнения состояния. В [13] было также показано, что если период пульсара меньше, чем некоторое критическое значение, то в течение года период возрастет и достигнет этого значения. Для простой модели пульсара, рассмотренной в [13], критическое значение составляет примерно 20 мс, что соответствует измерениям для пульсара в Крабе. Таким образом, для ГВ, порождаемых пульсаром, имеется верхний предел частоты, равный $(5 – 20) \cdot 10^2$ с$^{-1}$. Сегодня известны сотни пульсаров. У некоторых из них есть планеты [14,15], массы и размеры которых больше, чем у Юпитера. Предположительно у этих планет могут быть обширные атмосферы.

Следующий возможный источник ГВ – двойные системы. С учетом предлагаемых наблюдений изменения электромагнитного сигнала наиболее подходящими являются короткопериодные, а значит, тесные двойные. Они могут представлять собой пары: гелиевая звезда – белый карлик/нейтронная звезда, двойной белый карлик, белый карлик – нейтронная звезда/черная дыра, двойная нейтронная звезда, двойная черная дыра. Орбитальный период таких двойных имеет характерное значение. Когда расстояние между компонентами двойной станет достаточно малым, начинается процесс переноса массы. Этот перенос не ограничивает излучение ГВ, и двойные с начавшимся массопереносом могут представлять дополнительный интерес, поскольку изменения в их периоде, связанные с излучением ГВ, могут приводить к последствиям, которые можно связать с результатами предлагаемых здесь наблюдений. Объекты с начавшимся массопереносом и малым периодом известны как AM CVn и ультракомпактные рентгеновские двойные (UCXB's). Исчерпывающий обзор этих двойных проведен в [16-18]. В частности, встречаются такие двойные, как RX J1914.4+2456, V407 Vul [19] с периодом 9,5 минут, KUV 01584-0939, ES Cet [20] с периодом 10,3 минут и XTE J1807-294 [21] с периодом 40,1 минут. Двойная с самым коротким периодом, известная на сегодняшний день, это RX J0806.3+1527 [22], имеющая период в 5,3 минуты и удаленная от Земли всего на 100 пс. Таким образом, двойные дают частоты ГВ в 10^{-4}-10^{-3} с$^{-1}$. Радиус орбиты двойной может быть оценен с помощью закона Кеплера $R^3 = \frac{GMT^2}{4\pi^2}$

2. Источники ЭМИ

Число известных космических мазеров превышает тысячу. Их излучение соответствует различным переходам в атомах и молекулах, находящихся в космосе. Можно выделить следующие типы космических мазеров.

Районы звездообразования. Как отмечалось в [23-25], некоторые космические мазеры возникают в облаках, из которых формируется протозвезда. Если на подходящем расстоянии имеется источник ГВ, то такие мазеры могут представлять интерес.

Околозвездные. Плотность газа в пространстве около звезды может соответствовать условиям возникновения мазерного процесса. Возможны два интересных случая: а) звезда сама является источником ГВ (пульсар или двойная) и б) имеется удаленный источник ГВ, действующий на этот мазер. В соответствии с [23], плотность газа, необходимая для начала сильного мазерного процесса для таких молекул, как OH, H_2O и SiO, составляет 10^7-10^9 см$^{-3}$. Для оценки интенсивности такого мазера следует учитывать механизм накачки. Если накачка обусловлена только резонансным излучением, то постоянная распада γ соответствует естественной ширине с учетом действия сильного поля. Если в накачке участвуют и атомные столкновения, то значение постоянной распада должно быть исправлено учетом частоты столкновений, т.е. с учетом плотности газа.

Межзвездные. В пространстве имеются и мазерные источники, не связанные со звездами. Часто упоминается водородная линия λ = 21 см. В областях, удаленных от звезд, радиационная накачка доминирует над столкновительной [23]. Заметим, что коэффициент Эйнштейна A, характеризующий постоянную распада γ, может быть весьма малым. Например, для водородного перехода $\lambda = 21$ см он равен $A = 2.85\cdot10^{-15}$с$^{-1}$.

Поскольку из частотного условия ОМПР (2) следует $D \sim \alpha$ и $\alpha = \mu E/\hbar$ – частота Раби системы «атом-поле», необходимо учесть напряженность электрического поля в космических источниках ЭМИ, т.е. их интенсивности. Обычно интенсивности космических источников ЭМИ измеряют в терминах яркостной температуры T_b. Для изотропного распределения интенсивности мазера связь электрической напряженности с яркостной температурой дается соотношением [26]

$$E = 8\pi\nu_0\sqrt{\frac{kT_b\delta\nu}{c^3}} \tag{5}$$

где $\nu_0 = \omega/2\pi$ – частота перехода, $\delta\nu = \gamma/2\pi$ – ширина линии, $k = 1.38\cdot10^{-16}$ эрг/К – постоянная Больцмана. В отсутствие внешнего поля естественная ширина равна коэффициенту Эйнштейна $\gamma = A = \dfrac{32\pi^3\nu_0{}^3\mu^2}{3\hbar c^3}$. Но в присутствии сильного поля (насыщенный мазер) время, которое атом проводит на верхнем уровне, уменьшается в $\dfrac{kT_b}{\pi\hbar\nu_0}$ раз [26], и ширина полосы соответственно увеличивается в то же число раз $\delta\nu = \dfrac{\gamma}{2\pi}\dfrac{kT_b}{\pi\hbar\nu_0}$. Для получения нижнего предела значения яркостной температуры мазера, обеспечивающей ОМПР в случае излучательной накачки и в пренебрежении столкновениями, т.е. когда $\delta\nu = \dfrac{A}{2\pi}\dfrac{kT_b}{\pi\hbar\nu_0}$, учтем соотношения $E = \dfrac{\hbar\alpha}{\mu}; \alpha = D; \nu_0 = \dfrac{\omega}{2\pi}$ и подставим их в (5). Получим

$$T_b = \frac{\sqrt{3\pi}}{8}\frac{\hbar^2c^3}{k}\frac{D}{\omega^2\mu^2} \approx 10^{-7}\cdot\frac{D}{\omega^2\mu^2} \tag{6}$$

Если $\omega \sim 10^9$ с$^{-1}$ и $\mu \sim 3\cdot10^{-19}$ CGS$_q$·см, то минимальная необходимая яркостная температура мазера составит $T_b \sim 10^{14}$К для наблюдений, связанных с пульсаром ($D \sim 10^2$ с$^{-1}$), и $T_b \sim 10^8$К для наблюдений, связанных с двойной ($D \sim 10^{-4}$ с$^{-1}$). Если $\omega \sim 10^{15}$ с$^{-1}$ и $\mu \sim 3\cdot10^{-19}$ CGS$_q$·см, то минимальная необходимая яркостная температура лазера составит T_b ~ 10^2К для наблюдений, связанных с пульсаром ($D \sim 10^2$ с$^{-1}$), и $T_b \sim 10^{-4}$К для наблюдений, связанных с двойной ($D \sim 10^{-4}$ с$^{-1}$). Обычно яркостные температуры пятен в космических мазерах составляют 10^9К для метанольных мазеров, 10^{12}К для гидроксильных мазеров и 10^{15}К для H_2O-мазеров. Таким образом, существующие мазеры в принципе имеют интенсивности ЭМИ, необходимые для ОМПР. Если в накачке участвуют столкновения, то γ возрастает, и яркостная температура, соответствующая тому же значению напряженности понижается (5). Если интенсивность мазера выше, чем необходимо для ОМПР, это означает, что в глубине мазера существует область, где условия ОМПР выполнены [2]. Тогда именно эта область и станет источником нестационарного сигнала, характерного для ОМПР.

3. Расстояния

Оценим теперь расстояния между источником ГВ и областью ОМПР в мазере, которые приводят к реализации условий ОМПР. Условие (3) предполагает учет следующих факторов: 1) постоянная распада γ, характеризующая атомный переход и концентрацию атомов, 2) частоту атомного перехода, т.е. частоту мазера ω, 3) интенсивность поля мазера, выражаемую его частотой Раби α, 4) частоту D источника ГВ, 5) амплитуду ГВ h, связанную с расстоянием между источником ГВ и мазером r_S с помощью соотношения (4).

Используя определения и условия ОМПР, найдем соотношение между наблюдаемой частотой мазера D и соответствующими значениями μ, ω, T_b и γ

$$D = \alpha; \alpha = \frac{\mu E}{\hbar}; E = 8\pi\nu_0\sqrt{\frac{kT_b\delta\nu}{c^3}}; \delta\nu = \frac{\gamma}{2\pi}\frac{2kT_b}{2\pi\hbar\nu_0} \Rightarrow D = f(\mu, \omega, T_b, \gamma)$$

Тогда
$$D^2 = \frac{16k^2}{\pi c^3\hbar^3}\gamma\mu^2\omega T_b{}^2 \tag{7}$$

Подставим условие ОМПР $h = \gamma/\omega$ и условие (7) в уравнение (4). Получим

$$r_S = \frac{16Gk^2}{\pi c^7\hbar^3}(MR^2 g_e)(\mu^2\omega^2 T_b{}^2) \tag{8}$$

Первый множитель представляет собой комбинацию мировых констант, второй – характеризует источник ГВ, третий – характеризует источник ЭМИ. Для дальнейших оценок запишем

$$r_S \approx 10^{-31}\cdot(MR^2 g_e)(\mu^2\omega^2 T_b{}^2)\,\text{см}$$

и используем $M \sim 10^{33}$ г, $R \sim 10^{10}$ см, $g_e \sim 10^0$ для двойной, $M \sim 10^{33}$ г, $R \sim 10^5$ см, $g_e \sim 10^{-3}$ для пульсара, $\mu \sim 3\cdot10^{-19}$ CGS$_q$·см, $\omega \sim 10^9$ с$^{-1}$ для космического мазера, $\mu \sim 3\cdot10^{-19}$ CGS$_q$·см, $\omega \sim 10^{15}$ с$^{-1}$ для космического лазера. Сведем результаты в таблицу

Таблица 1

	Мазер	Лазер
Двойная	$r_S \sim 10^3\ T_b{}^2$ см	$r_S \sim 10^{15}\ T_b{}^2$ см
Пульсар	$r_S \sim 10^{-10}\ T_b{}^2$ см	$r_S \sim 10^2\ T_b{}^2$ см

Эти соотношения означают, что для данной яркостной температуры источника ЭМИ источник ГВ, пригодный для наблюдений, связанных с ОМПР, должен находиться на указанном расстоянии. Оценим необходимые яркостные температуры источников ЭМИ для околозвездных и межзвездных расстояний между источниками ЭМИ и источниками ГВ. Полагая $r_S \sim 10^{14}$см (порядка 10^0-10^1 а.е.) для околозвездного источника ЭМИ в окрестности источника ГВ, находим

Таблица 2

Околозвездный	Мазер	Лазер
Двойная	$T_b \sim 10^6$ K	$T_b \sim 10^0$ K
Пульсар	$T_b \sim 10^{12}$ K	$T_b \sim 10^{11}$ K

Полагая r_S ~ 10^{18}см (порядка 10^0-10^1 пс) для межзвездного источника ЭМИ вдали от источника ГВ, находим

Таблица 3

Межзвездный	Мазер	Лазер
Двойная	$T_b \sim 10^8$ K	$T_b \sim 10^2$ K
Пульсар	$T_b \sim 10^{14}$ K	$T_b \sim 10^{13}$ K

Сравнивая эти результаты со значениями минимальных необходимых температур, полученных в конце предыдущего раздела, получаем основной результат настоящей работы:

Космический мазер, расположенный на межзвездном расстоянии от источника ГВ, может быть использован для наблюдений ОМПР для обоих видов источников ГВ, в то время как мазер, расположенный поблизости от источника ГВ, непригоден для наблюдений ОМПР. Космический лазер подходит во всех случаях, но его относительно большое значение γ приводит к нарушению амплитудного условия (3), когда расстояние становится большим и h убывает в соответствии с (4). Таким образом, космический лазер может выявить действие ГВ, только когда он расположен в окрестности источника ГВ. Представляется вероятным, что лазерный эффект может возникать не только в звездных оболочках (как в η Carinae), но и в атмосферах гигантских планет в окрестностях пульсаров.

Таким образом, анализируя нестационарные составляющие сигналов космических мазеров, можно построить элементы ГВ-картины звездного неба. Сигналы космических лазеров, модулированные ГВ, (если они будут обнаружены) могут быть использованы для исследования соответствующих ГВ источников.

4. Задача об ОМПР для космических лазеров

В [10] был обнаружен *лазерный* эффект ($\omega \sim 10^{14}$ с$^{-1}$) в облаке, окружающем горячую звезду η Carinae. Он проявлялся в результате перехода между возбужденными состояниями в FeII, причем верхний уровень накачивался сильным излучением HLyα. Здесь нас не будет интересовать детальный механизм этого эффекта (он рассматривается в [10]). Важным является то, что в космосе могут иметься и пары обычных уровней (не метастабильных), которые могут принимать участие в ОМПР. Это означает, что уравнения, использованные в [1] для описания ситуации с космическим мазером, следует модифицировать с учетом того, что нижний уровень может быть и не основным.

В [1] задача об ОМРПР для атома мазера в поле ГВ рассматривалась для случая, когда нижний уровень двухуровневого атома был основным или мог считаться таковым (метастабильный уровень). Тогда постоянная распада соответствует либо естественной ширине линии, либо линии в сильном резонансном поле, либо может быть оценена с учетом столкновений. Решение описывает свойства поглощения/усиления системы «атом+поле». Во всех случаях ведущий член асимптотического разложения по степеням ε ($\varepsilon = \gamma/\alpha_1 2^{1/2}$) проявлял осциллирующее поведение и имел порядок единицы, т.е. амплитуда нестационарной компоненты члена, пропорционального потоку энергии, была сравнима со значением стационарной (обычной) компоненты.

Теперь модель вновь представляет собой двухуровневый атом, скорость которого осциллирует вдоль волнового вектора ЭМИ с частотой D, равной частоте ГВ. Этот атом взаимодействует с ЭМИ, состоящим из двух компонент: первая является сильной и представляет собой излучение мазера с частотой Ω_1, близкой к частоте атомного перехода ω, вторая, Ω_2 , является слабой и возникает в результате воздействия ГВ на электромагнитное поле, $\Omega_2 - \Omega_1 = D$. Тогда уравнения Блоха для компонент матрицы плотности, описывающей динамику атома, таковы

$$
\begin{aligned}
&\frac{d}{dt}(\rho_{22}-\rho_{11}) = -\gamma(\rho_{22}-\rho_{11}) + 4i[\alpha_1\cos(\Omega_1 t - k_1 y) + \alpha_2\cos(\Omega_2 t - k_2 y)](\rho_{21}-\rho_{12}) + \Lambda \\
&\frac{d}{dt}\rho_{12} = -(\gamma_{12}+i\omega)\rho_{12} - 2i[\alpha_1\cos(\Omega_1 t - k_1 y) + \alpha_2\cos(\Omega_2 t - k_2 y)](\rho_{22}-\rho_{11}) \\
&\rho_{12} = \overline{\rho}_{21} \\
&\frac{d}{dt} = \frac{\partial}{\partial t} + V\frac{\partial}{\partial z}; V = V_0 - V_1\cos Dt \\
&V_1 = hc
\end{aligned}
\tag{9}
$$

Здесь ρ_{22} и ρ_{11} – населенности уровней, ρ_{12}, ρ_{21} – поляризационные члены, γ – продольная постоянная распада, γ_{12} – поперечная постоянная распада, $\alpha_1 = \mu E_0/\hbar$, $\alpha_2 = \mu E_0 h\omega/D\hbar$ – параметры Раби основной (сильной) и дополнительной (слабой) волн такие же, как в [1], k_1, k_2 – волновые вектора этих волн (в нашем случае их можно считать равными и равными k), Λ – описывает некогерентную накачку произвольной природы. В приближении вращающейся волны воспользуемся обозначениями

$$
\begin{aligned}
&\rho_{21} = R_{21}\exp[i(\Omega_1 t - ky)]; \\
&\rho_{12} = R_{12}\exp[-i(\Omega_1 t - ky)]; \\
&R = 2^{-1/2}(\rho_{22}-\rho_{11})
\end{aligned}
$$

Используя обозначения, приведенные в Приложении, получим

$$
\begin{aligned}
&\frac{d}{d\tau}R = -\varepsilon R - iR_{12} + iR_{21} - iR_{12}a\varepsilon e^{i\delta_d\tau} + iR_{21}a\varepsilon e^{-i\delta_d\tau} + \lambda \\
&\frac{\partial}{\partial\tau}R_{12} = -\{\Gamma\varepsilon + i\delta + i\kappa V_0 - i\kappa v_1\varepsilon\cos(\delta_d\tau)\}R_{12} - i[1 + a\varepsilon e^{-i\delta_d\tau}]R
\end{aligned}
\tag{11}
$$

или в матричной форме

$$
\frac{\partial}{\partial\tau}W = (Q_0 + \varepsilon Q_1(\tau))W + C \tag{12}
$$

где

$$
\begin{aligned}
&Q_0 = i\begin{pmatrix} 0 & -1 & 1 \\ -1 & -\sigma & 0 \\ 1 & 0 & \sigma \end{pmatrix}; W = \begin{pmatrix} R \\ R_{12} \\ R_{21} \end{pmatrix}; C = \begin{pmatrix} \lambda \\ 0 \\ 0 \end{pmatrix}; \\
&Q_1 = \begin{pmatrix} -1 & -ia\exp(i\delta_d\tau) & ia\exp(-i\delta_d\tau) \\ -ia\exp(-i\delta_d\tau) & -\Gamma + i\kappa v_1\cos(\delta_d\tau) & 0 \\ ia\exp(i\delta_d\tau) & 0 & -\Gamma - i\kappa v_1\cos(\delta_d\tau) \end{pmatrix}; \sigma = \delta + \kappa V_0
\end{aligned}
\tag{13}
$$

Уравнение (12) может быть решено с помощью метода асимптотического разложения по малому параметру $\varepsilon = \gamma/\alpha_1 2^{1/2}$, характеризующему силу поля. Математический метод решения – тот же, что и в [1]. Основной момент – использование условий ОМПР

$$
\sqrt{\sigma^2 + 2} \equiv F = \delta_d + \varepsilon\nu = \delta + \varepsilon\nu; \nu = O(1) \tag{14}
$$

где ν – параметр настройки на ОМПР.

«Нестационарная компонента с большой амплитудой», упомянутая во Введении, пропорциональна ImR_{21} и равна

$$
\begin{aligned}
&\operatorname{Im}R_{21} = -\frac{\lambda\sigma H\sqrt{B^2+\nu^2}}{\varepsilon F[(\sigma^2+2\Gamma)(B^2+\nu^2)+2BH^2]}\cos(Dt + \arctan\frac{\nu}{B}) \\
&H = F(\frac{a}{F-\sigma)} + 2\kappa v_1) = 2^{1/2}\frac{\omega D}{\alpha_1\gamma}\frac{V_1}{c}(1 - h\frac{\alpha_1^{\ 2}}{D\omega}\frac{c^2}{V_0 V_1}) \\
&B = 1 + \Gamma(\sigma^2+1) = 1 + \frac{\gamma_{12}}{\gamma}(1 + \frac{1}{2\alpha_1^{\ 2}}(D + \frac{\omega V_0}{c})^2)
\end{aligned}
\tag{15}
$$

Ведущий член разложения пропорционален ε^{-1}, благодаря наличию ε в знаменателе. В соответствии с (3), $\varepsilon \sim h\omega/D$ и, поскольку все остальные параметры в (15) имеют порядок единицы, можно видеть, что амплитуда нестационарной компоненты велика. Этот результат доказывает возможность использования и других переходов, помимо тех, что характерны для космических мазеров, например, таких, как в [10], где рассматривается лазерный эффект в звездной оболочке.

5. Обсуждение

Результаты настоящей работы доказывают, что метод, основанный на использовании ОМПР, пригоден для прямого наблюдения ГВ и для развития гравитационной астрономии периодических источников ГВ. Этими источниками являются нейтронные звезды с ненулевой гравитационной эллиптичностью и короткопериодные двойные. Чувствительными элементами удаленных детекторов ГВ являются атомы достаточно распространенных в космосе (насыщенных) космических мазеров и атомы лазеров, наблюдаемых в звездных оболочках. Уже наблюдаемые интенсивности э-м излучения этих космических мазеров оказываются достаточными для того, чтобы эффект можно было наблюдать с Земли с помощь уже существующих инструментов (например, радиотелескопов), которые для регистрации ОМПР должны быть дополнительно оборудованы устройством с отпирающими импульсами.

Прежде, чем приступить к наблюдениям, необходимо ответить на два основных вопроса: а) возможно ли уверенно выделить сигнал, причиной которого является ГВ, и б) где наблюдать такой сигнал.

Во Введении было отмечено, что нестационарная компонента является такой деталью, которая не может быть замечена в обычном эксперименте. Более того, «обычные» возмущения в сигнале будут незаметными при наблюдениях, модифицированных для регистрации нестационарного поглощения/усиления потока энергии. Уравнение (15) и его аналог в [1] показывают, что с точки зрения чувствительности существующих инструментов нестационарная компонента излучения мазера может наблюдаться, если наблюдается обычный сигнал мазера. Но процедура наблюдения должна быть изменена таким образом, чтобы избежать усреднения по времени. Эффект ОМПР не означает, что мазерный сигнал просто содержит низкочастотную компоненту. Она присутствует, но ее амплитуда мала по сравнению с высотой пика основного сигнала мазера. Дело же заключается в том, что существует значение частоты, близкой к частоте мазера, на которой ЭМИ периодически ослабляется и усиливается с периодом ГВ, в то время как обычная (стационарная) компонента мазерного сигнала остается без изменений. Наблюдение нестационарной компоненты было бы прямым свидетельством существования ГВ.

Результаты раздела 4 показывают, что имеются довольно широкие возможности для наблюдения ОМПР. Мазеры всех типов (районы звездообразования, околозвездные, межзвездные) могут испытывать воздействие ГВ со стороны пульсаров или двойных. (Космический лазер также можно использовать для наблюдений ГВ с помощью ОМПР. Для этого потребуется лазер, расположенный вблизи пульсара или двойной). Частоты таких воздействий соответствуют частотам вращений пульсаров или двойных. Практически можно использовать для наблюдений почти любой мазер в надежде, что на него воздействует какой-либо источник ГВ.

Таким образом, возможный эксперимент по прямому наблюдению ГВсостоит в следующем. Принимая во внимание поперечный характер ГВ, следует постараться удовлетворить условию на геометрическое расположение астрофизической системы: источник ГВ и Земля располагаются на концах диаметра, а мазер – на самой окружности. Выбрав подходящую пару «источник ГВ – мазер», наблюдатель захватывает сигнал. Обычно сигнал космического мазера состоит из набора узких пиков, частоты которых расположены близко друг к другу. Ширина импульса дополнительного устройства детектора делается равной половине периода источника ГВ, промежуток между импульсами имеет такую же величину. Затем наблюдатель выбирает один из пиков и изменяет фазу импульса устройства детектора до тех пор, пока видимый сигнал не обнаружит рост. Затем эта процедура повторяется для других пиков. Конечно, при выборе каждого конкретного источника ЭМИ для поиска ГВ следует проводить более точные оценки параметров с учетом частоты линии, дипольного момента, яркостной температуры и существования источника ГВ поблизости. В данной работе показано, что такие оценки приводят к разумным результатам.

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Приложение
Обозначения, использованные в уравнении (11)

$$2^{1/2}\alpha_1 t = \tau; \frac{D}{\alpha_1} = 2^{1/2}\delta_d; \frac{k}{\alpha_1} = 2^{1/2}\kappa; \frac{\Lambda}{2\alpha_1} = \lambda; \frac{\gamma_{12}}{\alpha_1} = 2^{1/2}\Gamma\varepsilon;$$

$$\frac{\omega - \Omega_1}{\alpha_1} = 2^{1/2}\delta; \frac{\alpha_2}{\alpha_1} = a\varepsilon; V_1 = v_1\varepsilon$$

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On the interpretation of the theory of relativity in Ukraine: the first half of the 20th century

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The crisis of Newtonian physics and the attempts to overcome it were peculiar to the scientific community of Ukraine in the 1900-1939th. The scientific community of Ukraine as well as the world scientific community was trying to solve it by using in its efforts both the ideas of classic physics, and ideas professed by the followers of relativity. These works are selectively mentioned in the main historical and physical researches of the Russian historians of science: mainly those, which were published in the Russian editions. The work of the famous historians of physics Vizgin and Gorelik is the most representative in this context. The problems of the perception of the theory of relativity in Russia and USSR are considered in this work [2].

Positivism required solving the problem of the existence of the ether based on concrete, positive, experimental knowledge. After Einstein's article up to 1921, there were no any experimental substantiations of the denial the existence of the ether, except Michelson's experiment. Therefore, considerable distribution was received by theoretical works, which had two exact priorities. The first block of works was directed to the substantiation of the validity of the relativity and its results. The second one, based on special assumption and conditions within the framework of classic physics and ideas about the existence of the ether, was directed on maintaining the stability of Newtonian mechanics as an integrated loop system that allowed to reject relativity, or to explain its results.

The formation and development of the idea of the theory of relativity in Ukraine, was associated the worldwide and all-Russian scientific process. This introduced a number of new strokes. It also demonstrated the peculiarities of the development of physics in Ukraine in the first half of the 20th century. Among those who in Ukraine worked on these problems were disciples of the world known physicists (Planck's, Lanzheven's, Lorentz's, Varburg's). It gives evidence of the close connections of Ukrainian physicists not only with Russian, but also with the world scientific community. The activity of physicists in different countries were important for the development and understanding of relativity at a new, non-classical level.

We analyzed the effect of worldwide tendencies, like the creation of the theory of relativity; the quantum theories on the process of formation of the physical science in Ukraine both in the times of the Russian empire, and the USSR. The works of many scientists, including the Ukrainians, were either forgotten or weren't appreciated properly because of some historical circumstances. The aim of this paper is the analysis of the original works of the different authors. On the basis of these works it is possible to determine the main problems, which were actual during that period.

The theory of the ether took not the last place in a problematic of researches, which were conducted in the Ukraine at the end of the 19th century. Gruzintsev and Shyller worked in this area. Shyller offered his own variant of the theory of a resilient ether (1890) [31]. The problem of existence of the ether was called in question after the appearance of the special relativity theory because of the absence of the more or less satisfactory theory of ether. This circumstance caused minimization of the further extending and development of the ether theories.

The appearance in 1905 of Einstein's paper «To the electrodynamics of moving bodies» became the turning point in the history of physics. Debates concerning the relativity theory touched scientists all over the world. They caused the open debates, which affected the main concepts of physics, in particular space and time. Grdina, Gruzintsev, Kordysh, Kasterin, Shtrum, Lashkarev, Grave and other were engaged in the problems of the relativity theory. These scientists can be divided into two camps. Some tried to substantiate a relativity and its results. Others tried to deny the principle and to explain its results, thus they remained within the framework of classic physics.

The postulates of a relativity theory were critical estimated in that works. To substantiate or refute the relativity theory, the interpretation of a Michelson's and Morly's experiment was critical estimated. This experience was interpreted according to relativity at the time of the **CTO's** formation. The speed of light in any inertial system of counting has the identical value "c" and does not depend on a wave direction. This statement has called many critical remarks in the scientific community.

Let us stop on some works of the indicated authors.

The professors Ekaterinoslav (Dnepropetrovsk) institute of Mines Jaroslav Grdina can be referred to the consistent opponents of a relativity theory.

Jaroslav Grdina devoted seven works to problems of a relativity theory. They were published since 1912 to 1927. The test data are examined in these works. On the basis of these data it is possible to calculate the ratios e/m_0, where e is the elementary charge, m0 - complete transversal (apparent) electronic mass at infinitesimal speeds (1912 « To a problem on electronic mass ») [8]. In the work « To a problem on a relativity » 1914 [9] author stated that the new positions of a relativity theory complicated the customary concepts which were experimentally checked in a classic mechanics. The relativity theory is insufficiently proved in the theoretical sense. The persistence of the speed of light was criticized (« Physical or restricted relativity » 1915) [11]. The scientist was the follower of the theory of ether. That's why he considered, that the moving of a light source regarding ether had have an effect for speeds of light in different directions. On long distances from a light source, the speed of light should depend only on properties of ether. A criticism of a Delariv's hypothesis appeared in 1915. Delariv offered to supplement the Hertz's theory about a complete diggings of a ether by the Earth on "... A gradual daggling the ether by earth on different distances... " [p. 3, 7]. In such way he tried to explain the negative outcome of a Michelson's experiment. Grdina pointed, that the hypothesis „ not only can not explain an phenomenon of a light aberration, but even caused its complete denying" [p. 13, 7]. In paper « The basic Laws of moving » (1924) the scientist sets up a fundamentals of the "Cartesian mechanics with reference to, according to the newest views, only to relative movements, in relation to inertial systems of coordinates" [p. 188, 10]. Negative attitude to a special relativity theory also is reflected in it. He criticized Einstein for his denial the existence of ether "Einstein within 15 years (since 1905 for 1920) terrorized a science by complete negation of the existence of ether" [p. 189, 10]. In same paper, there is a promising to state, „ The declaiming against common principles of relativity in the special paper" [p. 189, 10]. It also indicates aversion by the writer of ideas of the Special relativity theory and general theory of relativity. In 1927 the last work devoted to special relativity theory was published. It was «A Note on a relativity » [13]. In it, by the way, is expressed, that the theory had no internal logical inconsistencies, therefore it was possible to deny it only by experienced way.

In 1924 Shtrum published his paper «About speeds that are more than the speed of the light, for Special relativity theory» [34]. He analyzed Einstein's and Laue's conclusions. The writer came to output, that the high speeds did not contradict Special relativity theory. Thus he did not disclaim postulates of a relativity theory. He also stated the supposition, that apart from a relativity of simultaneity of events and expansion of periods, there was a relativity of a direction of time. In the same 1924 he published, prolonging the problem, paper « The newest outcomes of a Michelson's experiment and attempts of their hypothetical explanation» [p. 107, 32]. The author offered his own explanation of the positive takes of Miller's experiments. He supposed that "speed of light of the movable body, is a function of a speed of moving of a source". This paper called a number of the remarks from the side of Grdina who published them in the article «New about a Michelson's experiment and Einstein's relativity» [12]

L. Shtrum concerned the problem of speeds that are higher than speeds of light in work «Phase speed for kinematics in relativity theory» [33]. Analyzing the behavior of the quantity c^2 /v he comes to the following conclusion "the existence of processes which move with speed that is more than speed of light doesn't contrary to the statements of the relativity theory and also to the gnoseological principle of a causality"[p. 87, 32]. Shtrum pointed that in 1923 the similar calculations were conducted by Viennese physicist Bass R. In this connection he wrote that his thoughts about them are correct.

Though the relativistic corrections for the astronomical researches are not considerable, the relativity theory was interesting for an astrophysics Gerasimovich (later he was the head of the Pulkov's observatory). «Aberration of a light and a relativity theory» was one of his first works [4]. He wrote it during his studies at the Charkov University. In 1914 it was published not only in Russian, but also in the French magazine [35]. Gerasimovich's research «Universe at light of a relativity theory » [3] is dedicated to the analysis of the problems of space and time from the point of view both Special relativity theory and General theory of relativity, and described its application in astrophysical researches. Gerasimovich in his theoretical researches of the planetary nebula used the achievements of modern physics such as ideas from the theory of quantum up to a general theory of relativity. In D. Menzel's and Gerasimovich's co-operative work «Subatomic energy and star-shaped radiation » (was awarded with the premium of the New York academy of sciences) was said :"in a subsoil of stars a certain statistical equilibrium of processes could take place: the process of energy release at annihilation can partly be compensated by the reverse process. It is the transformation of quantum of radiation in substance" [35].

Vadim Djachenko's three works dedicated to the problem of the epicycles task in the special relativity theory are known for us (first two were reported by the academician D.Grave). With the help of the equations of classic mechanics and using the positions of a special relativity theory, author described the movement of a planet Mercury.

The academician D. Grave, who was famous as talented mathematician, also paid attention to the problems of natural sciences. In his article «About electromagnetic phenomena in solar system» [6] he proposed to organize a laboratory on learning the effects of electromagnetic phenomena on planets movement. As he said, it is also necessary to find a physical reason in deviation of a perihelion of Mercury. In paper «Electromagnetic foundation of mechanics » [5] author tried to return to the old Thomson's ideas concerning the interaction of ether and substance.

V. Lashkarev's work «To the theory of substance and light movement in gravitational field» [26] it is also very interesting in this direction. Like Grdina, he tried without the rejection of Einstein's theory of relativity to explain the deviation of a perihelion of Mercury and the deviation of light beams in a gravitational field of the Sun being based on dynamical equations. In this work author tried to „construct the theory of phenomena in gravitational field accepting for a postulate the invariance of the space and time being based on equation of dynamics." [p. 12, 26] He proved his theory by the following regulations:

- All material objects (material solids and the light beams) are submitted to the laws of impulse and saving of energy;
- Newtonian attractive force influences all solids;
- Potential energy of attraction has a mass like kinetic energy does (generalized principle Hasenohrl)" [26]

Having used these statements, he received the same quantitative data for the magnitude of the planets perihelion movement and for light deflection as for general theory of relativity. In 1926 he presented the report on this topic on the 5-th Congress of the Russian physicists association in Moscow.

M. Kasterin worked in Novorossiysk (Odessa) University in 1910-th. Even at his life his researches caused sharp discussions. Published in "Izvestiya AN SSSR" (1937) articles are the evidence of this process «About article Kasterin's M. P. "Generalization of the main equations of aerodynamics and electrodynamics"» (Blokhihtsev D. I., Leontovich M. A., Rumer Yu. B., Tamm I. E., Fock V.A., Frenkel Ya. I.) [1], and «About operations Kasterin M. P. on an electrodynamics and interfacing problems» (Tamm I. E.) [29]. By the way, M. Kasterin worked on probation his ideas under the management of professor E. Varburg in Berlin, and under the professors Lorentz and Kamerling-Onnes in Leiden. M. Kasterin tried in his theory to transfer concepts of aerodynamics to electrodynamics.

In the work «About groundlessness of Einstein's relativity» [18] he analyzed Buherer's experiment on β- rays speed determination. This work was published in Odessa in 1919. But already in 1917 he made a report on this topic in Moscow mathematical society. M. Kasterin pointed on the contradiction of the law of compensation, according to Lorentz's theory ($\frac{U}{c}\sin\varphi = \frac{E}{H}$), with the test data on the basis of „Buherer's curves". That is why he came to the following conclusion: β- elements could have speeds more than speed of light. "…derivation of this formula is intimately linked with the new kind of equations of electrodynamics, and thus these new equations are subordinate to a mechanical relativity (Galileo). Therefore the problem of the Einstein's formula at this way of calculation β (ratio $\frac{U}{c}$) passes by itself …" [p. 10, 18]. In 1919 N. Shaposhnikov, professor in Ivanovo - Voznesenskiy Polytechnic Institute published his article «To the M. Kasterin's paper "Sur la non concordance du principe de relativite d'Einstein " [30]. N. Shaposhnikov agreed with the statement about the mismatch of the experimental data. However, he refrained from the cardinal statement about illegality of the relativity theory. „The reason of the conclusions could depend on formula $\frac{U}{c}\sin\varphi = \frac{E}{H}$. In spite of the accordance of its particular kind ($\frac{U}{c} = \frac{E}{H}$) to Buherer's, Wolz's, Neumann's and Schafer's experiments, it does not demonstrate, that it is true. Kasterin makes a more particular conclusion: he speaks about an inconsistency in relativity. The experiments of the indicated researchers prevent us from joining this conclusion ..." [p. 4, 30].

Leonid Kordysh (Planck's follower), professor of theoretical physics who worked in Kiev Polytechnic Institute and Kiev University, devoted a number of his works to problems both common, and special relativity theory. In 1910 there appeared "… one of the first papers in Russian …" [2] - « Elementary derivation of basic formulas of a relativity theory » [24]. In «Gravitation and inertia» [19] he expounded the regulations of a general theory of relativity. «Gravitational theory of diffraction phenomena» [20] was dedicated to the application of the above-stated positions. In 1924 in «Relativity theory and theory quantum» [22] L. Kordysh made an assumption about the existence of gravitational intermolecular fields. In «Electromagnetic waves with speeds higher than the speed of light » [24] he considered the possibility of the existence of waves spreading with super light speeds and which are subordinated to Maxwell's equations. In « Characteristic features of matrix theory» [23] and «About some peculiarities of Broli's and Shredinger's wave theory» (1928) [21] the scientist researched the quantum conditions of Bohr and Sommerfeld, analyzed the main ideas of a matrix mechanics and its connection with wave mechanics.

At the beginning of the 20-th century Charkov Professor Aleksey Gruzintsev in his works evolved his opinion on the ether, as the frame, which was intimately connected with an electromagnetic field. In the paper «Lorentz's transformation laws and relativity» [14] A. Gruzintsev pointed out that the equations of electrodynamics saved their form at Lorentz's transformation rules. He showed an invariance of Maxwell's equations for the sphere concerning Lorentz's transformation rules. Such an original conclusion was very important for the relativity theory substantiation. Nevertheless, at the same time the scientist very cautiously expressed his thoughts about the negative interpretation of the results of Michelson experiment. He was a supporter of a detailed investigation of this problem.

N. Rosen's investigation « Plainly polarized waves in the general relativity theory » [28] is dedicated to a general theory of relativity. The case of progressive wave was surveyed in it. The author concluded that because of the nonlinearity of the equations gravitational or electromagnetic perturbation caused the collapse of the metrics. In «Elementary particles in the theory of field» (1939) [27] Rosen reconsidered the previous attempts to construct a classic field theory, in which the existence of elementary particles would be admitted. In the paper of build-up of is he admitted the possibility of constructing the classic theory of elementary particles. However, such theory had a disadvantage. It was the negative electronic mass. The author expected that "… perhaps with the transition from classical to quantum theory this difficulty will disappear." [p. 286, 27].

For the impartial evaluation of the place of the native physicists in the process of non classical physics foundation it is necessary to analyze their works in details in future.

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Perspectives of the Special Relativity theory

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С помощью обобщения пространственно-временных преобразований вращения (преобразований Лоренца) на область сверхсветовых скоростей получены соотношения специальной теории относительности и волновое уравнение для тахионов (сверхсветовых частиц) с вещественной массой.

Инвариантность уравнений математической физики относительно преобразований вращения и галилеевых преобразований широко используется при исследовании свойств решений этих уравнений [1,2]. Пространственно-временные преобразования вращения позволяют получить важные характеристики релятивистских частиц, в то числе и частиц, движущихся с сверхсветовой скоростью.

Сверхсветовые частицы с мнимой массой впервые были рассмотрены Я.П. Терлецким [3,4]. Американский физик Дж. Фейнберг предложил называть эти частицы тахионами [5]. Обширную библиографию научных публикаций по тахионам и описание их свойств можно найти в [6-8]. Основным вопросом при изучении тахионов является физическая интерпретация их мнимой массы.

В настоящей работе с помощью предложенного в [9,10] обобщения пространственно-временных преобразований вращения (сверхсветовых преобразований Лоренца) дано описание тахионов с вещественной массой.

Пусть в изучаемой области физического пространства зарегистрирована характерная скорость распространения сигнала $c>0$ (например, скорость звука или скорость света). Наличие характерной (критической) скорости c позволяет в случае двух независимых переменных - одной пространственной координаты x и времени t ввести переменные

$$z = x + ct, \quad z^* = x - ct \tag{1}$$

Эти переменные «следят» за волнами, распространяющимися со скоростью c в отрицательном и положительном направлениях оси x. Назовем z и z^* сопряженными характеристическими аргументами (числами).

Введем параметр $V = x/t$ с размерностью скорости и «число Маха» $M = V/c$. Тогда аргументы (1) можно записать в виде $z = ct(M+1)$, $z^* = ct(M-1)$.

Приведем «тригонометрическую» форму записи характеристического аргумента $z = x + ct$ через гиперболические косинус и синус

$$z = \rho(\operatorname{ch}\varphi + \operatorname{sh}\varphi) = \rho \exp\varphi , \tag{2}$$

где $$\rho^2 = z \cdot z^* = x^2 - c^2t^2, \ \varphi = \ln\sqrt{\frac{x+ct}{x-ct}}, \operatorname{ch}\varphi = \frac{x}{\rho}, \operatorname{sh}\varphi = \frac{ct}{\rho}.$$

Графически можно изобразить плоскость характеристических чисел с графиками гипербол $\rho^2 = x^2 - c^2t^2$ при $\rho = 1$ и $c = 1$. Величина ρ характеризует расстояние (интервал) между началом координат $(x = 0, t = 0)$ и точками окружности или «гиперболическое расстояние» (интервал) между началом координат и точками гипербол. Угол φ отсчитывается от горизонтальной оси x.

Подчеркнем следующую важную особенность аргументов (1), (2). Для них величина интервала $\rho = \sqrt{x^2 - c^2t^2}$ и угла φ являются действительными при $|V| \geq c$. Если параметр V считать скоростью движения материальной точки, то области правого I и левого II квадранта (между биссектрисами $x/t = \pm c$) отвечают областям сверхкритических скоростей материальных точек $|V| > c$. При $|V| < c$ величины $\rho = \sqrt{x^2 - c^2t^2}$ и φ для области сверхкритических скоростей становятся мнимыми. Для возможности рассмотрения областей докритических скоростей (при действительных значениях интервала ρ и угла φ) следует ввести характеристические переменные по формулам

$$z = ct + x, \quad z^* = ct - x, \tag{3}$$

$$\rho^2 = z \cdot z^* = c^2 t^2 - x^2, \; \varphi = \ln\sqrt{\frac{ct + x}{ct - x}}, \; \mathrm{ch}\varphi = \frac{ct}{\rho}, \; \mathrm{sh}\varphi = \frac{x}{\rho}.$$

В этом случае имеем две области докритических скоростей $|V| < c$ с действительной величиной интервала $\rho = \sqrt{c^2 t^2 - x^2}$ и угла φ, отсчитываемого в этом случае от вертикальной оси *t*. Эти области представляются верхним III и нижним IV квадрантами (между биссектрисами $x / t = \pm c$). Также можно изобразить гиперболы $\rho^2 = c^2 t^2 - x^2$ при $\rho = 1$ и $c = 1$. В работах [9,10] изложены свойства функций характеристического аргумента и описаны их приложения к линейным задачам акустики и электродинамики.

Рассмотрим преобразования независимых переменных *x* и *t*, оставляющие неизменными интервалы ρ в плоскости характеристического аргумента. Этими преобразованиями будут параллельные переносы и вращения системы координат. Параллельные переносы приводят к элементарному изменению начала координат ($x = 0, \; t = 0$) и не позволяют получить дополнительных интересных свойств пространства характеристического аргумента и его функций. В то же время преобразования вращения дают уникальную информацию о свойствах пространства - времени.

Переход от координат x, t к новым координатам X, T при применении преобразования поворота определяется формулами

$$X = x\,\mathrm{ch}\varphi - ct\,\mathrm{sh}\varphi, \quad T = -\frac{x}{c}\,\mathrm{sh}\varphi + t\,\mathrm{ch}\varphi, \tag{4}$$

где φ - угол поворота. При преобразованиях (4) величина интервала ρ не меняется.

Представим преобразования поворота (4) для двух случаев: квадрантов I и II сверхкритических областей и квадрантов III и IV докритических областей. Для первого случая аргументов (1), (2) имеем

$$\mathrm{th}\varphi = \frac{c}{V} = \frac{1}{M}, \mathrm{sh}\varphi = \frac{1}{\sqrt{M^2 - 1}} = \gamma, \; \mathrm{ch}\varphi = \frac{M}{\sqrt{M^2 - 1}} = \gamma M.$$

Подставляя последние формулы в (4), получаем «сверхкритический» (в частности, «сверхсветовой») аналог преобразований Лоренца

$$X = \gamma(Mx - ct), \quad cT = \gamma(Mct - x) \tag{5}$$

Для второго случая аргументов (3) пишем

$$\mathrm{th}\varphi = M, \mathrm{sh}\varphi = \frac{M}{\sqrt{1 - M^2}} = \gamma M, \; \mathrm{ch}\varphi = \frac{1}{\sqrt{1 - M^2}} = \gamma$$

и получаем обычные «докритические» (досветовые) преобразования Лоренца

$$X = \gamma(x - Mct), \quad cT = \gamma(ct - Mx). \tag{6}$$

Использованный метод получения преобразований (5) и (6) повторяет подход А. Эйнштейна [11], который изложен также в [12,13] при получении преобразований Лоренца (6) в Специальной Теории Относительности (СТО). Однако традиционная СТО постулирует невозможность превышения скорости света *c* в пустоте [11-14], в связи с чем в ней не рассматривались преобразования (5). Если отказаться от этого постулата, то СТО обобщается на случай сверхсветовых скоростей (со сверхсветовыми преобразованиями Лоренца (5)) и сверхсветовых систем отсчета.

Впервые преобразования (6) были получены в 1887 г. В. Фогтом [15], как преобразования, оставляющие инвариантным волновое уравнение Даламбера. В 1900 г. в монографии

Дж. Лармора [16] было показано, что уравнения электродинамики Максвелла в вакууме также инвариантны относительно линейных преобразований пространственно-временных координат типа (6). Эти преобразования были получены и Г. Лоренцем в 1904 г. [17] и в последующих публикациях стали, следуя А. Пуанкаре [18], именоваться преобразованиями Лоренца. Установление свойств инвариантности уравнений Максвелла относительно преобразований Лоренца было решающим шагом на пути к созданию СТО [10] и к введению единой геометрии пространства - времени (псевдоевклидовой геометрии) Г. Минковским [14]. Обобщению преобразований Лоренца на область сверхсветовых скоростей посвящены также работы [19, 20].

Традиционная СТО [9-12] постулирует конечную скорость распространения взаимодействий, причем полагает эту скорость максимально возможной и равной скорости света в пустоте ($c = 2{,}998 \cdot 10^8$ м/с). А. Эйнштейн в своих лекциях, прочитанных в Принстонском университете в мае 1921 г., говорит: «Скорости материальных тел, превышающие скорости света, невозможны, что вытекает из появления радикала $\sqrt{1-V^2/c^2}$ в формулах частного преобразования Лоренца» (см. [11], стр. 31). В настоящей работе в качестве «частных» преобразований Лоренца возьмем наряду с преобразованиями (6) при $|M| < 1$ преобразования (5) при $|M| > 1$ и тем самым снимем ограничения по диапазону скоростей материальных тел и связанных с ними систем отсчета. Опираясь на сказанное, построим аналог СТО при $|M| > 1$.

Имеем «сверхсветовые» преобразования Лоренца (см. (5)) для системы координат X, Y, Z, T, которая движется относительно исходной системы координат x, y, z, t вдоль оси x со скоростью $V_0 > c$,

$$x = \gamma(MX + cT),\ y = Y,\ z = Z,\ ct = \gamma(McT + X), \tag{7}$$

где $\gamma = 1/\sqrt{M^2 - 1},\ M = V_0 / c$.

Из (7) имеем

$$dx = \gamma(MdX + cdT),\ dy = dY,\ dz = dZ,\ dt = \gamma(McdT + dX)$$

Разделив первые три равенства на dt, получим формулы преобразования скоростей

$$u = \frac{MU + c}{M + U/c},\ v = \frac{V\sqrt{M^2 - 1}}{M + U/c},\ w = \frac{W\sqrt{M^2 - 1}}{M + U/c} \tag{8}$$

При $M \to \infty$ имеем $u = U, v = V, w = W$.

При изучении сверхсветовых движений релятивистских материальных частиц с вещественной массой m будем исходить как и в традиционной СТО [11-14], из принципа наименьшего действия. Действие для свободной сверхсветовой частицы имеет вид

$$S = mc\int_a^b ds = \int_{t_1}^{t_2} L dt, \tag{9}$$

где L - функция Лагранжа. При $|M| > 1$ имеем $\rho = ds = cdt\sqrt{M^2 - 1}$. Тогда

$$S = mc\int_{t_1}^{t_2} c\sqrt{M^2 - 1}dt,\ L = mc^2\sqrt{M^2 - 1}.$$

Импульс частицы определяется как $\overline{p} = dL/d\overline{V}_0$, ее энергия $e = \overline{p}\overline{V}_0 - L$. Из (9) имеем

$$\overline{p} = \frac{m\overline{V}_0}{\sqrt{M^2 - 1}},\ E = \frac{mc^2}{\sqrt{M^2 - 1}}. \tag{10}$$

Из уравнений (10) получим систему

$$\begin{cases} E^2 - p^2c^2 = -m^2 \cdot c^4, \\ E \cdot M = p \cdot c. \end{cases} \tag{11}$$

Из системы (11) следует, что при $M^2 > 1$

$$E^2 \cdot (M^2 - 1) = m^2 \cdot c^4 > 0,$$

т.е. $m^2 > 0$, откуда получаем, что масса тахиона вещественна.
Система (11) была получена ранее в [3,4] введением мнимой массы тахиона. В данном случае оно получается естественным образом, т.е. при наличии вещественной массы тахиона.

Релятивистское волновое уравнение для частицы с нулевым спином было получено Клейном – Гордоном – Фоком. Приведем аналогичное уравнение для случая сверхсветовой частицы (тахиона) с вещественной массой m. Исходя из правил соответствия Шредингера $E \mapsto i \cdot \frac{\partial}{\partial t}$, $p \mapsto -i \cdot \nabla$ и первого соотношения (11) имеем следующее уравнение для волновой функции ψ

$$-\frac{\partial^2 \psi}{\partial t^2} = -c^2 \cdot \Delta\psi - m^2 c^4 \cdot \psi .$$

Запишем его в виде

$$(\Box - m^2 \cdot c^2) \cdot \psi = 0, \ \Box = \frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2} - \Delta . \qquad (12)$$

Уравнение (12) рассматривалось ранее, в частности, в работе [5] методом формального введения мнимой массы в уравнение Клейна – Гордона – Фока. Волновое уравнение (12) в настоящей работе получено для тахиона с вещественной массой m. При данном подходе автоматически снимаются вопросы физической интерпретации мнимой массы, которые возникают во всех работах, посвященных тахионам [3-8].

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Phase problem in problems of wave physics

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Фазовая проблема состоит в определении фазы волнового поля по его пространственному спектру мощности (ПСМ). Неоднозначность решения фазовой проблемы проявляется при исследовании одномерных волновых задач. При наличии априорной информации о характере искомого фазового распределения волнового поля определение решения фазовой проблемы может быть получено среди нетривиальных её решений, определяемых индексом функции ПСМ волнового поля. При исследовании двумерных и трехмерных волновых задач неоднозначность решения фазовой проблемы не отмечается. В докладе рассматривается несколько примеров однозначных и неоднозначных задач определения фазы волнового поля по его функции ПСМ из различных задач волновой физики (волны де Бройля, определение апертурного распределения электромагнитных волн СВЧ диапазона, лазерная интерферометрия, запись голограмм без опорного пучка, чистка радиоастрономических карт и др.).

Для решения фазовой проблемы используются методы регистрации пространственного распределения интенсивности волновых полей, после обработки которого можно сделать вывод о наличии или отсутствии изменения фаз исследуемого волнового поля, о структуре фазового распределения волновых фронтов, а также о свойствах их источников. Последнее свойство решения фазовой проблемы является следствием разработанного авторами доклада алгоритма решения фазовой проблемы, на основе численного решения задачи Римана–Гильберта, к которой можно свести решение фазовой проблемы.

Программная реализация предлагаемого алгоритма решения фазовой проблемы сводится к последовательному применению нескольких преобразований Фурье к массиву значений измеряемого ПСМ волнового поля. Использование современных эффективных алгоритмов быстрого преобразования Фурье позволяет достичь высокого быстродействия расчёта распределения фазы волновых полей по ПСМ, позволяющего применять разработанный алгоритм в реальном режиме времени непосредственно в соответствующих экспериментальных.

В докладе обсуждается применение разработанного алгоритма решения фазовой проблемы в задачах юстировки и диагностики зеркал и линз экспериментальных установок лазерной интерферометрии, а также зеркальных антенн, фазированных антенных решёток, радиоастрономических радиотелескопов, устройств прецизионной и адаптивной оптики.

What do very nearly flat detectable cosmic topologies look like?

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Abstract

Recent high precision data by WMAP and other Cosmic Microwave Background and high redshift surveys have produced a wealth of information concerning the early universe and provided strong evidence for a nearly spatially–flat universe with a primordial spectrum of adiabatic, Gaussian and nearly scale–invariant density perturbations, in line with the predictions of the inflationary cosmology. Such observations are also making it possible to probe the topology of the universe. I report on recent results [1, 2] which demonstrate that in the inflationary limit, i.e. $|\Omega_0 - 1| \ll 0$ a generic detectable spherical or hyperbolic topology is locally indistinguishable from either a cylindrical ($\mathbb{R}^2 \times \mathbb{S}^1$) or toroidal ($\mathbb{R} \times \mathbb{T}^2$) topology, irrespective of its global shape.

References

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Staton gravity as equivalent of antiscalar approach (a fresh view on the nature of gravitational interaction).

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Under general term 'staton field' one should comprehend here just the neutral superposition of quasi-static gravitating electric potentials $\phi = \phi^{+} + \phi^{-}$ with all the real fermions of the real Universe to be the sources of those. At the sub-quantum level these fields are supposed to have the super-rapid (highly superluminal) carriers to be named 'statons', so that these so to say 'preons' should not practically violate the custom long-range (instant) action of static fields. The manifestation of that 'radical' heuristic conjecture proves to be fairly productive for the present comprehension of the gravity nature without any crucial changes in the basic equations of underlying theory.

In general, the statons should have the almost evanescent (that is negligible for the most usual tests of electrodynamics and gravitation, excepting the cosmological applications) but non-zero tachyon mass which subsequently proves to be $m = 10^{-33} eV = 10^{-65} g$. Next, the scalar field thermodynamics – on the one hand, and static (scalar) limit of the Einstein-Maxwell equations – from the other, both give rise to the crucial property named 'antiscalarity'. This means that any scalar or pseudo-scalar EMT (energy-momentum tensor, never mind – electrically charged or neutral) should enter into the field equations with a sign opposite that of the usual matter, and also this leads inevitably to the negative cosmological Λ-term, if the last to take as a part and parcel of the full neutral massive scalar field.

It was notified at the previous PIRT-conference that the antiscalar approach as a whole has a deep geometrical foundation based on introduction from the very beginning of so-called tensor of space-time deformations which allows a fixing the main symmetries in the problem under consideration. Besides the evident convenience, this method (at first used in a simple case by Hawking and Ellis) gives rise to remarkable possibility of avoidance of any using the Gilbert Lagrangian in derivation of the Einstein equations. A subsequent attachment of the staton conception, that is of a well-defined scalar field having 'statons' in the capacity of its carriers, leads not only to conformity with experiment and not only to independent classical justification of the black-hole thermodynamics (under condition of replacement of BH by the more realistic compact objects), but also to definite theoretical conclusions of principle.

So, because of replacement of the vacuum (in a sense of General Relativity) Einstein-Gilbert equations by equations with antiscalar staton field, there are no more solutions of type 'black holes' or 'gravitational waves, propagating with speed o light'. But what is of special interest, it is conclusion that there are no more problems with definition of energy of gravitational field, because of such energy is now reduced to the energy of well-defined gravitating antiscalar staton field, only by means of which we can perceive the gravity proper.

Taking this conclusion as a guiding principle it is not difficult to show that the antiscalar background can be completely geometrized. Indeed, the usual EMT of scalar field $T_{\mu\nu}(\phi)$ has a standard dimension of energy density when scalar field ϕ has a dimension of the electric (that is of the Coulon-type) potential. From the other hand, we always can replace such an electric-type potential ϕ by the natural Newtonian-type potential φ_N in dimensionless form $\varphi = \varphi_N / c^2$ (by dividing it by c^2). Then energy-momentum tensor $T_{\mu\nu}(\phi)$ transforms just into the geometrical complex $T_{\mu\nu}(\varphi)$ having the pure geometrical dimension $[cm^{-2}]$ of the inverse square of length, coinciding with dimension of the Ricci tensor and of the (geometrical) Λ-term. So as a result from the Einstein equations with antiscalar field the dimensional constant $8\pi G / c^4$ is really eliminated. This operation does mean the true or full geometrization what at his time Einstein has dreamed about.

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